



*INF4820: Algorithms for  
Artificial Intelligence and  
Natural Language Processing*

Context-Free Grammars & Parsing

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Language Technology Group (LTG)

November 3, 2016

## Last Time

- ▶ Sequence Labeling
- ▶ Dynamic programming
- ▶ Viterbi algorithm
- ▶ Forward algorithm

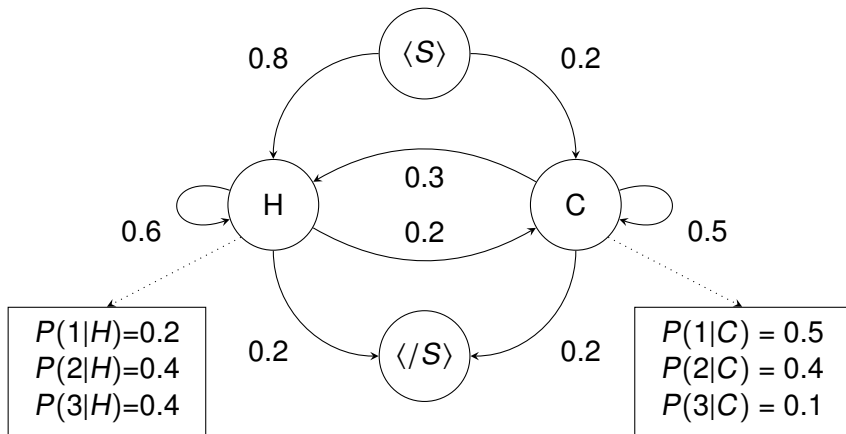
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- ▶ Sequence Labeling
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## Today

- ▶ Grammatical structure
- ▶ Context-free grammar
- ▶ Treebanks
- ▶ Probabilistic CFGs

# Recall: Ice Cream and Global Warming



# Recall: Viterbi Algorithm



- ▶ To find the best state sequence, **maximize**:

$$P(s_1 \dots s_n | o_1 \dots o_n) = P(s_1 | s_0) P(o_1 | s_1) P(s_2 | s_1) P(o_2 | s_2) \dots$$

- ▶ The value we cache at each step:

$$v_i(s) = \max_{k=1}^L [v_{i-1}(k) \cdot P(s|k) \cdot P(o_i|s)]$$

- ▶ The variable  $v_i(s)$  represents the maximum probability that the  $i$ -th state is  $s$ , given that we have seen  $O_1^i$ .
- ▶ At each step, we record backpointers showing which previous state led to the maximum probability.

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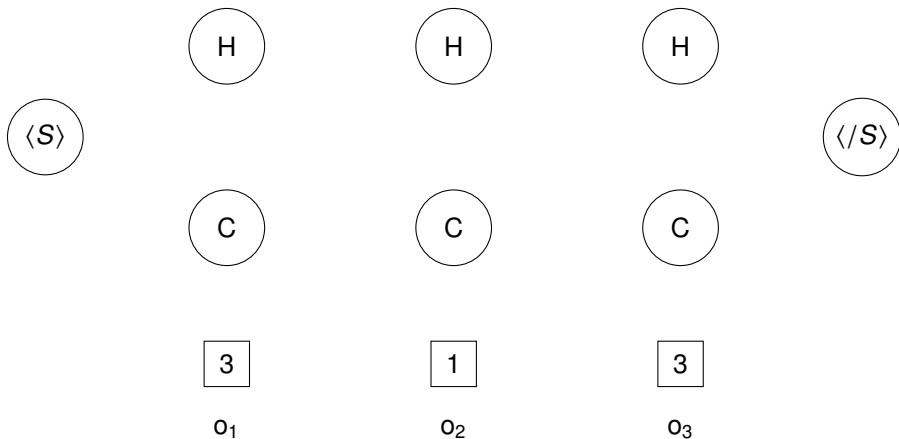
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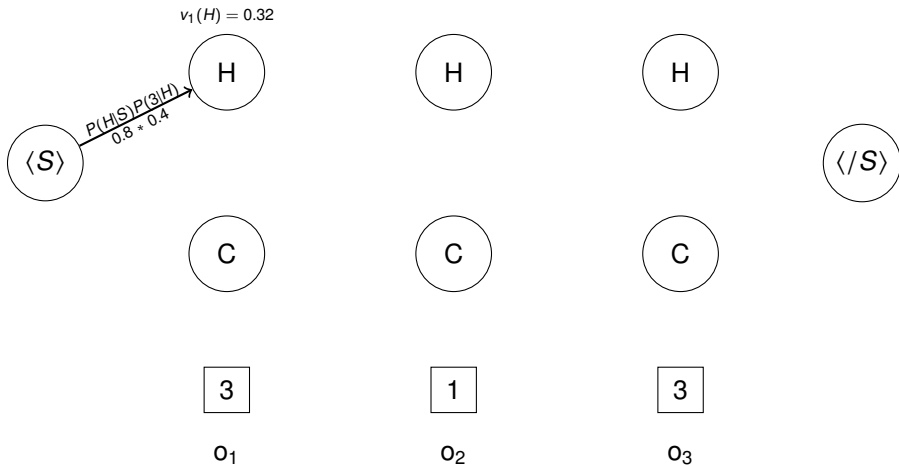
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- ▶ Examples:
  - ▶ Dijkstra's shortest path
  - ▶ minimum edit distance
  - ▶ longest common subsequence
  - ▶ Viterbi decoding



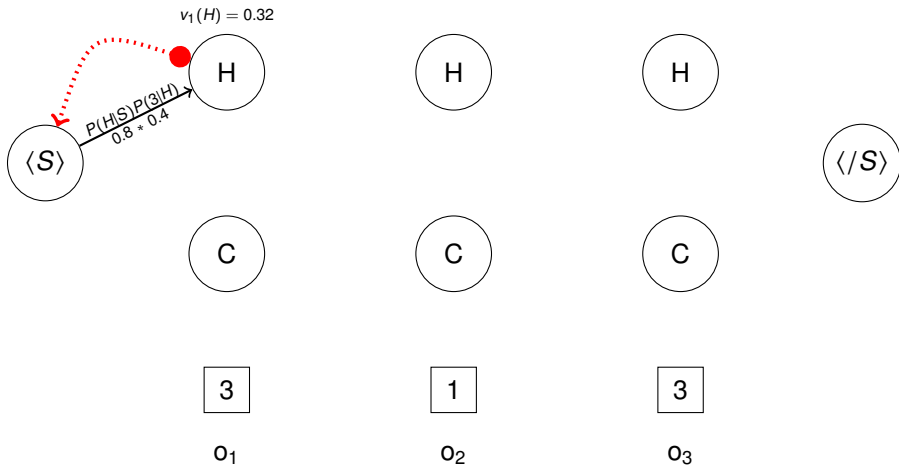
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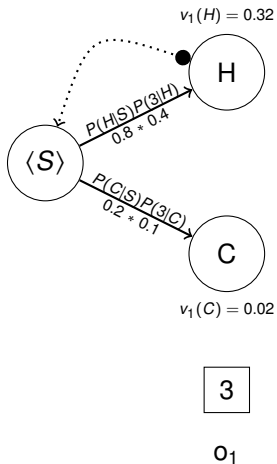
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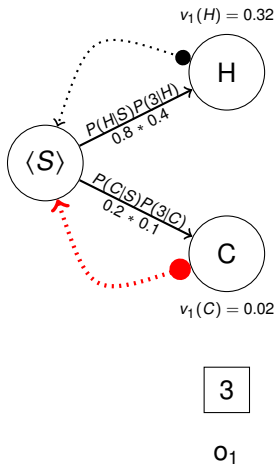
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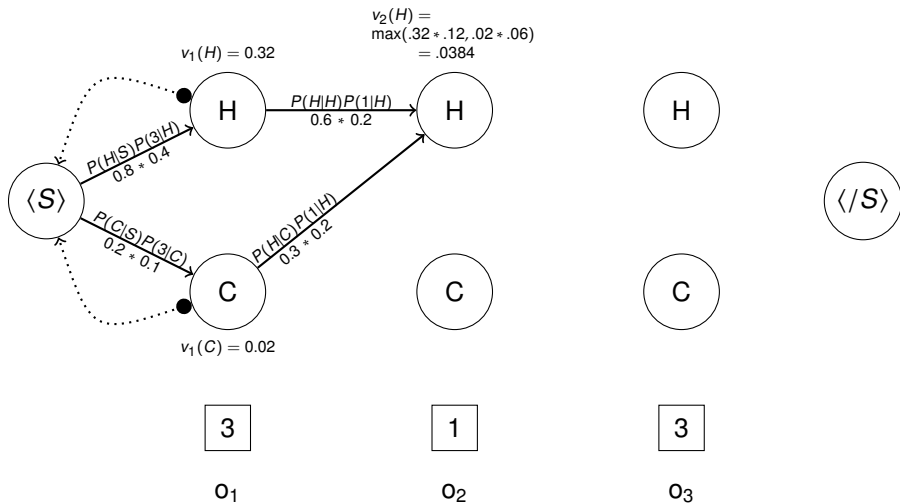
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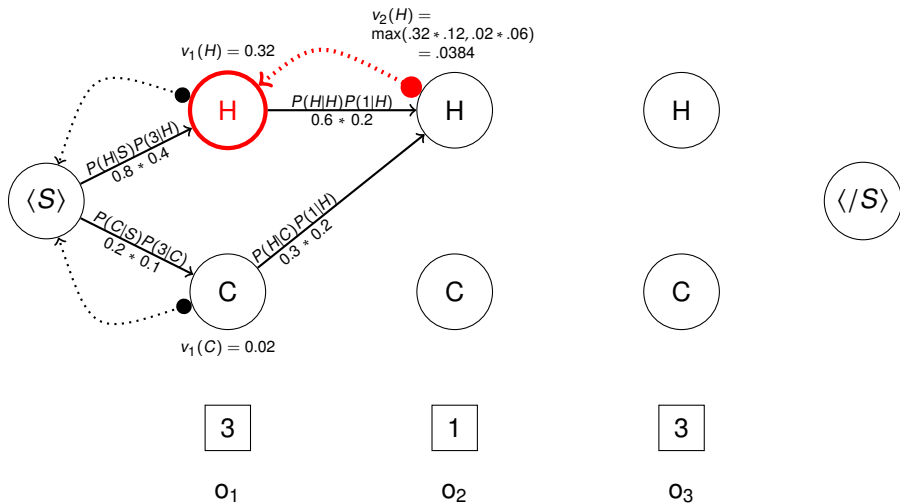
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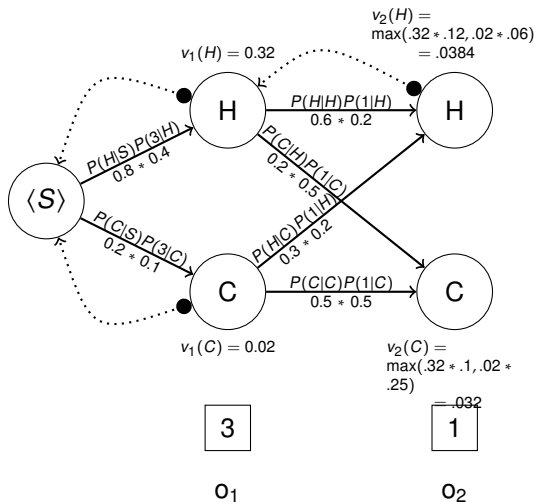
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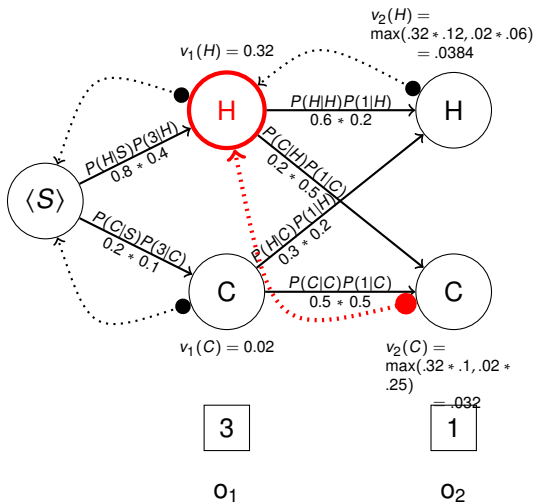


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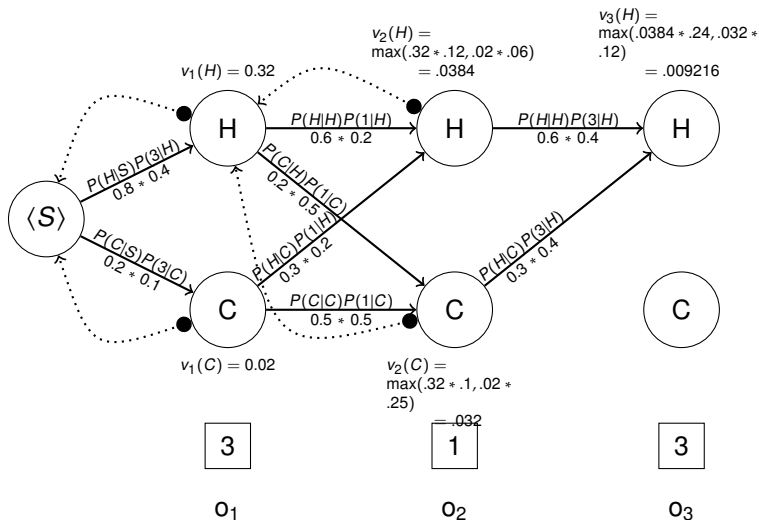




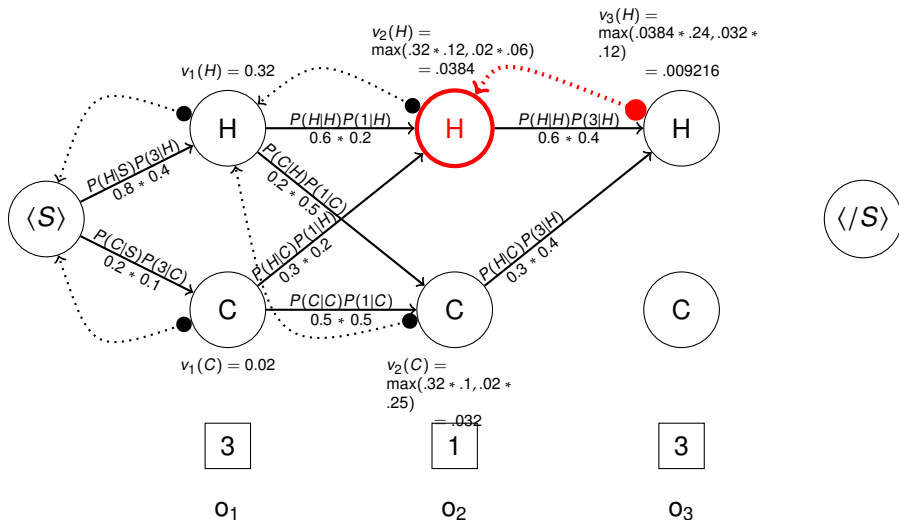
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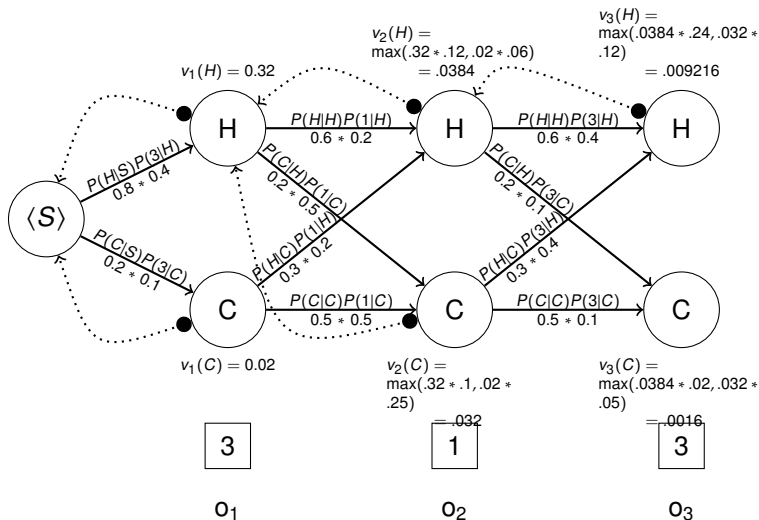
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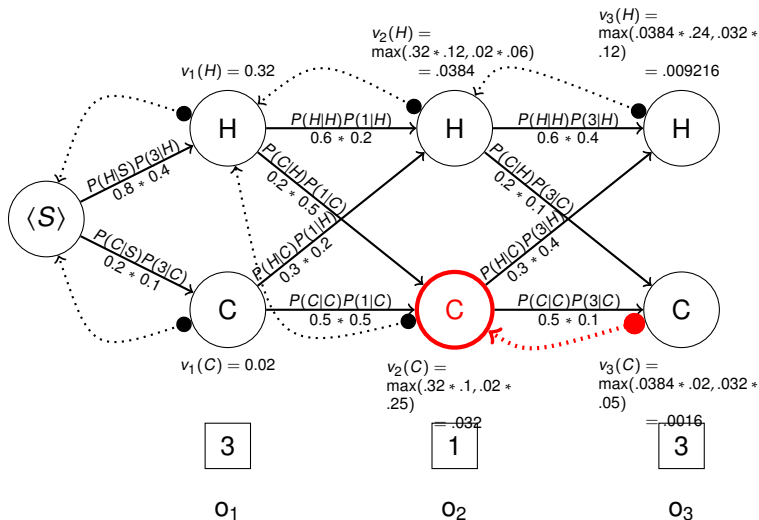
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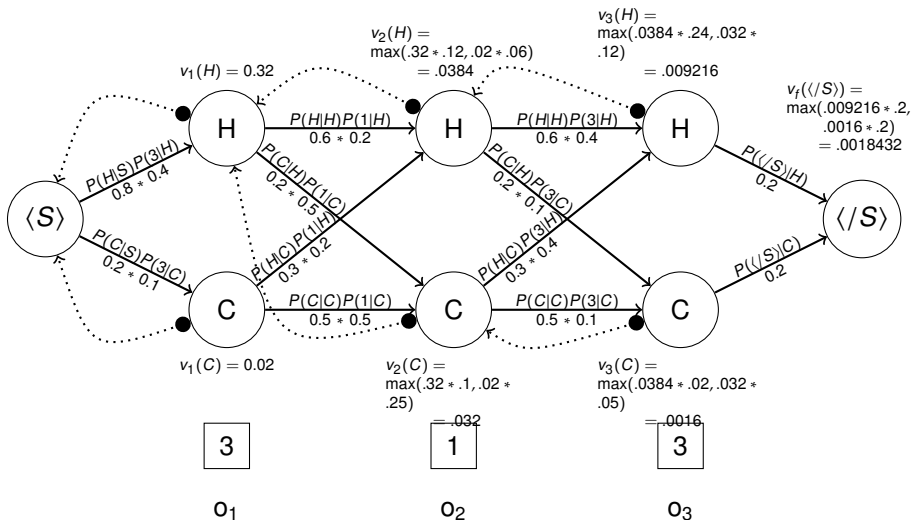
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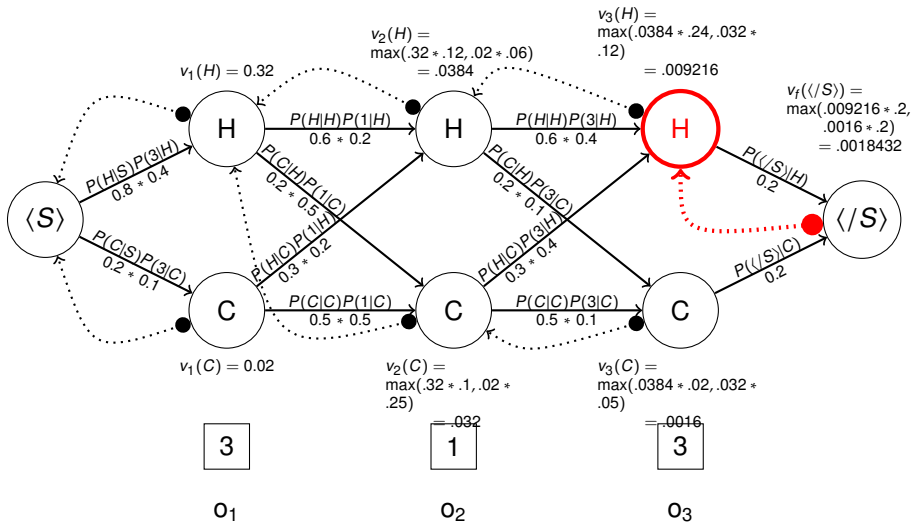
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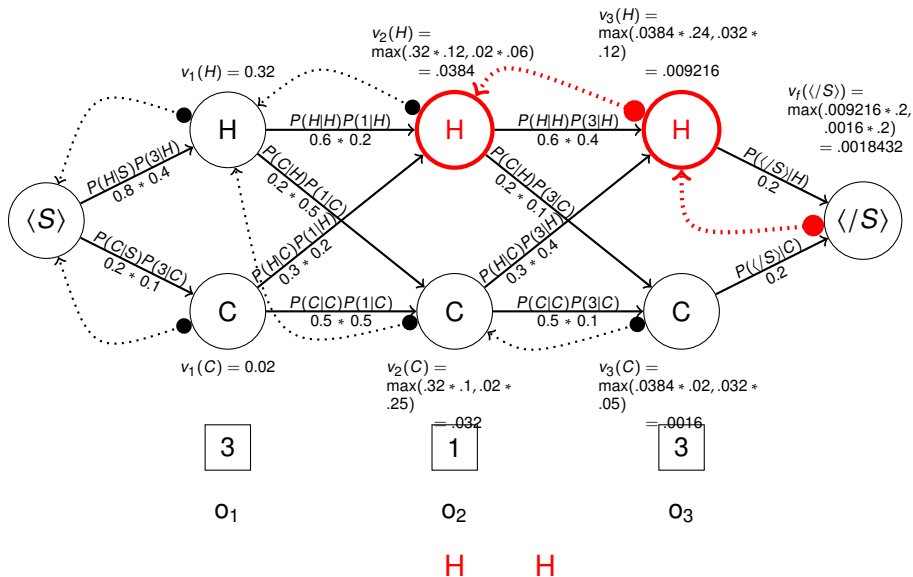


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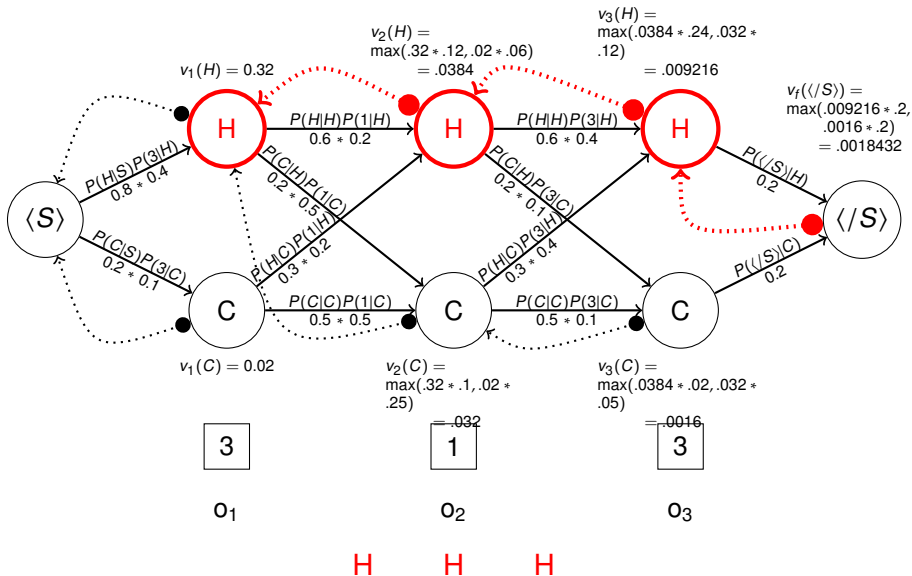
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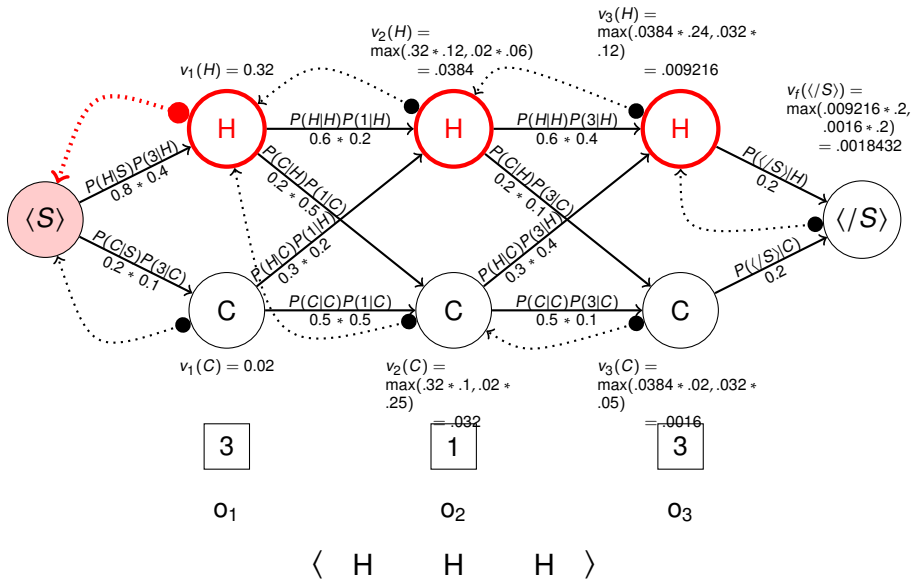




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# Recall: Using HMMs

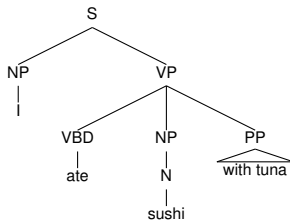
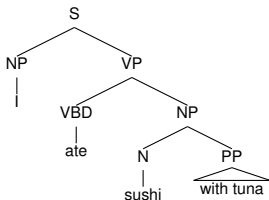


The HMM models the process of generating the labelled sequence. We can use this model for a number of tasks:

- ▶  $P(S, O)$  given  $S$  and  $O$
- ▶  $P(O)$  given  $O$
- ▶  $S$  that maximizes  $P(S|O)$  given  $O$
- ▶  $P(s_x|O)$  given  $O$
- ▶ We learn model parameters from a set of observations.

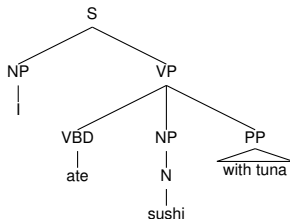
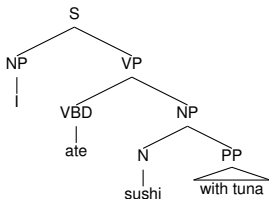
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- ▶ which string is most likely:
  - ▶ *How to recognize speech vs. How to wreck a nice beach*
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  - ▶ **NNS VB NNS** vs. **VBZ P NNS**
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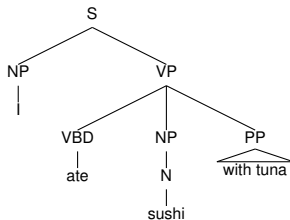
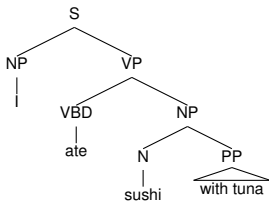
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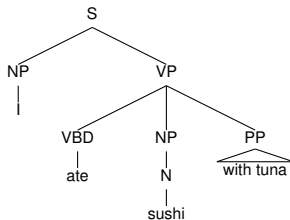
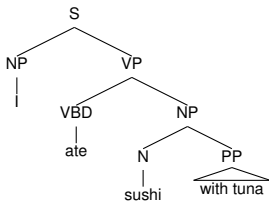
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# From Linear Order to Hierarchical Structure



- ▶ The models we have looked at so far:
  - ▶  $n$ -gram models (Markov chains).
    - ▶ Purely linear (sequential) and surface-oriented.
  - ▶ sequence labeling: HMMs.
    - ▶ Adds one layer of abstraction: PoS as hidden variables.
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    - ▶ Still only sequential in nature.
- ▶ **Formal grammar** adds hierarchical structure.
  - ▶ In NLP, being a sub-discipline of AI, we want our programs to '*understand*' natural language (on some level).
  - ▶ Finding the grammatical structure of sentences is an important step towards '*understanding*'.
  - ▶ Shift focus from *sequences* to *grammatical structures*.

## Constituency

- ▶ Words tends to lump together into groups that behave like single units: we call them *constituents*.
- ▶ *Constituency tests* give evidence for constituent structure:
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- (4) Kim read [a very interesting book about grammar]<sub>NP</sub>.  
Kim read [it]<sub>NP</sub>.
- (5) Kim [read a book]<sub>VP</sub>, [gave it to Sandy]<sub>VP</sub>, and [left]<sub>VP</sub>.
- (6) [Read the book]<sub>VP</sub> I really meant to this week.

# Why We Need Structure (2/3)



## Constituency

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- ▶ Constituents are theory-dependent, and are not absolute or language-independent.
- ▶ A constituent usually has a *head* element, and is often named according to the type of its head:
  - ▶ A noun phrase (NP) has a nominal (noun-type) head:

(9) [ a very interesting book about grammar ]<sub>NP</sub>

- ▶ A verb phrase (VP) has a verbal head:

(10) [ gives books to students ]<sub>VP</sub>

# Why We Need Structure (3/3)



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The decision of the Nobel committee members surprises most of us.

- ▶ Why would a **purely linear** model have problems predicting this phenomenon?
- ▶ Verb agreement reflects the **grammatical structure** of the sentence, not just the sequential order of words.



# Grammars: A Tool to Aid Understanding



Formal grammars describe a language, giving us a way to:

- ▶ judge or predict well-formedness

Kim was happy because \_\_\_\_\_ passed the exam.

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- ▶ derive abstract representations of meaning

Kim gave Sandy a book.

Kim gave a book to Sandy.

Sandy was given a book by Kim.

# A Grossly Simplified Example



## The Grammar of Spanish

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$VP \rightarrow VP PP$

$PP \rightarrow P NP$

$NP \rightarrow \text{"nieve"}$

$NP \rightarrow \text{"Juan"}$

$NP \rightarrow \text{"Oslo"}$

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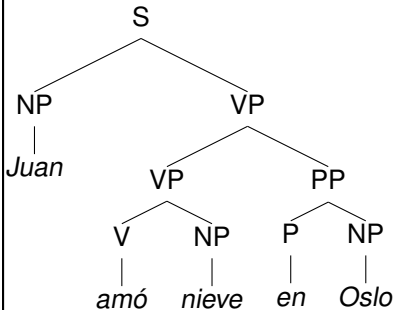
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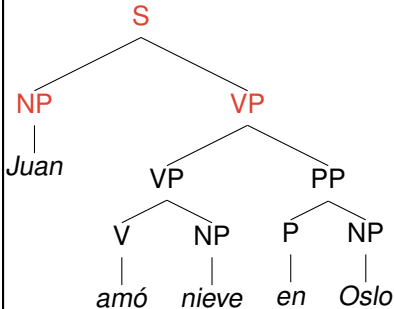
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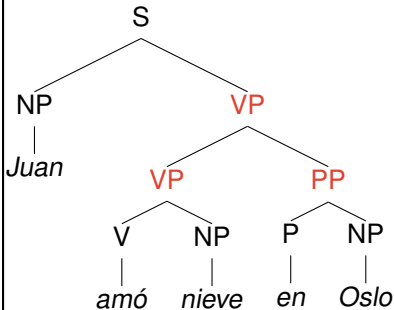
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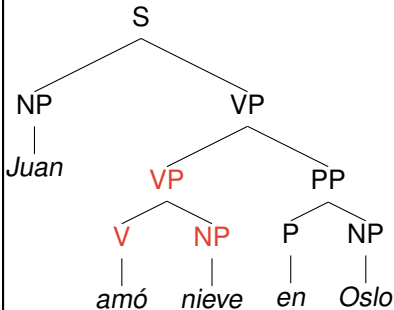
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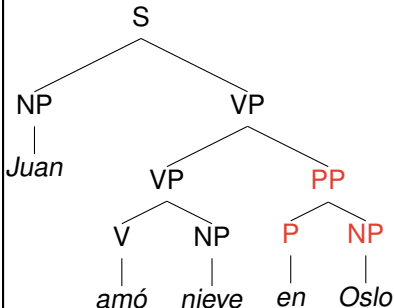
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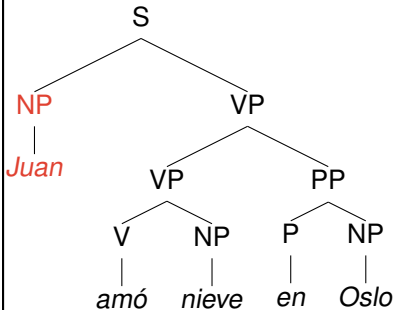
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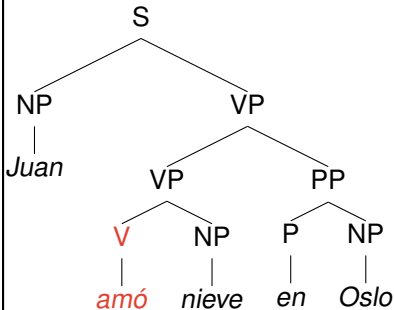
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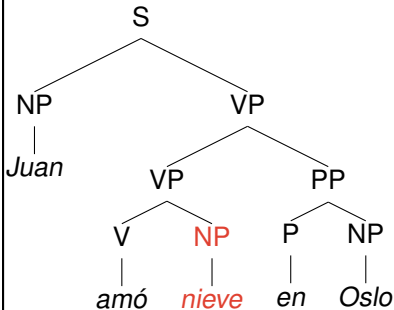
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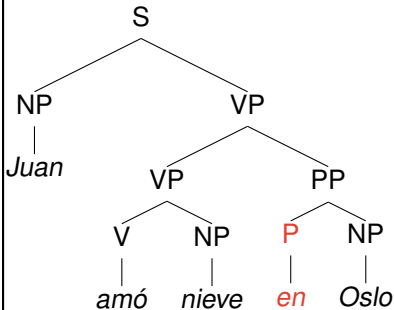
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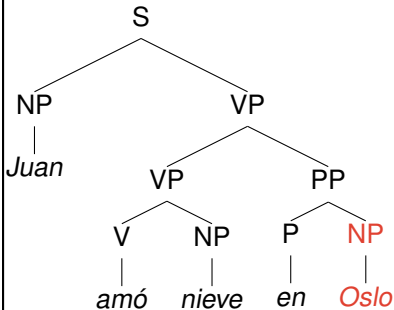
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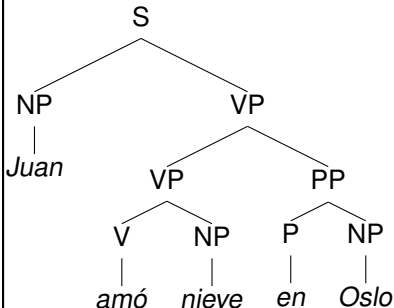
$NP \rightarrow \text{"nieve"}$  { snow }

$NP \rightarrow \text{"Juan"}$  { John }

$NP \rightarrow \text{"Oslo"}$  { Oslo }

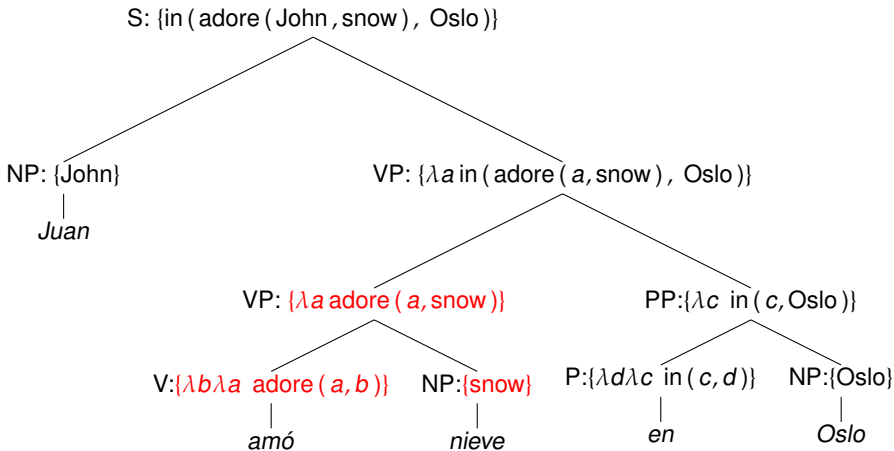
$V \rightarrow \text{"amó"}$  {  $\lambda b \lambda a \text{ adore}(a, b)$  }

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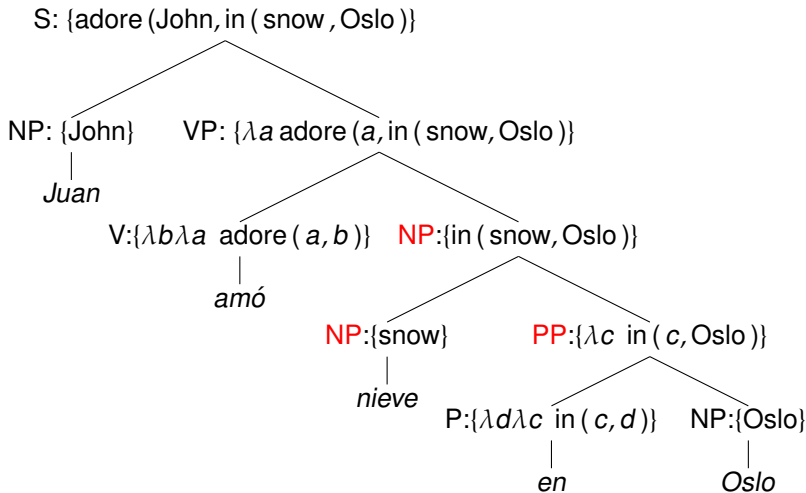
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# Meaning Composition (Still Very Simplified)



VP  $\rightarrow$  V NP    { V (NP) }

# Another Interpretation



$NP \rightarrow NP \text{ PP } \{ PP (NP) \}$

# Context Free Grammars (CFGs)



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- ▶ Defined in terms of a lexicon and a set of rules
- ▶ Formal models of 'language' in a broad sense
  - ▶ natural languages, programming languages, communication protocols, . . .

# Context Free Grammars (CFGs)



- ▶ Formal system for modeling constituent structure.
- ▶ Defined in terms of a lexicon and a set of rules
- ▶ Formal models of 'language' in a broad sense
  - ▶ natural languages, programming languages, communication protocols, . . .
- ▶ Can be expressed in the 'meta-syntax' of the Backus-Naur Form (BNF) formalism.
  - ▶ When looking up concepts and syntax in the Common Lisp HyperSpec, you have been reading (extended) BNF.

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- ▶ Can be expressed in the 'meta-syntax' of the Backus-Naur Form (BNF) formalism.
  - ▶ When looking up concepts and syntax in the Common Lisp HyperSpec, you have been reading (extended) BNF.
- ▶ Powerful enough to express sophisticated relations among words, yet in a computationally tractable way.

# CFGs (Formally, this Time)



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- ▶  $S \in C$  is the *start symbol*, a filter on complete results;
- ▶ for each rule  $\alpha \rightarrow \beta_1, \beta_2, \dots, \beta_n \in P$ :  $\alpha \in C$  and  $\beta_i \in C \cup \Sigma$

Top-down view of generative grammars:

- ▶ For a grammar  $G$ , the language  $\mathcal{L}_G$  is defined as the set of strings that can be derived from  $S$ .
- ▶ To derive  $w_1^n$  from  $S$ , we use the rules in  $P$  to recursively rewrite  $S$  into the sequence  $w_1^n$  where each  $w_i \in \Sigma$

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- ▶ The grammar is seen as **generating** strings.
- ▶ *Grammatical* strings are defined as strings that can be generated by the grammar.
- ▶ The 'context-freeness' of CFGs refers to the fact that we rewrite non-terminals without regard to the overall context in which they occur.



## Generally

- ▶ A *treebank* is a corpus paired with ‘gold-standard’ (syntactico-semantic) analyses
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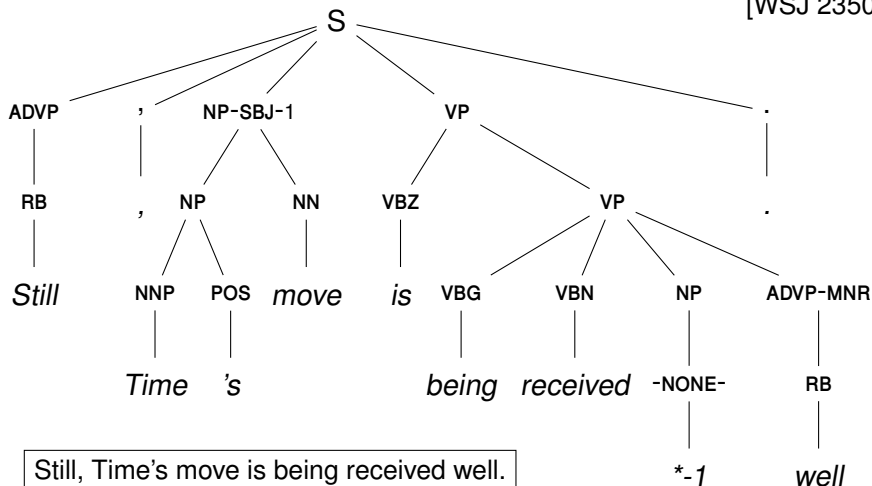
## Penn Treebank (Marcus et al., 1993)

- ▶ About one million tokens of Wall Street Journal text
- ▶ Hand-corrected PoS annotation using 45 word classes
- ▶ Manual annotation with (somewhat) coarse constituent structure

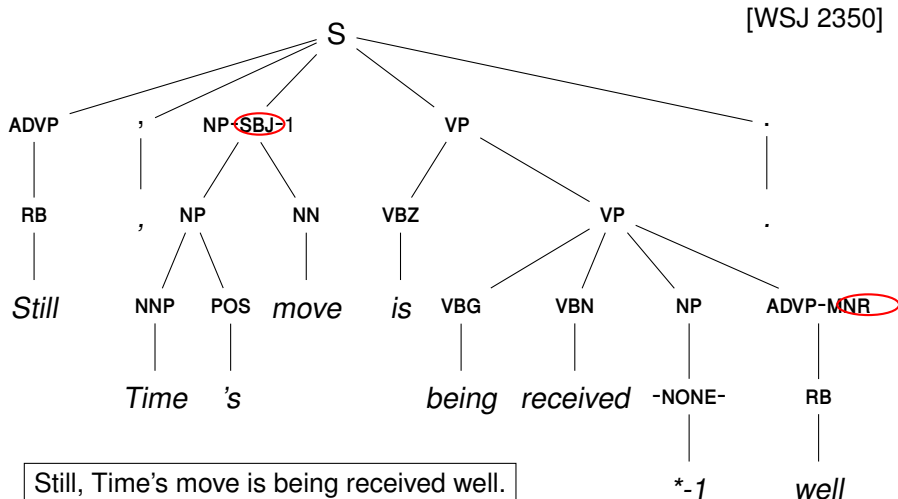
# One Example from the Penn Treebank



[WSJ 2350]



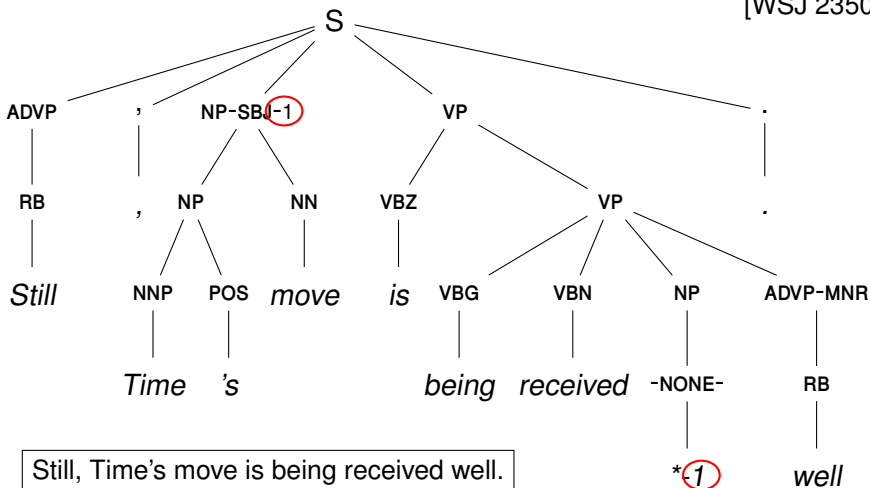
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# One Example from the Penn Treebank



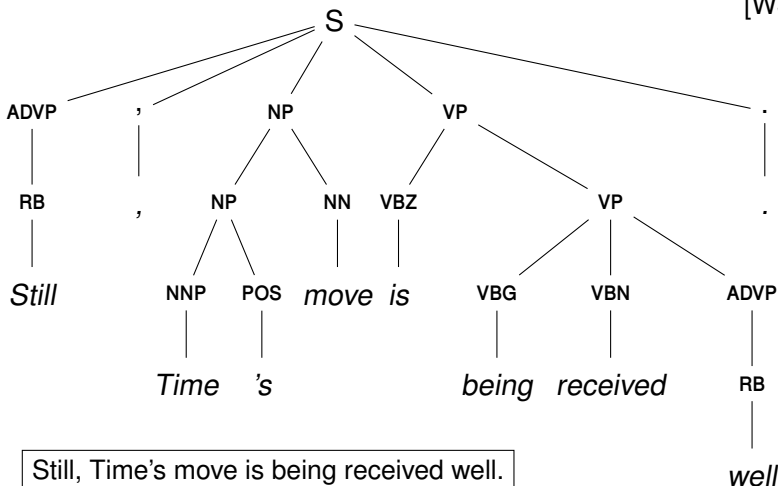
[WSJ 2350]



# Elimination of Traces and Functions



[WSJ 2350]



# Probabilistic Context-Free Grammars



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  - ▶  $S \rightarrow NP VP$                       0.6
  - ▶  $S \rightarrow NP VP PP$                       0.4
- ▶ These are conditional probabilities — the probability of the right hand side (RHS) given the left hand side (LHS)
  - ▶  $P(S \rightarrow NP VP) = P(NP VP|S)$



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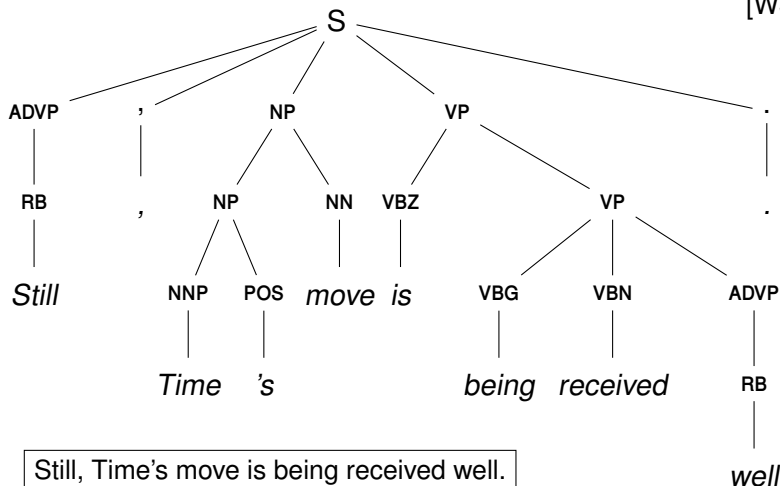


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- ▶ These are conditional probabilities — the probability of the right hand side (RHS) given the left hand side (LHS)
  - ▶  $P(S \rightarrow NP VP) = P(NP VP|S)$
- ▶ We can learn these probabilities from a treebank, again using Maximum Likelihood Estimation.

# Estimating PCFGs (1/3)



[WSJ 2350]



## Estimating PCFGs (2/3)



```
(S
  (ADVP (RB "Still"))
  (|,| ",")
  (NP
    (NP (NNP "Time") (POS "'s"))
    (NN "move"))
  (VP
    (VBZ "is")
    (VP
      (VBG "being")
      (VP
        (VBN "received")
        (ADVP (RB "well"))))))
  (\. "."))
```

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  (\. "."))
```

# Estimating PCFGs (2/3)



RB → Still	1
ADVP → RB	1

```
(S
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        (VBN "received")
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```

RB → Still	1
ADVP → RB	1
,  → ,	1

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```

RB → Still	1
ADVP → RB	1
,  → ,	1
NNP → Time	1

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RB → Still	1
ADVP → RB	1
,  → ,	1
NNP → Time	1
POS → 's	1



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```

RB → Still	1
ADVP → RB	1
,  → ,	1
NNP → Time	1
POS → 's	1
NP → NNP POS	1

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RB → Still	1
ADVP → RB	1
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POS → 's	1
NP → NNP POS	1
NN → move	1

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RB → Still	1
ADVP → RB	1
,  → ,	1
NNP → Time	1
POS → 's	1
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NN → move	1
NP → NP NN	1

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RB → Still	1
ADVP → RB	1
,  → ,	1
NNP → Time	1
POS → 's	1
NP → NNP POS	1
NN → move	1
NP → NP NN	1
VBZ → is	1

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RB → Still	1
ADVP → RB	1
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RB → Still	1
ADVP → RB	1
,  → ,	1
NNP → Time	1
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```

RB → Still	1
ADVP → RB	1
,  → ,	1
NNP → Time	1
POS → 's	1
NP → NNP POS	1
NN → move	1
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VBZ → is	1
VBG → being	1
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      (VP
        (VBN "received")
        (ADVP (RB "well")))))
  (\. "."))
```

RB → Still	1
ADVP → RB	2
,  → ,	1
NNP → Time	1
POS → 's	1
NP → NNP POS	1
NN → move	1
NP → NP NN	1
VBZ → is	1
VBG → being	1
VBN → received	1
RB → well	1



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RB → Still	1
ADVP → RB	2
,  → ,	1
NNP → Time	1
POS → 's	1
NP → NNP POS	1
NN → move	1
NP → NP NN	1
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VBG → being	1
VBN → received	1
RB → well	1
VP → VBN ADVP	1

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RB → Still	1
ADVP → RB	2
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NNP → Time	1
POS → 's	1
NP → NNP POS	1
NN → move	1
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VBZ → is	1
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RB → well	1
VP → VBN ADVP	1
VP → VBG VP	1

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RB → Still	1
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VBG → being	1
VBN → received	1
RB → well	1
VP → VBN ADVP	1
VP → VBG VP	1
\. → .	1
S → ADVP  ,  NP VP \.	1

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RB → Still	1
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RB → well	1
VP → VBN ADVP	1
VP → VBG VP	1
\. → .	1
S → ADVP  ,  NP VP \.	1
<b>START → S</b>	<b>1</b>

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$S \rightarrow \text{ADVP }  ,   \text{ NP VP } \backslash.$	50	$S \rightarrow \text{NP VP } \backslash.$	400
$S \rightarrow \text{NP VP PP } \backslash.$	350	$S \rightarrow \text{VP } !$	100
$S \rightarrow \text{NP VP S } \backslash.$	200	$S \rightarrow \text{NP VP}$	50

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$S \rightarrow \text{NP VP PP } \backslash .$	350	$S \rightarrow \text{VP } !$	100
$S \rightarrow \text{NP VP S } \backslash .$	200	$S \rightarrow \text{NP VP}$	50

$$P(S \rightarrow \text{ADVP } |, | \text{ NP VP } \backslash .) \approx \frac{C(S \rightarrow \text{ADVP } |, | \text{ NP VP } \backslash .)}{C(S)}$$



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$S \rightarrow \text{NP VP S } \backslash .$	200	$S \rightarrow \text{NP VP}$	50

$$\begin{aligned} P(S \rightarrow \text{ADVP } |, | \text{ NP VP } \backslash .) &\approx \frac{C(S \rightarrow \text{ADVP } |, | \text{ NP VP } \backslash .)}{C(S)} \\ &= \frac{50}{1150} \\ &= 0.0435 \end{aligned}$$