— INF4820 —
Algorithms for AI and NLP

Classification

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Language Technology Group (LTG)

September 29, 2016

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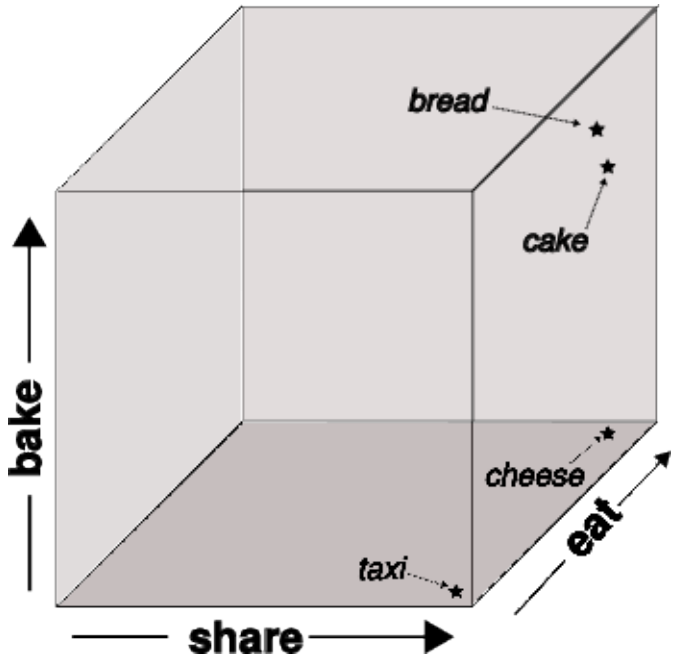
- Each word o_i is represented by a (length-normalized) n-dimensional feature vector $\vec{x}_i = \langle x_{i1}, \ldots, x_{in} \rangle \in \Re^n$.
- We can now measure, say, the Euclidean distance of words in the space, $d(\vec{x}, \vec{y})$.
- Semantic relatedness $\approx$ distributional similarity $\approx$ spatial proximity



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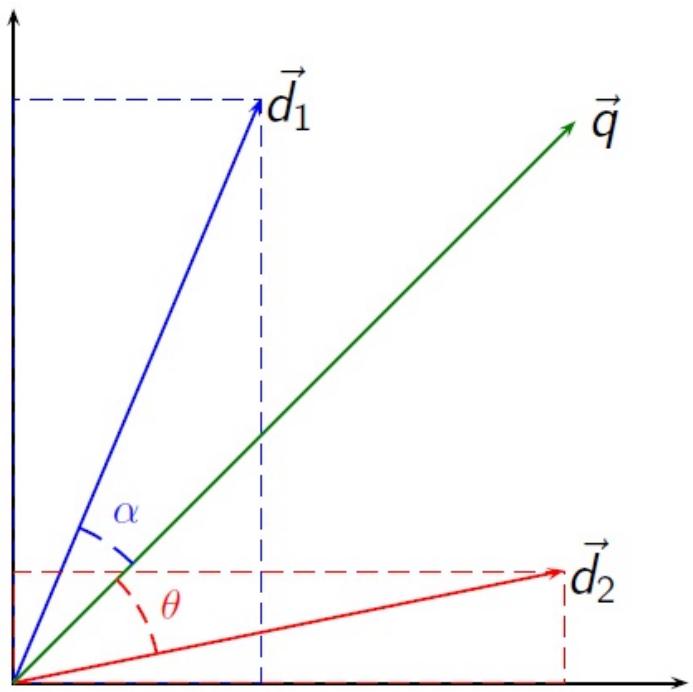
- Each document d_j is represented by a feature vector, with features corresponding to the terms $t_1, \ldots, t_n$ occurring in the documents.
  - Spatial distance $\approx$ similarity of content.
  - Can also represent a search query as a vector:
  - The relevance of documents given by their distance to the query.



Today's main topic

- Machine learning: Classification
- Representing classes and membership
- Rocchio classifiers
- kNN classifiers

Two categorization tasks in machine learning

Classification

- Supervised learning, requiring labeled training data.
- Train a classifier to automatically assign *new* instances to *predefined* classes, given some set of examples.
- (Topic for today.)

Clustering

- Unsupervised learning from unlabeled data.
- Automatically group similar objects together.
- No predefined classes or structure, we only specify the similarity measure.
- (The topic for the next lectures.)

Examples of classification tasks

Some examples of classification tasks

- Spam filtering
- Named entity recognition
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- Authorship attribution
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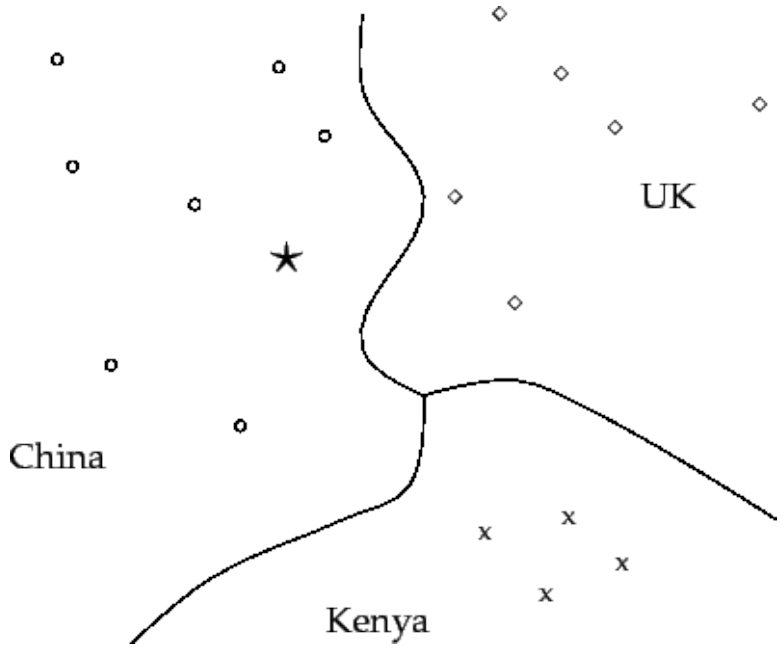
- We'll look at two simple examples of vector space classifiers:
  - Rocchio
  - kNN

Classes and classification

- A class can simply be thought of as a collection of objects.
- In our vector space model, objects are represented as *points*, so a class will correspond to a collection of points; a region.
- Vector space classification is based on the contiguity hypothesis:



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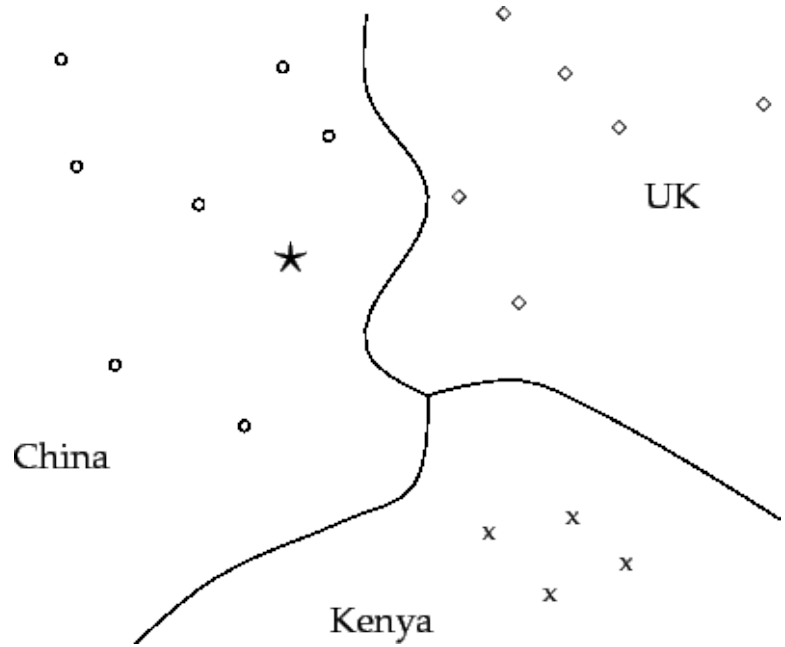

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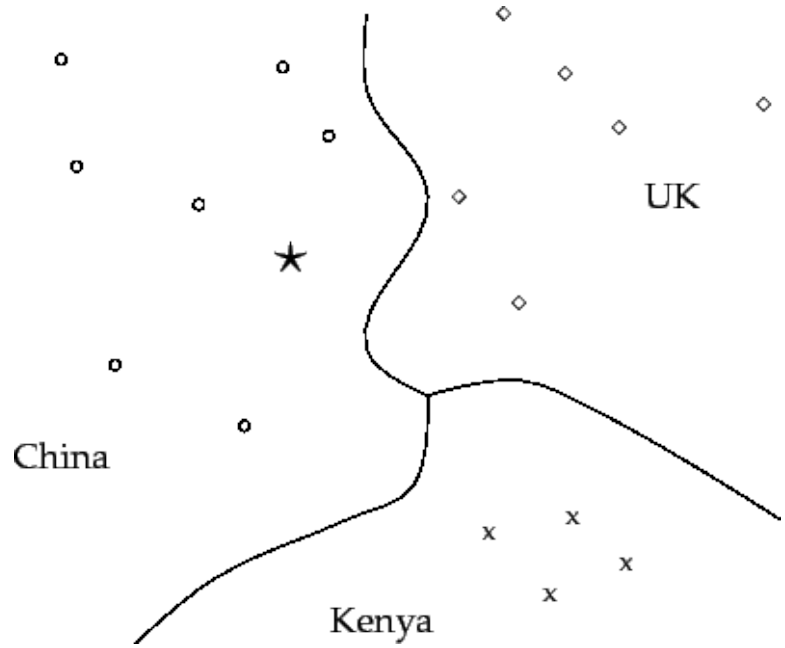

- Objects in the same class form a contiguous region, and regions of different classes do not overlap.
- Classification amounts to computing the boundaries in the space that separate the classes; *the decision boundaries*.
- How we draw the boundaries is influenced by how we choose to represent the classes.

Different ways of representing classes

Exemplar-based

- No abstraction. Every stored instance of a group can potentially represent the class.
- Used in so-called *instance based* or *memory based learning* (MBL).
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Centroid-based

- The average, or the *center of mass* in the region.
- Given a class c_i, where each object o_j being a member is represented as a feature vector $\vec{x}_j$, we can compute the class centroid $\vec{\mu}_i$ as

$$\vec{\mu}_i = \frac{1}{|c_i|} \sum_{\vec{x}_j \in c_i} \vec{x}_j$$

Different ways of representing classes (cont'd)

Some more notes on centroids, medoids and typicality

- Both *centroids* and *medoids* represent a group by a single prototype.
- But while a *medoid* is an actual member of the group, a *centroid* is an *abstract* prototype; an average.
- *Typicality* can be defined by a member's distance to the prototype.
- The centroid could also be distance weighted:
  Let each member's contribution to the average be determined by its average pairwise similarity to the other members of the group.
- There are parallel discussions on how to represent classes and determine typicality within linguistic and psychological prototype theory.

Representing class membership

Hard classes

- Membership considered a Boolean property: a given object is either part of the class or it is not.
- A *crisp* membership function.
- A variant: disjunctive classes. Objects can be members of more than one class, but the memberships are still crisp.

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Soft classes

- Class membership is a graded property.
- Distance weighted.
- Probabilistic: The degree of membership for a given object restricted to $[0, 1]$, and the sum across classes must be 1.
- Fuzzy: The membership function is still restricted to $[0, 1]$, but without the probabilistic constraint on the sum.

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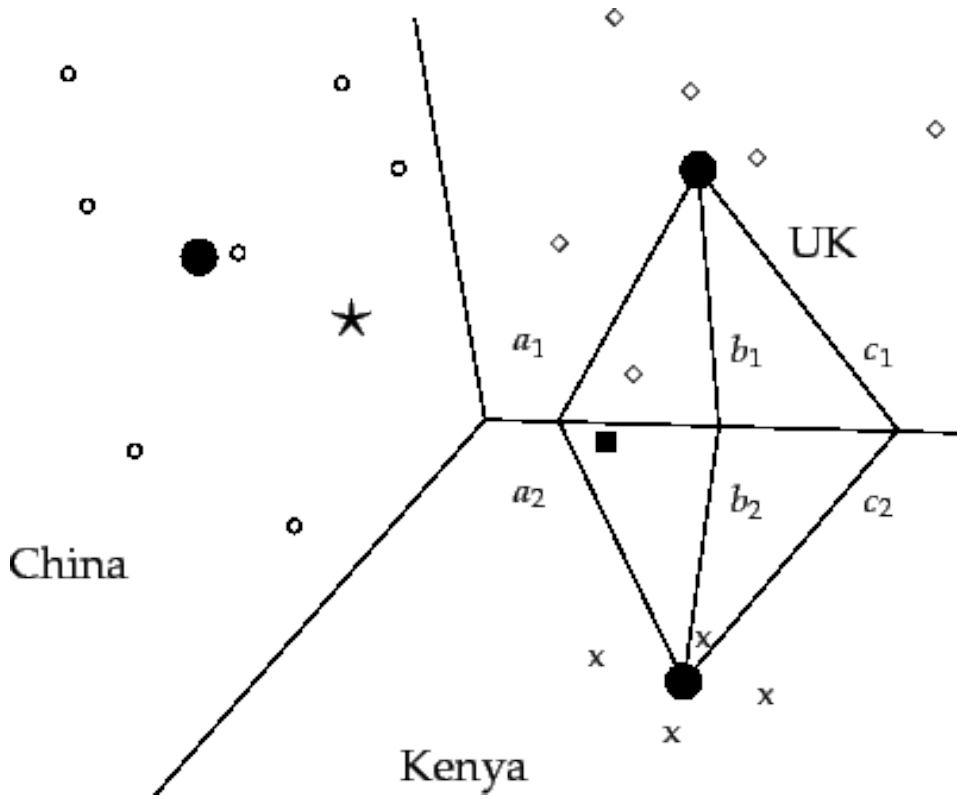
- The centroids define the boundaries of the class regions.

The decision boundary of the Rocchio classifier

- Defines the boundary between two classes by the set of points equidistant from the centroids.
- In two dimensions, this set of points corresponds to a *line*.
- In multiple dimensions: A line in 2D corresponds to a *hyperplane* in a higher-dimensional space.
- The boundaries are not computed explicitly.



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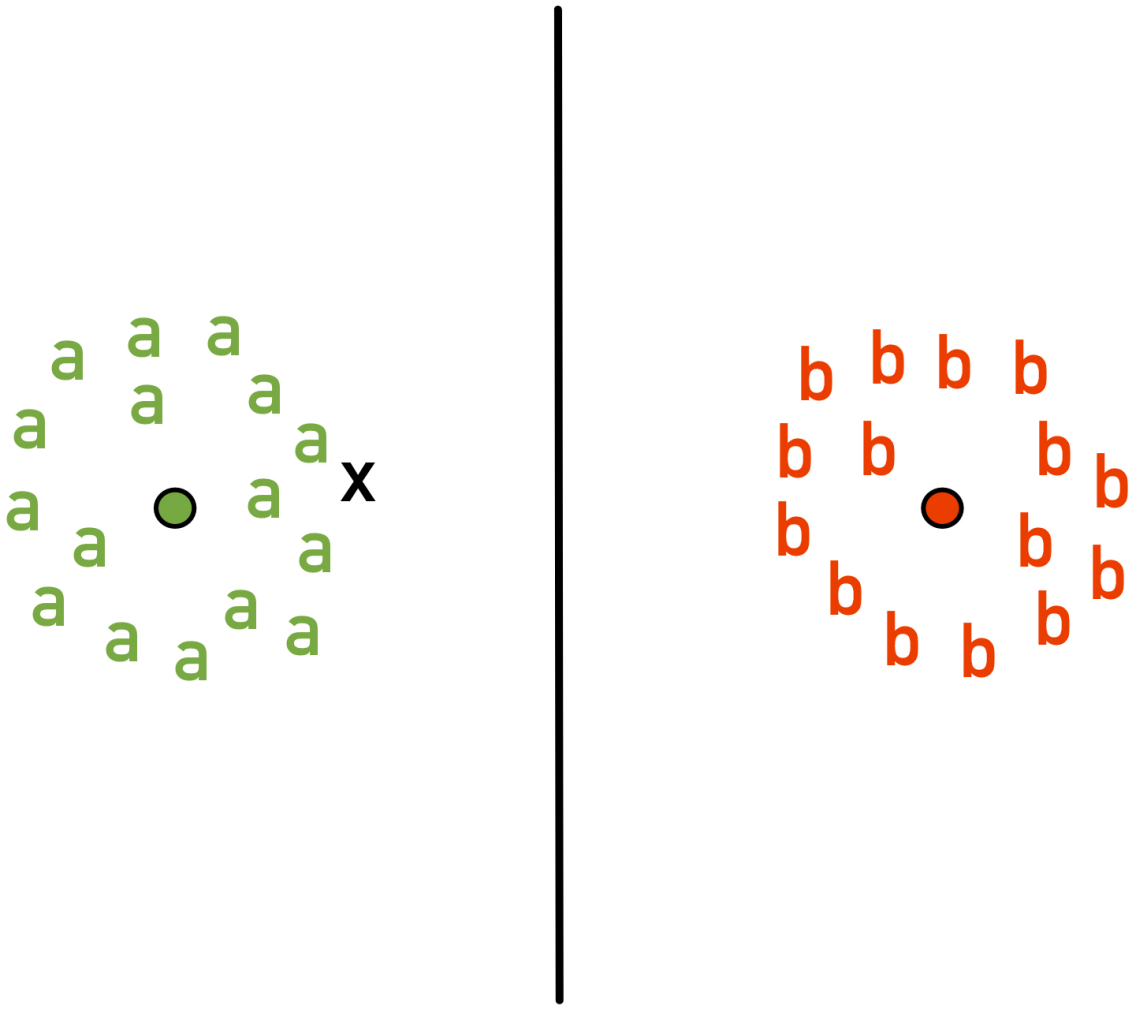
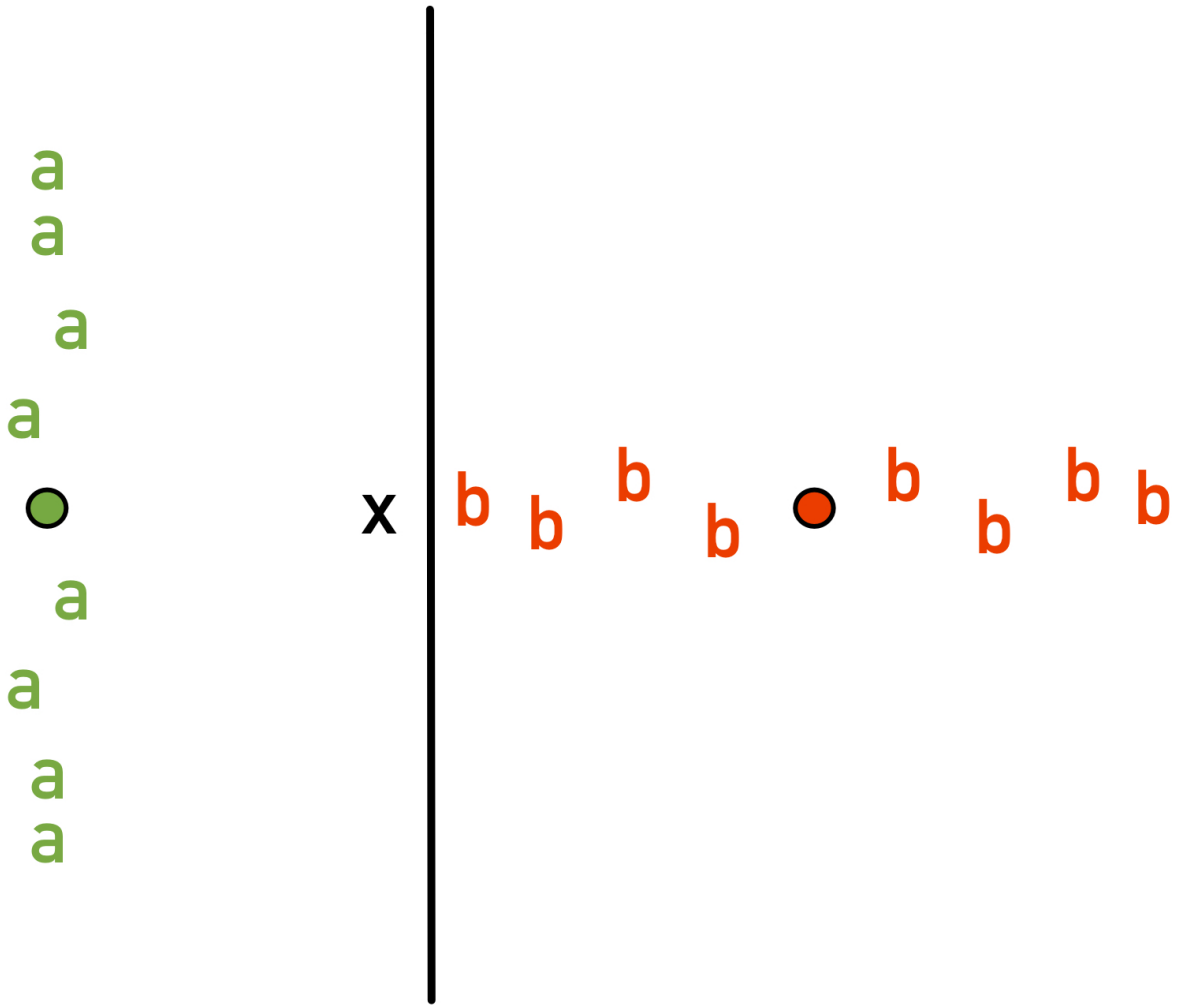
- Does not work well for classes than cannot be accurately represented by a single prototype or center (e.g. disconnected or elongated regions).
- Because the Rocchio classifier defines a linear decision boundary, it is only suitable for problems involving *linearly separable* classes.

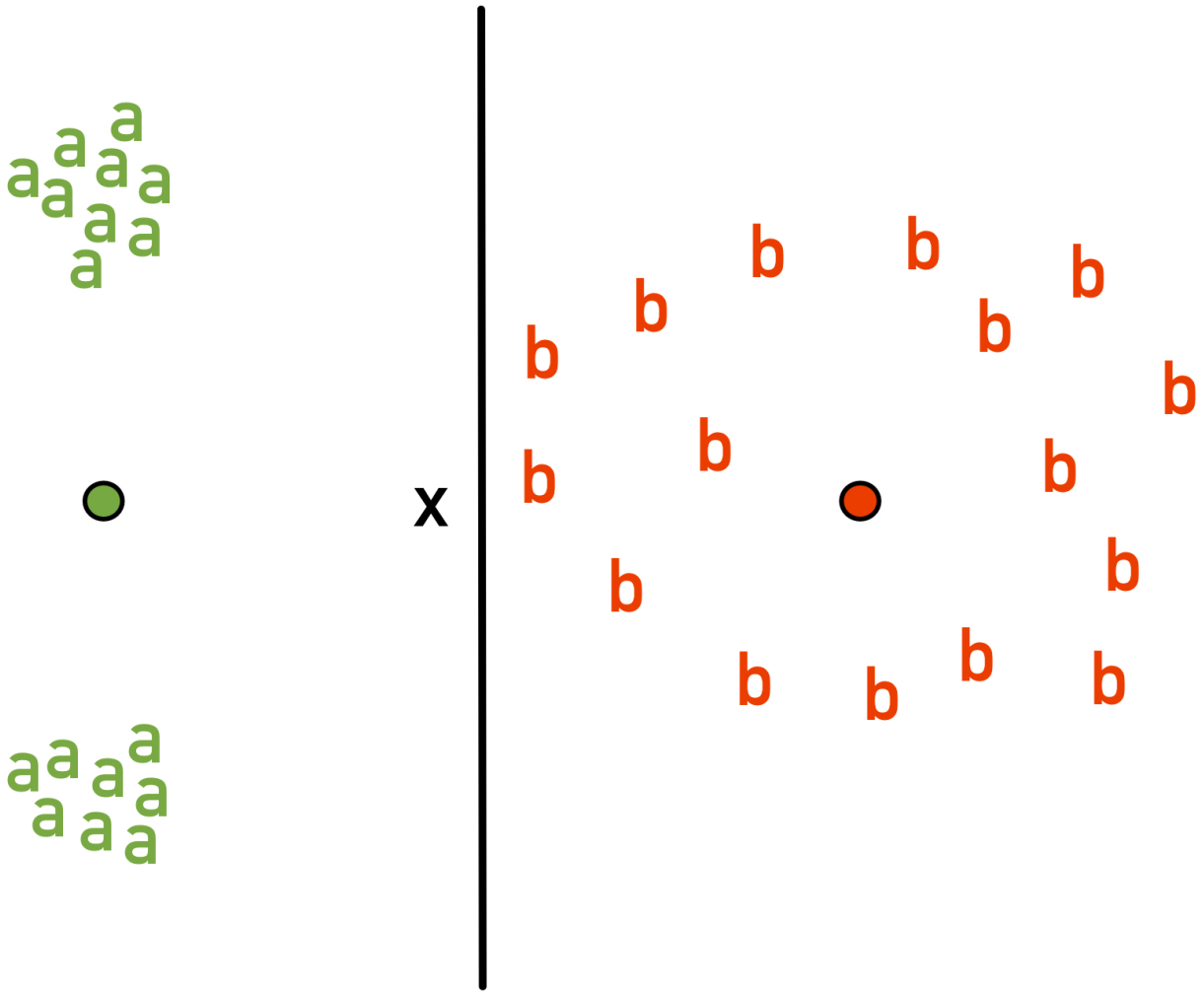
Ideal



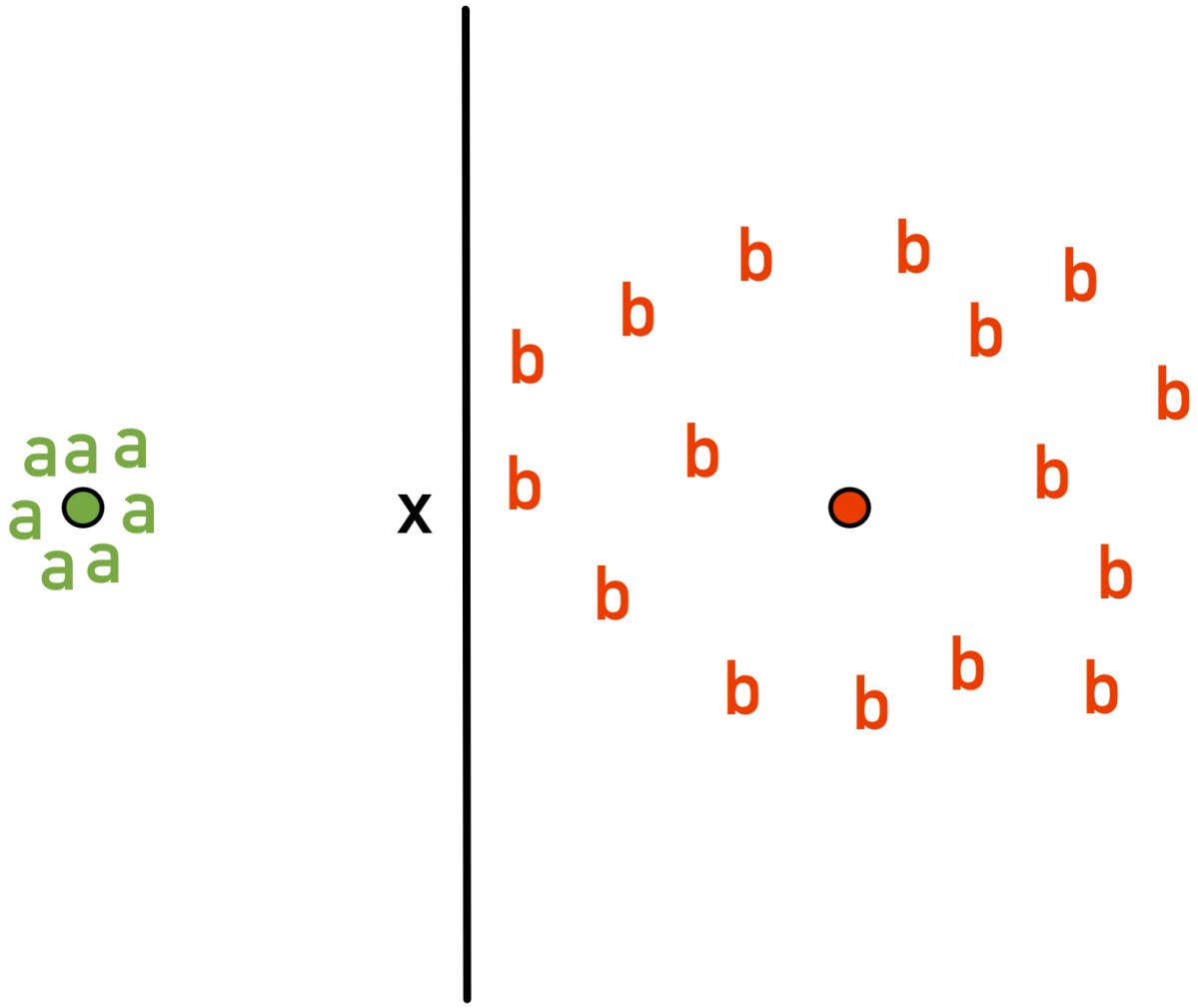
Problematic: Elongated regions



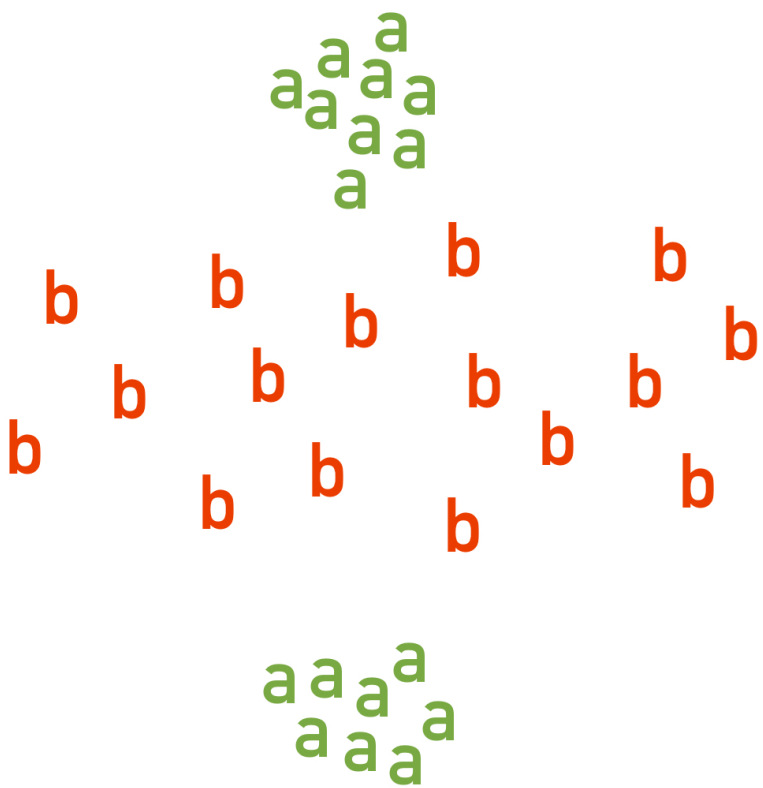
Problematic: Non-contiguous regions



Problematic: Different sizes



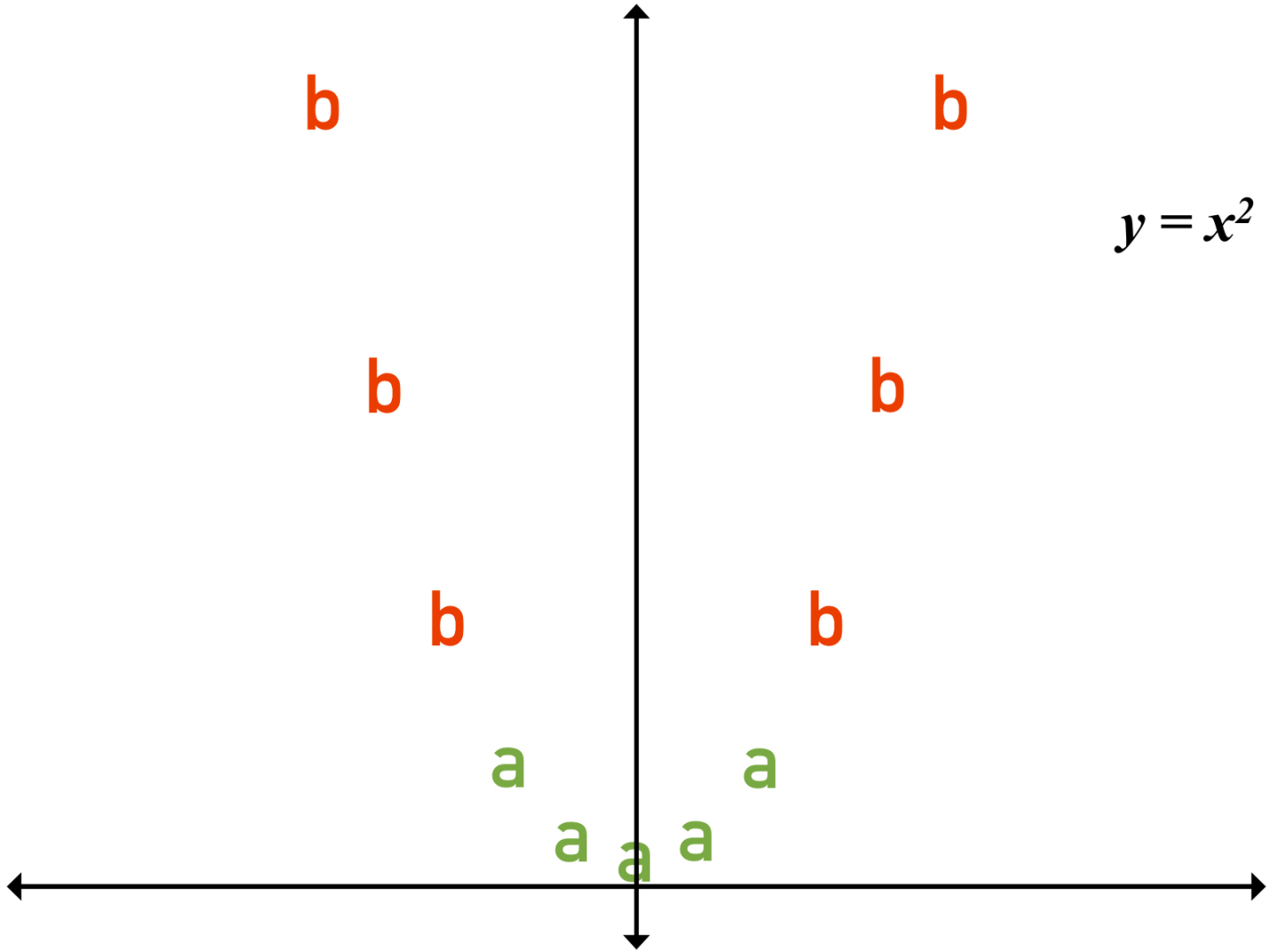
Problematic: Nonlinear boundary



A side-note on nonlinearity

- Before we turn to talk about non-linear classifiers, note that:
  Classes that are not linearly seperable in a given feature space. . .

A side-note on nonlinearity



- . . . may become linearly separable when the features are mapped to a higher-dimensional space (this is the basis for so-called kernel methods).

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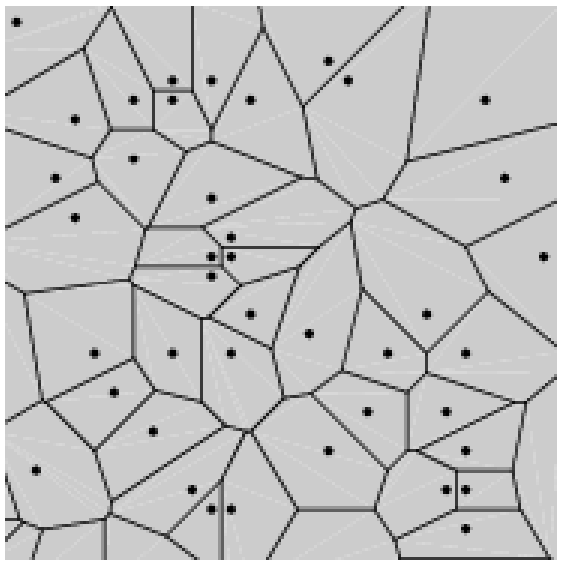
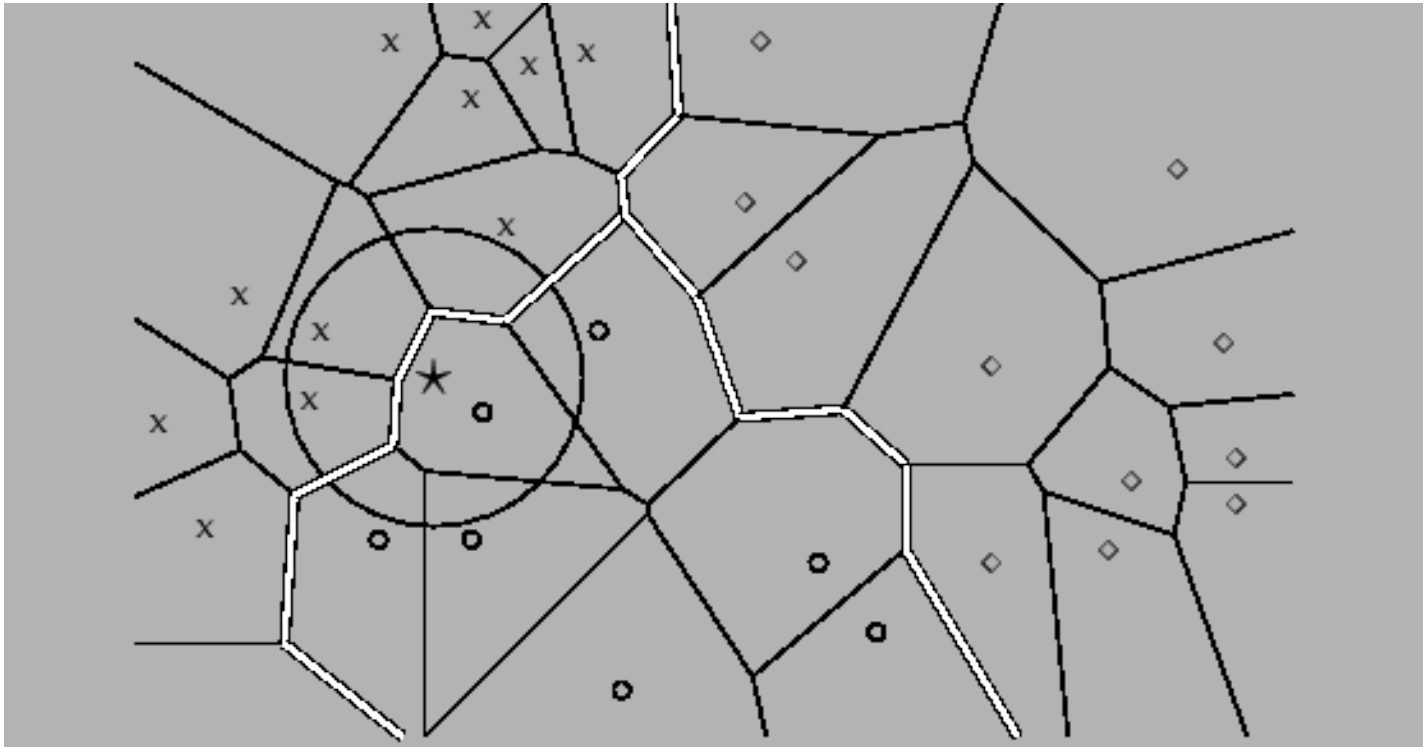
- The parameter k must be specified in advance.
- Unlike Rocchio, the kNN decision boundary is determined locally.
  - The decision boundary defined by the Voronoi tessellation.

Voronoi tessellation

- Assuming $k = 1$: For a given set of objects in the space, let each object define a cell consisting of all points that are closer to that object than to other objects.
- Results in a set of convex polygons; so-called Voronoi cells.
- Decomposing a space into such cells gives us the so-called Voronoi tessellation.



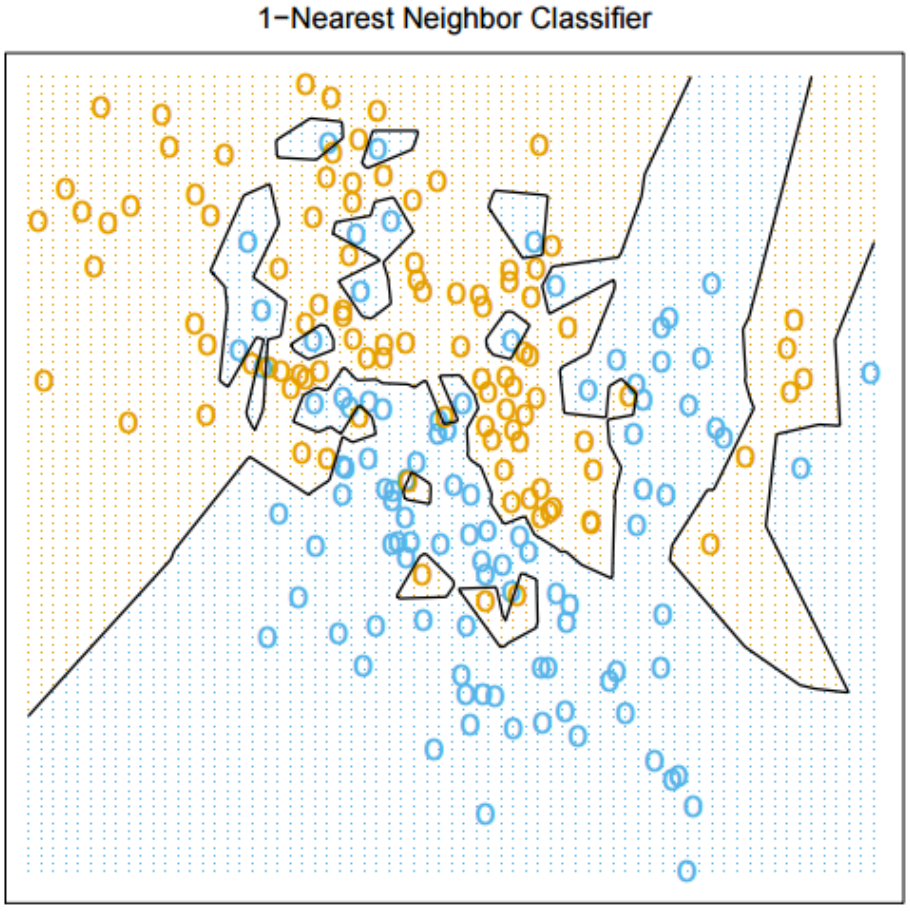
Voronoi tessellation for 1NN

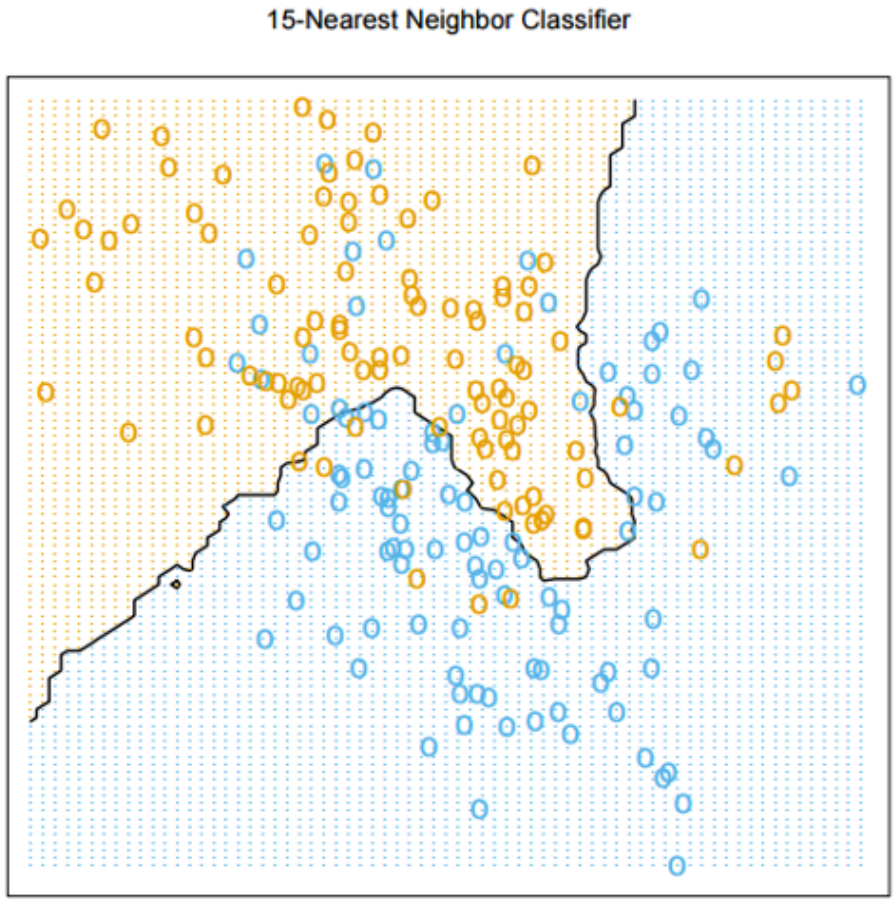


► **Figure 14.3** Voronoi tessellation and decision boundaries (double lines) in 1NN classification. The three classes are: X, circle and diamond.

Decision boundary for 1NN: defined along the regions of Voronoi cells for the objects in each class. Shows the non-linearity of kNN.

The effect of K





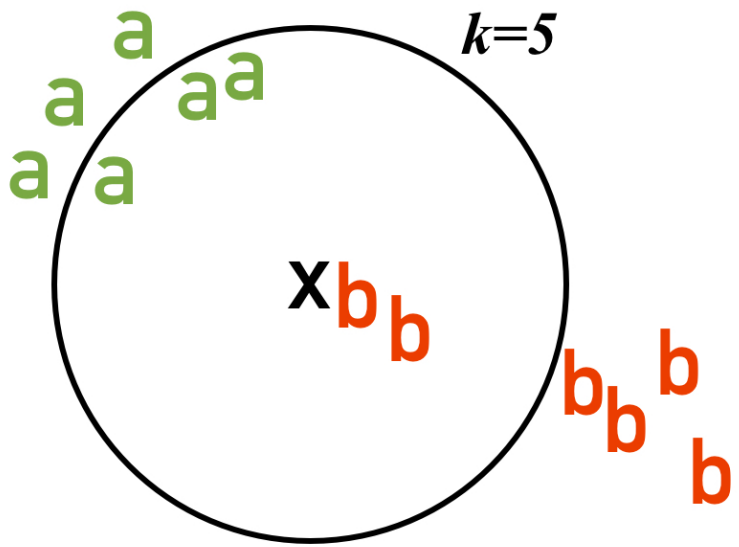
Figures from Elements of Statistical Learning

- What would happen if $K = N$?

"Softened" kNN-classification

A probabilistic version

- The probability of membership in a class c given by the proportion of the k nearest neighbors in c.



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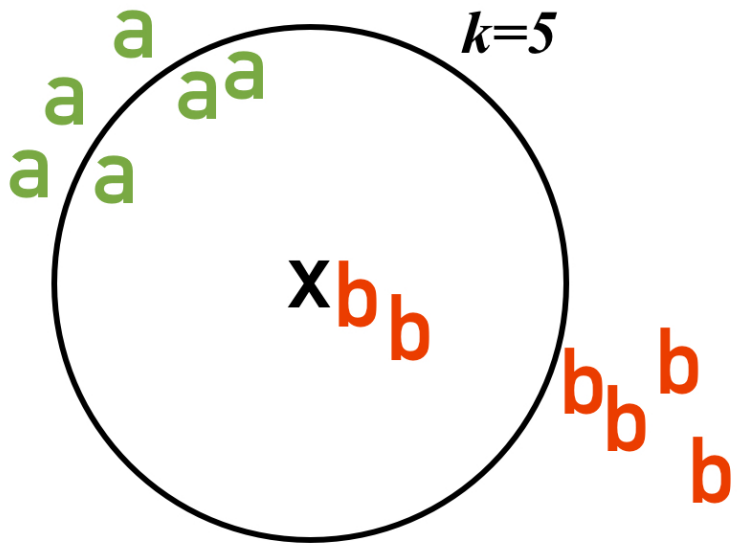

Distance weighted votes

- The score for a given class c_i can be computed as

$$\text{score}(c_i, o_j) = \sum_{\vec{x_n} \in \text{knn}(\vec{x}_j)} \text{I}(c_i, \vec{x}_n)\, \text{sim}(\vec{x_n}, \vec{x_j})$$

  where $\text{knn}(\vec{x}_j)$ is the set of k nearest neighbors of $\vec{x}_j$, sim is the similarity measure, and $\text{I}(c_i, \vec{x}_n)$ is 1 if $\vec{x}_n \in c_i$ and 0 otherwise.

- Can give more accurate results, and also help resolve ties.

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- but independent of the number of classes.
- A potential advantage for problems with many classes.
- Notice the similarity to the problem of ad hoc retrieval (e.g., returning relevant documents for a given query);
  - Both are instances of finding nearest neighbors.

Obligatory assignment 2b

- Builds on oblig 2a: Vector space representation of a set of words based on BoW features extracted from a sample of the Brown corpus.
- For 2b we provide class labels for most of the words.
- Train a Rocchio classifier to predict labels for a set of unlabeled words.

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Label	**Examples**
FOOD	*potato, food, bread, fish, eggs . . .*
INSTITUTION	*embassy, institute, college, government, school . . .*
TITLE	*president, professor, dr, governor, doctor . . .*
PLACE_NAME	*italy, dallas, france, america, england . . .*
PERSON_NAME	*lizzie, david, bill, howard, john . . .*
UNKNOWN	*department, egypt, robert, butter, senator . . .*

Testing a classifier

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- To evaluate the boundary, we measure the number of correct classification predictions on unseeen test items.
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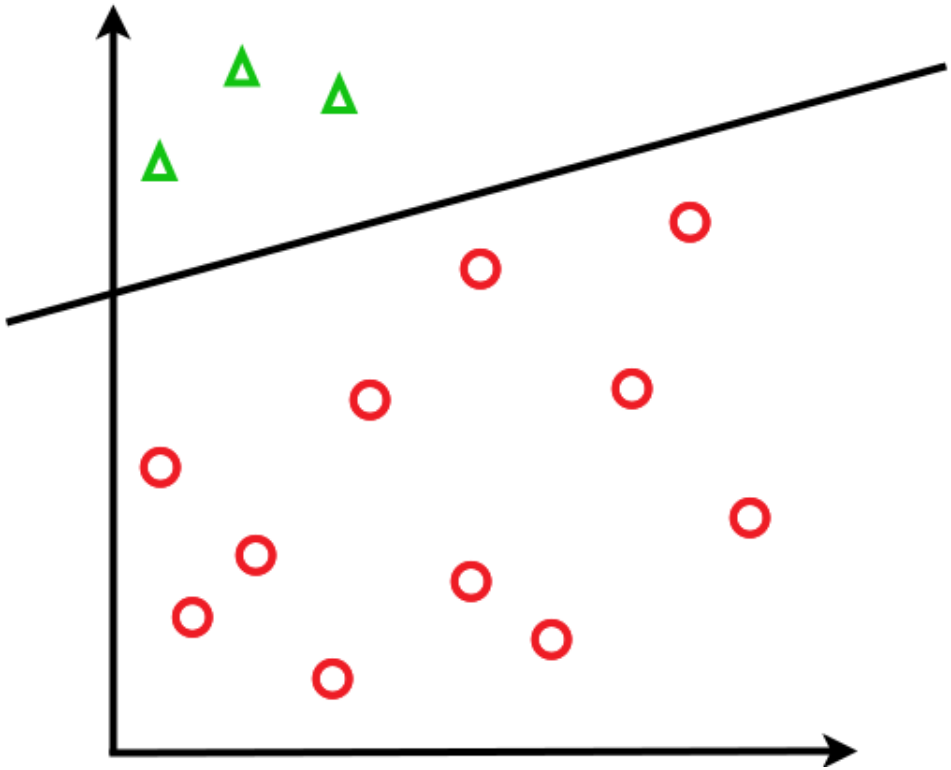
  - Many ways to do this. . .
- We want to test how well a model *generalizes* on a held-out test set.
- (Or, if we have little data, by n-fold cross-validation.)
- Labeled test data is sometimes refered to as the gold standard.
- Why can't we test on the training data?

Example: Evaluating classifier decisions



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prediction = positive	true positive (TP)	false positive (FP)
prediction = negative	false negative (FN)	true negative (TN)

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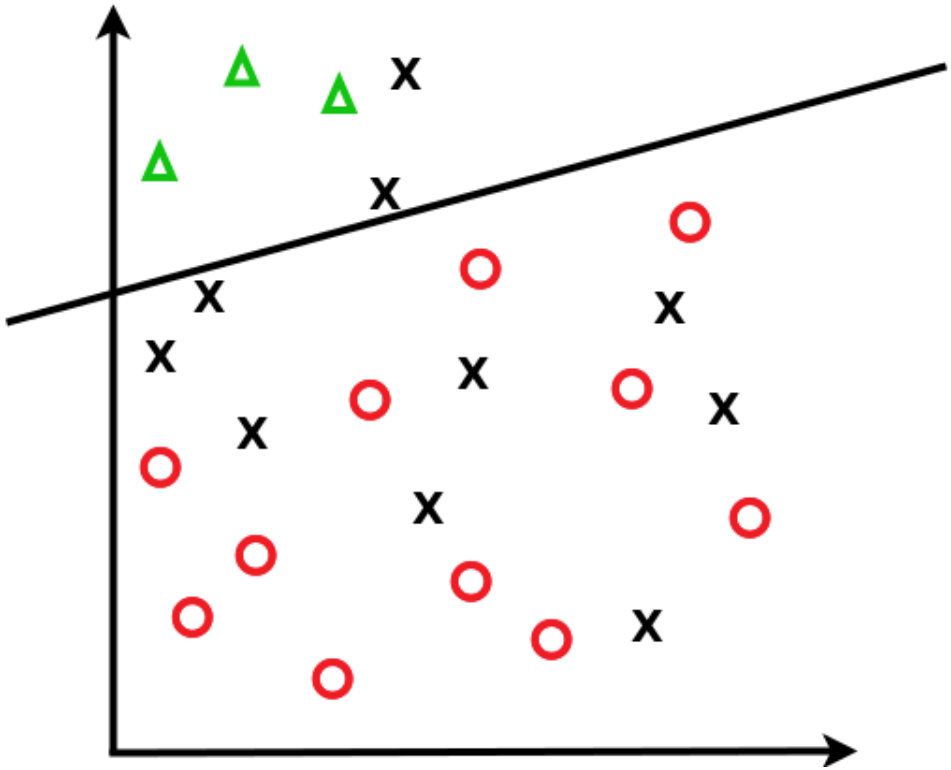

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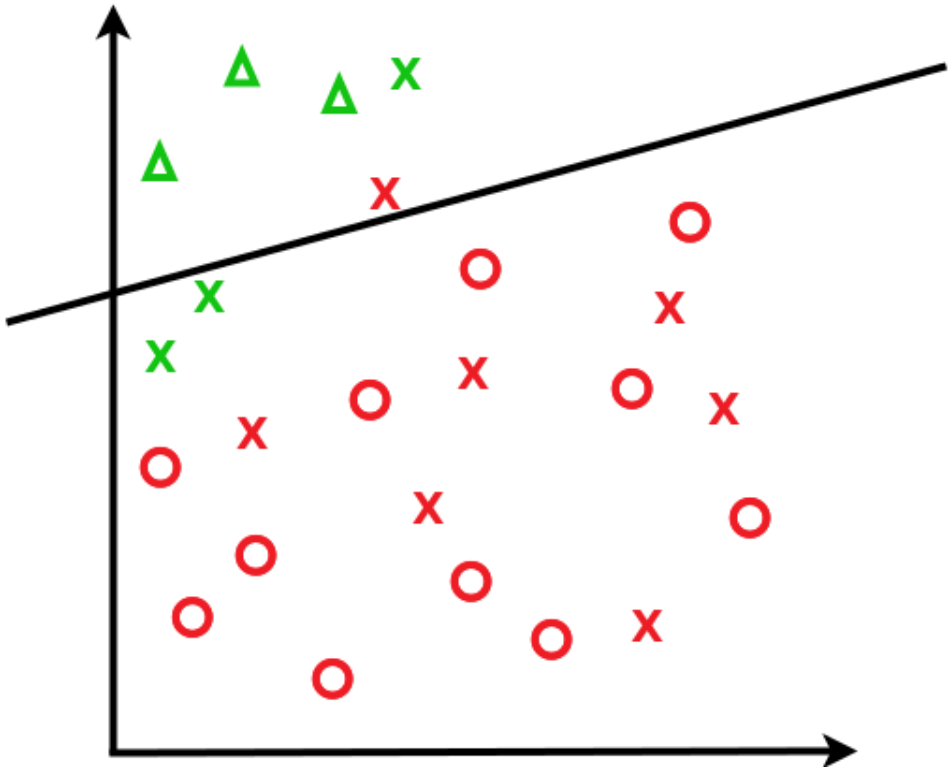

- Predictions for a given class can be wrong or correct in two ways:

	gold = positive	gold = negative
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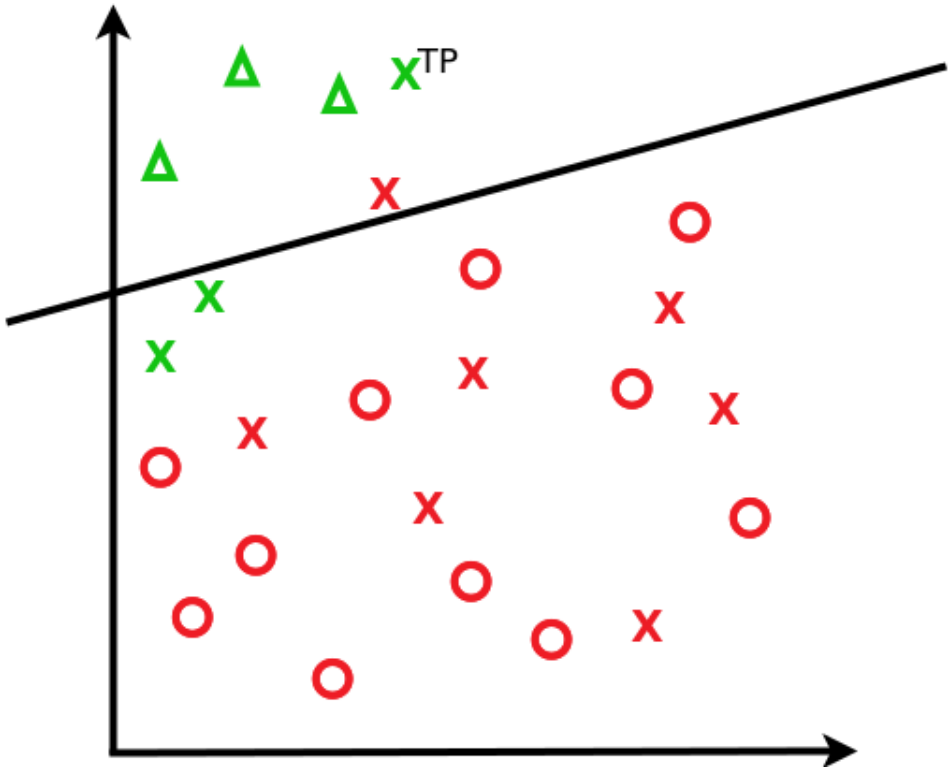

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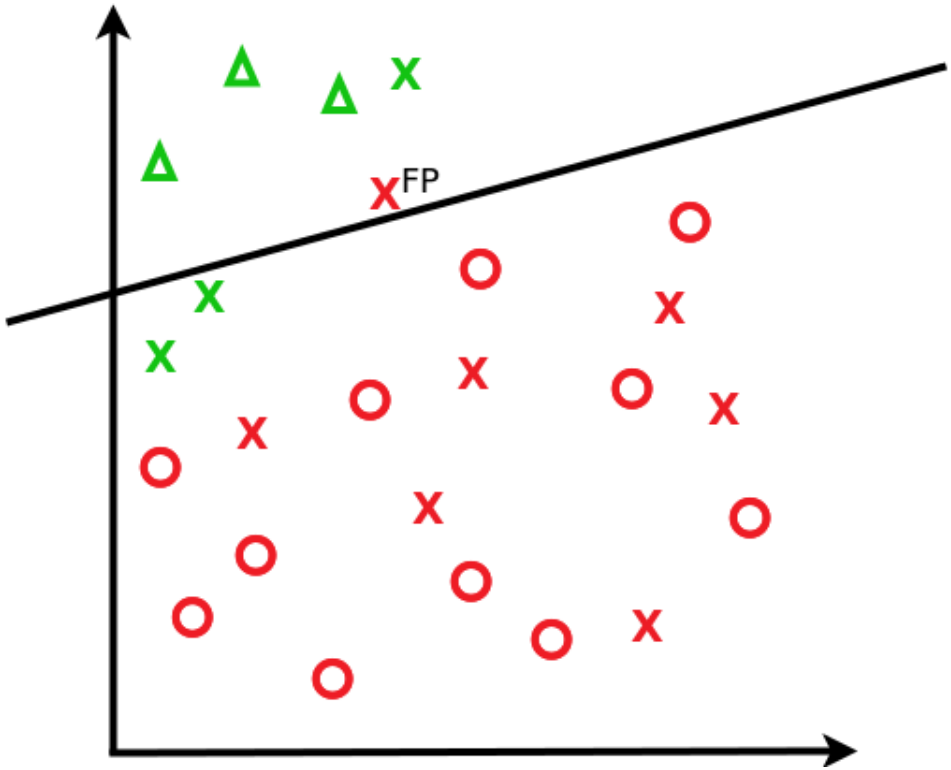

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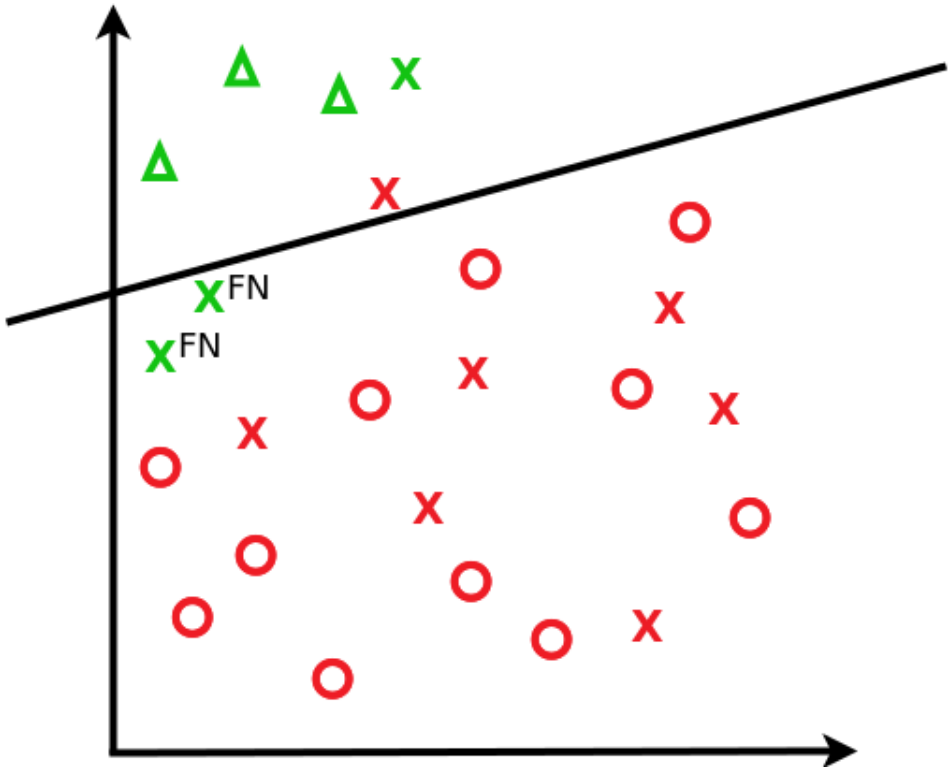

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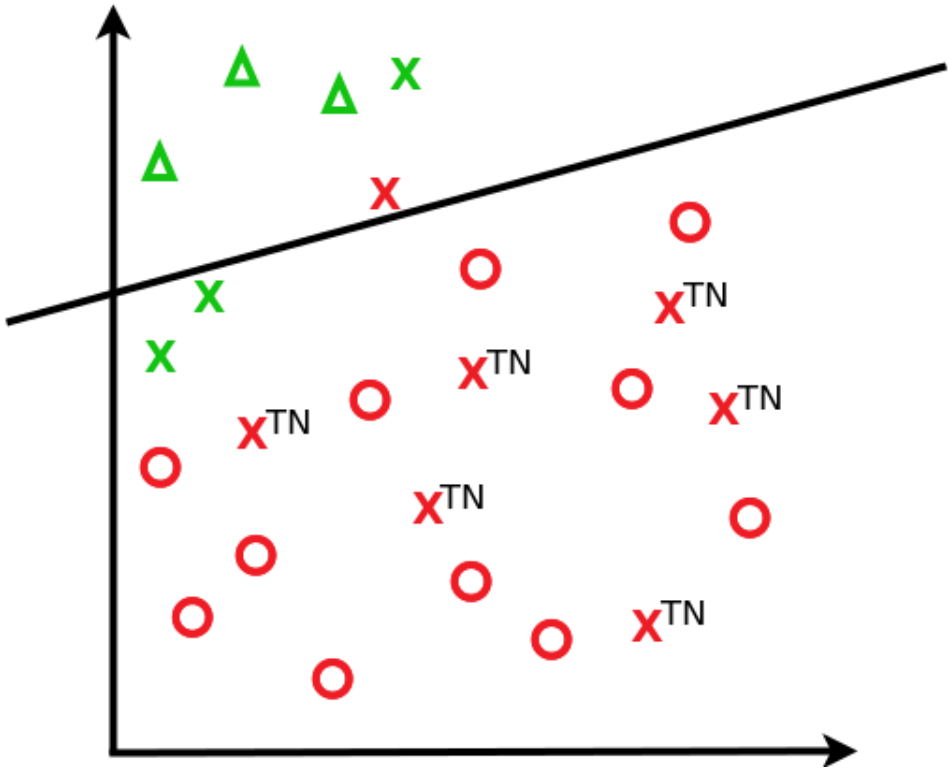

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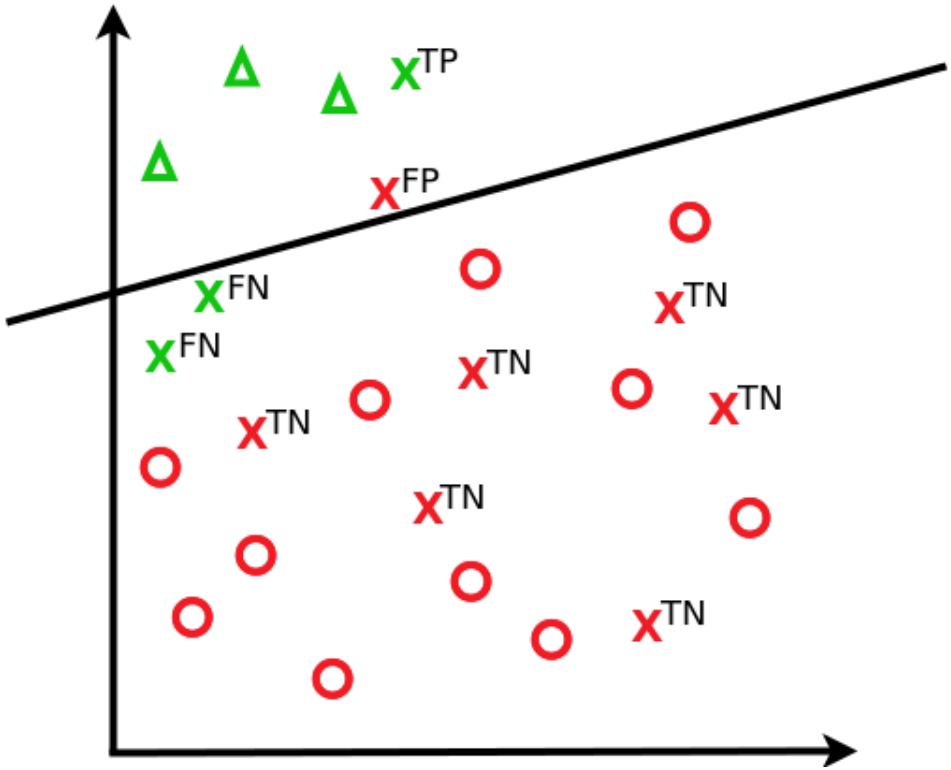

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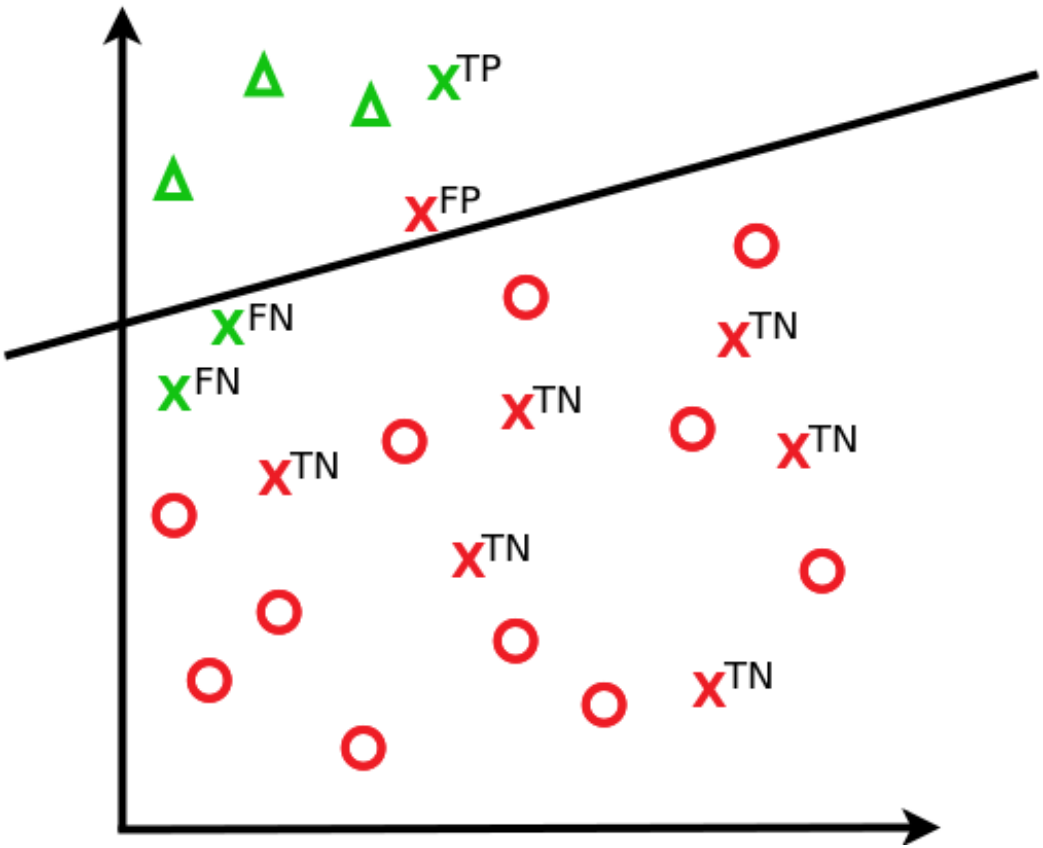
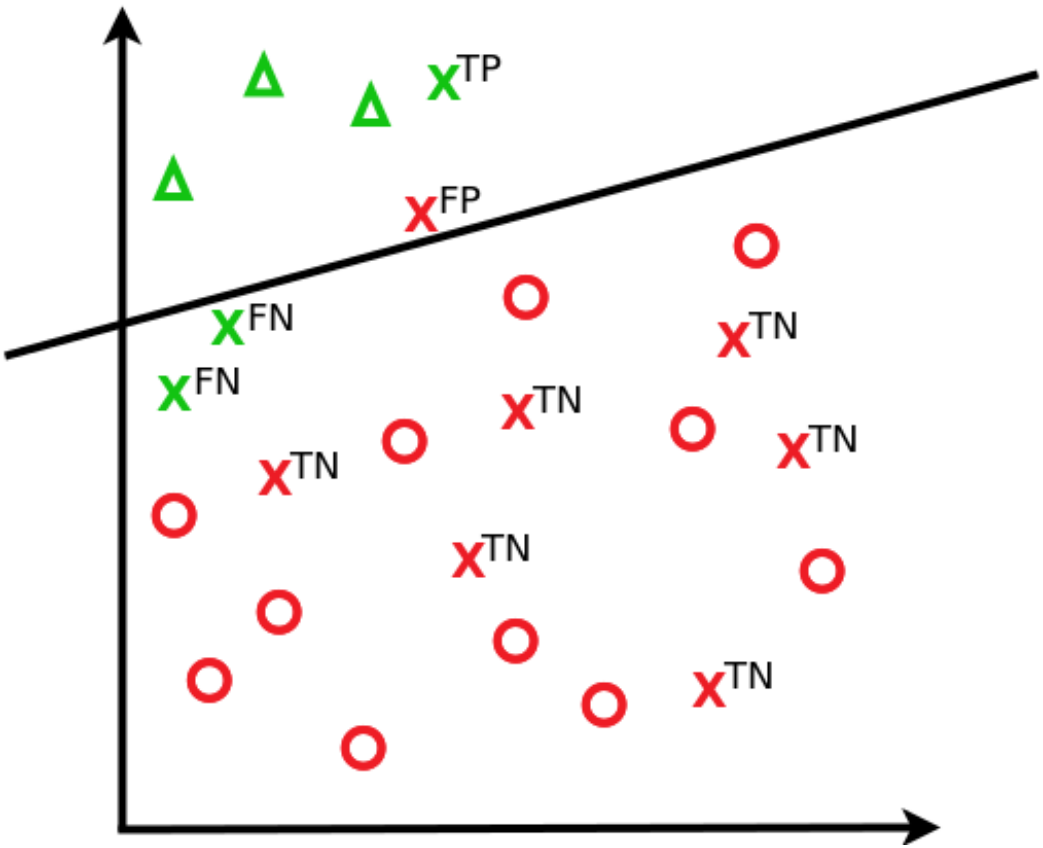


$$accuracy = \frac{TP+TN}{N}$$

Example: Evaluating classifier decisions



$$accuracy = \frac{TP+TN}{N}$$
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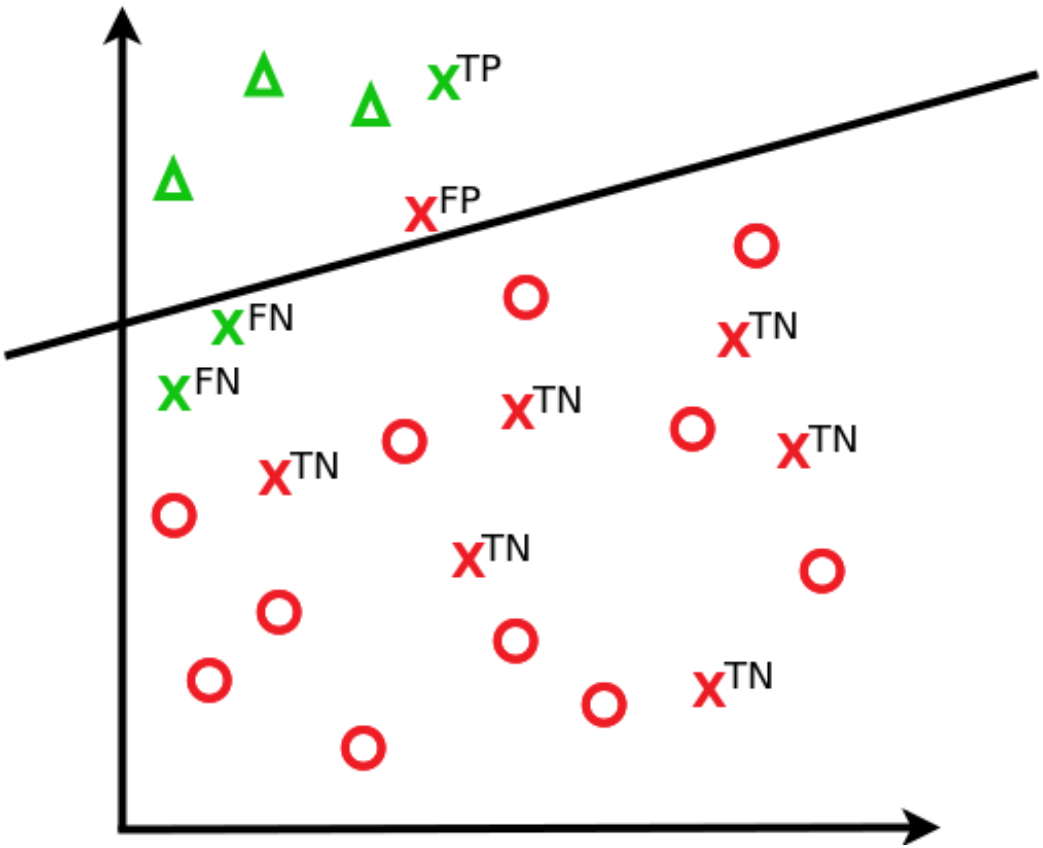
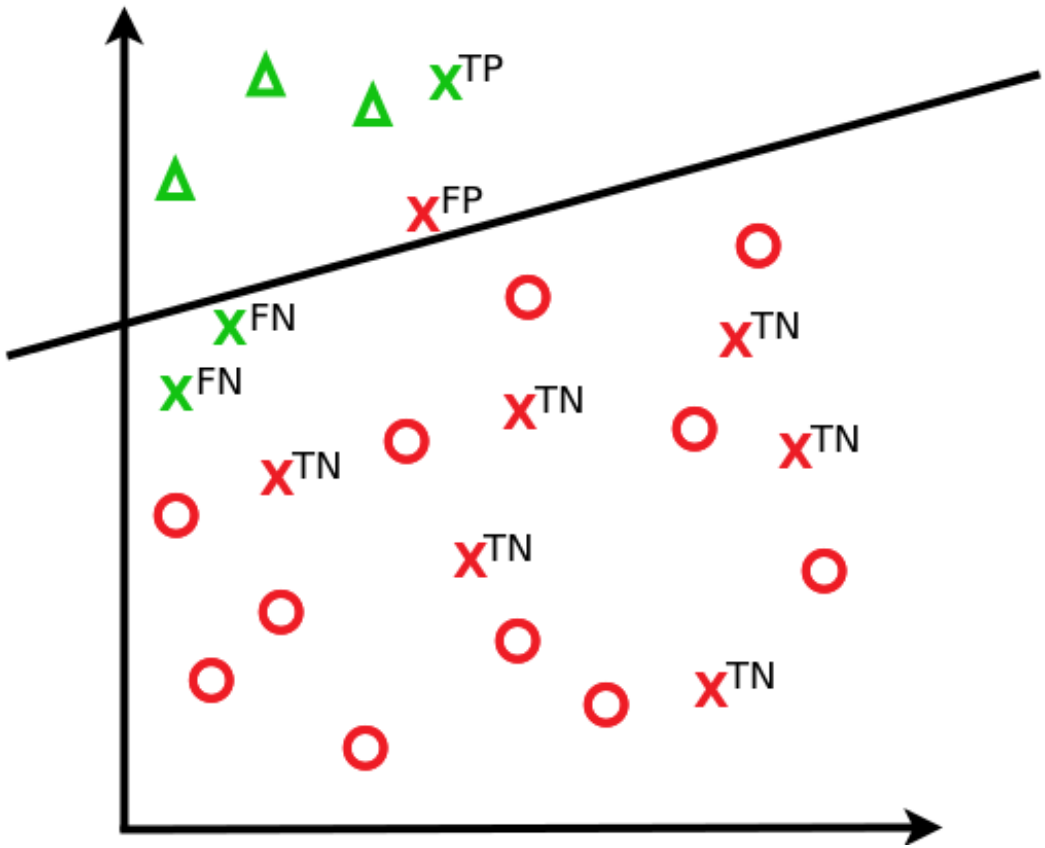


$$accuracy = \frac{TP+TN}{N} = \frac{1+6}{10} = 0.7$$

$$precision = \frac{TP}{TP+FP}$$

$$recall = \frac{TP}{TP+FN}$$

Example: Evaluating classifier decisions

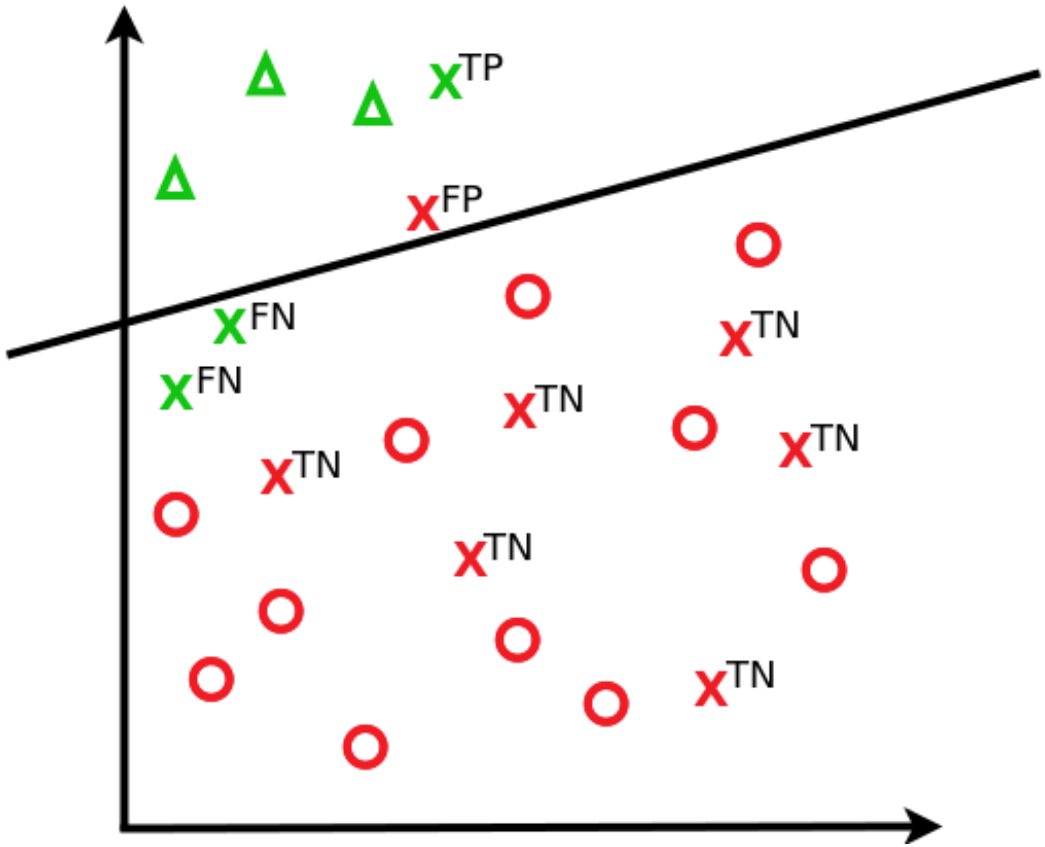


$$accuracy = \frac{TP+TN}{N} = \frac{1+6}{10} = 0.7$$

$$precision = \frac{TP}{TP+FP} = \frac{1}{1+1} = 0.5$$

$$recall = \frac{TP}{TP+FN} = \frac{1}{1+2} = 0.33$$

Example: Evaluating classifier decisions



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$$F\text{-}score = \frac{2 \times precision \times recall}{precision+recall} = 0.4$$

Evaluation measures

- $accuracy = \frac{TP+TN}{N} = \frac{TP+TN}{TP+TN+FP+FN}$
  - The ratio of correct predictions.
  - Not suitable for unbalanced numbers of positive / negative examples.
- $precision = \frac{TP}{TP+FP}$
  - The number of detected class members that were correct.
- $recall = \frac{TP}{TP+FN}$
  - The number of actual class members that were detected.
  - Trade-off: Positive predictions for all examples would give 100% recall but (typically) terrible precision.
- $F\text{-}score = \frac{2 \times precision \times recall}{precision+recall}$
  - Balanced measure of precision and recall (harmonic mean).

Evaluating multi-class predictions

Macro-averaging

- Sum precision and recall for each class, and then compute global averages of these.
- The macro average will be highly influenced by the small classes.

Evaluating multi-class predictions

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Micro-averaging

- Sum TPs, FPs, and FNs for all points/objects across all classes, and then compute global precision and recall.
- The micro average will be highly influenced by the large classes.

Next lecture

- Unsupervised machine learning for class discovery: Clustering
- Flat vs. hierarchical clustering.
- C-Means Clustering.
- Reading: Chapters 16 and 17 in Manning, Raghavan & Schütze (2008) (see course page for the relevant sections).