Project Euler Solutions

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Problem 587: Concave Triangle

Solution: 2240

Let the bottom-left of the rectangle be at $(0, 0)$ and the circles to have radius 1.

The area of the L-section is $1 - \frac{\pi}{4}$.

The diagonal line has slope $\frac{1}{n}$, and the intersection of the line with the first circle has coordinate:

$$x = \frac{-\sqrt{2} \cdot n^{\frac{3}{2}} + n^2 + n}{n^2 + 1}$$
$$y = \frac{x}{n}$$

The concave triangle can be split into a triangle and a curved section.

The area of the triangle is $\text{TriangleArea} = \frac{1}{2}xy$.

Let:

$$\begin{aligned} \text{CircleInt}(a, b) &= \int_a^b \left(\sqrt{1 - x^2}\right) \mathrm{d}x \\ &= \frac{1}{2}\left(x\sqrt{1 - x^2} + \sin^{-1}(x)\right)\Big|_a^b \end{aligned}$$

Then the area of the curved section is $\text{SectionArea} = (1 - x) - \text{CircleInt}(0, 1 - x)$.

And so the ratio is:

$$\frac{\text{TriangleArea} + \text{SectionArea}}{1 - \frac{\pi}{4}}$$

Problem 932: 2025

Solution: 72673459417881349

Let d be the number of digits in b. We have:

$$10^d a + b = (a + b)^2$$
$$b^2 + (2a - 1)b + a^2 - 10^d a = 0$$
$$b = \frac{1 - 2a + \sqrt{4 \cdot 10^d a - 4a + 1}}{2}$$

Similarly:

$$a = \frac{\left(10^d - 2b\right) + \sqrt{10^{2d} - 4 \cdot 10^d b + 4b}}{2}$$

Then search through the numbers up to half of the required length, where the corresponding other number is obtained using the formulas.