

What do you mean by effective mass of an electron Derive expression for $m = \dots$

Positive effective mass implies that the object react to applied external force in the normal way.

Negative effective mass implies that the object decelerates in the direction of applied force. This can happen if the applied force is overmatched by internal forces due to crystal lattice, when an external force tries to impel a particle into a region dominated by repulsive potential whose magnitude increases as the particle penetrates further into the region.

The infinite effective mass implies that the object cannot be accelerated by external force. This can be due to the internal force of crystal which cancels the effect of applied force.

Hence effective mass allow us to describe the response of crystal to the particles in terms of external force applied.

Effective Mass of an Electron

For a free particle, the relationship between energy

$$\text{and momentum is given by } E = \frac{P^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$\frac{dE}{dk} = \frac{\hbar^2 k}{m} = \hbar \cdot \frac{\hbar k}{m}$$

$$= \frac{\hbar P}{m} = \frac{\hbar mv}{m} = \hbar v$$

$$v = \frac{1}{\hbar} \frac{dE}{dk} \quad \dots(1)$$

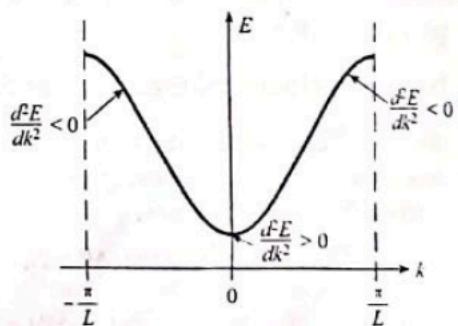


Figure : Electron motion in the conduction band of a conductor

$$\text{Here, } \frac{dE}{dk} = \frac{\hbar^2 k}{m} \Rightarrow \frac{d^2E}{dk^2} = \frac{\hbar^2}{m}$$

$$\text{Therefore, } m = \frac{\hbar^2}{d^2E/dk^2}$$

This shows that E versus k plot is parabolic. For a free particle this curvature d^2E/dk^2 is constant. But for electron in solid the curvature d^2E/dk^2 is not always constant and hence mass is also not constant.

For solid we can write.

$$m^* = \frac{\hbar^2}{d^2E/dk^2} \quad \dots(2)$$

Where m^* represents effective mass of electron in the crystal.

The curvature d^2E/dk^2 is positive when the E - k curve is concave upward and is negative when the curve is concave downward as shown in figure.

The effective mass of electron is positive at $k = 0$ since the curve is concave upward. But the effective mass is negative in the vicinity of $k = \pm \pi/L$.

This is the perfectly consistent conclusion, but our experiences are with positive masses. It is also worth noting that there are values of k for which the curvature is zero, implying the mass is infinite.

Electrical Conduction in Liquid ✓

When electric field is applied through liquid dielectric there is no current initially. As we go on increasing electric field, the impurity, air or liquid molecules get discharged giving small current. When the electric field strength is large enough called breakdown voltage, we get large amount of current suddenly and the liquid acts as conducting medium. This phenomenon is called *Electric breakdown in liquid*.

The electrical conduction in liquid can be understood in terms of ionic conduction as in electrolysis or electroplating. In electrolysis process we take aqueous media containing the metal to be plated in which two electrodes called cathode and anode are placed when an external potential is applied cations and anions are formed. The positively charged cations move towards cathode and the negatively charged anions move toward anode thus transferring electric current through liquid electrolyte.

The ionic conductivity is given by

$$\sigma = n e \mu \quad \dots(1)$$

Where, n = number of ions formed, μ = mobility of ion.

$$\text{From Einstein's relation, } \frac{D}{\mu} = \frac{KT}{e} \Rightarrow \mu = \frac{eD}{KT} \quad \dots(2)$$

The diffusion coefficient varies with temperature as

$$D = D_0 e^{-\frac{Q}{KT}} \quad \dots(3)$$

Thus equation (1) becomes, $\sigma = n e \frac{e}{KT} D_0 e^{-Q/KT}$

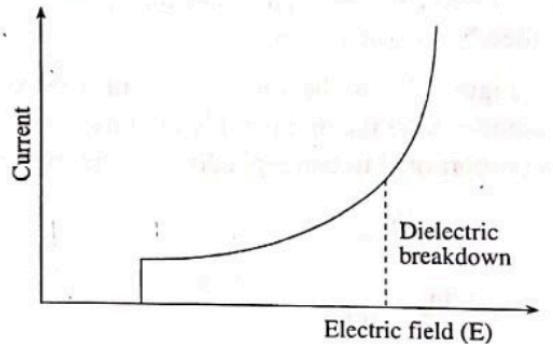


Figure 1: Conduction current versus electric field

$$\text{Or, } \sigma = \frac{n e^2 D_0}{K T} e^{-\frac{Q}{K T}}$$

$$= \sigma_0 e^{-\frac{Q}{K T}} \text{ where } \sigma_0 = \frac{n e^2 D_0}{K T}$$

Taking \ln on both side $\ln \sigma = \ln \sigma_0 + \ln (e^{-\frac{Q}{K T}})$

$$\text{or } \ln \sigma = \ln \sigma_0 - \frac{Q}{K T} \quad \dots(4)$$

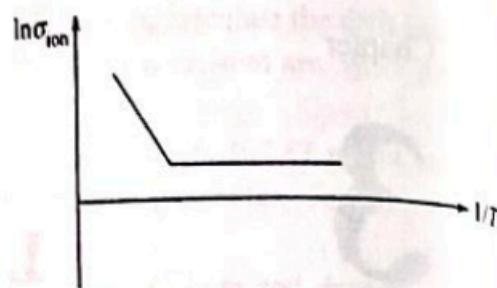


Figure 2: Dependence of ionic conductivity with inverse of temperature.

This relation shows that ionic conductivity depends inversely on temperature, as shown in figure below.

Electrical Conduction in Gases

A gas in its normal state is almost a perfect insulator. However when a high voltage is applied between two electrodes immersed in gaseous medium, the electric breakdown occurs and the gas becomes a conductor. The maximum voltage applied at the time of electrical breakdown is called breakdown voltage.

In Figure, let n_0 be the number of electrons produced by ultraviolet radiation incident on cathode. n_x be the number of electrons at a distance x from cathode. Then concentration gradient is proportional to corresponding number of electrons produced.

$$\text{ie. } \frac{dn_x}{dx} \propto n_x$$

$$\frac{dn_x}{dx} = \alpha n_x \quad \dots(1)$$

Where α is the average ionizing collision made by electrons per centimeter called Townsend's first ionization coefficient.

$$\frac{dn_x}{n_x} = \alpha dx, \text{ integrating,}$$

$$\ln(n_x) = \alpha x + c$$

$$\text{at } x = 0, n_x = n_0$$

$$c = \ln(n_0)$$

$$\ln(n_x) = \alpha x + \ln(n_0) \Rightarrow \ln \frac{n_x}{n_0} = \alpha x$$

$$n_x = n_0 e^{\alpha x} \quad \dots(2)$$

Then the number of electrons reaching the anode (i.e. $x = d$) will be, $n_d = n_0 e^{\alpha d}$... (3)
and the current produced by those electrons is,

$$I = I_0 e^{\alpha d} \quad \dots(4)$$

Where I_0 is initial current at cathode.

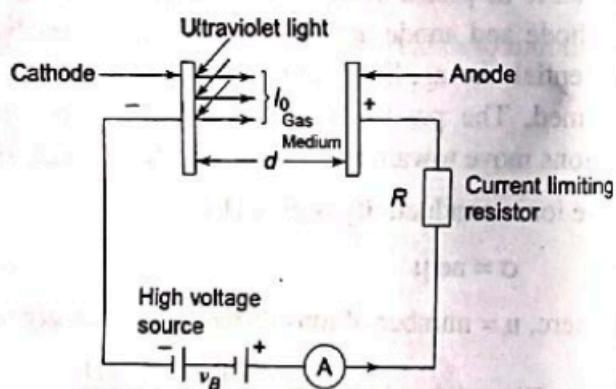


Figure : Arrangement for Townsend discharge

Clausius - Massotti Equation:

(For electronic polarizability, α_e)

The actual field experienced by a molecule in dielectric is called local field E_{local} . This is the sum of applied field and field due to polarization.

$$E_{local} = E + \frac{P}{3\epsilon_0}$$

And induced polarization is now given by,

$$P = \alpha_e E_{local}$$

Hence, polarization due to N, molecules

$$P = Np = N\alpha_e E_{local} = N\alpha_e \left(E + \frac{P}{3\epsilon_0}\right)$$

$$\Rightarrow P = N\alpha_e E + \frac{N\alpha_e P}{3\epsilon_0}$$

$$P \left(1 - \frac{N\alpha_e}{3\epsilon_0}\right) = N\alpha_e E$$

$$P = \frac{N\alpha_e E}{1 - \frac{N\alpha_e}{3\epsilon_0}} \quad \dots(1)$$

Also, $P = \epsilon_0 \chi E \dots\dots(2)$

$$\text{Comparing equation (1) and (2)} \quad \epsilon_0 \chi E = \frac{N\alpha_e E}{1 - \frac{N\alpha_e}{3\epsilon_0}}$$

$$\epsilon_0 \chi \left(1 - \frac{N\alpha_e}{3\epsilon_0}\right) = N\alpha_e$$

$$\epsilon_0 \chi - \frac{N\alpha_e \chi}{3} = N\alpha_e$$

$$N\alpha_e + \frac{N\alpha_e \chi}{3} = \epsilon_0 \chi \Rightarrow N\alpha_e \left(1 + \frac{\chi}{3}\right) = \epsilon_0 \chi$$

$$\Rightarrow N\alpha_e \left(\frac{\chi+3}{3}\right) = \epsilon_0 \chi$$

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\chi}{\chi+3} = \frac{\epsilon_r - 1}{\epsilon_r - 1 + 3} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\chi}{\chi+3} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \quad \dots(3)$$

Equation (3) is called Clausius - Massotti equation, which relates relative permittivity and electronic polarizability more accurately.

In terms of optical frequency, the Clausius-Massotti equation is given by

$$\frac{N\alpha}{3\epsilon_0} = \frac{n^2 - 1}{n^2 + 2} \quad [\text{Since, } \epsilon_r = n^2, n \text{ is the optical frequency}]$$

Tropic Polarization:

2. Hysteresis Loss (Hysteresis Loop)

Most of the magnetic materials are polycrystalline having microstructure consisting of many *grains* of various sizes and orientations. Each grain possess domains. When a field is applied in particular direction, the domain in that direction grows, which increases magnetization. As we increase the field, the domain motion extend to large distance until the domain wall stuck at an imperfection (due to crystal defect, impurities). Domain wall cannot move until the field applied is sufficient to break it. This sudden jerks in wall motion leads to a small jump in magnetization of specimen. This phenomenon is called *Barkhausen effect*. As we continue to increase the field to H_{sat} , Magnetization reaches to saturation value M_{sat} [point b].

If we decrease the field, domains in each grain will rotate to align in nearest easy direction. In some grain additional small domains may develop that reduce the magnetization. There exist some finite value of magnetisation even at zero field. This value of magnetization is called *remanent or residual magnetization* (M_r) [point c]

If now magnetizing field is applied in the reverse direction, the magnetization decreases and gradually becomes zero at a sufficiently large applied field. This magnetizing field required to totally demagnetize the sample is called *coercive field* H_c [point d]

If the field is further increased in reverse direction, the similar loop is obtained in reverse direction. The complete 'M' versus 'H' curve obtained by changing magnetizing field in both positive and negative direction is a closed loop called Hysteresis loop.

The area enclosed with in B versus H curve is the measure of energy dissipated per unit volume per cycle of applied field variation. It is given by

$$E = \oint H dB$$

Steinmetz developed an empirical relation for hysteresis power loss as,

$$P_h = K_h f B_m^k$$

Where, P_h = Hysteresis power loss in watt/m³

K_h = Hysteresis coefficient (Joules /m³)

f = Frequency of magnetization (Hz)

B_m = Maximum flux density in Wb/m² (or T)

k = Steinmetz coefficient

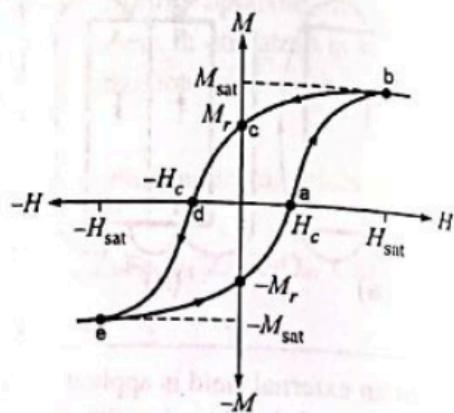


Figure : Hysteresis loop

Soft and Hard Magnetic Materials

On the basis of Hysteresis loop magnetic materials are classified into soft magnetic materials and hard magnetic materials.

1. Soft Magnetic Materials

- These are the materials having small hysteresis loop area.
- Power loss/ volume is small
- They are easy to magnetize and demagnetize so require low field.
- They have high susceptibility and high permeability.
- low residual magnetization.
- low coercivity i.e. having small value of H_c .
- These are suitable for the application where cycles of magnetization and demagnetization involved. Such as in Electric motors, transformers, inductors etc.
- They are composed of very large grain and the anisotropy energy is small. So small field is sufficient to magnetize them to saturation as a single domain.

2. Hard Magnetic Materials

- These are the materials having large hysteresis loop area.
- So power loss per unit volume is large.
- They are hard to magnetize and demagnetize so require large field.
- They have low value of susceptibility and permeability.
- They have large residual magnetization.
- They have large coercivity, coercive field (H_c) for these material is millions of times greater than that of soft magnetic material.
- They are suitable to make permanent magnet, suitable for magnetic storage of data.
- They are composed of very small grain and the anisotropy energy is high. So sufficiently large field is required to magnetize to saturation as a single domain.

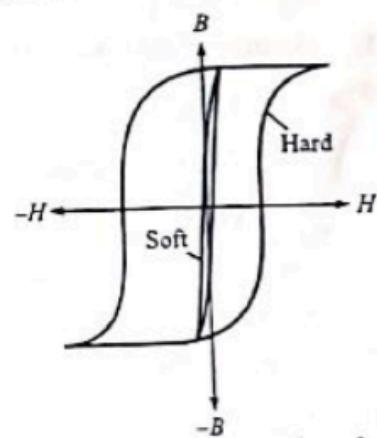


Figure : Hysteresis curve for Soft and Hard magnetic material

Types of Magnetic Material

1. Diamagnetic Material

- Diamagnetic substance have no magnetic moment in the absence of external field.
- They are feebly magnetized in the direction opposite to the applied field so they are weakly repelled by magnet or magnetic field.
- So they have small negative value of susceptibility and relative permeability ($\mu_r = \chi + 1$) is slightly less than unity. (The average value of $\chi = -10^{-6}$)
- The diamagnetic properties of a substance is independent of temperature so they do not obey Curie's law.

Examples: Superconductor, Bismuth, Mercury, Silver, Diamond, Lead, Copper, Ethyl alcohol, Nitrogen, Gold, Antimony, Water, Hydrogen etc.

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2. Paramagnetic Materials

- Paramagnetic substance have no magnetic moment (or magnetization) in the absence of external field, this is because the atomic moments are randomly oriented in the absence of field.
- When placed in magnetic field, they are feebly magnetized in the direction of applied field.
- So they have small positive value of susceptibility (E.g. $\chi = 1.2 \times 10^{-5}$ for metals)
- The susceptibility decreases with rise in temperature. Because of higher temeprature, the molecular collision destroy the alignment of molecular magnetic moments with applied field. i.e. Curie law ($\chi = C/T$) is obeyed.

Examples: Aluminium, Chromium, Platinum, Oxygen, Tungsten, Magnesium, Solution of Ferro magnetic materials etc.

3. Ferromagnetic Materials:

- Ferro magnetic material have large permanent magnetization even in the absence of field.
- They are strongly magnetized in the direction of applied field.
- So they have positive and very large value of susceptibility (χ)
- The susceptibility decreases with rise in temperature at certain temperature T_c , Ferro magnetism is lost and the substance becomes paramagnetic. This value of temperature is called critical or curie temperature.

Examples: Iron, Cobalt, Nickel and their alloys such as AlNiCo.

4. Antiferromagnetic Materials:

- Anti Ferromagnetic materials have permanent magnetic moments but have opposite alignment on alternating atoms.
- So they have no net magnetization in the absence of external field.
- They are feebly magnetized when subjected to a strong magnetic field.

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- They are feebly magnetized when subjected to a strong magnetic field.
- So they have small positive value of susceptibility.
- A temperature increases, antiferromagnetism is lost at certain temperature called Neel temperature above this they are paramagnetic.

Examples: Cr, Mn, MnS, MnO, MnO₄, NiO, CoO, FeO, FeCl₃ etc.

5. Ferrimagnetism:

- Ferri magnetic materials have some magnetic moment even in the absence of external field. They exhibit magnetic behaviour similar to Ferromagnetic material. In such

materials, there are permanent magnetic moments having opposite alignment of alternating atoms. However, the value of magnetic moment in one atom is higher than in its alternating atom. Therefore there is net magnetization even in the absence of field.

- They have higher value of susceptibility than antiferromagnetic materials and lower than that of Ferromagnetic material.
- Examples: MgFe₃O₄, CuFe₃O₄, MnFe₃O₄, NiFe₃O₄, ZnFe₃O₄, CdFe₃O₄ [i.e. ferrites of Mg, Cu, Mn, Ni, Zn, Cd]

Note: These are non conducting.

1. Calculate the permeability and susceptibility of an iron bar of cross sectional area 0.2 cm² when a magnetizing field of 1200 Am⁻¹ produces magnetic field of 24 micro-Weber.

Solution:

$$\text{Here, } \phi = 24 \times 10^{-6} \text{ Wb} = 2.4 \times 10^{-5} \text{ Wb}$$

$$A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2$$

$$H = 1200 \text{ Am}^{-1}$$

Using the relation, $\phi = B \cdot A$

$$\Rightarrow B = \frac{\phi}{A} = \frac{2.4 \times 10^{-5}}{0.2 \times 10^{-4}} = 1.2 \text{ T}$$

The permeability is given by, $B = \mu H$

$$\Rightarrow \mu = \frac{B}{H} = \frac{1.2}{1200} = 1 \times 10^{-3} \text{ H/m}$$

The susceptibility is given by,

$$\chi = \mu_r - 1 = \frac{\mu}{\mu_0} - 1 = \left(\frac{1 \times 10^{-3}}{4 \pi \times 10^{-7}} - 1 \right) = 795.2$$

Electron Concentrations in Intrinsic Semiconductor

In an intrinsic semi conductor every electron transferred to the conduction band leaves behind a hole in the valence band. Therefore, the total number of electrons in the conduction band is equal to the total number of holes in the valence band.

If n is the density of electrons i.e. the number of electrons per unit volume in the conduction band, then in the equilibrium condition,

$$n = \int_{E_r}^{\infty} Z(E) F(E) dE \quad \dots(1)$$

Where, $Z(E) = \frac{4\pi}{h^3} (2m^*)^{3/2} (E)^{1/2} = \frac{4\pi}{h^3} (2m^*)^{3/2} (E - E_c)^{1/2}$... (2) is the density of states in conduction band. As the bottom of the conduction band has an energy E_c and not zero we substitute $(E - E_c)$ for E . The electrons in the conduction band are not completely free but interact with periodic potential of the semiconductor crystal we substitute effective mass m^* instead of m .

The Fermi-Dirac distribution function is given by

$$F(E) = \frac{1}{1 + e^{\frac{(E-E_F)}{KT}}} \approx e^{\frac{(E-E_F)}{KT}} \text{ for } (E - E_F) \gg KT$$

$$\text{Therefore, } n = \frac{4\pi}{h^3} (2m^*)^{3/2} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{\frac{(E-E_F)}{KT}} dE$$

$$n = \frac{4\pi}{h^3} (2m^*)^{3/2} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{\frac{(E_F-E_c)}{KT}} \cdot e^{\frac{-(E-E_c)}{KT}} dE$$

$$n = \frac{4\pi}{h^3} (2m^*)^{3/2} e^{\frac{-(E_c-E_F)}{KT}} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{\frac{-(E-E_c)}{KT}} dE$$

$$\text{Let, } \frac{E-E_c}{KT} = x \Rightarrow E - E_c = KTx$$

$$dE = (KT) dx$$

$$\text{When, } E = E_c, x = 0$$

$$E = \infty, x = \infty$$

$$\text{And, } (E - E_c)^{1/2} = (KT)^{1/2} x^{1/2}$$

$$\text{Now, } n = \frac{4\pi}{h^3} (2m^*)^{3/2} e^{\frac{-(E_c-E_F)}{KT}} \int_0^{\infty} (KT)^{1/2} x^{1/2} e^{-x} KT dx$$

$$n = \frac{4\pi}{h^3} (2m^* KT)^{3/2} e^{\frac{-(E_c-E_F)}{KT}} \int_0^{\infty} x^{1/2} e^{-x} dx$$

Using integration table

$$n = \frac{4\pi}{h^3} (2m^* KT)^{3/2} e^{\frac{-(E_c-E_F)}{KT}} \frac{\sqrt{\pi}}{2}$$

$$n = 2 \left(\frac{2\pi m^* KT}{h^2} \right)^{3/2} e^{\frac{-(E_c-E_F)}{KT}}$$

$$n = N_c e^{\frac{-(E_c-E_F)}{KT}} \quad \dots (3)$$

Where $N_c = 2 \left(\frac{2\pi m^* kT}{h^2} \right)^{3/2}$ is called effective density of states at the conduction band edge.

So by multiplying effective density of states at conduction band edge by Boltzmann's statistics we can calculate the electron concentration 'n' at E_c .

1. A heavily doped p-side with acceptor concentration of 10^{18} cm^{-3} is connected to n-side with donor concentration of 10^{16} cm^{-3} . Calculate the built in potential, depletion width in n-side and p-side and overall depletion width. Assume $T = 300 \text{ K}$ and $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$.

Solution:

Here, $N_a = 10^{18} \text{ cm}^{-3}$, $N_d = 10^{16} \text{ cm}^{-3}$, $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$, $T = 300 \text{ K}$.

1. $V_o = ?$

$$\begin{aligned} \text{We have, } V_o &= \frac{KT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right) \\ &= \left(\frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \right) \cdot \ln \left[\frac{10^{18} \times 10^{16}}{(1.45 \times 10^{10})^2} \right] \\ &= 0.815 \text{ volt} \end{aligned}$$

2. The total depletion width is given by,

$$\begin{aligned} w &= \sqrt{\frac{2\epsilon V_o}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right)} \\ &= \sqrt{\frac{2\epsilon_r \epsilon_0 V_o}{e} \left(\frac{N_a + N_d}{N_a N_d} \right)} \\ &= \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-12} \times 0.815}{1.6 \times 10^{-19}} \left(\frac{10^{24} + 10^{22}}{10^{24} \times 10^{22}} \right)} \\ &= 3.3 \times 10^{-7} \text{ m} \end{aligned}$$

3. Now the depletion width in n-region is,

$$\begin{aligned}
 x_n &= \left[\frac{2\epsilon V_c}{e N_d \left(1 + \frac{N_d}{N_a} \right)} \right]^{1/2} \\
 &= \left[\frac{2\epsilon_r \epsilon_0 V_c N_a}{e N_d (N_a + N_d)} \right]^{1/2} \\
 &= \left[\sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-12} \times 0.815}{1.6 \times 10^{-19}}} \frac{10^{24}}{10^{22} (10^{24} + 10^{22})} \right]^{1/2} \\
 &= 3.26 \times 10^{-7} \text{ m}
 \end{aligned}$$

Now the depletion width in p-region is,

$$\begin{aligned}
 x_p &= w - x_n \\
 &\approx 3.3 \times 10^{-7} - 3.26 \times 10^{-7} \\
 &= 0.04 \times 10^{-7} \text{ m} \\
 &= 4 \times 10^{-9} \text{ m}
 \end{aligned}$$

5. An n-type Si semiconductor containing 10^{16} Phosphorous (donor) atoms per cm^{-3} has been doped with 10^{17} boron (acceptor) atoms cm^{-3} . Calculate electron and hole concentration in this semi conductor ($n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$)

Solution:

$$\text{Here, } N_d = 10^{16} \text{ cm}^{-3}, N_a = 10^{17} \text{ cm}^{-3}.$$

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Thus the semiconductor has been compensation doped with excess acceptors over donor, so

$$p = N_a - N_d = 10^{17} - 10^{16} = 9 \times 10^{16} \text{ cm}^{-3}$$

The electron concentration

$$n = \frac{n_i^2}{p} = \frac{(1.45 \times 10^{10})^2}{9 \times 10^{16}} = 2.34 \times 10^3 \text{ cm}^{-3}$$

6. Find the resistance of a 1cm³ pure Silicon crystal. What is the resistance when the crystal is doped with one Arsenic in 10⁹ Silicon atoms? Given: Atomic concentration is Si is $5 \times 10^{22} \text{ cm}^{-3}$, $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$, $\mu_e = 1350 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1}$ and $\mu_h = 450 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1}$.

Solution:

Case I, for pure silicon, $\sigma = n_e \mu_e + n_h \mu_h$

$$\begin{aligned}\sigma &= n_i e (\mu_e + \mu_h) \\ &= 1.45 \times 10^{16} \times 1.6 \times 10^{-19} (0.1350 + 0.0450) \\ &= 4.176 \times 10^{-4} \Omega^{-1} \text{m}^{-1}\end{aligned}$$

Since, $L = 1 \text{ cm} = 10^{-2} \text{ m}$ and $A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$,

$$\text{The resistance is, } R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{10^{-2}}{4.176 \times 10^{-4} \times 10^{-4}} = 210 \times 10^3 \Omega = 210 \text{ k}\Omega$$

Case II,

When the crystal is doped with 1 As in 10⁹ Si atoms,

Then, 10⁹ silicon atoms are doped with 1 As

1 silicon atom is doped with $\frac{1}{10^9}$ As.

5×10^{22} silicon's atoms are doped with $\frac{5 \times 10^{22}}{10^9}$ As

Therefore, $N_d = 5 \times 10^{13} \text{ cm}^{-3}$

At, room temperature all the donors are ionized so,

$$n = N_d = 5 \times 10^{13} \text{ cm}^{-3}$$

The hole concentration is now,

$$p = \frac{n_i^2}{N_d} = \frac{(1.45 \times 10^{10})^2}{5 \times 10^{13}} = 4.2 \times 10^6 \text{ cm}^{-3}$$

Here $p \ll n_i$ or $p \ll n$

Therefore,

$$\begin{aligned}\sigma &= n_e \mu_e \\ &= 5 \times 10^{19} \times 1.6 \times 10^{-19} \times 0.1350 \\ &= 1.08 \Omega^{-1} \text{m}^{-1}\end{aligned}$$

$$\text{The resistance, } R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{10^{-2}}{1.08 \times 10^{-4}} = 92.6 \Omega$$

Czochralski Crystal Growth

The Czochralsky process is a method of crystal growth used to obtain single crystal of semiconductors. A Silicon wafer is a thin piece of semi conductor material which is used in fabrication process in integrated circuits. The following is the summary of steps in Czochralski process of Silicon wafer manufacturing.

1. Preparation of high purity of molten silicon

In this process high purity of silicon is encouraged to be used as molten form single crystal Silicon. SiO_2 can be used to prepare high purity molten Silicon. Then the substance will be heated to its melting point into a crucible (pot) made of quartz. The supersaturated molten silicon will become the source of silicon wafer.

2. Dipping the seed crystal

A small piece of single crystal material known as seed crystal will be dipped into the saturated molten silicon solution. Seed crystal is equipment used to grow a large crystal of same material. The large crystal will grow when the seed crystal dipped into the melt will then be cooled.

3. Pulling the seed upward

The seed will extract from the molten Silicon pool and rod will be pulled upward and rotated at the same time. During this time the rod and crucible rotated in opposite direction to minimize the effect of contraction in the melt. In manufacturing single crystal silicon, the temperature gradient, pulling rate and rotation speed influences the size of crystal. As the seed crystal is slowly raised upward, the molten silicon will solidifies as same as the seed. This is why this process is called growing which is producing a new crystal of silicon from molten silicon. The large cylindrical crystal silicon is called ingot which can be grown 300mm to 400 mm in diameter.

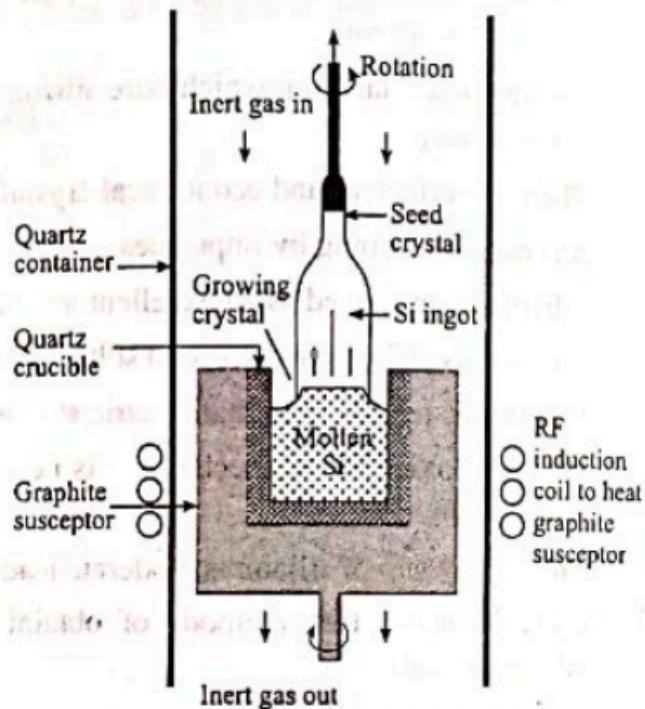


Figure: Single crystal growth by Czochralski method

Ion Implantation

Ion implantation is an engineering process by which ions of a material are accelerated in an electric field and impacted into a solid. This process is used to change the physical, chemical or electrical properties of solid. Ion implantation is used in semiconductor device fabrication and in metal finishing as well as in various applications of material science research.

An ion implantation equipment consists of an ion source where plasma of desired impurity are produced, an accelerator (the accelerating

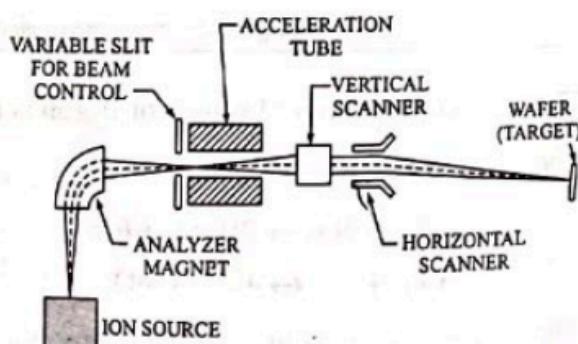


Figure 1: Ion implanter

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voltage may be from 20 kV to as much as 250 kV) where the ions are accelerated to a high energy and a target chamber where the ions impinge on a target which is the material to be implanted. An analyzer magnet bends the ion beam through a right angle to select the desired impurity ion. Scanning system consist of a vertical scanner and a horizontal scanner which provides necessary deflection to give a uniform implantation and to build up the desired dose.

The centrifugal force is balanced by magnetic force.

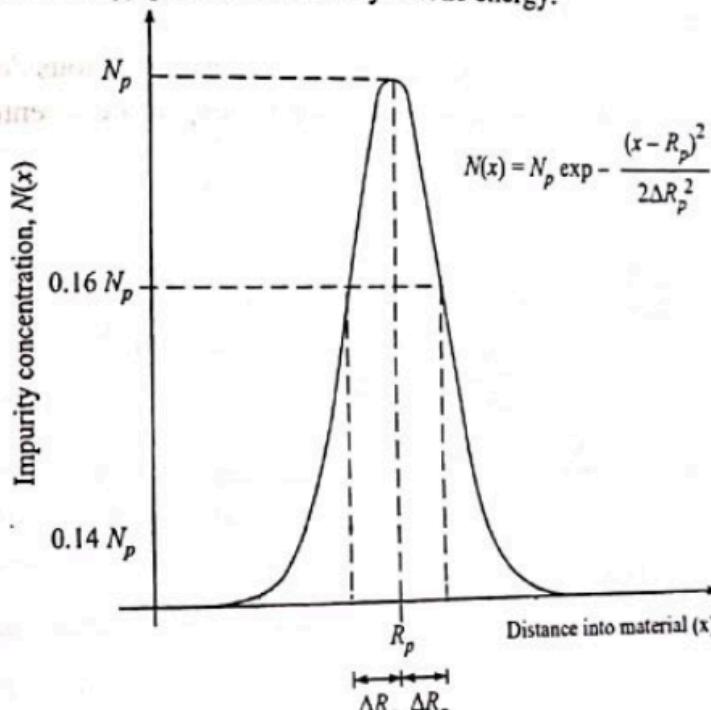
$$Bqv = \frac{mv^2}{r}$$

$$B = \frac{mv}{qr}$$

$$r = \frac{mv}{Bq} \quad \dots(1)$$

This means the impurities having different mass have different radius. This allows us to pass desired impurity species by providing slit on the path of beam.

And, the electrostatic energy provides necessary kinetic energy.



$$qV = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$B = \frac{mv}{qr} = \frac{m}{qr} \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2qV}{m} \times \frac{m^2}{q^2 r^2}}$$

$$B = \sqrt{\frac{2mV}{qr}} \quad \dots(2)$$

So the magnitude of magnetic field can be adjusted for a required ion of mass 'm'.

The target chamber is maintained at relatively low temperature during the implantation which prevents undesired spreading of impurities by diffusion. It is very important in VLSI (very large scale integration) fabrication. A wide range of impurities can be implanted by Ion implantation process as compared to that of diffusion. There is precise control of dose and depth profile.

Ion implantation usually follows a Gaussian distribution as given by

$$N(x) = N_p \exp \left[-\frac{(x - R_p)^2}{2\Delta R_p^2} \right] \quad \dots(3)$$

Where, $N(x)$ is the impurity concentration

N_p is the peak concentration

R_p is the projected range and ΔR_p is the standard deviation called *straggle* (straggle: to spread out from others in a disorganized way)

The ion implantation is often used to form shallow pn junctions for various device applications. The *junction depth* is calculated from the point at which the impurity concentration equals bulk concentration.

$$\text{i.e. } N(x_j) = N_B$$

$$\text{or, } N_p \exp \left[-\frac{(x_j - R_p)^2}{2\Delta R_p^2} \right] = N_B$$

$$\exp \left[-\frac{(x_j - R_p)^2}{2\Delta R_p^2} \right] = \frac{N_p}{N_B} \Rightarrow \frac{(x_j - R_p)^2}{2\Delta R_p^2} = \ln \left(\frac{N_p}{N_B} \right)$$

$$(x_j - R_p)^2 = 2 \Delta R_p^2 \ln \left(\frac{N_p}{N_B} \right)$$

$$x_j - R_p = \pm \Delta R_p \sqrt{2 \ln \left(\frac{N_p}{N_B} \right)}$$

$$x_j = R_p \pm \Delta R_p \sqrt{2 \ln \left(\frac{N_p}{N_B} \right)} \quad \dots(4)$$

Lattice Damage and Annealing in Ion Implantation

The incident ions during implantation produce many defects in the target crystal on impact such as vacancies and interstitials. This is called *damaging* the implanted region in the crystal.

Vacancies are crystal lattice points unoccupied by an atom. In this case the incident ion transfer sufficient amount of energy to the target atom such that the target atom leaves crystal site. This target atom then itself becomes a projectile in the solid and can successive collision events. Interstitial results when the incident ion come to rest in solid but finds no vacant space in the

lattice to reside. These point defects can migrate and cluster with each other resulting in dislocation loops and other defects.

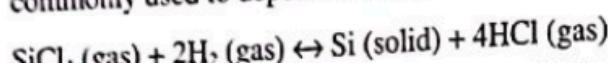
This damage recovery is done by heating the wafer to a temperature ~ 900°C for approximately 30 minutes and then cooling slowly. This process is called *annealing*. At this temperature, the Si atoms can move back into lattice site and impurity atoms can enter substitutional sites. The larger amount of crystallographic damage can be enough to completely amorphize the surface of the target that is it can become an amorphous solid. With heavier impurity lower dose will be required to create an amorphous layer. An amorphous is a solid that lacks the long range order that is characteristic of metal.

In some cases, the complete amorphization of a target is preferable to a highly defective crystal and regrown by annealing.

Epitaxial Growth

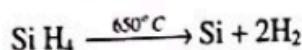
Epitaxy refers to the deposition of a crystalline over layer on a crystalline substrate. The overlayer is called an epitaxial film or epitaxial layer. If the over layer either forms a random orientation with respect to the substrate or does not form an ordered over layer, it is termed as non epitaxial growth. The term epitaxy comes from Greek roots 'epi' meaning 'above' and 'taxis' meaning 'an ordered manner'.

1. The vapour phase epitaxy (VPE) is a modification of chemical vapour deposition is commonly used to deposit Silicon.



This reaction takes place at approximately 1200 to 1250°C and is reversible.

The Silicon VPE may also use pyrolytic (Pyrolytic: Chemical change because of heat) decomposition of silane.



This reaction is not reversible and takes place at lower temperature.

2. In liquid phase epitaxy (LPE), the substrate is dissolved into the melt of the material to be deposited. This happens at temperature well below the melting point of the substrate material. The substrate acts as a seed for material crystallizing directly from the melt.
3. Solid phase epitaxy (SPE) is usually done by first depositing a film of amorphous material on a crystalline substrate. The substrate is then heated to crystallize the film.
4. In molecular beam epitaxy (MBE), a source material is heated to produce an evaporated beam of particles. These particles travel through a very high vacuum to the substrate where they condense. Substrate temperature during this process ranges from 400 - 900°C.

Einstein's Relation between Mobility and Diffusion Coefficient

In the presence of electric field, the total current density in a conductor is given by,

$$\begin{aligned} J &= J_{\text{drift}} + J_{\text{diffusion}} \\ &= \sigma E + eD \frac{dn}{dx} = ne\mu E + eD \frac{dn}{dx} \end{aligned}$$

Under equilibrium condition, The total current must be zero.

$$ne\mu E + eD \frac{dn}{dx} = 0 \Rightarrow eD \frac{dn}{dx} = -ne\mu E$$

$$\text{Therefore, } \frac{dn}{n} = -\frac{\mu E}{D} dx$$

Integrating,

$$\ln(n) = -\frac{\mu Ex}{D} + \ln(A) \text{ (constant)}$$

$$\text{or } \ln\left(\frac{n}{A}\right) = -\frac{\mu Ex}{D}$$

$$n = A e^{-\frac{\mu Ex}{D}} \quad \dots(1)$$

From Boltzmann's statistics, for an electric field we have

$$n = A e^{-e \frac{Ex}{KT}} \quad \dots(2)$$

$$\text{Comparing equations (1) and (2)} \frac{\mu Ex}{D} = \frac{eEx}{KT}$$

$$\frac{\mu}{D} = \frac{e}{KT}$$

$$\left[\frac{D}{\mu} = \frac{KT}{e} \right] \quad \dots(3)$$

Equation (3) is the relation between diffusion coefficient and electron mobility called Einstein's relation.

$$\text{For electron it is given by } \left[\frac{D_e}{\mu_e} = \frac{KT}{e} \right] \quad \dots(4)$$

$$\text{For holes it is given by } \left[\frac{D_h}{\mu_h} = \frac{KT}{e} \right] \quad \dots(5)$$

