

How to really get rid of side  
effects completely?  
Introduction to Free Monad.

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# Hi

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- Worked in WOOMOO
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- A **Fullstack Software Engineer**, including
  - iOS, Android, Web backend/Frontend
- Learn Haskell as a hobby :D



# What is a Monad?

- $(\gg=) :: m\ a \rightarrow (a \rightarrow m\ b) \rightarrow m\ b$
- Monad Laws
- Do syntax!

```
main = do
  a <- readLn :: IO Int
  b <- readLn :: IO Int
  print $ a + b
```

What is Free Monad?

# What is Free Monad

- A **free monad** generated by a **functor** is a special case of the more **general free** (algebraic) structure over some underlying structure. For an explanation of the general case, culminating with an explanation of free monads, see the article on free structures.
- from: [https://wiki.haskell.org/Free\\_monad](https://wiki.haskell.org/Free_monad)
- WAT ????????

What can  
Free Monad do?

Entirely  
Side Effect Free  
Program!



# Ordinary Program with Side Effect

```
main :: IO ()  
main = do  
    a <- readLn :: IO Int  
    b <- readLn :: IO Int  
    print $ a + b
```

# Side Effect -> Inverse of Control!!

```
program :: IS ()  
program = do  
  a <- readInt  
  b <- readInt  
  output $ show $ a + b
```

```
program = do
  a <- readInt
  b <- readInt
  output $ show $ a + b
```

Run it like normal IO Monad

```
main = runProgram program
```

```
program = do
  a <- readInt
  b <- readInt
  output $ show $ a + b
```

Or test it like pure function

```
testProgram program ["1", "2"] == ["3"]
```

How to write  
an Interpreter  
with  
self-defined instruction set?

```
data IS a = Input (String -> IS a)
          | Output String (IS a)
          | Return a
```

```
p :: IS ()
p =
  (Input (\s ->
    Output ("Hello " ++ s)
    (Return ())))
```

```
run :: IS () -> IO ()
run (Input f) = do
  s <- getLine
  run $ f s
```

```
run (Output s next) = do
  putStrLn s
  run next
```

```
run (Return a) = return a
```

Try to make it a  
Monad!

```
instance Monad IS where
  (Input genNext) >>= f =
    Input $ (\s -> genNext s >>= f)
```

```
  (Output s next) >>= f =
    Output s (next >>= f)
```

```
  (Return a) >>= f = f a
```

```
input = Input (\s -> Return s)
output s = Output s (Return ())
```

```
p :: IS ()
```

```
p =
```

```
  (Input (\s ->
    Output ("Hello " ++ s)
      (Return ())))
```



```
instance Monad IS where
  (Input genNext) >>= f =
    Input $ (\s -> genNext s >>= f)
```

```
  (Output s next) >>= f =
    Output s (next >>= f)
```

```
  (Return a) >>= f = f a
```

```
input = Input (\s -> Return s)
output s = Output s (Return ())
```

```
p :: IS ()
p = do
  s <- input
  output $ "Hello " ++ s
```

Cool!

But there is a problem

# Don't Repeat Yourself!

- We need to define/refine monad every time we create new DSL or add new instruction to existed DSL.

How?

By extracting the recursive  
structure

Fix Point !!

Review: **y combinator**  
(oversimplified version)

Recursive function can be generated by finding  
the **fix point** of recursive function generator

$yCom\ g = f\ where\ f = g\ f$

$genFib\ f\ 0 = 0$

$genFib\ f\ 1 = 1$

$genFib\ f\ n = f\ (n - 1) + f\ (n - 2)$

$fib = yCom\ genFib$

a  
Fx a  
Fx (Fx a)

The type of Recursive  
Construct can be obtained by  
finding the fix point of ???

Lets place some holes  
for now

data  $A = Fx A$



# How to extract?

```
data A = Fx A
```

```
data IS a  
  = Input (String -> IS a)  
  | Output String (IS a)  
  | Result a
```

# How to extract?

```
data A = Fx (f A)
```

```
data IS a  
  = Input (String -> a)  
  | Output String a  
  | Result a
```

# How to extract?

```
data A = Fx (f A)
      | Result a
```

```
data IS a
  = Input (String -> a)
  | Output String a
```

# How to extract?

```
data Free f a = Fx (f A)
               | Result a
```

```
data IS a
  = Input (String -> a)
  | Output String a
```

# How to extract?

```
data Free f a = Fx (f (Free f a))  
               | Result a
```

```
data IS a  
  = Input (String -> a)  
  | Output String a
```

```
-- ex: Free IS ()
```

# Before define the monad

```
data Free f r =  
    Free (f (Free f r)) | Pure r  
    deriving (Functor)
```

```
data IS next = Input (String -> next)  
             | Output String next  
             deriving (Functor)
```

# Define the monad!!

```
data Free f r =  
    Free (f (Free f r)) | Pure r  
    deriving (Functor)
```

```
data IS next = Input (String -> next)  
             | Output String next  
             deriving (Functor)
```

```
instance (Functor f) => Monad (Free f) where  
    (Free x) >>= f = Free (fmap (>>= f) x)  
    (Pure r) >>= f = f r
```

```
instance (Functor f) => Monad (Free f) where
  (Free x) >>= f = Free (fmap (>>= f) x)
  (Pure r) >>= f = f r
```

```
p = (Free (Input (\s -> (Pure s))))
  >>= (\s -> (Free (Output s (Pure ())))))
```



```
instance (Functor f) => Monad (Free f) where
  (Free x) >>= f = Free (fmap (>>= f) x)
  (Pure r) >>= f = f r
```

```
p = (Free (Input (\s -> (Pure s)))) >>= f
  where f =
    (\s -> (Free (Output s (Pure ())))))
```

```
instance (Functor f) => Monad (Free f) where
    (Free x) >>= f = Free (fmap (>>= f) x)
    (Pure r) >>= f = f r
```

```
p = (Free
      (fmap (>>= f)
            (Input (\s -> (Pure s)))))
where f =
    (\s -> (Free (Output s (Pure ())))))
```

```
instance (Functor f) => Monad (Free f) where
    (Free x) >>= f = Free (fmap (>>= f) x)
    (Pure r) >>= f = f r
```

```
p = (Free
      (Input (\s -> ((Pure s) >>= f))))
where f =
      (\s -> (Free (Output s (Pure ())))))
```

```
instance (Functor f) => Monad (Free f) where
    (Free x) >>= f = Free (fmap (>>= f) x)
    (Pure r) >>= f = f r
```

```
p = (Free
      (Input (\s -> (f s))))
where f =
      (\s -> (Free (Output s (Pure ())))))
```

```
instance (Functor f) => Monad (Free f) where
  (Free x) >>= f = Free (fmap (>>= f) x)
  (Pure r) >>= f = f r
```

```
p = (Free
      (Input (\s ->
              (Free (Output s (Pure ()))))))
```

# liftF

```
input = (Free (Input (\s -> Pure s)))  
output s = (Free (Output s (Pure ())))
```

```
liftF i = Free (fmap Pure i)
```

```
input = liftF $ Input (\s -> s)  
output s = liftF $ Output s ()
```

```
readInt :: Free IS Int  
readInt = read <$> input
```

```
program = do  
  a <- readInt  
  b <- readInt  
  output $ show $ a + b
```

```
runProgram program
```

```
testProgram program ["1", "2"] == ["3"]
```

# Complete Interpreter

```
runProgram :: Free IS () -> IO ()
runProgram (Free (Input genNext)) = do
  s <- getLine
  runProgram $ genNext s

runProgram (Free (Output s next)) = do
  putStrLn s
  runProgram next

runProgram (Pure r) = return r
```



So What is Free Monad?  
Given any Functor, we  
can get a monad for free!

```
instance (Functor f) => Monad (Free f) where  
  (Free x) >>= f = Free (fmap (>>= f) x)  
  (Pure r) >>= f = f r
```

Use 3rd Party Free Monad Lib

```
import Control.Monad.Free
  ( Free(Free, Pure)
  , liftF)

data IS a = Input (String -> a)
          | Output String a
          deriving (Functor)

input = liftF $ Input (\s -> s)

output :: String -> Free IS ()
output s = liftF $ Output s ()

p :: Free IS ()
p = do
  s <- input
  output $ "Hello " ++ s

run :: Free IS () -> IO ()
run (Free (Input f)) = do
  s <- getLine
  run $ f s

run (Free (Output s next)) = do
  putStrLn s
  run next

run (Pure a) = return a
```

# Free Monad + Interpreter Pattern

# Cool Example: Software Thread!!

```
interleave (Atomic m1) (Atomic m2) = do
  next1 <- atomic m1
  next2 <- atomic m2
  interleave next1 next2
```

```
interleave t1 (Return _) = t1
interleave (Return _) t2 = t2
```

```
runThread (interleave thread1 thread2)
```

# References

- <http://www.haskellforall.com/2012/06/you-could-have-invented-free-monads.html>
- <https://github.com/ekmett/free/>