

School of Computer Science
Final Examination

Examiner: Prof. D. Thérien
Associate Examiner: Prof. N. Friedman

Monday, December 5, 2005
14:00 – 17:00

Instructions:

Answer in the examination booklets provided.

NO notes or texts allowed.

Calculators are not allowed.

Dictionaries are allowed.

You may keep the exam.

This exam comprises 2 pages, including the cover page.

1. (4 points) If L_1 is a regular language, what can you say about $L_1 \cap L_2$ when:
 - (a) L_2 is regular?
 - (b) L_2 is context-free?
 - (c) L_2 is turing-decidable?
 - (d) L_2 is turing-recognizable?
2. (12 points)

- (a) State the pumping lemma for context-free languages.
- (b) We say $x = x_1 \dots x_k$ is a *subsequence* of y if $y \in \Sigma^* x_1 \Sigma^* x_2 \Sigma^* \dots x_k \Sigma^*$, i.e., y can be factorized as $y = y_0 x_1 y_1 \dots x_k y_k$ where for all i we have $x_i \in \Sigma$ and $y_i \in \Sigma^*$. Show that the language:

$$L = \{x\$y : x \text{ is a subsequence of } y\}$$

is not context-free.

3. (12 points) Give an algorithm to decide whether a given CFG G generates at least one string $w \in \{a, b, c\}^*$ such that no a occurs after a b or a c , and no b occurs after a c .
4. (10 points). Recall that for words $w_1, w_2 \in \Sigma^*$, the shuffle $s(w_1, w_2)$ of w_1 and w_2 is the set of words w that can be formed by inserting the letters of w_1 into w_2 (or vice-versa) while preserving the order of the letters in both strings, e.g., $s(ab, cd) = \{abcd, acbd, acdb, cabd, cadb, cdab\}$. Recall also that for languages L_1, L_2 we define $s(L_1, L_2) = \bigcup_{w_1 \in L_1, w_2 \in L_2} s(w_1, w_2)$. Show that the class of turing-decidable languages is closed under shuffle.
5. (12 points) Let $L_{pal} = \{x \in \Sigma^* : x \text{ is a palindrome}\}$. Show that the following language is undecidable:

$$L = \{\langle M \rangle : L(M) \subseteq L_{pal}\}.$$