COMPUTER SCIENCE - COMP 330.
Theoretical Aspects of Computer Science

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Monday, December 6, 2004

14:00 – 17:00

Instructions

1. There are 6 problems in the exam together worth of 100 points.

2. The number of points for each problem is indicated in parenthesis.

3. Answer in the exam book.

4. NO notes or texts are allowed.

5. You are permitted TRANSLATION dictionaries ONLY.

6. Return the exam book. You may keep this examination paper.

7. Read all instructions carefully.

8. This exam comprises 2 pages, including the cover page.

Note: Throughout the exam you may assume that we already showed that the language $A_{TM} = \{< M, w >; M$ is a TM and M accepts $w\}$ is Turing-recognizable but not Turing-decidable and that the language $E_{TM} = \{< M >; M$ is a TM and $L(M) = \emptyset\}$ is co-Turing-recognizable but not Turing-decidable.

Problem 1 (15 pts.) - True or false. (If false, give a counter-example.)

a) If a class of languages is closed under union and not closed under intersection, then it cannot be closed under complement.

b) If a class of languages is closed under union but not closed under complement, then it cannot be closed under intersection.

c) If L_1 is Turing-decidable and L_2 is context-free, then $L_1 \cap L_2$ is context-free.

d) If A mapping reduces to B and B is Turing-undecidable then A is Turing-undecidable.

Problem 2 - Let $L = \{0^n 1^m; n, m \geq 0$ and $(n = 0$ or $m = 0$ or $n > m)\}$.

a) (10 pts.) - Is L regular, context-free, non-regular, non-context-free? Write all which apply—you do not have to justify your answer. If L is regular construct a DFA for it. If L is context-free construct a context-free grammar for it.

b) (10 pts.) - Give an example of an *infinite* proper subset of L which is
   i) regular.
   ii) non-regular.
(A is a proper subset of B if A is a subset of B and $A \neq B$.)

Problem 3 (25 pts.) - State the Pumping Lemma for context-free languages and use it to show that the following language is not context-free: $L = \{uu^R u; u \in \{0, 1\}^*\}$. Here u^R means the *reverse of* u, i.e. $u^R = u_k u_{k-1} \cdots u_1$, for $u = u_1 \cdots u_k$.

Problem 4 (10 pts.) - Are Turing-decidable languages closed under operation star? Justify your answer.

Problem 5 (10 pts.) - Prove that the language $SUBSET_{DFA} = \{< M_1, M_2 >$; M_1, M_2 are deterministic finite automata and $L(M_1) \subseteq L(M_2)\}$ is Turing-decidable.

Problem 6 (20 pts.) - Prove that $Fin = \{< M >: M$ is a TM that accepts only finite number of inputs$\}$ is not *Turing-recognizable*.