Math 488, Assignment 1

1. Read the list of the ZFC axioms at
`http://en.wikipedia.org/wiki/Zermelo-Fraenkel_set_theory`

2. Prove that there is no set x such that $x \in x$.

3. Prove that for any ordinals α, β, γ we have
(1) $\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma$
(2) $\alpha \cdot (\beta \cdot \gamma) = (\alpha \cdot \beta) \cdot \gamma$

4. Find two ordinals α, β such that $\alpha \cdot \beta \neq \beta \cdot \alpha$.

5. Suppose that $\alpha > 0$ and γ are ordinals. Show that there exist unique β and $\rho < \alpha$ such that $\gamma = \alpha \cdot \beta + \rho$.

6. Show the following *Hartogs lemma* without using the Axiom of Choice: for any set X there exists an ordinal α such that there is no injection from α into X.

7.* Prove Cantor's theorem.

8.* Prove the Bernstein–Cantor–Schröder theorem.