McGill University MATH 323

Assignment 1

19 January, 2017

- The deadline for submission is 2 February 2017, 10 PM.
- Upload your paper in PDF format (scan it if using pen and paper) to the folder named "Assignment-1" located under "Assignments" tab in myCourses.
- Write your name and Student ID on top of the first page.
- Explain your argument clearly. Solutions without proper justification may not receive full credit.
- Answer all questions. The problems carry a total of 16 marks. Maximum you can score is 15.
- **1.** Three boys (B1, B2, and B3) and three girls (G1, G2, and G3) line up in a random order for a photograph.
- (a) What is the probability that all three girls stand next to each other (e.g., B2-G2-G3-G1-B1-B3)?
- (b) What is the probability that the boys and girls alternate (e.g., G2-B3-G3-B1-G1-B2)?

(2.5+2.5)

- **2.** Definitions of 'simple random sampling without replacement' (SRSWOR) and 'simple random sampling with replacement' (SRSWR) are given at the end of this document.
- (a) 10 cards were drawn WITH REPLACEMENT from a standard deck of cards. What is the probability that cards of both color (red and black) were selected?
- (b) 10 cards were drawn WITHOUT REPLACEMENT from a standard deck of cards. What is the probability that cards of both color (red and black) were selected?

(2.5+2.5)

3. Two cards were drawn WITHOUT REPLACEMENT from a standard deck of cards. Let A (resp. B) denote the event that the first card (resp. second card) chosen was a diamond.

- (a) Find $\mathbb{P}(B)$.
- (b) Show that *A* and *B* are **not** independent events.

(2+1)

4. In a town, 40% of the population has HIV. An HIV test for a person that actually has the disease comes back positive with probability 0.9. If a person does not have the disease, then the test comes back negative with probability 0.95.

If a person is chosen at random from the town population and his/her test comes back positive, then what is the probability that the person actually has HIV?

(3)

Simple random sampling WITH replacement (SRSWR):

- You have *n* items in a box, and you want to select a sample of *m* items.
- You select one item uniformly at random, note which item it is, and RETURN IT to the box.
- Repeat this *m* times.

Note: An item can be selected more than once in this sampling scheme.

Simple random sampling WITHOUT replacement (SRSWOR):

- You have *n* items in a box, and you want to select a sample of *m* items, where $m \le n$.
- You select one item uniformly at random, note which item it is, but then you DO NOT return it to the box.
- Next, select one item uniformly from the REMAINING items.
- Repeat until you have *m* items.