McGill University
Department of Mathematics and Statistics
MATH 254 Analysis 1, Fall 2015
Assignment 2

You should carefully work out **all** problems. However, you only have to hand in solutions to **problems 2,4.**

This assignment is due **Monday, October 5, at the end of the class**. **Late assignments will not be accepted.**

1. Let $A \subseteq \mathbb{R}$, $B \subseteq \mathbb{R}$ be two sets bounded from above. The sum of A and B is the set
$$A + B = \{a + b \,:\, a \in A, b \in B\}.$$
Prove that $A + B$ is bounded from above and that
$$\sup(A + B) = \sup(A) + \sup(B).$$
2. Using only field axioms of $\mathbb{R}$ (Definition 2.1.1 in the book), prove that
$$(-1) \cdot (-1) = 1.$$
Write every step of the proof carefully indicating which field property you are using.
3. Show that there does not exist a rational number r such that $r^2 = 3$.
4. Let $x, y, z \in \mathbb{R}$. Show that $|x - y| + |y - z| = |x - z|$ if and only if $x \leq y \leq z$ or $x \geq y \geq z$.
5. If $a \in \mathbb{R}$, $a > -1$, prove using Mathematical Induction that
$$(1 + a)^n \geq 1 + na$$
for all $n \in \mathbb{N}$.
6. For any $A \subseteq \mathbb{R}$ we define
$$-A = \{-a \,:\, a \in A\}.$$
Suppose that A is bounded from above. Prove that $-A$ is bounded from below and that
$$\inf(-A) = -\sup(A).$$
7. If $x \in \mathbb{R}$ is irrational and $r \in \mathbb{R}$, $r \neq 0$, is rational, show that $x + r$ and $x \cdot r$ are irrational.