Assignment 5: 5th Nov 2015

Prakash Panangaden
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There are **5** questions for credit and one for your spiritual growth. Sorry, there are no alternate questions this time. The homework is due in class at the beginning of the class.

Question 1[20 points] Give a context-free grammar for the language

$$\{a^m b^n c^p d^q \mid m + n = p + q\}.$$

Question 2[20 points] Describe *briefly and informally* a pushdown automaton that recognizes the context free language $\overline{L}$ where $L = \{ww | w \in \Sigma^*\}$. **Do not just use the algorithm for converting a CFG to a PDA**! I want an explanation of the idea and not a detailed transition table. I (and the TAs) would much prefer not to see a transition table or diagram. We want a high-level description of the idea in the same way that I described how to recognize the complement of $\{a^n b^n c^n \mid n \geq 0\}$ with a PDA in class.

Question 3[15 points] Suppose that we have a language L defined over the alphabet $\{a, b, c\}$ and suppose that L is context-free. We define a new language $\mathsf{perm}(L)$ to be the set of all permutations of all words in L. For example, if $L = \{abc, aab\}$ then $\mathsf{perm}(L) = \{abc, acb, bac, bca, cab, cba, aab, aba, baa\}$. Show that $\mathsf{perm}(L)$ need not be context-free by giving an example of a language L that is context-free but where $\mathsf{perm}(L)$ is not context-free. You need not give a pumping lemma proof *if your example is just like one we have seen in class.*

Question 4[20 points] For each of the following assertions give *brief* arguments indicating whether they are true of false. In each case I am talking about sets of positive integers.

a. For each $n \in \mathbb{N}$ we have a computable set C_n. The set $\bigcup_n C_n$ is computable. We assume that the collection of computable sets is *effectively given*: this means that there is an algorithm that reads a natural number n as input and outputs a description of a Turing machine that decides the set C_n.

b. For each $n \in \mathbb{N}$ we have a computably enumerable set C_n effectively given as described above. The set $\bigcup_n C_n$ is computably enumerable.

Remark: You will find loads of rubbish written about this on the internet. If you copy from the internet without thinking you will, with high probability, write some nonsense.

Question 5[25 points] For this question the alphabet is $\{a, b\}$. Suppose that the language L is CE but not computable; this means that $\overline{L}$ cannot be CE. We define a new language as follows:

$$K = \{aw|w \in L\} \bigcup \{bv|v \in \overline{L}\}.$$

1. Show that K is not computable. [5 points]
2. Show that K is not CE. [10 points]
3. Show that K is not co-CE. [10 points]

Question 6[0 points] Show that over a one-letter alphabet the context-free languages are regular.