FACULTY OF SCIENCE McGill University
FINAL EXAMINATION for Fall 2014

COMPUTER SCIENCE COMP 330

Theory of Computing

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16th December 2014
6 pm to 9 pm

Instructions:

This exam has 6 questions. Please answer all questions. Each question is worth 10 points. This is a **closed book exam; however you may have three cheat sheets double-sided in your handwriting, or typed using 10 point font. You may not use small fonts to get many sheets to print on a single page.** You have three hours in all. You may **not** use calculators, computers, cell phones or electronic aids of any kind. Please answer all questions **in the official answer book**. You may keep the questions. The questions appear on page 1; this title page is not numbered. There are a total of three pages including this title page.

Question 1
For this question the alphabet is $\{a, b\}$. Give the state-transition diagram for the minimal DFA that recognizes the language that contains an even number of a's and between *every pair* of a's there is an odd number of b's. Please remember that zero is an even number. You must show dead states.

Question 2
Consider the following language defined over the alphabet $\Sigma = \{a, b\}$:

$$L = \{ww^{\text{rev}}w \mid w \in \Sigma^*\}.$$

Classify this language as

1. regular,
2. not regular but context-free or,
3. not context-free.

You must prove your answer. If you think it is regular it is sufficient to give an NFA. If you think that it is context-free but not regular you have to give a PDA or CFG **and** a pumping lemma proof that it is not regular and for the last case you need to give a pumping lemma proof.

Question 3
In this question we are working with the alphabet $\{a, b\}$. The language

$$\{a^n b^m a^n b^m \mid n, m \geq 1\}$$

is definitely not context-free; you are **not** being asked to prove this. However, its complement *is* context-free. Prove this by giving a *high-level* description of how you would recognize this using a PDA.

Question 4
One of the following questions is decidable and the other is undecidable.

1. Given a CFL L and a regular language R, is $L \cap R = \emptyset$?
2. Given a CFL L and a regular language R, is $\overline{L} \cap R = \emptyset$?

For the one that is decidable give an algorithm, for the other give an undecidability proof. Your descriptions should be high-level. Remember $\overline{L}$ means the complement of L.

Y1?

Question 5
One of the following sets is c.e. and the other is not.

1. $\{M : |L(M)| \leq 330\}$,
2. $\{M : |L(M)| \geq 330\}$,

where M is a Turing machine description, $L(M)$ means the language recognized by M and $|L(M)|$ is the size of this language. Identify which set is c.e. and which is not and *give proofs for both.* There is no credit for guessing, you must give the proofs. You can take it for granted that neither set is computable.

Question 6
Here are 5 true/false questions. You are not required to give any reasons for your answer. In these questions we assume the same fixed alphabet for *all* the languages.

1. Every infinite context-free language contains an infinite regular subset.
2. It is undecidable whether a regular language is co-finite. Recall, co-finite means that the *complement* of the language is finite.
3. If L_1 and L_2 are languages over the same alphabet and $L_1 \cap L_2$ is context-free then at least one of them must be context-free.
4. Suppose L is any language and R is a regular language. If $L \cap R$ is context-free then L is context-free.
5. Turing machines are so-called because they are useful for travelling from place to place.

Happy Holidays!