McGill University
math-315 (2015-Fall): Ordinary Differential Equations

Written Assignment # 3

Due date: Nov. 17, Hand-in in class
(Total 60 point)

(1) (10 Points) Given the following inhomogeneous equation:

$$P(D) = D^3 + 2D^2 + 4D + 8 = b(x) = 3xe^x + 2e^{-2x} + x\sin 2x.$$

(a) Find the annihilator for the above inhomogeneous term.
(b) Determine the form of a particular solution and derive its final expression.
(c) Find the general solution for the equation.

(2) (10 Points) By changing the independent variable x and using differential operator method to solve the following equation:

$$x^2y'' + \alpha xy' + 3y = 0, \quad x > 0,$$

where α is a real number.
(a) Find all values of α for which all non-zero solutions approach to zero as $x \to \infty$.
(b) Find all values of α for which no non-zero solution is bounded as $x \to \infty$.
(c) Find all values of α for which all non-zero solution are oscillatory as $x \to \infty$.

(3) (10pt.) Find the general solution for the following equations by using the method of variation of parameters:

$$x^2y'' - 4xy' + 6y = x^4\sin x, \quad (x > 0).$$

(4) (10pt.) Given that $y_1 = \cos(x)/\sqrt{x}$, $y_2 = \sin(x)/\sqrt{x}$ are linearly independent solutions of the differential equation

$$x^2y'' + xy' + (x^2 - 1/4)y = 0, \quad (x > 0),$$

find the general solution of the equation

$$x^2y'' + xy' + (x^2 - 1/4)y = x^{5/2}, \quad (x > 0).$$

(5) (10pt.) Find the general solution of the equation

$$(1-x^2)y'' - 2xy' + 2y = 0, \quad (-1 < x < 1)$$

given that $y = x$ is a solution.

(6) (10pt.) Given the differential equations:

$$(\sin x)y'' + xy' + \big(x - \frac{1}{2}\big)y = 0.$$

- Determine all the regular singular points for the Eq.;
- Derive the indicial equation corresponding to each regular point;
- Determine the form of two linearly independent solutions near each of regular singular points.
- Give the behavior of all non-zero solutions as $x \to 0$.

Markers information:

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