

# Useful facts for the final exam

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1. A DFA can be converted to a minimal form using the splitting algorithm.
2. The Myhill-Nerode theorem, which implies that the minimal form is unique for DFAs. There is no analogous statement for NFAs.
3. An NFA with  $\epsilon$  moves is equivalent to an NFA which is equivalent to a DFA.
4. Regular expressions define exactly the same languages as DFAs.
5. Equality of regular expressions is decidable, but only by going through DFAs and minimization. You cannot assume that regular expressions can be tested for equality directly.
6. Regular languages are closed under the following operations: union, intersection, complement, star and concatenation. There are other operations under which they are closed.
7. The pumping lemma for regular languages.
8. Context-free languages are recognized by pushdown automata.
9. Pushdown automata cannot always be made deterministic.
10. Pumping lemma for CFLs.
11. All regular languages are CFLs.
12. There is an algorithm to decide if the language of a CFG is empty.
13. There is an algorithm to decide if a given word is accepted by a given grammar, the CKY dynamic programming algorithm.

14. There is an algorithm to put a grammar  $G$  into Chomsky normal form  $G'$  so that  $L(G) = L(G') \cup \{\epsilon\}$ .
15. The intersection of a regular language and a CFL is a CFL.
16. The complement of a CFL may not be a CFL.
17. The union of two CFLs is a CFL.
18. The intersection of two CFLs may not be a CFL.
19. Turing machines are equivalent to **while** programs, to RAM machines, to machines with two stacks, to machines with two counters, to multi-tape Turing machines, to nondeterministic Turing machines, to multidimensional Turing machines, to Post production systems, to  $\lambda$ -calculus and to any of the common programming languages.
20. The halting problem for any of the above algorithmic frameworks is unsolvable.
21. The acceptance problem for Turing machines is unsolvable.
22. Learn the following definitions: computable, decidable, CE, co-CE.
23. Rice's theorem.
24. All the theorems on the computability handout are important.
25. If  $P \leq_m Q$  and  $P$  is undecidable then so is  $Q$ . If  $P$  is not CE then neither is  $Q$ .
26. The PCP is unsolvable. How this is proved is not important.
27. It is undecidable whether  $L(G) = \Sigma^*$  for a CFG  $G$ .
28. It is undecidable whether  $L(G_1) \cap L(G_2) = \emptyset$ , where  $G_1, G_2$  are context-free languages.
29. Any mapping reduction is a Turing reduction but the converse is not true.
30. Turing reductions are good for proving undecidability results but they are not good for showing the difference between CE and co-CE.
31. Finally, please read the questions carefully **especially** pay attention to the quantifiers that appear in the statement.