McGill University

MATH 323, Winter 2017

Assignment 4

9 March, 2017

- The deadline for submission is 22 March 2017, 10 PM.
- Answer all questions. The problems carry a total of 16 marks. Maximum you can score is 15.
- Upload your paper as a single PDF file (scan it if using pen and paper) to the folder named "Assignment-4" located under "Assignments" tab in myCourses. Do not upload multiple pages separately. Make sure pages are not upside down.
- Write your name and Student ID on top of the first page.
- Explain your argument clearly. Solutions without proper justification may not receive full credit.
- Simplify your answer as much as possible.

1.

(a) Suppose $Z \sim N(0,1)$. Compute $\mathbb{E}\big(|Z|\big)$.

(b) Suppose $\alpha, \beta > 0$, $\theta > -\alpha$, and $X \sim \text{Gamma}(\alpha, \beta)$. Compute $\mathbb{E}\big(X^\theta\big)$.

(2+2)

2. Suppose $X \sim N(\mu, \sigma^2)$.

(a) Show that the assumption in Q2 of Assignment 3 is satisfied by the density of X. This implies that $\mathbb{E}\big((X-\mu)^{2n+1}\big) = 0$ for any integer $n \geq 1$.

(b) Show that for any integer $n \geq 1$,

$$\mathbb{E}\big((X-\mu)^{2n}\big) = \sigma^{2n} \frac{2n!}{2^n n!}.$$

(1+3)

3. For each integer $n \geq 1$, consider the random variable X_n with pmf

$$\mathbb{P}\left(X_n = \frac{k}{n}\right) = \frac{1}{n}, \quad k = 1, 2, \ldots, n.$$

(a) Find the cdf of X_n.

(b) Show that for any $u \in \mathbb{R}$,

$$\lim_{n \to \infty} \mathbb{P}(X_n \leq u) = \mathbb{P}(U \leq u),$$

where $U \sim \text{Unif}[0,1]$.

(2+2)

4. Suppose X is a real-valued random variable with pdf

$$f(x) = \frac{c}{1+x^2}, \quad x \in \mathbb{R}.$$

(a) Find c.

(b) Show that $\mathbb{E}(X)$ does not exist.

(Note: The distribution of X is called the standard Cauchy distribution.) (2+2)