McGill University

MATH 323, Winter 2017

Assignment 2

10 February, 2017

- The deadline for submission is 24 February 2017, 10 PM.
- Answer all questions. The problems carry a total of 16 marks. Maximum you can score is 15.
- Upload your paper as a single PDF file (scan it if using pen and paper) to the folder named "Assignment-2" located under "Assignments" tab in myCourses. Do not upload multiple pages separately. Make sure pages are not upside down.
- Write your name and Student ID on top of the first page.
- Explain your argument clearly. Solutions without proper justification may not receive full credit.
- Simplify your answer as much as possible. For example, the expression for the final answer should not be an infinite series.

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1. Jim keeps playing an online chess game until he wins once, and then he will stop. He has been promised an amount $\$\frac{100}{n}$ if he wins for the first time in his $n-$th attempt. Assume that Jim wins each game with probability p, and that the outcomes of different games are independent.

Let X denote the amount of Jim's winnings.

(a) Write down the probability mass function of X.

(b) Find $\mathbb{E}(X)$.

(1+3)

2. A factory produces a machine that has $m \geq 2$ components which function independently. Probability of each component functioning properly is p, and the machine operates effectively if **not more than one** of its components stop functioning.

An engineer's job is to check the machines produced for defects.

(a) What is the probability that he has to inspect at least k machines before he finds the first non-operational machine?

(b) The engineer inspects 10 machines per hour. What is the probability that the first non-operational machine will be found in the fourth hour?

(2+2)

3. A coin is tossed n times independently. Assume that probability of landing heads in each toss is p. Define a random variable X as follows:

$$X = \begin{cases} 2^{n+1}, & \text{if none of the } n \text{ tosses results in a head,} \\ 2^{i}, & \text{if the first head appears in the } i-\text{th toss, where } 1 \le i \le n. \end{cases}$$

(a) Write down the probability mass function of X.

(b) Find $\mathbb{E}(X)$.

(2+2)

4. A box contains n objects labeled $1, 2, \ldots, n$. Jim selects n objects one after another **with replacement** from the box. Let X_i denote the label of the i-th object selected, $1 \le i \le n$.

(a) Find the probability that X_i is different from $X_1, \ldots, X_{i-1}$.

(b) Let D_n denote the number of distinct labels in the sample. Show that

$$\lim_{n} \frac{\mathbb{E}(D_n)}{n} = 1 - e^{-1}.$$

(2+2)