

Homework 1

COMP 527 Logic and Computations

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Exercise 1: Natural deduction proofs(45 pts)

25 pts Give proofs in natural deduction using the proof tutor Tutch:

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proof easy0 : (A & (B & C)) => (A & B);
proof easy1 : (A => B) => ((B => C) => (A => C));
proof d1a   : ((A | B) => C) => (A => C) & (B => C);
proof d1b   : (A => C) & (B => C) => (A | B) => C;
proof d1c   : ((A => B) & (A | C)) => (B | C);
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8 pts Annotate your proofs d1a and d1b with proof terms.

12 pts Simply write down the proof terms for easy0, easy1, d1c; you do not have to give a fully annotated proof.

Exercise 2: De Morgan's Law (40 pts)

In this exercise we try to prove the de Morgan's laws in constructive logic. One of the following conjectures only holds in classical logic. Give constructive proofs for all the conjectures which you believe are true in constructive logic and identify the one conjecture, which is not provable in constructive logic. Give your proofs using the proof tutor Tutch.

Provide a classical proof using the excluded middle for the conjecture which only holds in classical logic.

1. $\neg A \vee \neg B \supset (\neg(A \wedge B))$.
2. $\neg A \wedge \neg B \supset \neg(A \vee B)$.
3. $\neg(A \vee B) \supset \neg A \wedge \neg B$.
4. $\neg(A \wedge B) \supset (\neg A \vee \neg B)$.
5. $\neg\neg\neg A \supset \neg A$

Exercise 3: Alternative elimination rules for \wedge (15 pts)

Assume someone defines conjunction with the following two rules:

$$\frac{\begin{array}{c} \neg u \quad \neg v \\ A \quad B \\ \vdots \\ C \end{array} \quad \frac{A \wedge B}{C} \quad \wedge E^{u,v}}{\quad} \quad \frac{A \quad B}{A \wedge B} \wedge I$$

Are these rules sound and complete? – Show local soundness and completeness.