Math 488, Assignment 3

1. Let κ be an uncountable regular cardinal and let L be a countable language. Suppose $(M_\alpha : \alpha < \kappa)$ is an increasing and continuous (i.e. $M_\gamma = \bigcup_{\alpha<\gamma} M_\alpha$ for γ limit) sequence of L-structures and let $M = \bigcup_{\alpha<\kappa} M_\alpha$. Show that $\{\alpha \in \kappa : M_\alpha \prec M\}$ is a club set.

2. Using a coding of formulas as elements of HF (via their parsing trees), write "φ is a formula and $x \models \varphi(y)$" as a Δ_1 property over HF in the language of set theory.

3. Write the sentence "V=WF" in the language of set theory.

4. Write the sentence "V=L" in the language of set theory.

5. Show that if M is a transitive class such that the Comprehension Axiom holds in M and for every subset $x \subseteq M$ there exists a set $y \in M$ with $x \subseteq y$, then $M \models ZF$.

6. Let κ be a regular uncountable cardinal. Show that $H(\kappa) \models ZF - P$, where P stands for the Power Set Axiom.

7. Let κ be a regular uncountable cardinal. Show that $L_\kappa \models ZF - P$.