Math 318, Assignment 1

Due date: September 19, in class

1. (4 points) For each statement below choose if it is true or false:
   (1) $1 \in 1$,
   (2) $1 \subseteq 1$,
   (3) $1 \in \{1, 2\}$,
   (4) $1 \subseteq \{1, 2\}$

2. (2 points) Draw the following sets in $\mathbb{R}^2$:
   (1) $\{(x, y) \in \mathbb{R}^2 : x > y + 1 \text{ and } x^2 > y\}$,
   (2) $\{(x, y) \in \mathbb{R}^2 : (x^2 + y^2 \leq 1 \text{ and } x \geq y) \text{ or } x^2 + y^2 \geq 2\}$

3. (1) (2 points) Compute the transitive closure T of the following relation R on $\{1, 2, 3, 4\}$:
   $$R = \{(1, 2), (2, 3), (3, 1), (4, 4)\}.$$
   (2) (1 point) Is T an equivalence relation?
   (3) (1 point) If T is an equivalence relation, then compute the equivalence class of 1.

4. (3 points) For each of the relations below decide if it is an equivalence relation:
   (1) E on $\mathbb{N}$ defined as follows: $x \; E \; y$ if $x < y$,
   (2) E on $\mathbb{N}$ defined as follows: $x \; E \; y$ if $x^2 = y^2$,
   (3) E on $\mathbb{R}$ defined as follows: $x \; E \; y$ if $x - y \notin \mathbb{Q}$,

5. (2 points)
   (1) Compute the composition of the following functions: (warning: do not confuse with the composition of relations!):
   $$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 1 & 4 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 3 & 6 & 2 & 4 & 5 \end{pmatrix}$$
   (2) Compute the inverse of the following permutation:
   $$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 4 & 1 & 6 & 3 \end{pmatrix}$$