COMP 360, FINDING THE DUAL OF A LINEAR PROGRAMS

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Consider the following maximization linear form that is not necessarily in standard form.

$$\begin{array}{lrl} \max & x_1 + 2x_2 + 3x_3 & \\ \text{s.t.} & x_1 + 2x_2 & \leq 1 \\ & x_1 - 4x_3 & \geq 5 \\ & x_1 + x_2 - x_3 & = 6 \\ & x_1 & \geq 0 \\ & x_2 & \leq 0 \\ & x_3 & \text{free} \end{array}$$

Recall that the dual comes from multiplying the constraints by new variables and adding them up. We want to be able to conclude the following:

$$\begin{array}{lrl} y_1 \times & (x_1 + 2x_2 & \geq 1) \\ y_2 \times & (x_1 - 4x_3 & \leq 5) \\ + \quad y_3 \times & (x_1 + x_2 - x_3 & = 6) \\ \hline & (y_1 + y_2 + y_3)x_1 + (2y_1 + y_3)x_2 + (-4y_2 - y_3)x_3 & \leq y_1 + 5y_2 + 6y_3 \end{array}$$

For this to be true, we need $y_1 \geq 0$, $y_2 \leq 0$, and y_3 can be either positive or negative.

Next we want to be able to say that the left hand side of the above inequality is an upper-bound on the original objective function:

$$(y_1 + y_2 + y_3)x_1 + (2y_1 + y_3)x_2 + (-4y_2 - y_3)x_3 \leq x_1 + 2x_2 + 3x_3.$$

Since $x_1 \geq 0$, $x_2 \leq 0$ and $x_3 \in (-\infty, +\infty)$, in order to guarantee this, we need $y_1 + y_2 + y_3 \geq 1$, $2y_1 + y_3 \leq 2$, and $-4y_2 - y_3 = 3$. Hence the dual linear program will be the following:

$$\begin{array}{lrll} \min & y_1 + 5y_2 + 6y_3 & & \\ \text{s.t.} & y_1 + y_2 + y_3 & \geq 1 & \text{(since } x_1 \geq 0) \\ & 2y_1 + y_3 & \leq 2 & \text{(since } x_2 \leq 0) \\ & -4y_2 - y_3 & = 3 & \text{(since } x_3 \text{ is free)} \\ & y_1 & \geq 0 & \text{(since the 1st constraint in LP is standard)} \\ & y_2 & \leq 0 & \text{(since the 2nd constraint in LP is the opposite of standard)} \\ & y_3 & \text{free} & \text{(since the 3rd constraint in LP is equality)} \end{array}$$

References

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