COMP 360, MODELING PROBLEMS AS LINEAR PROGRAMS

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Example 0.1. A small firm produces bookcases and tables. The following table shows the revenue obtained from selling a bookcase or a table, and the number of hours required for their cutting time, assemly time, and finishing time.

	cutting time	assembley time	finishing time	revenue
Bookcase	6/5	1	3/2	\$ 80
Table	1	1/2	2	\$ 55

The firm has maximum of 72 hours per day for cutting, 50 hours for assmbley, and 120 hours for finishing. The following linear program decides how many bookcases and tables must the firm produce to maximize its profit: There are two variables, x_1 is the number of bookcases to be produced and x_2 is the number of tables to be produced.

$$\begin{array}{ll} \max & 80x_1 + 55x_2 \\ \text{s.t.} & \frac{6}{5}x_1 + x_2 \leq 72 \\ & x_1 + \frac{1}{2}x_2 \leq 50 \\ & \frac{3}{2}x_1 + 2x_2 \leq 120 \\ & x_1, x_2 \geq 0 \end{array}$$

Note that unfortunately we cannot impose a constraint that forces the variables to take only integer values. That would lead to an *integer program* which is difficult to solve (it is NP-hard). However here fortunately the optimal solution turns out to be integer valued: $x_1 = 35$ and $x_2 = 30$. ■

Example 0.2. A meat packing plant produces 480 hams, 400 pork bellies, and 230 picnic hams every day; each of these products can be sold either fresh or smoked. The toal number of hams, bellies, and picnic hams that can be smoked during a normal working day is 420; in addition up to 250 products can be smoked on overtime at a higher cost. The net profits are as follows:

	Fresh	smoked on regular time	smoked on overtime
Hams	8	14	11
Bellies	4	12	7
Picnics hams	4	13	9

The objective is to find the schedule that maximizes the total net profit. To formulate this as an LP problem, we use variables $x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3$. Here x_1, x_2, x_3 are respectively the number of fresh hams, smoked on regular time hams, and smoked on overtime hams. Similarly y_1, y_2, y_3 correspond to pork bellies, and z_1, z_2, z_3 to picnic hams.

$$
\begin{array}{ll}
\min & 8x_1 + 14x_2 + 11x_3 + 4y_1 + 12y_2 + 7y_3 + 4z_1 + 13z_2 + 9z_3 \\
\text{s.t.} & x_1 + x_2 + x_3 = 480 \\
& y_1 + y_2 + y_3 = 400 \\
& z_1 + z_2 + z_3 = 230 \\
& x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3 \geq 0
\end{array}
$$

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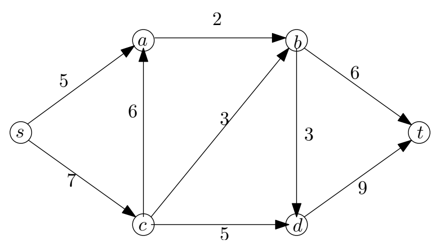
Example 0.3. We can formulate the max flow problem for a flow network as a linear program. For example consider the following flow network.



Here the variables are $f_{sa}, f_{sc}, f_{ca}, f_{ab}, f_{cb}, c_{cd}, f_{bd}, f_{bt}, f_{dt}$, and the linear program is the following:

$$
\begin{array}{ll}
\max & f_{sa} + f_{sc} \\
\text{s.t.} & f_{sa} + f_{ca} - f_{ab} = 0 \\
& f_{ab} + f_{cb} - f_{bd} - f_{bt} = 0 \\
& f_{sc} - f_{ca} - f_{cb} - f_{cd} = 0 \\
& f_{cd} + f_{bd} - f_{dt} = 0 \\
& f_{sa} \leq 5 \\
& f_{sc} \leq 7 \\
& f_{ca} \leq 6 \\
& f_{cb} \leq 3 \\
& f_{cd} \leq 5 \\
& f_{ab} \leq 2 \\
& f_{bd} \leq 3 \\
& f_{bt} \leq 6 \\
& f_{dt} \leq 9 \\
& f_{sa}, f_{sc}, f_{ca}, f_{ab}, f_{cb}, c_{cd}, f_{bd}, f_{bt}, f_{dt} \geq 0
\end{array}
$$

Note that this linear program is quite flexible and we can easily modify it to solve other variants of the max flow problem: We can add lower-bounds for the flow values on the edges. For example adding the constraint $f_{cb} \geq 2$ finds a max flow that will assign at least two units of flow to the edge cb, or adding the constraint $f_{ab} + f_{cb} \geq 2$ will find the max flow that satisfied $f^{\text{in}}(b) \geq 2$ (in other words, it assigns a capacity 2 on the incoming flow of the vertex b). We can also assign costs to edges, and find the max flow that has total cost equal to some number, etc.

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