MATH 340: Discrete Structures II. Winter 2017.

OPTIONAL.
Submit on myCourses or by e-mail to snorin@math.mcgill.ca by Monday, April 17th, 5PM.

Assignment #6: Generating functions.

1. *Fruit salad.* Let $s(n)$ be the number of ways to make a fruit salad with n pieces of fruit, given that we must use strawberries by the half-dozen, an even number of apples, at most five bananas and at most one pineapple.

a) Evaluate the ordinary generating function for s.

b) Use this to find $s(n)$.

2. *The Round table.* Let $r(n)$ be the number of different ways to seat n people around a round table. Find the exponential generating function for r.

3. *Sum of cubes.*
Use generating functions to evaluate

$$\sum_{k=0}^{n}(k-1)k(k+1)$$

4. *Alternating Permutations.* A permutation $\pi_1, \pi_2, \ldots, \pi_n$ of numbers $1, 2, \ldots, n$ is *alternating* if

$$\pi_1 > \pi_2 < \pi_3 > \pi_4 < \ldots.$$

Let $a(n)$ be the number of alternating permutations of size n.

a) Find a recurrence relation for $a(n)$.

b) Evaluate the exponential generating function for a.