

# COMP 360 - Some duality examples

## 1 Questions

Let  $G = (V, E, \{c_e\})$  be a graph where every edge  $e$  has a cost  $c_e$ . Write the duals of the following Linear Programs:

1.

$$\begin{array}{ll} \min & \sum_{uv \in E} c_{uv} x_{uv} \\ \text{s.t.} & \sum_{uv \in C} x_{uv} \geq 1 \quad \text{for every cycle } C \text{ in } G \\ & x_{uv} \geq 0 \quad uv \in E \end{array}$$

2.

$$\begin{array}{ll} \max & \sum_{uv \in E} c_{uv} (x_u + x_v) \\ \text{s.t.} & \sum_{u \in V} x_u \leq 100 \\ & x_u \leq 1 \quad u \in V \\ & x_u \geq 0 \quad u \in V \end{array}$$

3. An independent set in  $G$  is a set of vertices, no two of which are adjacent. Let  $\mathcal{I}$  denote the set of all independent sets in  $G$ .

$$\begin{array}{ll} \min & \sum_{S \in \mathcal{I}} x_S \\ \text{s.t.} & \sum_{S: u \in S} x_S \geq 1 \quad \forall u \in V \\ & x_S \geq 0 \quad S \in \mathcal{I} \end{array}$$

## 2 Solutions

1.

$$\begin{array}{ll} \max & \sum_{C \in \mathcal{C}} y_C \\ \text{s.t.} & \sum_{C \in \mathcal{C}: e \in C} y_C \leq c_e \quad \forall e \in E \\ & y_C \geq 0 \quad C \in \mathcal{C} \end{array}$$

2.

$$\begin{array}{ll}
 \min & 100y' + \sum_{u \in V} y_u \\
 \text{s.t.} & y' + y_u \geq \sum_{v:uv \in E} c_{uv} \quad \forall u \in V \\
 & y' \geq 0 \\
 & y_u \geq 0 \quad u \in V
 \end{array}$$

3.

$$\begin{array}{ll}
 \max & \sum_{u \in V} y_u \\
 \text{s.t.} & \sum_{u \in S} y_u \leq 1 \quad \forall S \in \mathcal{I} \\
 & y_u \geq 0 \quad \forall u \in V
 \end{array}$$