Math 488, Assignment 2

1. Prove that the following statements are equivalent on the basis of ZF
   (i) Axiom of Choice,
   (ii) Well Ordering Principle (saying that every set can be well-ordered),
   (iii) Zorn's Lemma

2. Let κ be a regular uncountable cardinal. Say that a set $C \subseteq \kappa$ is *closed unbounded* (*club*) if C is closed under taking limits of transfinite sequences of elements in C and C is not bounded. Show that
   (1) the intersection of two club sets is club
   (2) the intersection of any family of less than κ club sequences is a club

3. Suppose $(C_\alpha : \alpha < \kappa)$ is a sequence of club subsets of a regular uncountable κ. Prove that the set $\Delta_{\alpha<\kappa} C_\alpha = \{\beta < \kappa : \beta \in \bigcap_{\gamma<\beta} C_\gamma\}$ is a club.

4. Let κ be a regular uncountable cardinal. Say that a subset $S \subseteq \kappa$ is *stationary* if $S \cap C \neq \emptyset$ for every club C. Find two disjoint stationary subsets in κ.

5. Say that a function $f : \kappa \to \kappa$ is *regressive* if $f(\alpha) < \alpha$ for all $\alpha > 0$. Prove Fodor's lemma: for any regressive function $f : \kappa \to \kappa$ (κ regular unctbl) there exists a stationary set $S \subseteq \kappa$ such that f is constant on S.