An Attempt at the Replication of Yongsung Chang and Sun Bin-Kim's Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations

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Abstract

This paper adresses the low correlation between hours and productivity in the business cycle literature along with the large cyclical movement in the wedge derived for the intratemporal choice of commodity consumption and hours worked in the business cycle literature. Using only a technology shock as the aggregate disturbance, the authors obtained a low correlation between hours worked and productivity. The interaction between incomplete capital markets and indivisible labor breaks the link between employment and wages at the aggregate level. Aggregate employment is not highly correlated with productivity because individual optimality conditions do not aggregate well.

Keywords optimal consumption and labor, heterogeneous-agents, incomplete capital markets, indivisible labor

JEL codes D31, E32, J22, J24, J31

Original Papper: American Economic Association (Paywall)
GitHub: Replication Project

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1 Introduction

Chang and Kim (2007) is a broken paper with an aesthetic in code writing reminiscent of Korean art (no, that's not a compliment). Basically, we really just want to replicate Figure 1 shown below, specifically the third line as written in the legend box. While we understand that an honest replication should aim for the first line, we have realized that that particular line is beyond our current capabilities to reproduce at this moment at given the time. So there you go. We will show you what we managed in these few grueling months.

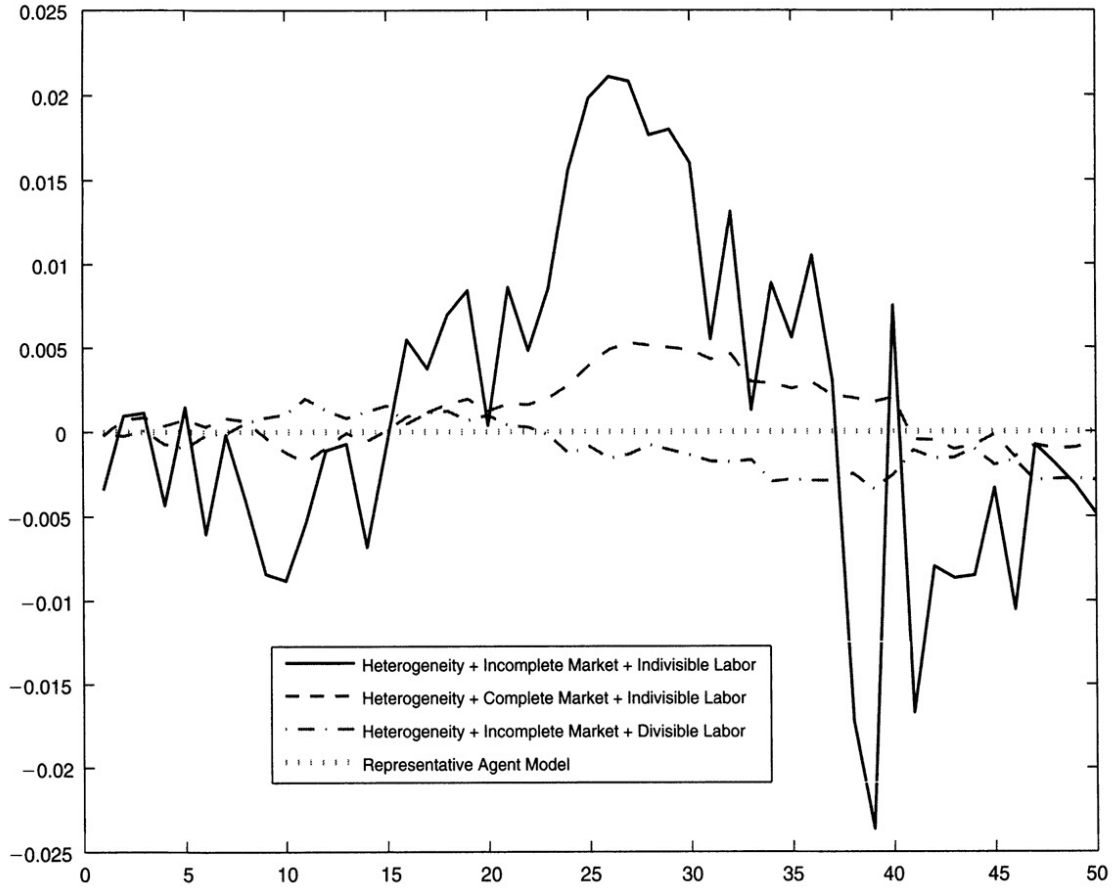

Figure 1 Labor-Market Wedges From The Models

Source: Chang and Kim (2007)

$$U(C_t, H_t) \;=\; lnC_t \;-\; B\left[\frac{H^{1+1/\gamma}}{(1\;+\frac{1}{\gamma})}\right]$$

C_t : commodity consumption
H_t : hours worked

γ : compensated labor supply elasticity

B : constant

Assuming an aggregate Cobb-Douglas production function with labor-income share denoted by α, the Marginal Rate of Stubstitution (MRS) should be equal to the Marginal Productivity of Labor (MPL):

$$B\frac{H_t^{1/\gamma}}{C_t^{-1}} \;=\; \alpha\frac{Y_t}{H_t}$$

Nevertheless, as we can see from Figure 2, the MRS is more volatile that hours and often moves in opposite direction to productivity, thereby violating the equilibrium.

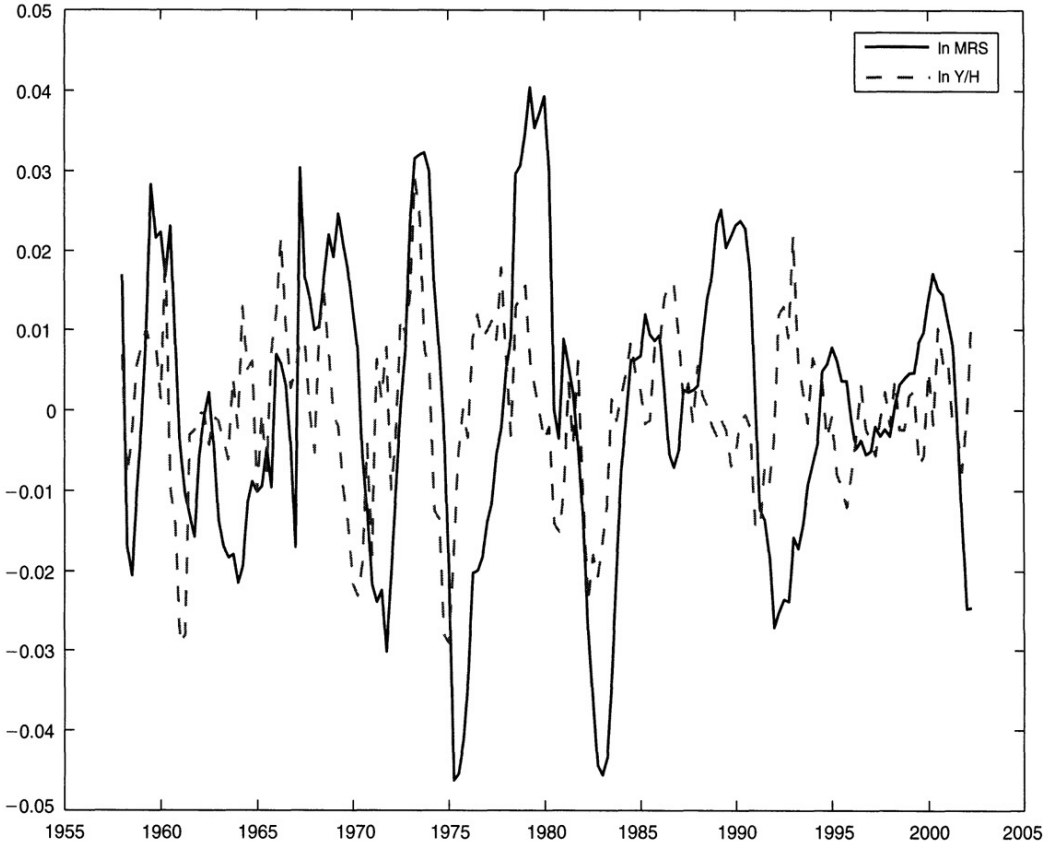

Figure 2 Cyclical Components of MRS and Labor Productivity

Source: Chang and Kim (2007)

On the other hand, the labor-market wedge is defines as:

$$ln\ Wedge_t \;=\; ln\ MRS_t \;-\; ln\frac{Y_t}{H_t} \;+\; constant$$

Figure 2 shows that the wedge is highly correlated with hours worked and its volatility

has similar magnitude as hours worked. The wedge arises because hours worked are not highly correlated with productivity.

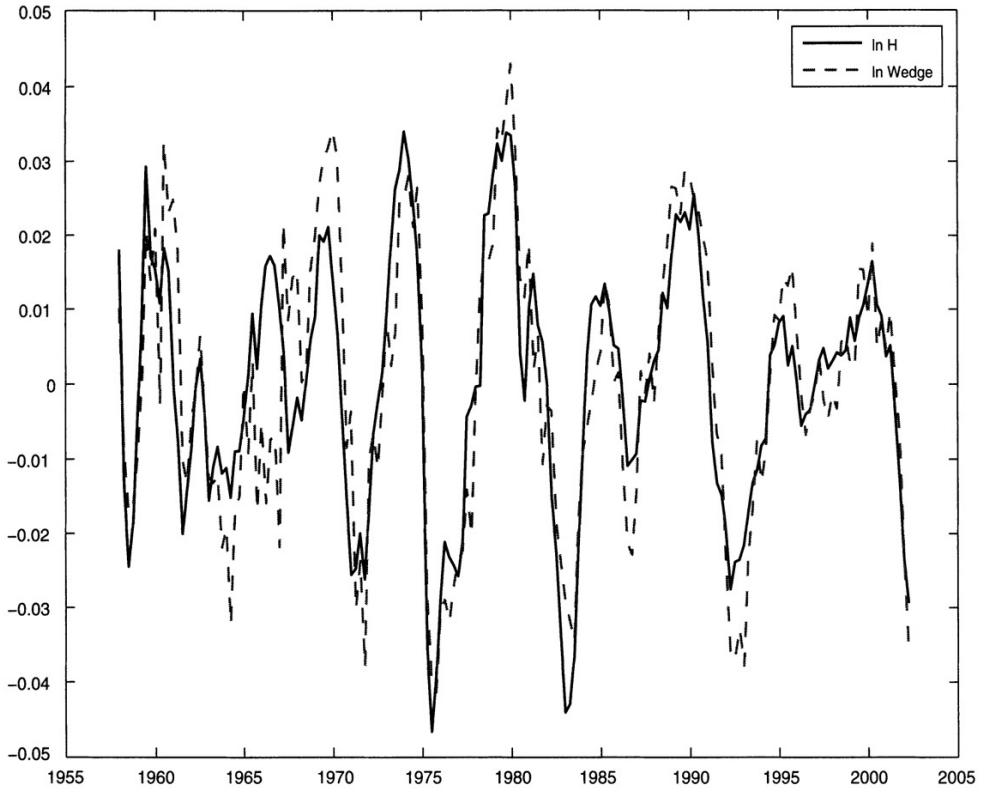

Figure 3 Cyclical Components of Hours and Labor-Market Wedge for The United States

Source: Chang and Kim (2007)

2 The Model

2.1 Setup

The model is an extension of Krussel-Smith's (1998) heterogeneous agent model with incomplete capital markets to indivisible labor supply. A continuum of workers have identical preferences but different productivity along with a separable preference over consumption and hours worked. Capital markets are incomplete. Workers trade claims for physical capital, a_t that yields a return of r_t Workers face a borrowing constraint $a_t \geq \bar{a}$, for all t. Labor supply is indivisible such that if employed, a worker supplies $\bar{h}$ units of labor and earns $w_t x_t \bar{h}$ where:

- w_t: wage rate per effective unit of labor
- x_t: individual productivity

w_t varies exogenously according to a stochastic process with a transition probability distribution function defined as:

- $\pi_x(x'|x) \ = \ Pr(x_{t+1} \leq \ x'|x_t \ = \ x)$
- x_t is the only idiosyncratic risk faced by the agents in the model and the only source of heterogeneity

Output is produced through a Cobb-Douglas Production function defined by: $Y_t \ = F(L_t, K_t, \lambda_t) \ = \ \lambda_t L_t^{\alpha} K_t^{1-\alpha}$

- K_t: capital which depreciates at rate δ each period
- L_t: effective units of labor
- μ: distribution of workers
- λ: aggregate productivity

λ evolves with a transition probability distribution function:

$$\pi_\lambda(\lambda'|\lambda) \ = \ Pr(\lambda_{t+1} \leq \lambda'|\lambda_t \ = \ \lambda)$$

The value function for an employed worker is:

$$\begin{aligned} & V^E(a, \ x; \ \lambda, \ \mu) \\ & \quad = \max_{a' \ \in \ A} \Bigg\{ ln \ c \ - \ B\frac{h^{1 \ 1/\gamma}}{1 \ + \ 1/\gamma} \\ & \qquad + \ \beta E[max\{V^E(a', x'; \lambda', \mu'), \ V^N(a', x'; \lambda', \mu')\}|x, \lambda] \Bigg\}, \end{aligned}$$

subject to:

$$c \ = \ w(\lambda, \mu)x\bar{h} \ + \ (1 \ + \ r)\lambda, \mu))a \ - \ a',$$

$$a' \geq \bar{a},$$

$$\mu' \ = \mathbb{T}(\lambda, \mu)$$

where $\mathbb{T}$ is a transition operator defining the law of motion for the distribution of worker μ. The value function of non-employed worker (h = 0) is defined by:

$$V(a, x; \lambda, \mu) \ = \max_{h \in \{0, \bar{h}\}} \{V^E(a, x; \lambda, \mu), \ V^N(a, x; \lambda, \mu)\}$$

2.2 Equilibrium Conditions

Equilibrium consists of:

1. A set of value functions

- $V^E(a, x; \lambda, \mu),\ V^N(a, x; \lambda, \mu),\ V(a, x; \lambda, \mu)$

2. A set of decision rules for consumption, asset holdings and labor supply
   - $\{c(a, x; \lambda, \mu),\ a'(a, x; \lambda, \mu),\ h(a, x; \lambda, \mu)\}$
3. A set of decision rules for aggregate inputs
   - $\{K(\lambda, \mu), L(\lambda, \mu)\}$
4. A set of decision rules for factor prices
   - $\{w(\lambda, \mu), r(\lambda, \mu)\}$
5. Law of motion for the distribution of workers
   - $\mu' \ = \ \mathbb{T}(\lambda, \mu)$

such that:

1. Individuals optimizes given $w(\lambda, \mu)$ and $r(\lambda, \mu)$.
2. Representative firms maximizing profits.

$$w(\lambda, \mu) \ = \ F_1(L(\lambda, \mu), K(\lambda, \mu), \lambda)$$

$$r(\lambda, \mu) \ = \ F_2(L(\lambda, \mu), K(\lambda, \mu), \lambda) \ - \ \delta$$

3. The goods market clears.

$$\int \{a'(a, x; \lambda, \mu) \ + \ c(a, x; \lambda, \mu)\} d\mu$$

$$= \ F(L(\lambda, \mu), K(\lambda, \mu), \lambda) \ + \ (1 \ - \ \delta)$$

4. Factor market clears.

$$L(\lambda, \mu) \ = \ \int x h(a, x; \lambda, \mu) \ d\mu$$

$$K(\lambda, \mu) \ = \ \int a \ d\mu$$

5. Individual and aggregate behaviours are consistent.

$$\mu'(A^0, \ X^0)$$

$$= \int_{A^0, X^0} \int_{A, \chi} \mathbb{1}_{a' = a'(a, x; \lambda, \mu)} d\pi_x(x'|x) d\mu\} da' dx'$$

3 Quantitative Analysis

3.1 Calibration

The unit of time is business quarter and parameters are obtained from the Panel Study of Income Dynamics (PSID) along with the empirical labor supply literature. Parameters are summarized in Table 4 below:

Parameter description	
$\alpha = 0.64$	Labor share in production function
$\beta = 0.98267$	Discount factor
$\gamma = 0.4$	Individual labor-supply elasticity with divisible labor
$B = 166.3$	Utility parameter
$\bar{h} = 1/3$	Labor supply if working
$\bar{a} = -2.0$	Borrowing constraint
$\rho_x = 0.929$	Persistence of idiosyncratic productivity shock
$\sigma_x = 0.227$	Standard deviation of innovation to idiosyncratic productivity
$\rho_\lambda = 0.95$	Persistence of aggregate productivity shock
$\sigma_\lambda = 0.007$	Standard deviation of innovation to aggregate productivity

Figure 4 Parameters of the Benchmark Model Economy

Source: Chang and Kim (2007)

3.2 Cross-Sectinal Distribution for Earnings, Wealth and Reservation Wages

	Quintile					
	1st	2nd	3rd	4th	5th	Total
PSID–primary households						
Share of wealth	1.03	7.07	13.01	21.10	57.76	100
Group average/population average	0.05	0.36	0.64	1.06	2.97	1
Share of earnings	14.29	14.67	20.08	25.07	25.86	100
Participation rate	0.86	0.84	0.83	0.87	0.79	1
Benchmark model						
Share of wealth	−2.46	3.27	12.21	26.05	60.93	100
Group average/population average	−0.12	0.16	0.61	1.30	3.08	1
Share of earnings	13.52	17.87	20.50	22.65	25.46	100
Participation rate	0.86	0.63	0.56	0.50	0.43	1

Notes: The PSID statistics reflect the family wealth and earnings in the 1984 survey. The statistics of "primary households" are those for household heads whose education was 12 years and whose age is between 35 and 55. The participation rate is based on individual employment status (household heads and spouse) for the same group.

Figure 5 Characteristics of Wealth Distribution

Source: Chang and Kim (2007)

Table 5 summarizes the PSID and the model's information on wealth and earnings. For each quintile group of wealth distribution, the authors calculated:

1. Wealth share
2. ratio of group average to economy-wide average
3. Earnings share

In the model, labor-market participation is determined by market opportunity (wage) and wealth (asset holdings). The steady-state reservation wage schedule is shown in Figure 6 below.

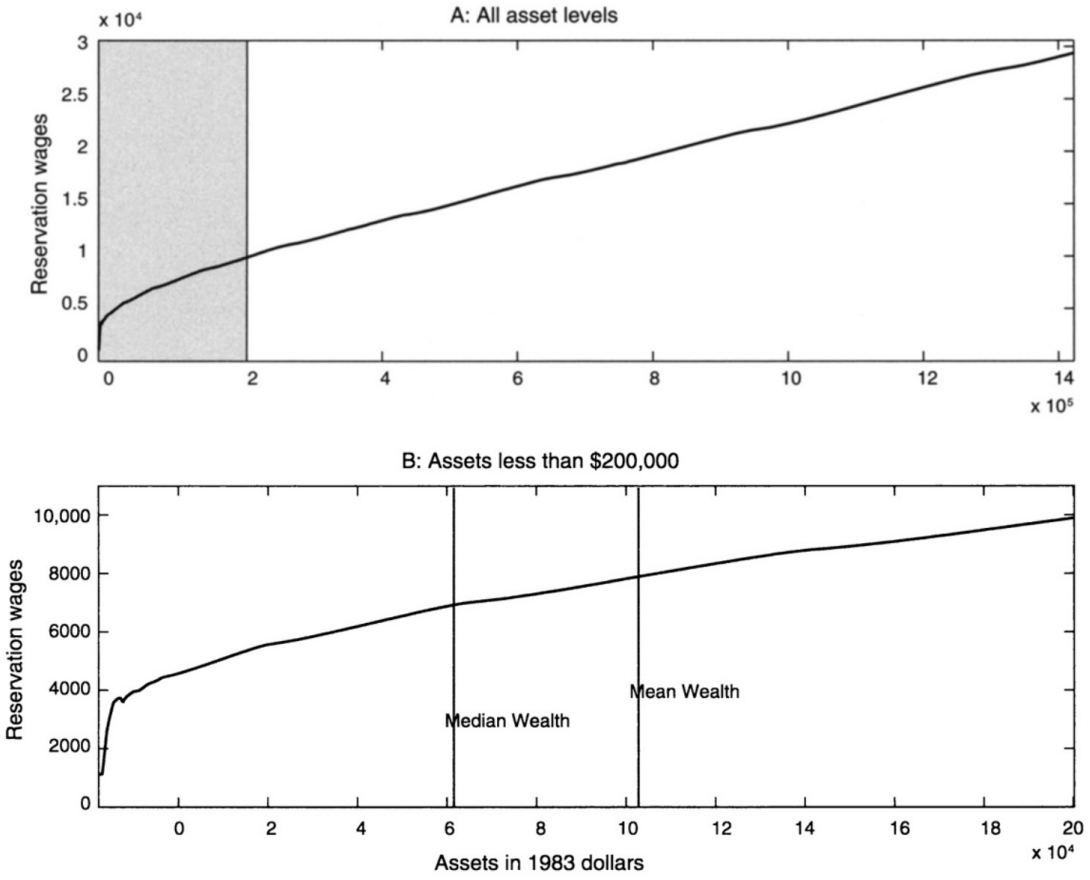

Figure 6 Reservation Wages From the Benchmark Model

Source: Chang and Kim (2007)

3.3 Cyclical Properties of the Model

The equilibrium of the model is solved using the "bounded rationality" method developed by Krussel Smith (1998) where agents uses a finite set of moments of μ in forecasting aggregate prices. Table 7 below presents the volatility of key aggregate variables in the model economy.

These main observations obtained from Table 3 are:

Variable	US data	Model
σ_Y	2.06	1.28
σ_C/σ_Y	0.45	0.39
σ_I/σ_Y	2.41	3.06
σ_H/σ_Y	0.82	0.76
σ_L/σ_Y	—	0.50
$\sigma_{Y/H}/\sigma_Y$	0.50	0.50
$\sigma_H/\sigma_{Y/H}$	1.64	1.72
σ_{MRS}/σ_Y	0.90	0.83
$\sigma_{\text{wedge}}/\sigma_Y$	0.92	0.76

Figure 7 Volatilities of Aggregate Variables

Source: Chang and Kim (2007)

- Ouput volatility is less than 2/3 of actual output volatility, similar to that of the standard representative agent models.
- Volatility of hours relative to output and the volatility of labor productivity relative to output is the same as that in the data
- The realtive volatility of hours to productivity is very colse to that in the data.

Table 8 below shows the cyclicality of key aggregate variables.
The main observations obtained from Table 8 are:

- The correlation between output, consumption, investment and labor productivity are higher than the data, a common feature of standard RBC models.
- The correlation of hours with output is close to the data
- Hours worked and labor productivity has low correlation despite aggregate productivity shock being the sole driving force in the simulation.

4 Conclusion

The fact that hours worked are not strongly correlated with labor productivity has been considered as a shortcoming of equilibrium business cycle theory. Using a heterogeneous-agent economy with incomplete capital markets and indivisible labor, the authors show that they can generate a low employment-productivity correlation. When the optimality condition implied by the representative agent is applied to the model, the authors were

Variable	Data	Model
Corr(Y, C)	0.69	0.84
Corr(Y, I)	0.90	0.98
Corr(Y, H)	0.86	0.87
Corr(Y, L)	—	0.92
Corr(Y, Y/H)	0.57	0.68
Corr(H, Y/H)	0.08	0.23
Corr(Y/H, MRS)	0.25	0.43
Corr(Y, wedge)	0.55	0.56
Corr(H, wedge)	0.85	0.87

Figure 8 Cyclicality of Aggregate Variables

Source: Chang and Kim (2007)

also able to find a time-varying wedge between MRS and MPL without the addition of distortion nor exogenous labor-supply shock.

Appendix

The replication of this paper utilizes the Econ-ARK/HARK toolkit to attempt at a reproduction of a figure produced in the original paper, specifically Figure 5. In order to do so, the main HARK tool used here is `ConsAggShockModel` and `ConsLaborModel` class, portions of which we've combined to create a new class named `ConsLaborAggMarkov` in which agents have the utility function described in the main body of the paper and face idiosyncratic shocks their productivity.

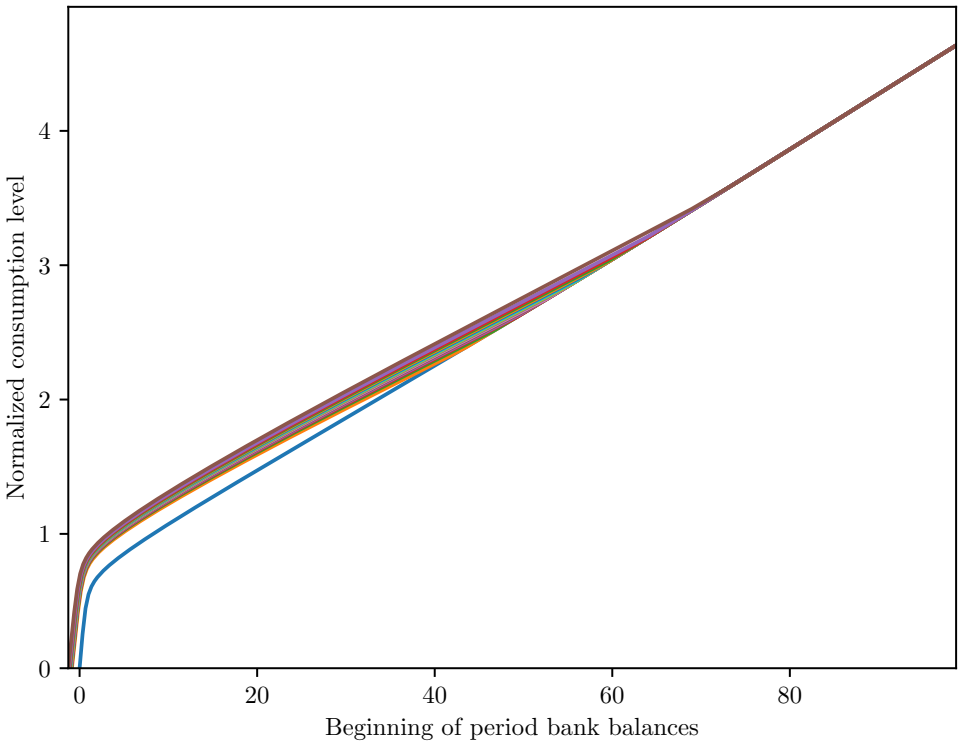

Figure 9 Placemat of what is produced by a Python Code

Source: Econ-Ark

Given the current limitations of the `ConsLaborModel`, we were not able to incorporate the artificial borrowing constraint into the final replication model.

References

Chang, Yongsung, and Sun-Bim Kim (2007): "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuation," *American Economic Review*, 97(5), 1939–1956.