Singular Value Decomposition

for recommender systems

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Used in LSA/LSI, PCA…

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columns are orthonormal:

$$U^\top U = V^\top V = I$$

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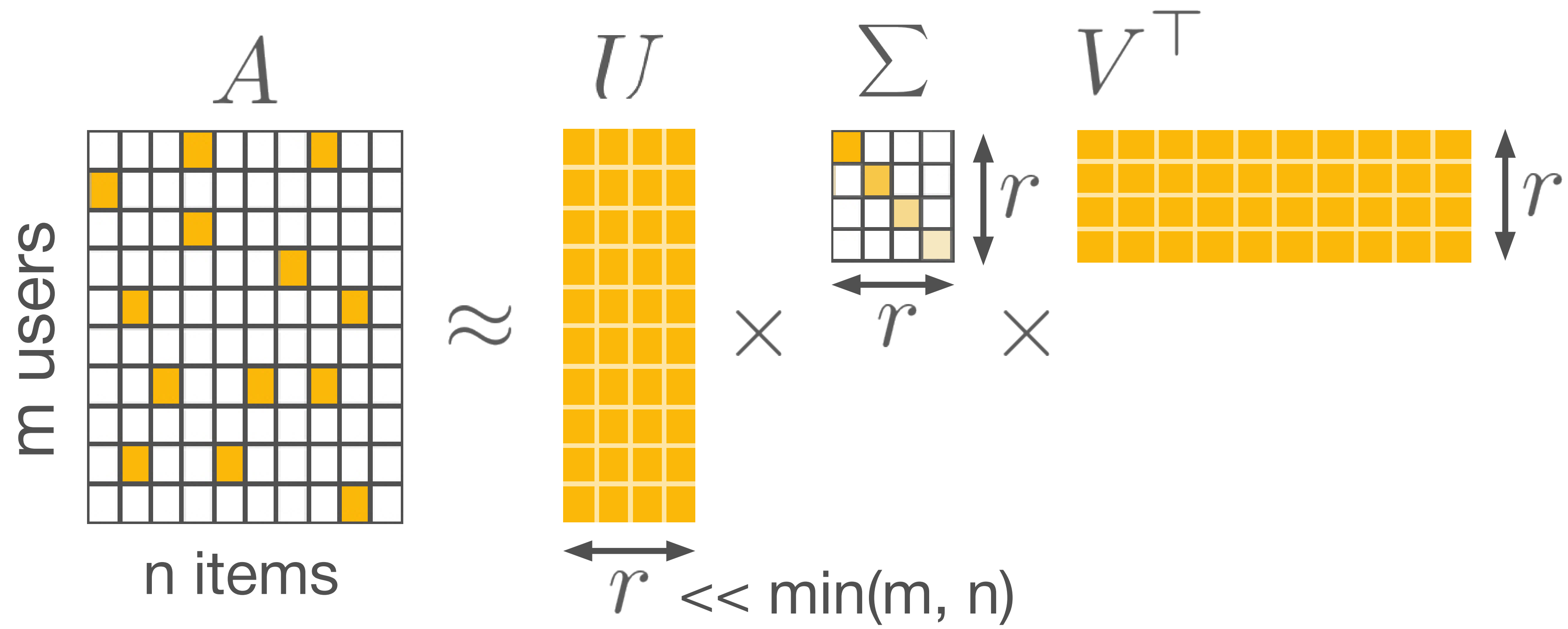
Truncated SVD of rank r



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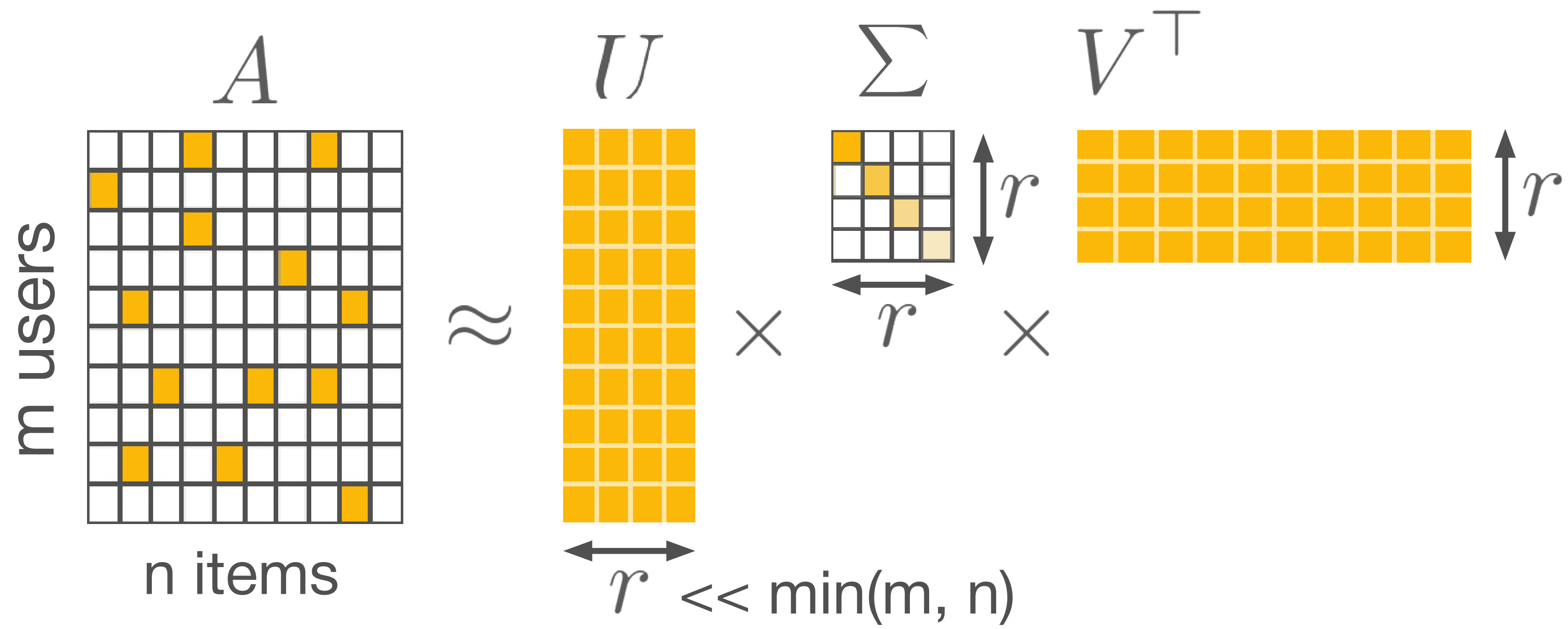

columns are orthonormal:
$$U^\top U = V^\top V = I$$

$$||A - A_r||_F^2 \to \min$$

$$A_r = U \Sigma V^\top$$

$$||X||_F^2 = \sum_{ij} x_{ij}^2$$

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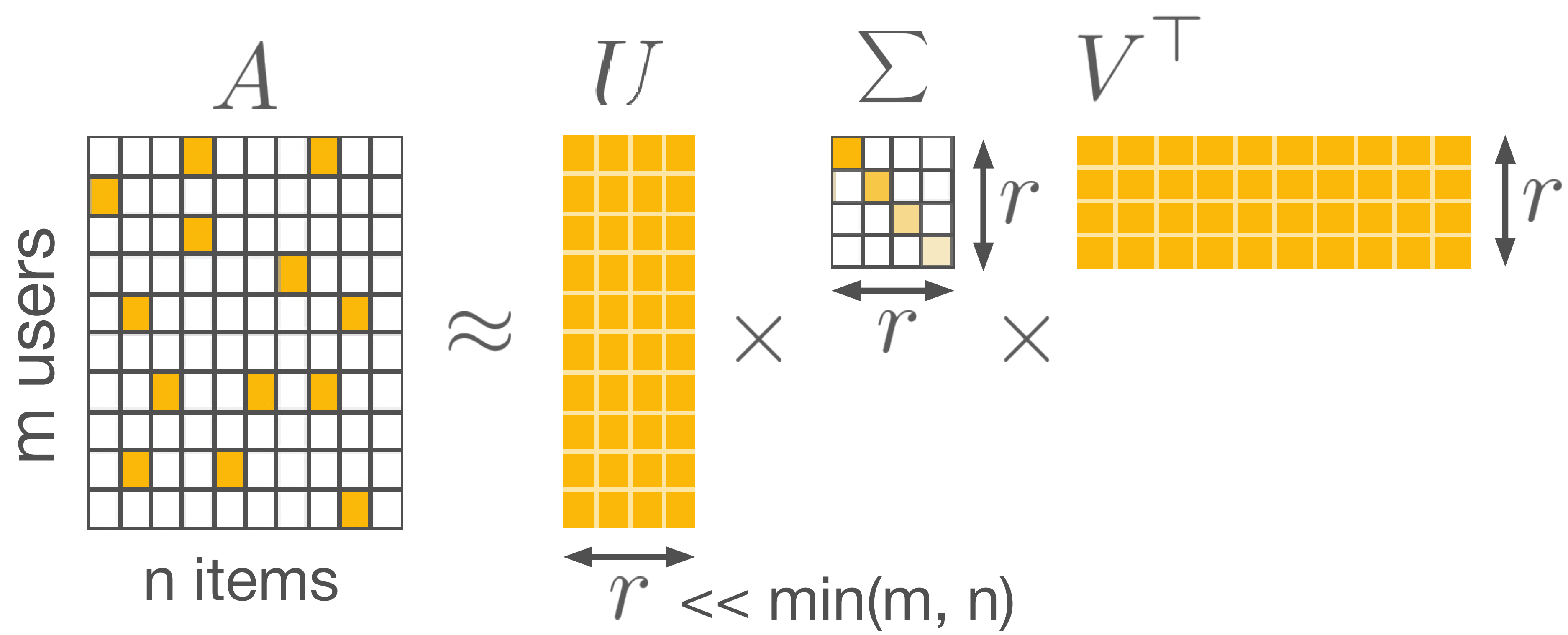

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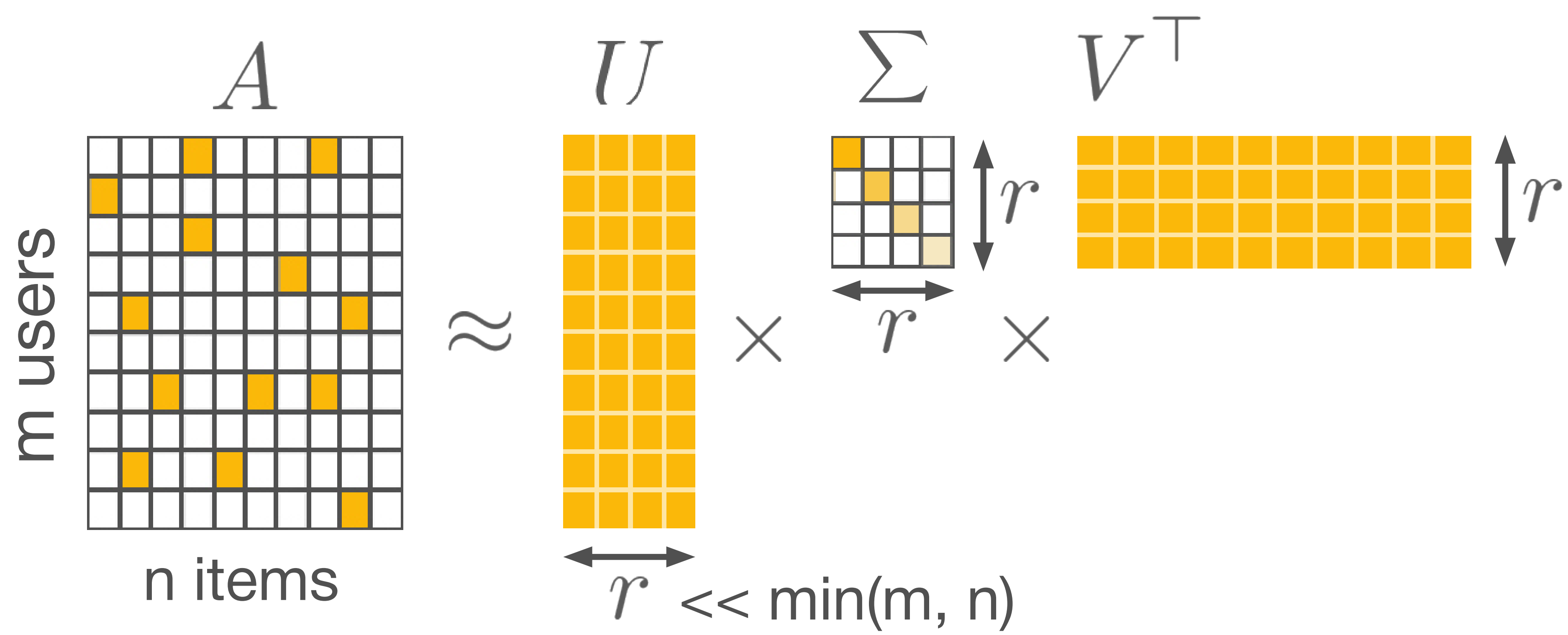

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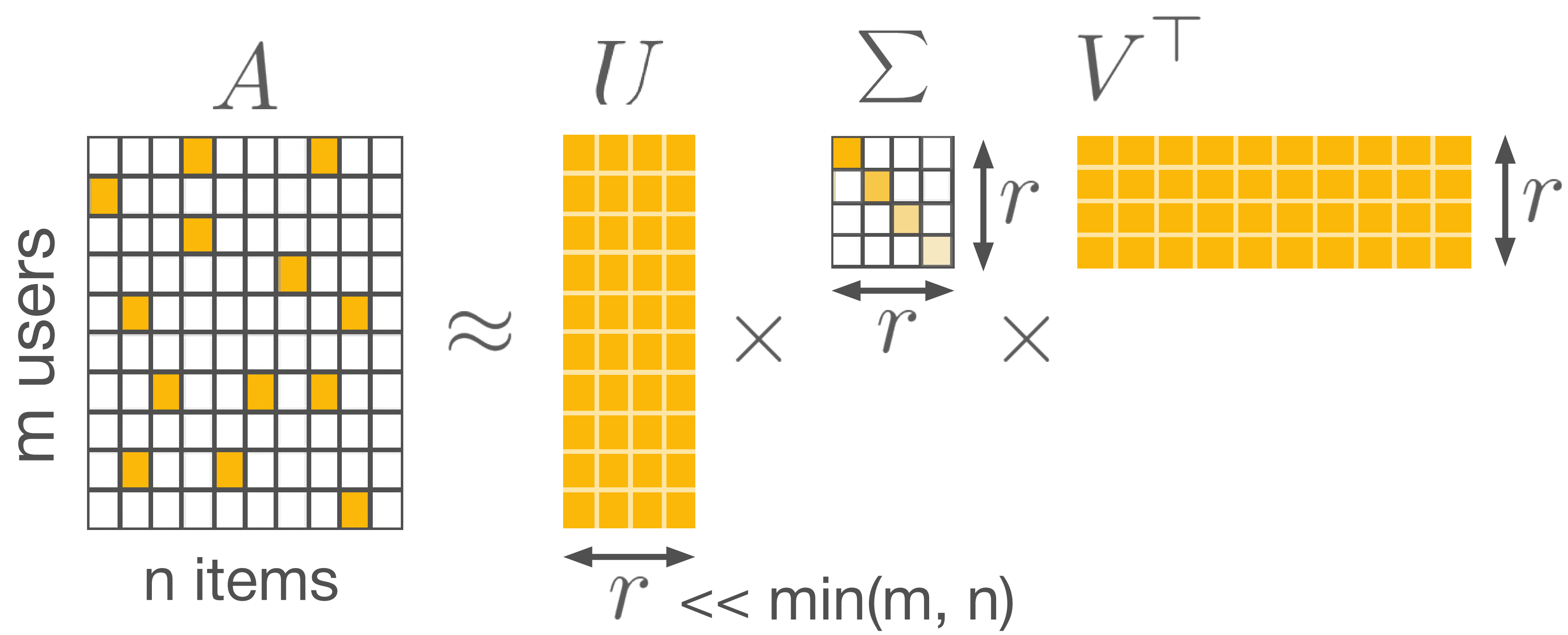

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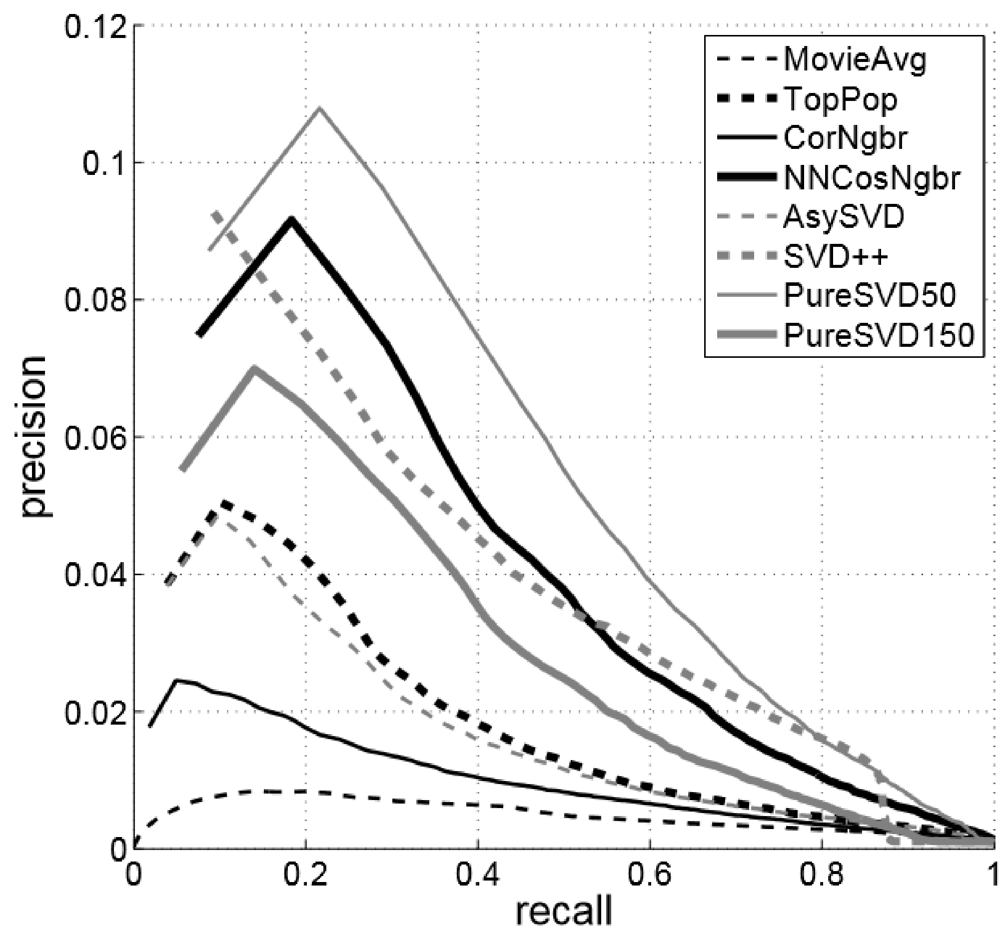
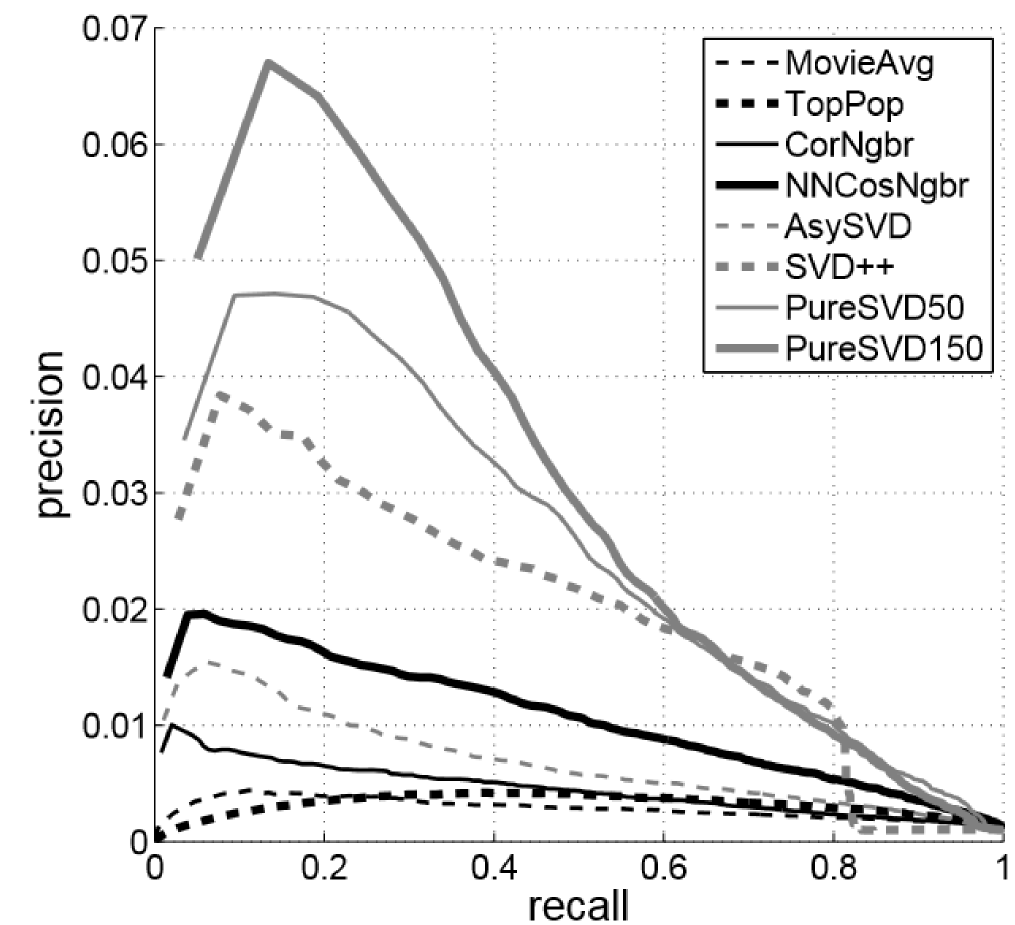
Let's impute zeros - PureSVD model.

- values are highly biased towards 0
- not good for rating prediction
- its not a big problem for ranking task

PureSVD – quality of recommendations



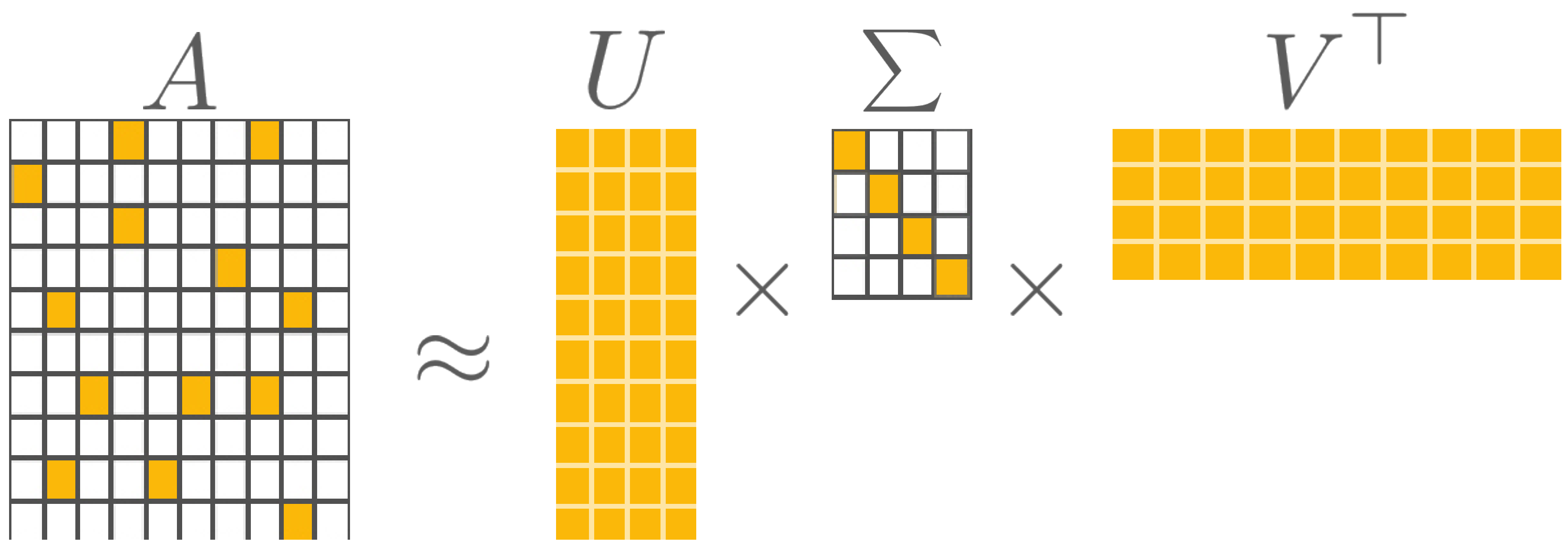


Netflix data: complete dataset (left) and “long-tail” (right).

P. Cremonesi, Y.Koren, R.Turrin, “Performance of Recommender Algorithms on Top-N Recommendation Tasks“,Proceedings of the 4th ACM conference on Recommender systems, 2011.

Note: Funk SVD, SVD++, TimeSVD++, Asymmetric SVD … **are not** the SVD!

PureSVD – recommending online



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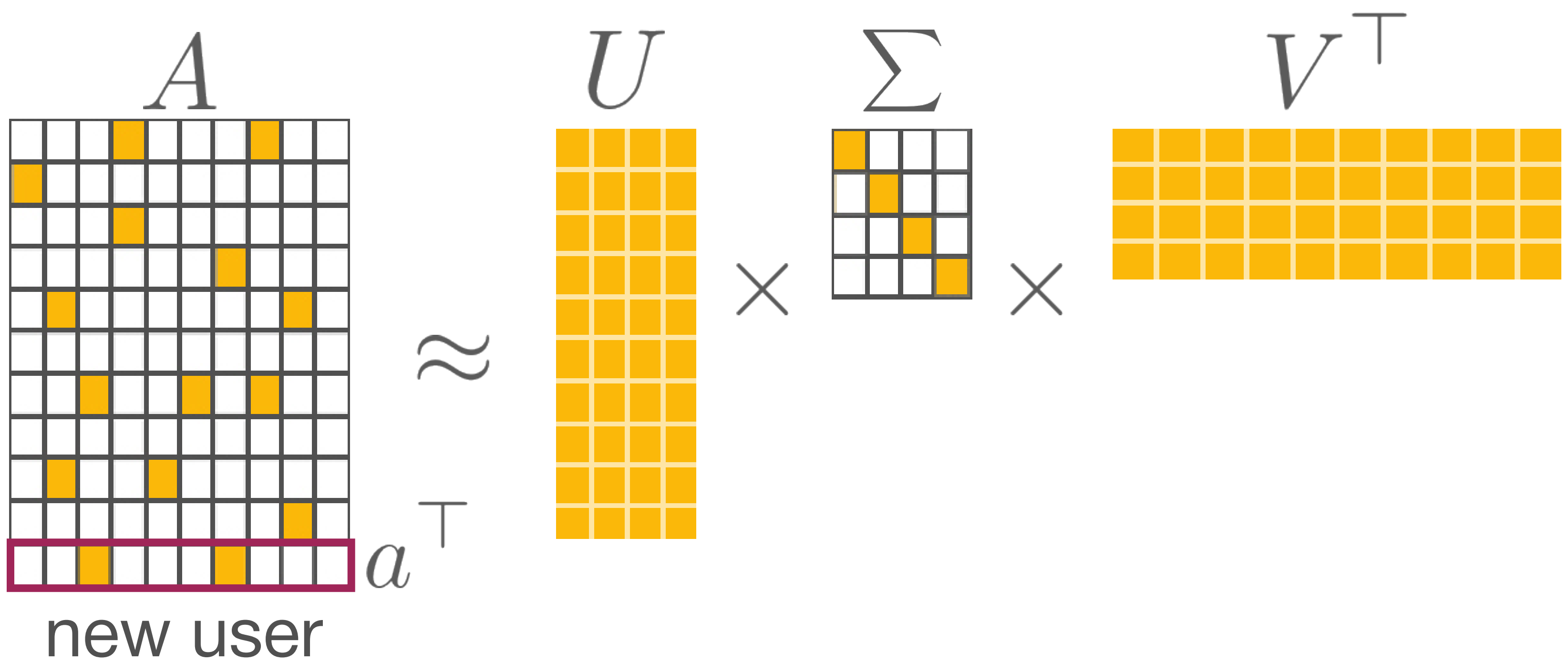

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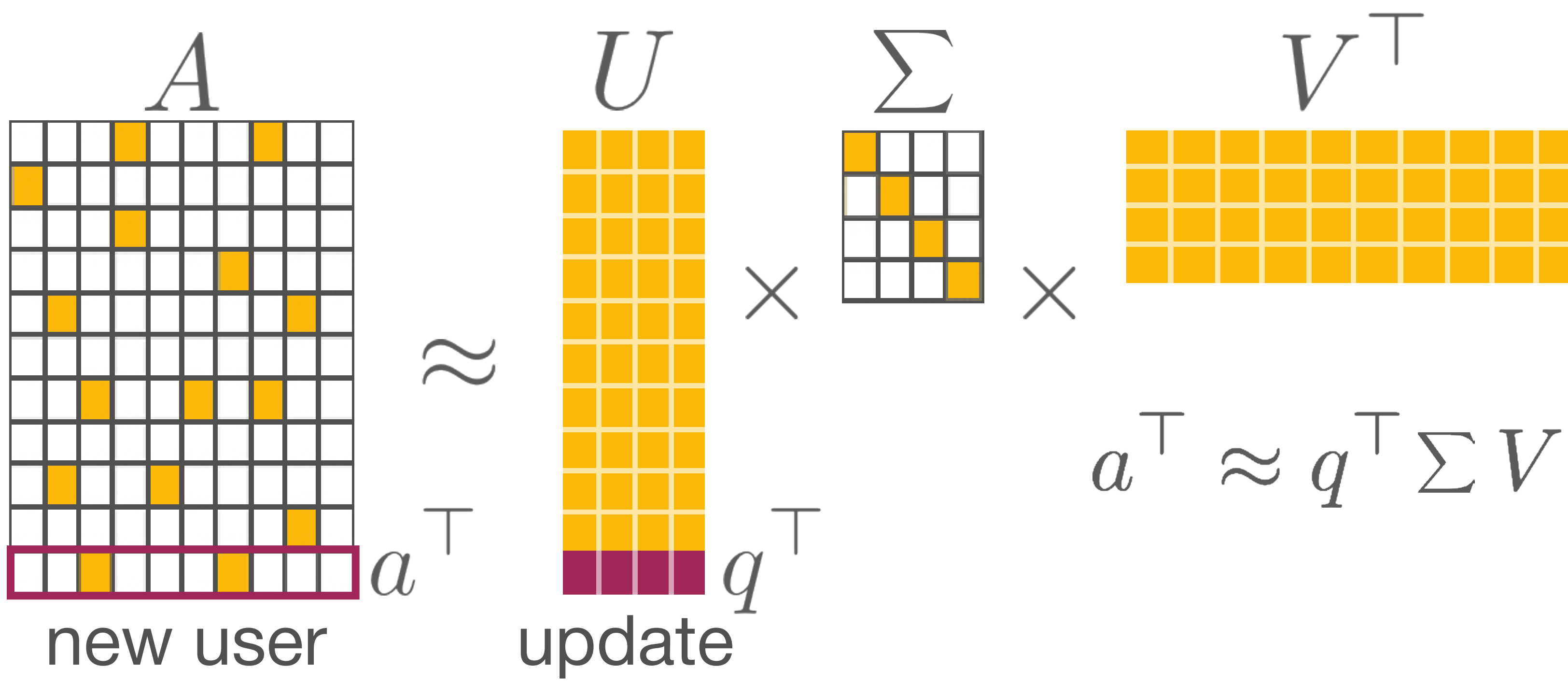

$V^\top V = I$ orthogonality of columns

$$a^\top \approx q^\top \Sigma V^\top \longrightarrow \boldsymbol{a}^\top \approx \boldsymbol{q}^\top \Sigma V^\top V = \boldsymbol{q}^\top \Sigma$$

$$\boxed{\boldsymbol{q}^\top \approx \boldsymbol{q}^\top V \Sigma^{-1}}$$ new user embedding

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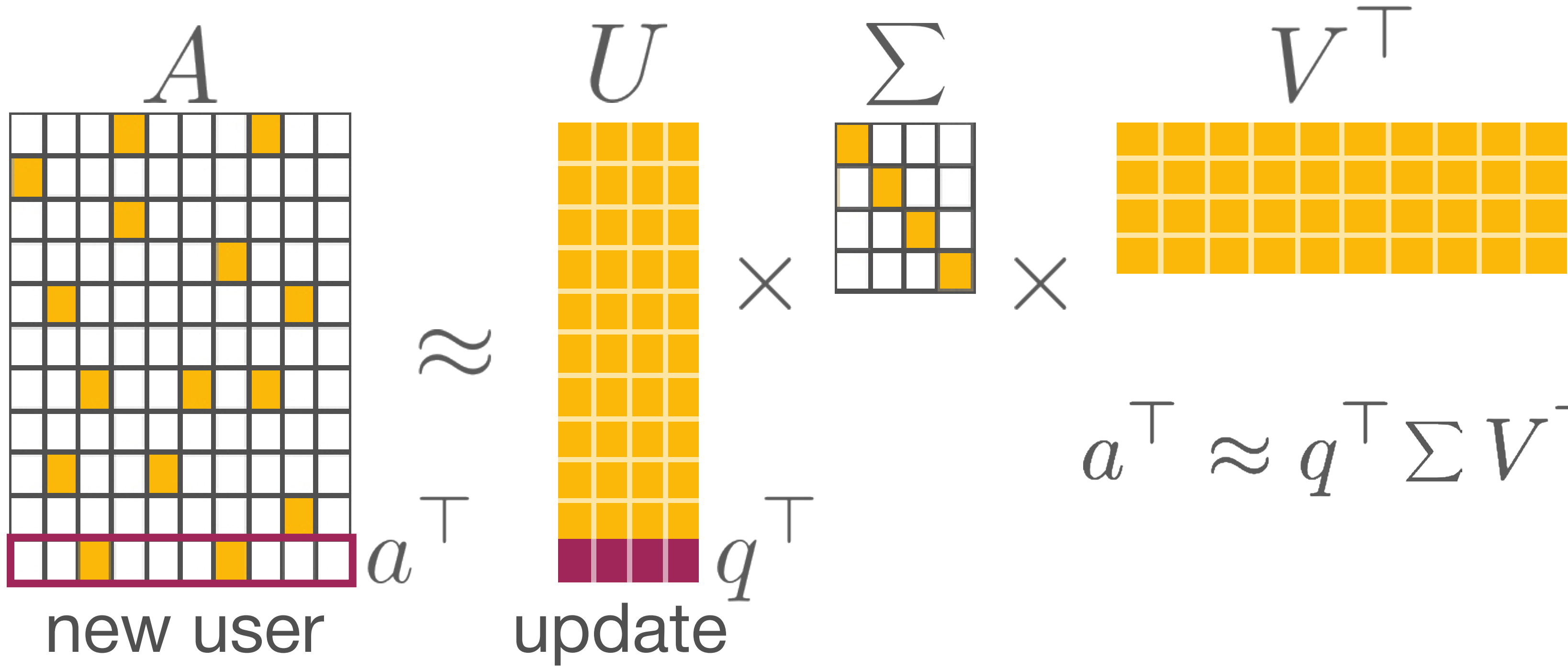

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*folding-in technique**

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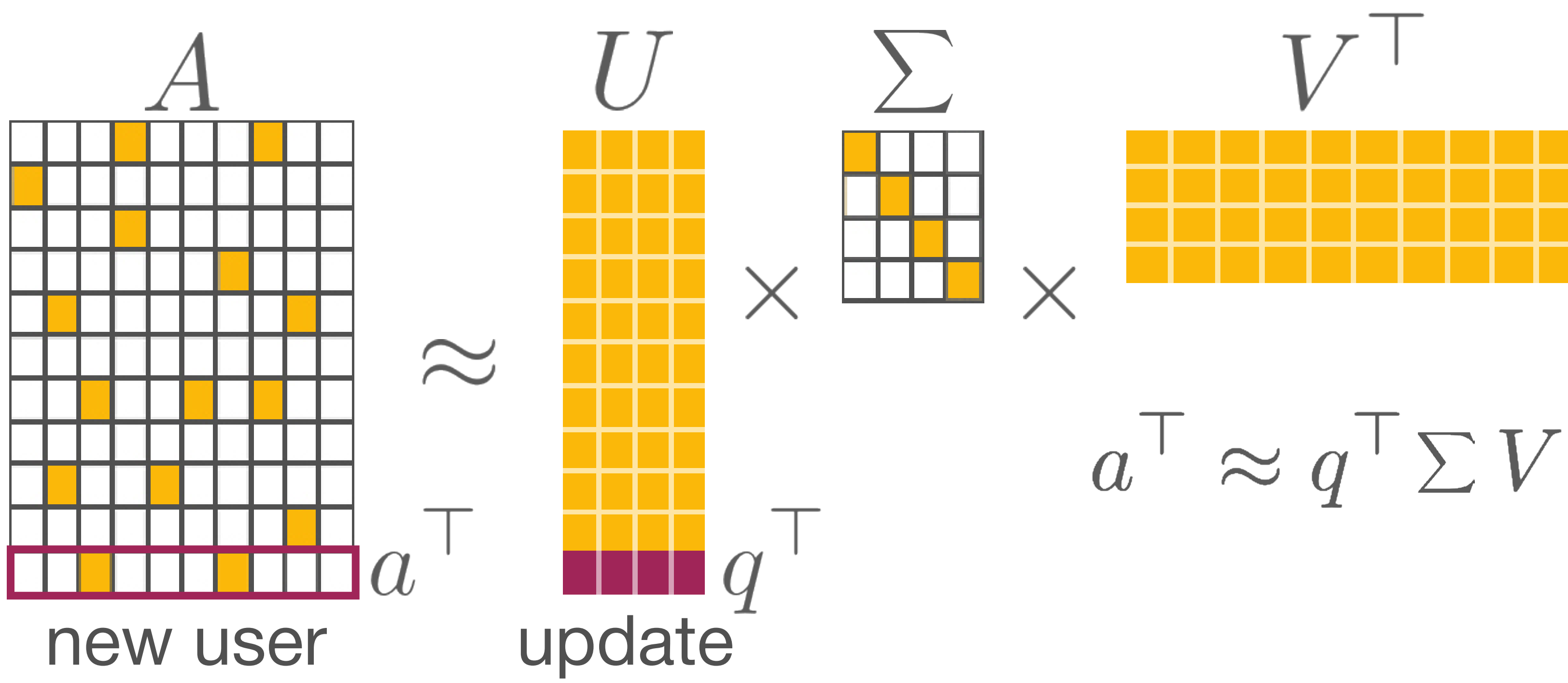

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$$r^\top = q^\top \Sigma V^\top \approx a^\top V \Sigma^{-1} V^\top = a^\top V V^\top$$

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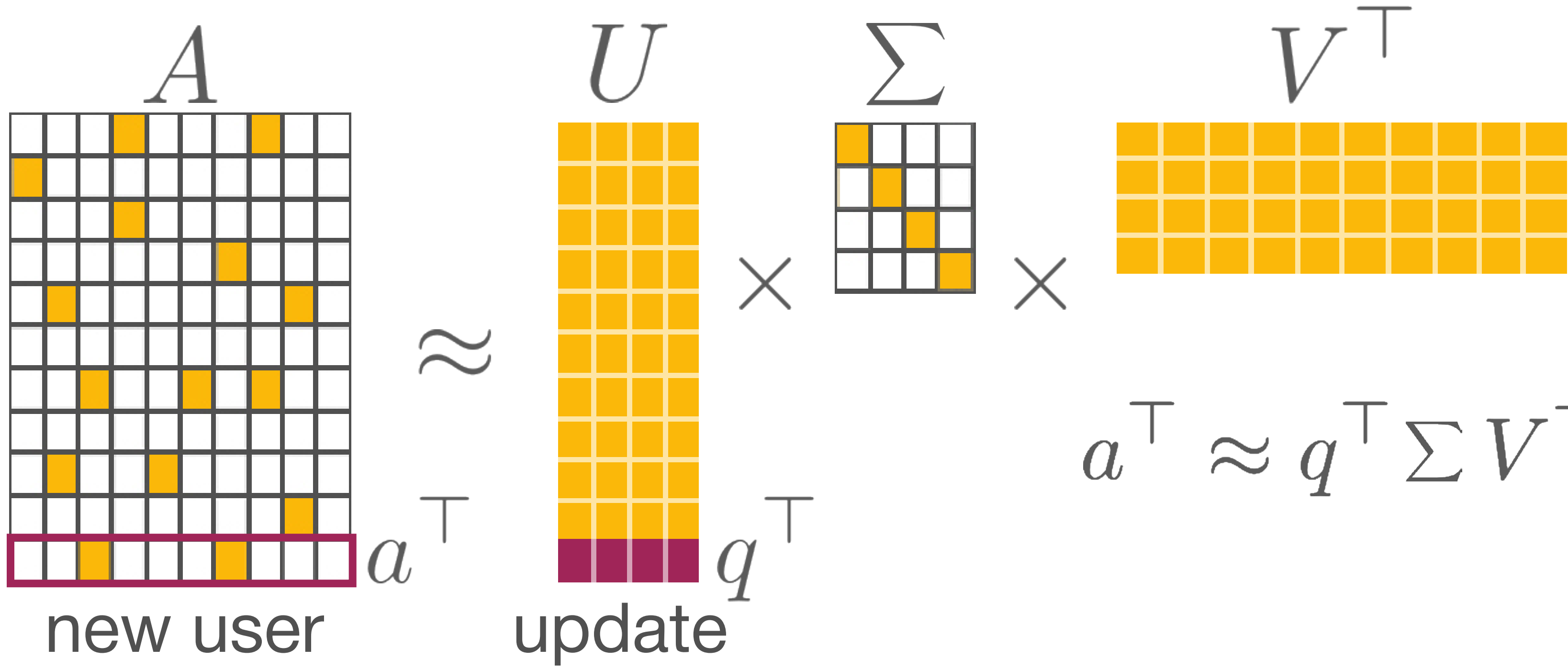

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vector of predicted item scores

$$\boldsymbol{r} \approx V V^\top \boldsymbol{a}$$

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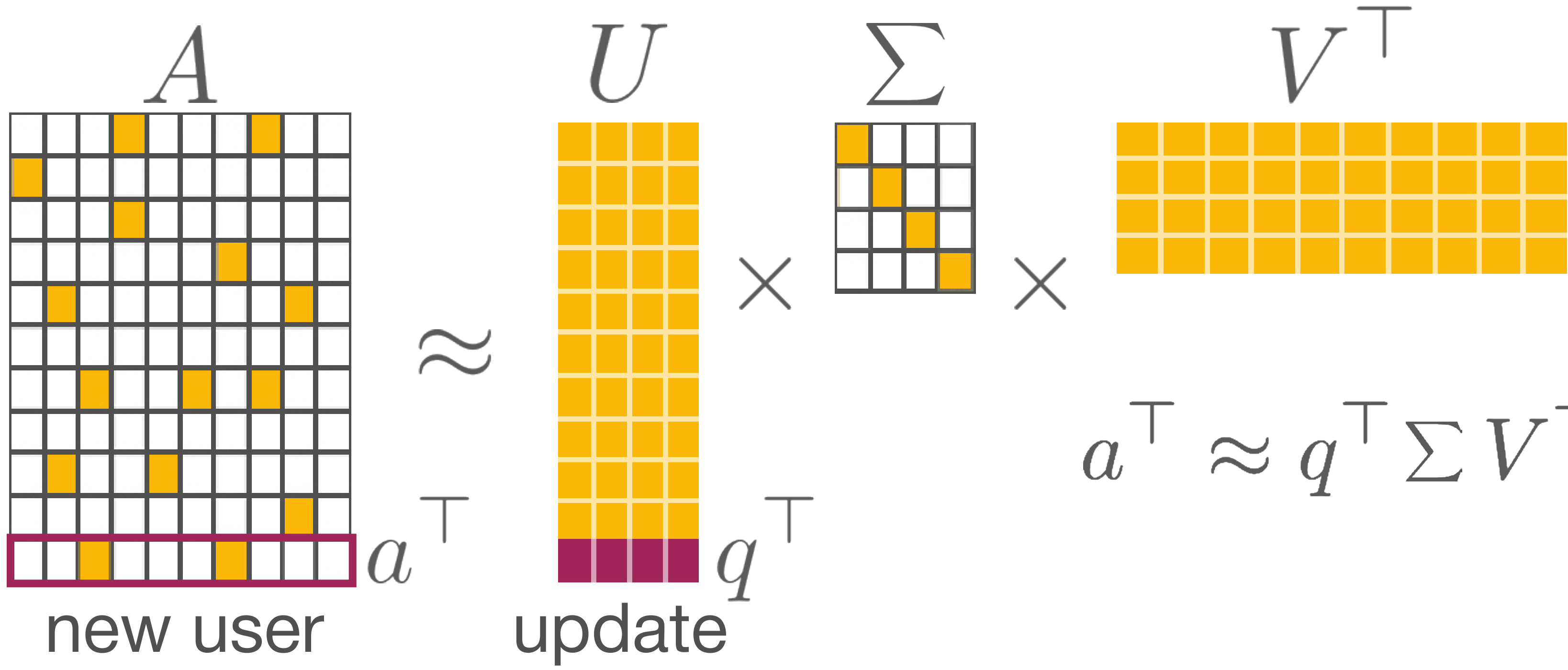

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SIGMA

allows for real-time recommendations
O(nr) complexity

vector of predicted item scores

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- You understand the role it plays in the **PureSVD** model
- You can explain how **folding-in** approach works and how to use it for online recommendations