

A survey of underwater docking guidance systems

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ABSTRACT

Autonomous underwater vehicles (AUVs) are increasingly being used for underwater survey and exploration missions. The expanding mission scope for AUVs highlights the need for a long-endurance operational capability, which mainly depends on propulsion system efficiency and battery capacity. The use of submerged docking stations permitting battery recharge and data download/upload offers a means of enabling persistence without compromising propulsion and payload power budgets, while also reducing associated deployment/recovery costs and risks. Autonomous docking with an underwater station is, however, complicated by the presence of currents and obstacles in the water, and by the relative dynamic differences in pose between the dock and the vehicle. A robust docking guidance system is identified as a core and crucial component for ensuring successful AUV docking. This paper presents a detailed literature review summarizing the current state-of-the-art in AUV docking guidance methodologies, identifying their relative merits and shortcomings, and revealing the docking guidance methodologies that seems to be the most prominent.

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1. Introduction

In recent years, oceanographic communities, oil and gas industries, and military agencies have paid significant efforts to expanding the use of autonomous underwater vehicles (AUVs) in place of remotely operated vehicles for a diverse range of undersea missions, including, bathymetric mapping, inspection, maintenance, mine detection, environmental monitoring, surveillance, and intervention [1–5]. Since, AUVs have no tether they are in principle able to perform such tasks as long-range surveys without the constant presence of a mothership to operate them. An AUV's operational endurance is limited by its

- energy storage capacity
- propulsion system efficiency
- hydrodynamic attributes of the vehicle hull
- hotel loads (the total power used for all non-propulsion devices, including, computers, navigation and guidance systems, communication devices and sensor payloads)
- vehicle guidance and navigation accuracy

In this context, the on-board energy storage is a key element that determines mission duration, AUV speed, as well as the

extent to which an AUV can be equipped with additional payloads such as mapping sonars that impose a high hotel load. One line of research to improve mission endurance takes advantage of recent advancements in battery technology, such as lithium-air and zinc-air batteries [6], however, the costs of such exotic batteries are prohibitively expensive. Most existing mid-size AUVs, such as REMUS, Bluefin, Hugin, and Explorer (see Fig. 1 and Table 1), which are typically used for large area survey applications [7–10], therefore rely on less energy dense but more cost-affordable battery solutions, such as lithium-ion, lithium-polymer, or even nickel metal-hydride and silver-zinc, although this comes at the cost of reduced mission endurance. Another line of research has been devoted to boosting the endurance capability of AUVs by using very low power propulsion systems and hotel loads. Long-range examples, such as Autosub Long-Range and Tethys, possess endurance of many days to months, however, these have limited sensor payload capabilities and are consequently constrained in terms of operational applicability [3]. Aside from energy consumption, for long-endurance missions AUVs will still need to regularly upload the large volumes of collected data collected during survey operations and to download updated mission scripts. In the absence of a tethered connection back to a mothership, data communication can only be done using low-bandwidth acoustic communications while subsurface or via high-bandwidth RF communications while on the surface. Neither of these options is ideal where large volumes of data

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are concerned. The alternative option is to recover the data via a wired connection once the AUV is back on board its mothership. Given both the energy storage and the data communication limitations, AUVs need to be periodically recovered and recharged by a mothership which poses a significant limitation on their efficiency.

AUV launch and recovery operations, are still forced to rely on human operator intervention, particularly for the recovery phase. The cost-overheads and potential risks to operators and equipment associated with man-in-the-loop deployments, have thus far stymied the accepted use of AUVs. Given these considerations, researchers must turn to other solutions to answer the question: *how can AUV mission persistence be improved?* Consequently, increasing attention is being paid to reduce AUV dependence on manned supporting vessels [11–13] as a way of improving their acceptability. To circumvent the aforementioned problems and limitations, a separate and complementary promising line of research considers the use of automated launch and recovery docking guidance systems that enable an AUV to operate fully autonomously from an underwater dock.

A docking system is an enabling technology which leverages the submerged endurance capability of AUVs for long-term operations while reducing operation cost and hazards (Fig. 2). To this end, the docking station (DS) may include facilities for battery recharge, and mission data download and upload without needing to continuously recover the AUV back to a ship. The DS may be attached to an above surface RF network or alternatively integrated into a subsea cabled observatory network, which would allow the vehicle to become a part of a larger system with the capacity for direct data transferring/processing and vehicle monitoring from the shore [14–16]. Various forms of DSs have been developed. Most implementations are either rigidly fixed to the seabed or integrated within a subsea station; *free floating* where the DS is buoyant and moored to the seabed or rigidly attached to a larger underwater vessel; or *mobile* where the AUV is towed behind a recovery ship. Each form has its advantages and disadvantages. Both fixed and free floating DSs can provide safe anchorage place in between missions while waiting to recharge, upload data and retrieve new mission commands. Towed DSs provide a means of retrieval of an AUV back onto a mother ship. While fixed stations do not move with currents, they are typically more expensive to install. In contrast free floating and mobile stations can easily be relocated, however, their orientations are susceptible to water currents and other disturbances. AUV docking guidance systems should ideally be able to facilitate docking with any type of dock irrespective of whether it is fixed, free-floating, or towed. A typical use case scenario for the oil and gas industry might have an AUV being deployed from a ship, assigned to operate unattended for many months in a zone equipped with fixed docks, connected to a wired network permitting recharging and data communication, and then recovered from a ship towed DS for maintenance purposes.

To benefit from the facilities that a DS provides, an AUV should be able to perform a range of fully autonomous, efficient, and reliable docking manoeuvres considering the constraints associated with the operating environment, vehicle dynamics, DS geometry, and type of DS. This leads to the need to develop a systematic and universal guidance framework, or equivalently a trajectory generator engine, enabling an AUV to perform optimal homing and docking operations in diverse operating environments. From the operational standpoint, the guidance framework should satisfy a series of requirements; namely it should

- accommodate a wide range of DS poses, both static and time varying, regardless of the initial vehicle pose
- consider the DS geometry constraints once the vehicle approaches the DS entrance
- provide smooth manoeuvring from the starting point all the way through to arrival into the DS
- consider the operating environment constraints, i.e., current disturbances, obstacles, and no-fly zones (NFZs)
- consider the physical limitations of the vehicle's states and actuators
- optimize the vehicle's energy expenditure, docking operation time, and any desired performance index
- be amenable to on-board computing in real-time so as to enable online trajectory refinement in response to situational awareness of the surrounding environment.

The development of such a universal guidance framework for underwater vehicle docking has received less attention in comparison with aerospace vehicle docking. With advances in computational performance and power consumption of state-of-the-art embedded systems, optimal control theory is increasingly being utilized to develop robust and universal guidance systems for complex aerospace problems, however, it still remains a very underdeveloped tool in relation to underwater vehicle guidance applications. The aim of this paper is to expand the use of optimal control theory for developing an underwater docking guidance system. The main contributions of this paper are threefold. First, the paper provides a detailed survey on the concept, elements, and requirements of the underwater docking problem with a particular focus on analysing existing docking guidance systems. Second, the paper aims to highlight the suitability of optimal control theory for underwater docking guidance applications by describing two major branches of numerical optimal control methods, namely indirect and direct methods. Third, the paper proposes a new universal guidance framework that exploits the use of optimal control theory to ensure reliable and cost-efficient docking operation.

The paper is organized as follows. Section 2 introduces the key elements required for autonomous underwater docking operations. In Section 3, the related developments in the field of underwater docking and substantial challenges faced in autonomous docking operations are presented. Section 4, reviews the use of optimal control theory for developing a docking guidance system, followed by Section 5 discussing the applicable numerical methods of optimal control theory used as guidance solutions. Section 6, introduces a proposed universal guidance framework that is capable of addressing the challenges of autonomous underwater docking. The conclusions are then presented in Section 7.

2. Key elements of autonomous underwater docking systems

The fundamental elements for autonomous docking operation are shown in Fig. 3. The main elements in this figure are the guidance, navigation, and control components, collectively known as the GNC architecture, which operate interactively with each other.

The guidance system is responsible for generating a set of feasible and/or quasi-optimal/optimal trajectories for the control system in response to the situational awareness of the operating environment and the destination target provided by the navigation system, the mission objectives, and the vehicle's dynamics. The guidance system can operate in both offline and online modes. In the offline mode, the DS location and pose are assumed constant, hence the trajectory is computed only once at the onset of the docking operation, whereas in the online mode, the DS position and pose are allowed to be constantly varying

- provide a significant degree of freedom to control initial, midcourse, and final vehicle pose, velocities, and accelerations

Table 1
Physical and operational specifications of AUVs in Fig. 1.

Vehicle type	Physical specifications		Operational specifications		
	Size	Weight	Depth	Speed	Endurance
REMUS-600	4.27 m × 0.32 m × 0.32 m	326 kg	600 m	1.5 m/s	50 h
Bluefin-12	3.77 m × 0.32 m × 0.32 m	204 kg	200 m	1.5 m/s	26 h
Hugin-1000	4.7 m × 0.75 m × 0.75 m	850 kg	3000 m	2.05 m/s	24 h
Explorer	4.5 m × 0.69 m × 0.69 m	750 kg	3000 m	2.5 m/s	36 h



Fig. 1. (a) REMUS-600 [7]; (b) Bluefin-12 [8]; (c) Hugin-1000 [9]; (d) Explorer [10], not shown to scale.

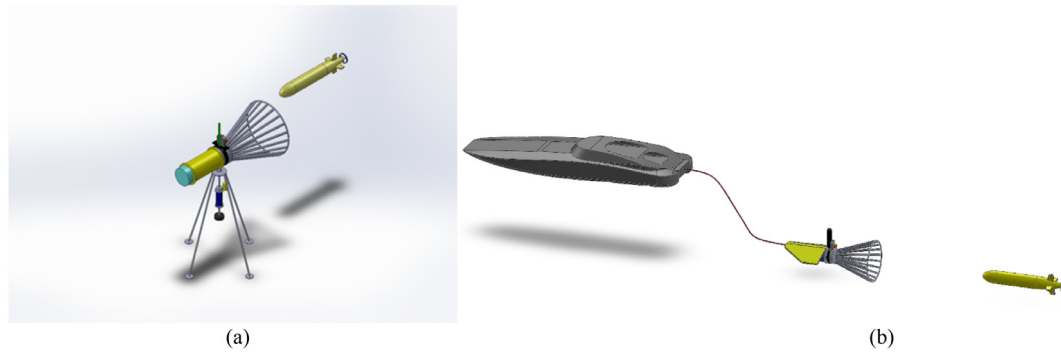


Fig. 2. Pictorial representation of an AUV docking operation with: (a) fixed; and (b) towed DSs.

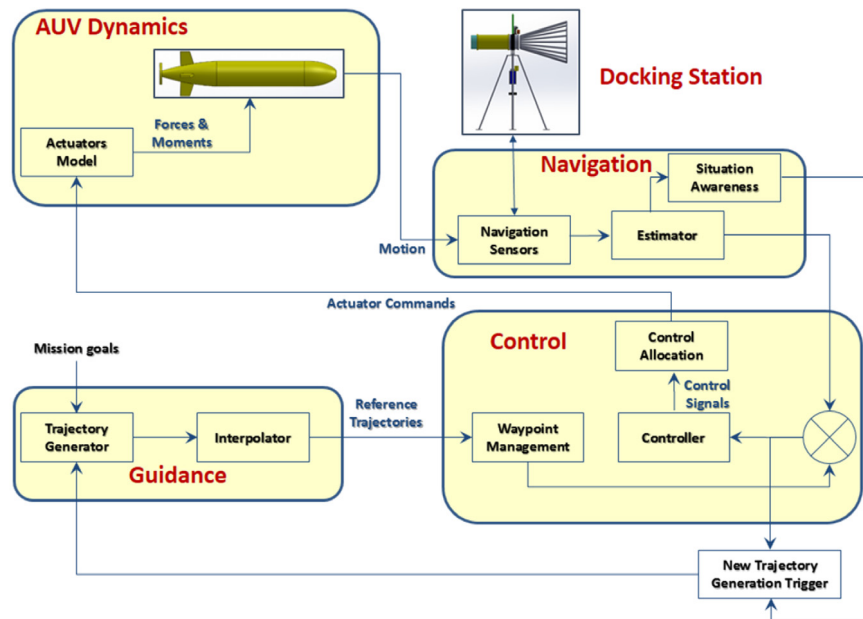


Fig. 3. A typical AUV docking GNC architecture.

requiring the trajectory to be continuously updated whenever a DS change is detected. The offline mode is suitable for fixed DS structures, whereas the online mode is necessary for both

free floating and towed DS structures. The navigation system utilizes its onboard sensors and often an observer subsystem to estimate/predict vehicle states such as position, orientation and



Fig. 4. (a) REMUS docking system; (b) Maridan Martin AUV with Eurodocking system; (c) Bluefin docking system; (d) Dorado docking system.

velocity, the current and future pose of the DS, and information regarding the underlying operating environment such as spatiotemporal ocean currents and probable locations of obstacles for the guidance system. The control system produces corrective control forces and moments for the vehicle with respect to the vehicle's states provided by the navigation system and reference trajectories generated by the guidance system to successfully complete the docking operation. As indicated in Section 1, the scope of this paper focuses on the docking guidance system and other related elements that directly impact on it.

2.1. Mechanical configurations of docking stations

The chief philosophy behind the design of most dock structures is to increase the likelihood of safely docking the AUV. This, in general, comprises a reduction of constraints on both approach direction and attitude provided by large horizontal and vertical capture apertures. In this regard, two principal forms of docking configuration, namely unidirectional and omnidirectional have been developed [17,18].

The unidirectional DS is the most common configuration usually used for recovering torpedo-shaped AUVs and typically comprises a funnel/cone-shaped entrance to provide a large cross-section area for the AUV capture mechanism. It is employed with fixed, free floating and towed docks. Once captured within the dock, the AUV is protected against environmental hazards. To perform docking with a funnel-shaped station, the AUV is required to approach and accomplish the terminal phase of docking process along the longitudinal axis of the funnel. This imposes a condition on the AUV that it must at all times know the DS pose in order to align itself with the centreline of the funnel during the final approach. In the case of a fixed dock, the DS pose and location will not change, however in the case of the free floating and towed DS, the pose and location are variable, hence some means of measuring the relative difference in pose and location between AUV and the DS will be required. Fig. 4 shows examples of funnel-shaped docking systems and their associated AUVs including REMUS [19], Eurodocking [20], Bluefin [21], and Dorado [15].

The omnidirectional DS is usually designed as a vertical structure composed of rigid poles or cables under tension that enable a vehicle to attach itself to a pole/cable using a latching device mounted on the nose of vehicle. The pole shaped arrangement is typically used with fixed docks whereas the tensioned cable arrangement is typically used with towed docks. For the omnidirectional DS, the vertical aperture is a function of the length of the docking segment on the pole/cable whereas the horizontal aperture is related to the width of the capture mechanism on the vehicle. This configuration is a truly omnidirectional station and allows the AUV to dock using a relatively simple navigational system that only requires knowledge of the pole/cables location. It is equally applicable anywhere along the water column between the surface and the sea floor. Utilization of the omnidirectional DS is limited for a certain class of AUV since latching onto such a structure needs a complex mechanical device implemented on the nose of the AUV, thus making it difficult to incorporate forward looking sonars and cameras in the nose section, particularly in smaller vehicles. The integration of battery charging and DS–AUV communication systems with pole/cable type DS structures is also complex [22,23]. Fig. 5 illustrates the pole-shaped DS developed as part of the AOSN project for the Odyssey AUV [24].

Aside from the diverse mechanical design choices, the DS functionality can be further distinguished by the type of power charging and data/command communication transfer facilities integrated in the DS. Recharging of the AUV batteries can generally be conducted by direct electrical power transfer using wet mateable connectors or by inductive electrical coupling [25]. Data transfer can be undertaken via inductive coupling, optical communication, radio frequency (RF) communication, and direct electrical coupling [25]. Key factors such as the volume of data to be transferred and communication bandwidth, battery capacity and charging time available, the AUV capturing and undocking mechanisms required, as well as the nature of the DS operating environment must be taken into account when deciding the choice of communication transfer and recharging technologies employed [25]. Table 2 summarizes the key features of the aforementioned power and communication transfer systems used with DSs.

2.2. Docking navigation system

In the case of free-floating and towed docks where the docks are allowed to freely orient themselves with the direction of the water flow, the docking guidance process requires regular AUV–DS communication to update the AUV about the DS's location and pose. The docking navigational system should include navigational sensors and communication systems installed on the DS as well as the AUV to enable the AUV to determine its relative position and pose to the DS. In general, three types of navigation systems are used for docking operations, these are acoustic, optical, and electromagnetic systems.

Acoustic based navigation systems operate based on the propagation of acoustic signals over long distances through a water medium between an AUV and a reference platform. In docking operations, the common configuration employed is for a transceiver mounted on the AUV to emit an interrogating ping to which a transponder mounted on the DS in turn sends a reply message. The round trip travel time of the acoustic signal is used to determine the range. In the case of inverted ultra-short baseline (iUSBL) acoustic systems, the returned signal is received on multiple precisely separated transceivers mounted on the AUV. The slant range, which is the distance between each transceiver and the transponder, is calculated based on the round-trip time. The relative azimuth and elevation angles between DS and AUV are determined with respect to the small differences of arrival

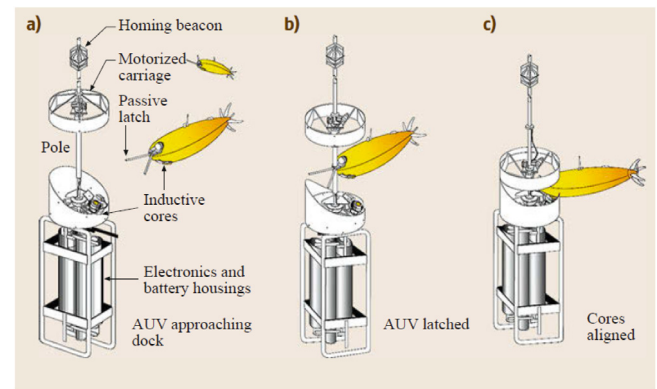


Fig. 5. Three stages of the Odyssey IIb AUV docking process with a pole-shaped station [25].

time between the respective receivers. The iUSBL-navigation aid is commonly used for AUV docking operations [15,19,24,26–28].

Optical docking systems are composed of multiple lights on the DS and a vision system on the AUV. The vision system can be as simple as a quadrant detector, although a camera and associated image processing algorithms can be used to better resolve the relative AUV pose. The relative position and pose of the AUV can be obtained either in a dock-coordinate system or in other frames considering the configuration of lights. This provides more flexibility for the vehicle in situations where the approach phase can only be accomplished from a particular direction. Compared to acoustic systems, the acquisition range of optical systems is considerably limited in ocean environments as the light sources are susceptible to scattering and absorption depending on the turbidity of the water. [29]. Nevertheless, there exist several documented studies employing optical systems for docking purposes [11,13,30,31].

Unlike the optical navigation aid whose performance is subject to water turbidity, the electromagnetic navigation system is more robust and accurate under almost all oceanographic conditions. This system consists of electromagnetic field generating coils on the DS and corresponding sensing coils on the vehicle. This arrangement enables the bearing angle of the vehicle relative to the DS to be determined. It was confirmed in a field-trial experiment that the accuracy of the electromagnetic navigation system is better than 20 cm but its sensing range may extend as much as 25–30 m [32].

Table 3 compares the performance of navigation aids employed for the docking operations based on key functional properties such as acquisition range, accuracy, update rate, latency, and environmental sensitivity.

2.3. Docking guidance system

The terminology AUV guidance systems or guidance laws refers to systems capable of producing feasible and applicable trajectory commands based on the information provided by the navigation system in accordance with the mission objectives to enable an AUV to travel from any arbitrary initial condition to a target of interest. In a general classification, these guidance laws can be divided into two main branches of local and global methods. Table 4, tabulates this classification and shows three major approaches utilized to develop either local or global guidance strategies.

The docking procedure can be divided into two major stages; in the initial stage, known as *homing stage*, the guidance system directs the vehicle from a far distance (approximately 1 km~100

Table 2

Applicable power and data transfer technologies proposed or in use with AUV DSs.

Type	Power transfer	Communication	Specifications	Example
Direct electrical coupling	✓	✓	Relatively simple — uses off-the-shelf wet mateable connectors; Requires careful alignment and significant mating forces; Has limited lifetime due to corrosion.	REMUS [26]
Inductive coupling	✓	✓	No exposed contacts — uses inductor coil arrangement; High charge rates of several kilowatts reported; Less sensitive to alignment and requires low mating force; Efficiency is a direct function of inductor size and permeability of materials used.	Dorado [15]
Mechanical coupling	✓	×	Simple power generation by allowing the AUV's propellers to spin freely in the water flow; No such system has been implemented in practice to date.	N/A
Material transfer	✓	×	Physical swapping of batteries or refuelling of chemical reactants; Could lead to faster refuelling time but has not yet been implemented in practice.	N/A
Optical system	×	✓	Does not require exposure of electrical connectors to seawater; Has much higher misalignment tolerance than electrical connectors.	Flying Plug system (33)
RF system	×	✓	Can take advantage off-the-shelf hardware; Can provide high bandwidth communication, for example up to 10 Mb/s at separations up to 6.25 cm, through the seawater.	Dorado [15]

m) to arrive in close vicinity (roughly 20 m~10 m) to the DS. In the terminal stage, known as *docking stage*, the guidance system drives the vehicle from close vicinity of the DS into the DS entrance and latches the vehicle in the DS. It is important that the guidance method is capable of handling both homing and docking stages properly. The following two categories of guidance methods are typically utilized for homing and docking procedures.

The first category contains point-to-point guidance laws that are usually simple in design and operate by pointing the vehicle directly into the DS. These methods normally are employed in the docking stage and more suitable for docking operations that can be approached from any direction, for instance docking with a pole-shaped DS. The simplest of these are the classic guidance laws such as line-of-sight (LOS) [30], linear-terminal guidance [31], and pure-pursuit guidance [33].

The second category encompasses optimization-based guidance laws. Unlike point-to-point guidance systems, optimization-based guidance laws are capable of being employed for both homing and docking stages. The objective for this group of guidance systems is to generate a geometric three-dimensional (3D) path together with associated time histories of the vehicle's states such as velocity and acceleration commands, to provide enhanced manoeuvrability. The major concern with these approaches is the computational burden that make them less suitable for real-time docking operations. Examples of these approaches include, AUV path planning using evolutionary algorithms such as genetic algorithm (GA), particle swarm optimization (PSO), etc., [34–37], and trajectory generation methods such as indirect/direct methods developed from optimal control theory [38,39].

3. Autonomous docking solutions

There exist numerous documented examples in the literature that attempt to address the complexities of autonomous docking, however these are mostly limited to docking with a static DS in defined or semi-defined operating environments. Relatively few research studies have so far been attempted to address the problem of operating in variable and spatiotemporal operating fields where the fidelity and tractability of the docking algorithm is of utmost significance. In the following subsection, a brief review of existing solutions is presented to illustrate the need for developing a universal docking framework.

3.1. Review of autonomous docking research studies

Documented studies on underwater vehicle docking indicate that existing solutions are still relatively immature when compared with solutions employed for automated spacecraft docking [47]. The seminal works on autonomous underwater docking conducted by the Oceanographic Systems Laboratory at Wood Hole Oceanographic Institute (WHOI) date back to 1997 [48], where a REMUS AUV was used to dock with a funnel-shaped DS. The experiment completely encapsulated all aspects of the docking problem, including mechanical design of the DS, battery charging and communication circuitry, AUV navigation system, and docking guidance. The AUV docking process in this experiment was divided into the following segments. In the first segment the REMUS manoeuvre comprised a straight motion followed by a turn to arrive at a position 50 m away from the DS with the AUV pointing straight into the DS funnel. In the next segment, the AUV attempted to follow a reference path along the DS centreline using a path-following guidance law. In the final segment, the AUV reached a 2m-threshold in front of the DS, at which point it straightened out its control fins and continued with a constant thrust for 15 s to enter through the DS funnel. This experiment used a USBL navigation system with a tolerance accuracy of 1–3° and an acoustic range of 2000 m. The methodology employed in this experiment did not consider the AUV dynamic model, the DS geometrical model, nor the impact of current disturbances. To minimize the impact of current disturbances, the DS was installed such that it pointed into the current. A success rate of 21% was initially recorded, however this was later improved to 91% following subsequent improvements to the overall system [19,26].

An optical based terminal guidance experiment was conducted by Naval Command, Control and Ocean Surveillance Centre using the Odyssey IIB AUV equipped with a camera on the nose and a DS with a mounted light [49]. A pure-pursuit guidance system directed the vehicle at a constant advance speed of 1–1.5 m/s towards to the light source. Decoupled horizontal and vertical PID controllers were used for depth and heading control. This experiment reported a 20–28 m acquisition range with a positioning accuracy of the order of 1 cm given the prevailing water turbidity condition. The effect of current disturbances was not considered, as current disturbances were negligible in the vicinity of the DS. The main drawback with the proposed approach was

Table 3

Performance comparison of navigation aids with application to AUV docking.

Modality	Advantages	Limitations	Versatility
Acoustic	<ul style="list-style-type: none"> Acquisition range in order of kilometres (up to 2 km); High resolution; Insensitive to oceanographic conditions. 	<ul style="list-style-type: none"> Susceptible to acoustic refractions, sound reflection reverberation, and ambient noise near sea-floor and sea surface; Latency due to physics of sound in underwater medium; Amplifying range and direction errors proportional to distance. 	<ul style="list-style-type: none"> USBL, SBL, acoustic imaging; Suitable for use with any type of DS.
Optical	<ul style="list-style-type: none"> Relatively high update rate (10 Hz); High lateral and temporal accuracy (2~10 cm); Simple implementation. 	<ul style="list-style-type: none"> Limited acquisition range (10~30 m); Susceptible to signal attenuation due to scattering and absorption of light caused by water turbidity; Susceptible to interference by other light sources. 	<ul style="list-style-type: none"> Active and custom-configured lighting systems; Camera and photodiode sensors; Suitable for use with a funnel type DS. Suitable for use with a funnel type DS.
Electromagnetic	<ul style="list-style-type: none"> High accuracy in order of centimetre (around 20 cm); Robust to almost all oceanographic conditions. 	<ul style="list-style-type: none"> Must operate at low frequencies; Relatively short acquisition range (25~30 m); Difficulties associated with size and implementation. 	<ul style="list-style-type: none"> Suitable for use with a funnel type DS.

Table 4

Classification of AUV guidance systems.

Guidance systems	
Local guidance laws	Global guidance laws
(1) Point-to-point methods <ul style="list-style-type: none"> LOS [30] Pure pursuit [33] Linear terminal [31] Proportional navigation [40,41] Artificial potential fields [42–44] 	(2) Graph search methods <ul style="list-style-type: none"> A* [45,46] Dijkstra [45,46] (3) Optimization-based methods <ul style="list-style-type: none"> Evolutionary algorithms (GA, PSO, etc.) [34–37] Indirect/direct method [38,39]

the direct impact of the water turbidity on the range variations of the docking operation. In addition, the proposed approach was susceptible to sunlight in shallow waters.

An electromagnetic navigation terminal guidance experiment was conducted by Monterey Bay Aquarium Research Institute (MBARI) using a funnel-shaped DS and an Odyssey IIB AUV [32]. In this experiment, the coils installed on the DS emitted horizontal and vertical magnetic fields while corresponding receiver coils on the vehicle detected the magnetic fields. A pure pursuit guidance system and a decoupled PID control loop directed and controlled the vehicle manoeuvre to the DS, respectively. The experiment was conducted in such a way that the AUV performed a 60 s straight manoeuvre out from the DS followed by a 180° turn back on the outbound path in order to point back into the DS centreline. The AUV continued to travel towards the DS using dead reckoning until it reached the active range of the magnetic fields. An accuracy of up to 20 cm from the DS centreline was achieved using this approach. The operating range of the electromagnetic system was, however, limited to 25–30 m. The experiment also reported that the vehicle was unable to successfully dock when there was more than 30° discrepancy in the AUV alignment with the DS centreline.

Unlike the previous unidirectional docking experiments, an omnidirectional docking trial was conducted by WHOI using a pole-shaped DS [24]. The authors of this study claimed that their approach could be tailored for both homing and docking stages using simple LOS guidance laws. The LOS guidance system worked on the principle of nullifying the bearing to the DS calculated from the azimuth and elevation angles obtained using a USBL navigation system. An inner–outer loop PID was used to enable the AUV to track the reference heading angle. The authors discussed the versatility of their approach as the homing stage could be performed independent of initial bearing of the DS to the vehicle. To this end, they used a technique to distinguish miss-docking occasions so that it could provide a chance for the AUV to retry the homing with a higher possibility of success. A layer-based hierarchical control architecture using

a high-level finite-state machine (FSM) model was employed to monitor and supervise the autonomous docking process. Although the omnidirectional AUV docking proposal was promising, its major disadvantage was the placement of the mechanical latching mechanism in the AUV nose for passive latching with the pole, such a location is typically used to accommodate forward looking sensors such as cameras and sonar.

MBARI conducted a docking experiment using a Dorado/Bluefin 21" AUV and a funnel-shaped DS in an open sea environment [15]. The docking approach in this experiment encapsulated both homing and docking stages in consecutive steps as follows: (i) locate and home to the DS; (ii) compute a position fix; (iii) travel to start of the final approach path; (iv) execute final approach; and (v) latching on. The Dorado AUV was equipped a USBL and Doppler velocity logger (DVL) as the navigation sensors and a pure pursuit law for the guidance system in the homing stage. A path following scheme was adopted for the docking stage in which a PID controller was tasked with keeping the vehicle travelling with a constant speed of 1 m/s along the centreline of the DS by minimizing the cross-track between the reference and actual heading angles. Neither the pure pursuit nor the path following guidance systems proved effective at compensating for current disturbance, instead the DS direction was set so as to minimize the impact of this disturbance.

A control solution for AUV homing and docking using a long baseline (LBL) navigation sensor was proposed by the University of Southampton, in [50,51]. The guidance system was based on the conventional artificial potential field (APF) method for path generation, a LOS guidance law for computing the reference heading, and a decoupled sliding mode controller (SMC) for the vehicle heading and depth control. A valid reduced-dynamic model of the AUV that assumed passive control of the roll direction was used in this study. The effectiveness of the docking solution was only ever tested in MATLAB simulation.

An evaluation of a visual navigation-based docking system was conducted by the Korea Advanced Institute of Science and Technology (KAIST) to direct the ISiMI AUV during its terminal

docking phase into a static funnel-shaped DS equipped with five lights [43]. A pure-pursuit guidance system was used to generate a reference heading angle and a decoupled PID control loop was employed to track the reference heading and depth parameters. Compensation for current disturbances was not considered in this study. This work was later expanded by introducing a modified linear terminal guidance system that applied a sideslip-angle for the AUV when approaching the DS in the presence of ocean currents [31,52–54]. In these studies, the authors assumed a priori known pose of the DS and they further used both known uniform distributed current disturbances and time-varying current fields monitored by an ocean current observer. The chief philosophy in these studies was to generate an intentional cross-track error to compensate for the impact of current disturbances and then to compute the reference heading angle enabling the vehicle to align itself with the DS centreline in the terminal phase of the docking process.

Ifremer and DCNS jointly developed a docking solution for the Asterx AUV, a 4.5m, 800 kg and 2800m depth rated vehicle to dock with a moving underwater vessel borne DS travelling at a speed of 2 knots [55]. The AUV was guided towards the funnel-shaped DS by active acoustic positioning until it was in optical range of the DS whereupon it then homed into the DS by recognizing LED patterns mounted on the DS. This work has since been enhanced by Naval Group (formerly DCNS) in combination with RTSys to develop a submarine-based launch and recovery system, complete with integrated induction charging and Wi-Fi data communication system and an acoustic terminal guidance system in place of the optical guidance system [56].

A two-step docking approach based on the integration of guidance and control strategies for an intervention type AUV was developed by University of Lisbon in [57]. The control law for the first step was derived by assuming an underactuated dynamic model for the vehicle operating at relatively high speed. It provided the AUV with a steering manoeuvre that would take it toward the final docking path with zero convergence for the errors associated with almost all initial conditions. In the second step, the vehicle moved at a lower speed for final approach into the DS, and a fully-actuated dynamic model was assumed to achieve accurate docking performance. In this step, an adaptive control law was proposed to account for the uncertainty in the hydrodynamic parameters and to guarantee desired accuracy for the vehicle docking profile. This two-step strategy was developed for the two-dimensional (2D) case in the horizontal plane without considering the impact of current disturbances. The authors however claimed the potential for the proposed approach to be extended to accommodate 3D docking operations.

A three-layer hierarchical control architecture was developed by University of Porto to enable docking of an AUV with a mobile DS mounted on an ROV [58]. The higher-level control layer employed hybrid automata to supervise the whole docking operation. This included interacting discrete events and continuous states represented by nonlinear ordinary differential equations. The supervisor layer was responsible for switching between manoeuvres while the middle layer was responsible for generating the necessary reference state, path following, and the round-about manoeuvre. Potential fields were also used for obstacle avoidance system. The lower level controller used sliding mode theory to support reference tracking in the presence of parametric uncertainties and external disturbances. The effectiveness of the proposed controller has thus far only been demonstrated using a MATLAB simulator.

More recently, a control law based on range-only measurements was developed by the University of Porto for robust homing of a MARES AUV to a beacon [59]. A guidance law developed from Lyapunov theory, rather than computationally costly state-estimation techniques, was derived to direct the AUV close in

toward the beacon without requiring initialization. The proposed approach was intended to minimize both computational complexity and sensor/equipment requirements. The results of the homing experiment conducted in the Douro River demonstrated the asymptotical convergence of the vehicle to the reference beacon.

The Lyapunov stability theory was similarly utilized by Harbin Engineering University to design a docking guidance controller [60]. The proposed guidance system, which used a visual positioning system, showed capability during pool testing to generate a reference heading and crabbing angle to compensate for horizontal and vertical deviation. The objective was to align the vehicle with the centreline of the DS during the terminal phase of the docking process. The explicit impact of current disturbances was not considered in this study. The pool test experiments achieved 80% successful docking rate.

A guidance system for docking a hybrid underwater glider with a fixed funnel-shaped DS was presented by Zhejiang University in [61]. The DS in this study was an active structure with actuators that enabled the DS to change orientation to accommodate vehicle approach directions. Additionally, the DS was equipped with an acoustic Doppler current profiler (ADCP) to measure water currents around the DS so that they may be communicated to the AUV and used by the vehicle's guidance system to compensate for the disturbance. The AUV was equipped with a USBL and a visual navigation system. A modified pure pursuit guidance with current compensation and tailored for the terminal phase of docking process was designed. The guidance system was responsible for directing the vehicle to the dock position as accurately as possible while the DS took care of orientating itself to face the AUV during the terminal docking stage. The experiment was conducted in a swimming pool and the results indicated the feasibility and effectiveness of the proposed cooperative docking approach.

A hybrid docking guidance system was developed by Norwegian University of Science and Technology (NTNU) to enable an AUV to dock with a funnel-shaped DS in the presence of cross-currents without applying a crab angle [62]. In this study, the terminal docking phase is divided into approaching and sliding paths. During the approaching path, the vehicle was supported by an integral line-of-sight guidance (ILOS) law to follow a straight path parallel to the DS centreline. Once the vehicle reached the sliding path, a combined ILOS and speed regulated guidance (SRG) law directed the vehicle to the DS entrance without applying a crab angle. The guidance system was derived based on a simple AUV kinematic model in the horizontal plane. It provided a vehicle heading parallel to the DS entrance together with desired surge speed for the arrival to the DS, in the presence of cross-currents.

Kongsberg Hydroid conducted a docking experiment using a REMUS-100 AUV and a mobile funnel-shaped DS towed behind a surface vessel [63]. The DS was fitted with depressor wings to stabilize its motion through the water. A four-step approach comprising detection and tracking, interception, docking, and undocking was utilized in this experiment. A digital USBL (D-USBL) navigation system, that calculates the rendezvous location and intercept course with LOS correction, was used to develop a 2D guidance system (based on prescribed depth) for approaching and latching with the mobile DS. During docking, the AUV positioned itself approximately 200 m behind the DS and aligned with the DS central axis based on horizontal angular measurement provided by the DUSBL system. Docking was tested with the AUV moving with the water current, opposite the water current, and across the water current. During trials the vehicle was given a maximum of ten attempts for each mission to achieve docking and managed to successfully dock itself within the first two attempts over 77 percent of the 11 tests conducted.

As an extension of the docking system reported in [63], Hydroid developed a Line Capture Line Recovery (LCLR) docking system [64] for the Remus 600. The vehicle is fitted with a D-USBL and a pair of retractable whiskers on the nose. The vehicle homes in on an acoustic transponder attached to a vertically suspended cable. As the vehicle approaches, it deploys its whiskers to increase the aperture for capturing the cable while simultaneously opening a latch mechanism located between the whiskers. Once the cable is intercepted, the latch hooks on securing the AUV. The LCLR system can be used in both stationary or towed deployment from a vessel, although recovery may be affected by seastate condition. Once captured, the AUV can be pulled down into a DS in the case of an undersea installation or hauled up onto the stern ramp of a moving vessel. A commercial version of the LCLR system, Sea Launcher, is available from Hydroid.

Florida Atlantic University demonstrated an alternative line capture system for a REMUS-100 AUV deployed from a catamaran style USV [65]. In this demonstration, the line was held taut by a depressor wing fitted with an acoustic transponder. The USV was responsible for communicating its position and pose to the AUV at regular intervals. D-USBL navigation and fuzzy logic guidance/control systems were then utilized to locate the vertical line. The results of sea trials demonstrated that under protected conditions three out of eight recovery attempts were successful and therefore the proposed taut cable can be a useful alternative to a mobile DS approach.

LOON-DOCK project, an extension of the Universitat de Girona's SUNRISE FP7 project, focused on the development of a docking system that enables remote deployment of survey missions through a web-based interface [66,67]. A two-stage navigation system based on acoustic range-only measurement and visual beacon detection was developed to enable a Sparus II AUV to localize and detect the pose of a DS. A 2D-LOS guidance system with two proportional heading and velocity controllers was used to direct the Sparus II AUV into the DS. Experiments conducted both in a water tank and at sea demonstrated that the docking guidance system effective under minimal current disturbance conditions. The presence of moderate and strong current disturbances, however, made docking difficult, especially when the DS heading was not aligned with the current direction. The visual navigation system was also found to be vulnerable to water clarity which made docking difficult under turbid conditions.

3.2. Challenges involved in autonomous docking

The above review of the literature related to autonomous docking reveals the limitations of existing autonomous docking approaches in both range and operating conditions. Most experiments assume docking operations in close vicinity to the DS with constraints associated with the vehicle kinematics and DS geometry, and minimal presence of current disturbance and obstacles. Table 5 shows an example list of docking trials reported in the above literature review together with the respective guidance systems employed — the role of most of these guidance systems is to try to minimize the drift and miss distance during the terminal phase. These trials assume the following simplifying characteristics:

- constant AUV forward velocity and acceleration
- fixed DS position and orientation
- applicability for the final docking stage, i.e., terminal phase
- the docking trajectory always commences from some point in front of the DS
- the presence of currents is not considered or considered to be negligible
- the controller is not required to implement smooth and collision-free arrival into the DS cone

- energy/time expenditure is not taken into consideration for generating the docking trajectory

The level of AUV's autonomy for AUV missions and more specifically for autonomous underwater docking operations mainly depends on the performance of the navigation, guidance and control systems. While, the navigation and control systems are sufficiently general-purpose to be used throughout the AUV mission, not just for docking operations, the docking guidance systems requires additional capabilities not necessary for other mission tasks. Based on the demands of the docking process, the design of guidance system can comprise diverse levels of sophistication. These levels should address requisites associated with reliable and efficient docking operations.

These guidance systems are relatively limited in that they are based only on the AUV's kinematics and can only operate reliably in a controlled environment with negligible current disturbances. Consequently, these systems cannot provide a closed-form solution assuring collision-free unsaturated-control motion. The only mechanisms available for satisfying the terminal conditions comprise limiting the final speed and acceleration, and the time of arrival. While these approaches might be useful in the ultimate stage of a docking operation when an AUV is within reach of and aligned with the DS, the process of the AUV arriving at this point should use a more elaborate guidance system.

The above analysis defines the need for a systematic and universal docking guidance framework with capabilities to generate feasible and tractable trajectories providing a broad range of manoeuvrability for the AUV to travel into the DS considering different constraints, while minimizing energy expenditure/operation time and allowing capacity to re-plan (update trajectory) on-the-fly when operating environment or objectives change. In practice, however, this level of autonomy has not thus far been used for docking AUVs [25].

4. Optimal-control-theory-based docking guidance

Although there have been a series of studies reported in the literature on the subject of underwater vehicle docking, the use of optimal control methods has thus far been relatively rare. Compared to the aerospace research realm, optimal control theory is still a very underdeveloped tool in relation to underwater vehicle guidance applications. For instance, in [68] time and energy efficient trajectory planning and collision avoidance using optimal control framework is developed and a numerical solution is provided by the nonlinear programming approach. In [38], an efficient trajectory is generated using a control strategy that minimizes the energy consumption of an underwater vehicle along a desired path. The limitations of the vehicle's thrusters are taken into account to ensure implementable trajectories on a real vehicle. The resulting optimal solution is achieved by significant reduction of multiple rapid switching of the thrusters. In [39], an analytical time-optimal trajectory solution for depth control is achieved in a closed-form using explicit second-order differential equations of depth motion. Depending on the robust behaviour of the tracking controller, the generated trajectory can provide the shortest manoeuvring time for the underwater vehicle. This solution, in essence, is suitable for manoeuvring the vehicle over a relatively short distance in an operating environment. In [69], energy minimization trajectory planning for a stable underwater vehicle using analytically derived relationships between energy consumption and number of thrusters is developed and eventually in [70], the optimal control framework is applied to derive the time optimal trajectories for a fully actuated underwater vehicle subject to constraint on input force.

Optimal control theory is indeed an influential tool for design, synthesis and analysis of complex nonlinear systems. It

Table 5

Summary of AUV docking research studies.

Year	AUV type	DS type	Environment	Docking methodology	Limitations
1997–2006	REMUS [19,26,48]	Fixed funnel-shaped DS	Sea	Two-phase docking manoeuvre; USBL navigation aid, path following guidance system, and PID controller; Success rate varies between 21% and 91%.	The DS axis should be aligned with the current; 2D guidance system; Does not consider AUV dynamics, DS geometry, and the impact of current disturbances.
1997	Odyssey IIB AUV [49]	Non-stationary funnel-shaped DS	Sea	Terminal phase docking with constant speed; Optical navigation aid, pure pursuit guidance system and decoupled PID controllers; Docking accuracy in order of 1 cm in the acquisition range.	Applicable in a limited acquisition range and certain operating environments; 2D guidance system; Does not consider AUV dynamics, DS geometry, and the impact of current disturbances.
2001	Odyssey IIB AUV [32]	Fixed funnel-shaped DS	Sea	Two-phase docking manoeuvre; Electromagnetic navigation aid, pure pursuit guidance system, and a decoupled PID control loop; Accuracy up to 20 cm from the DS centreline.	Limited operation range; The guidance system cannot guarantee docking success in more than 30° discrepancy; 2D guidance system; Does not consider AUV dynamics, DS geometry, and the impact of current disturbances.
2001	N/A [24]	Fixed pole-shaped DS	Sea	An omnidirectional docking approach based on miss-docking strategy; USBL navigation aid, LOS guidance system, and inner–outer loop PID controllers; Tailored for both homing and docking stages.	Mechanical complexity in design and installation of the DS; Limited to a particular type of AUV with a mechanical latching tool on the nose of vehicle.
2008	Dorado [15]	Fixed funnel-shaped DS	Sea	A five-stage docking approach; USBL and DVL navigation aids, pure pursuit guidance system, and PID controller.	Applicable in a limited acquisition range in particular homing stage; 2D guidance system; Does not consider AUV dynamics, DS geometry, and the impact of current disturbances.
2007–2008	N/A [50,51]	Fixed funnel-shaped DS	Simulation	Artificial potential field (APF) method for path generation; LBL navigation aid, LOS heading guidance, and decoupled sliding mode controller.	Application requires LBL network; The guidance system is susceptible to local minima; No compensation for current disturbance.
2009–2011	ISiMI [31,52–54]	Fixed funnel-shaped DS	Basin	Vision based docking strategy; Optical navigation aid, pure-pursuit and modified linear terminal guidance systems, and decoupled PID controller; Accuracy of 10–20 cm in the acquisition range.	Applicable in a limited acquisition range in particular homing stage; 2D guidance system and limited to a priori known pose of the DS; Sensitive to water turbidity and sunlight.
2010	Light AUV [58]	Mobile funnel-shaped DS	Simulation	A three-layer hierarchical control docking approach; Depth sensor LBL navigation aids, APF guidance system, sliding mode controller;	Docking limited to the horizontal plane only and a priori known pose of the DS; Docking only possible from front direction; Limited effectiveness in practical operation.
2012	REMUS-100 [63]	Mobile funnel-shaped DS	Sea	A four-step docking approach; D-USBL navigation aid; LOS guidance system; Enables REMUS-100 for recovery in moderate sea conditions and in vicinity of the surface vehicle's propellers.	2D guidance system; Limited to fixed and prescribed depth operation; Performance limited by sea state conditions and current disturbance; Guidance system limited to terminal phase of docking;
2014 –2015	Ifremer Asterx AUV [55,56]	Mobile funnel-shaped DS	Sea	Combined USBL and vision-based docking strategy; Integrated induction charging and Wi-Fi data communication system; LOS and linear terminal guidance systems; Tolerance analysis algorithm to determining the dimensions of the DS entrance cone; Risk analysis and onboard exception handling design to secure the critical dive phase.	Mechanical complexity and non-general-purpose DS structure; Limited to a particular type of AUV (large-scale); Docking only possible from front direction; Applicable in a limited acquisition range of optical systems, in particular homing stage.

(continued on next page)

Table 5 (continued).

Year	AUV type	DS type	Environment	Docking methodology	Limitations
2015	Intervention type AUV [57]	Fixed DS	Simulation	A two-step docking approach; USBL navigation aid, adaptive integrated guidance-control systems; Good accuracy in the presence of sensor noise, uncertainty in the hydrodynamic damping parameters and control saturation.	Developed for the horizontal plane (2D); Does not take into consideration the impact of current disturbances.
2015	MARES AUV [59]	N/A	River	Control law based on range-only measurement homing approach; Acoustic beacon-transducer navigation aids, LOS guidance system, Lyapunov-based control systems; The homing strategy does not need to solve localization problem and it guarantees asymptotical convergence of the vehicle to the reference beacon.	2D guidance system; The homing strategy is limited to constant forward speed of the vehicle; Realistic constraints on the actuators are not considered in derivation of homing law; The homing law is limited to a small ratio of current velocity to the vehicle forward velocity.
2015	WL-3 AUV [60]	Fixed funnel-shaped DS	Pool	A two-layer control system that accommodates under-actuated features of AUV; Visual positioning and dead reckoning navigation aids, Lyapunov-based guidance system, PID controllers; Success rate up to 80%.	Has limited acquisition range, applicable to terminal stage; Relatively long computational delay needed for vision positioning; Sensitive to water turbidity and sunlight; Does not consider the explicit impact of current disturbances.
2016	Hybrid underwater glider [61]	Active stationary funnel-shaped DS	Pool	Active DS with adjustable orientation for final vehicle approach from any angle; USBL and camera navigation aids, pure pursuit guidance system with current compensation, heading and depth controllers.	Requires special mechanical design, and infrastructure for the active DS; 2D guidance system; Limited to terminal docking phase.
2016	Torpedo-shaped AUV [62]	Fixed funnel-shaped DS	Simulation	A hybrid 2-stage docking approach; USBL and DVL navigation aids, ILOS and SRG guidance systems, feedback linearizing controller; Enables vehicle to dock in presence of cross-current disturbances.	2D guidance system; Limited to terminal phase of docking; Does not consider AUV dynamic model; Less accurate in presence of cross-currents with magnitude > 0.2 m/s.
2016	REMUS-600 [64]	Line Capture	Sea	Uses a horizontal D-USBL to home in on a vertically suspended line; Line capture system offers a lower drag alternative to a towed conical DS – better suited to L&R from a small vessel;	Requires mechanical latching system and capture whiskers on AUV nose; L&R limited by sea state and current conditions;
2017–2018	REMUS-100 [65]	Line Capture	Sea	A two-step L&R operation.; USBL navigation aid; Fuzzy logic guidance/control system; OrcaFlex modelling supports the feasibility of the operations; Depressor wing fitted at end of line enables capture line to be held taut.	Requires mechanical latching system and capture whiskers on AUV nose; L&R limited by sea state and current conditions; Controller performance is dependent on true vehicle heading identification; The complexity of design and multiplicity of modules make control and diagnosis process hard.
2017–2018	Sparus II AUV [66,67]	Funnel-shaped DS	Water tank and Sea	A two-step docking approach; Web-based interface service for remotely program docking missions and retrieving the collected data; Cost-effective acoustic and visual navigation aids; LOS guidance system, proportional heading and velocity controllers;	DS structure is not universal and only works for Sparus II AUV; 2D guidance system; Vulnerable to water turbidity; Limited docking operations in presence of cross currents; No practical mechanism for compensating cross-track (due to the current disturbance) and AUV misalignment with the DS centre.

is able to provide a universal framework for developing a reliable and efficient trajectory optimization (generation) engine or equivalently guidance system. The goal of optimal control theory in general is to optimize, either minimize or maximize, a specified performance index (or cost/objective function) which is subjected to a series of constraints such as system dynamics, boundary conditions (endpoint constraints), path constraints, box constraints (including bounds on states and controls), and linkage constraints (known as phase continuity constraints). In

the context of trajectory optimization (generation), it is desired to generate a trajectory (solution) enabling a system (in our context an AUV) to transverse from the starting point to the destination point while all physical (vehicular) and environmental constraints are taken into account and a specified performance index (time, energy, time–energy, etc.) is minimized. However, obtaining an analytical optimal solution for most aerospace and underwater guidance problems is typically very difficult if not

impossible. Instead, an optimal control problem (OCP) is usually solved numerically.

Indirect methods, offer numerical solutions for an OCP, using the calculus of variations and Pontryagin's maximum principle (PMP) to construct the first-order optimality conditions and then convert an original OCP into a Hamiltonian boundary-value problem (HBVP) [71]. The term *indirect* stems from the fact that numerical solutions are indirectly obtained by solving the equivalent HBVP of an OCP. Indirect methods are able to provide the most accurate solutions for an OCP while ensuring the first-order optimality conditions are met. Indirect shooting and collocation methods are the most common branches of indirect methods. In the indirect shooting method, initial guesses are provided for the unknown boundary conditions at one end of the interval of the underlying OCP. Using the proposed guesses together with the known initial conditions, the HBVP is integrated either in a forward or backward manner to the other end of the interval. The resulting terminal conditions are then compared with the known terminal conditions, which are derived from first-order necessary conditions, and the difference with respect to the accuracy tolerance is calculated. In the case that the obtained terminal conditions are not within the maximum tolerance, the unknown initial conditions need to be adjusted and the integration process repeated until the desired tolerance accuracy is met.

The indirect shooting method has shown considerable numerical difficulties in dealing with hypersensitive OCPs mainly due to the ill-conditioning of the Hamiltonian dynamics [72–74]. This ill-conditioning stems from the fact that the divergence of the flow of trajectories should be zero and by performing the integration process in either direction of time, errors due to the unknown boundary conditions are amplified [69]. To alleviate these difficulties, the indirect multiple-shooting method has been developed [75].

The indirect collocation method is another branch of the indirect method family in which an OCP is solved based on the state and control parametrization techniques. For this purpose, piecewise polynomials are usually employed and the OCP is transcribed into a root-finding problem in which the unknown coefficients of the polynomials are decision variables. Then using an appropriate root-finding technique such as Newton's method, the new system is solved. The indirect collocation method is suitable to be applied to multipoint boundary value problems, for instance simple trajectory optimization problems. However, like the previous indirect methods, they cannot solely be used without solving the costate differential equations.

In essence, while indirect methods provide the most accurate numerical solution for OCPs, they are more often than not impractical for dealing with constrained OCP. Firstly, analytical derivations of cost function and constraints are required to derive a Hamiltonian Boundary Value Problem (HBVP), that is at the heart of indirect methods. This process is highly cumbersome and tedious for complicated problems. Secondly, finding an accurate solution for a constrained HBVP is not an easy task and is associated with a series of practical difficulties. For instance, the initial guess of co-state variables must be properly provided to start the iterative methods. With respect to the non-physical nature of co-states, it is cumbersome to determine a suitable initial guess and an improper one can result in a non-converged or non-optimal solution. There is also a possibility that even relying on a reasonable guess, the adjoint equations may turn out to be an ill-conditioned case. Finally, an insight into the switching functions of problems with active path constraints is required [76]. Therefore, direct methods are used to circumvent these difficulties.

In the subsequent section, the principles as well as the advantages and disadvantages of direct methods are presented to obtain insight and ultimately show the rationale for designing and

developing a systematic and universal docking guidance framework using direct methods. This approach is to the best of the authors' knowledge totally novel in relation to autonomous underwater docking, and there is no other similar work.

5. Numerical guidance solutions using direct methods

Direct methods, are essentially, different in nature to indirect methods as they transcribe the original OCP into a nonlinear problem (NLP) and then directly approximate states and/or controls in an appropriate manner. The proposed NLP can be solved either by gradient-based techniques such as sequential quadratic programming or by evolutionary-based techniques, for instance, genetic and simulated annealing algorithms [71].

In general, direct methods are categorized into two major branches; the first one is called *control parametrization method* in which controls are parametrized using specific functions and the state equations are kept untouched. In the second branch, called *control-state parametrization method*, both states and controls are parametrized simultaneously based on particular basis functions in which the unknown coefficients are considered as NLP variables. In what follows, the direct methods most applicable for AUV trajectory optimization problems are reviewed. These methods are summarized in Fig. 6.

5.1. Direct shooting methods

The direct shooting method is a control parametrization method and is established upon an implicit integration of a trajectory optimization problem. In this method, controls are computed based on piecewise, linear, or polynomial approximations with unknown coefficients in a series of small time intervals. The states with unknown initial values are also treated as optimization variables. The constraints associated with system dynamics are satisfied by using time-marching integration algorithms. The cost-function is approximated using a quadrature rule corresponding to the numerical integrator applied for system dynamic constraints. In other words, the NLP arising from the direct shooting method optimizes the cost function taking into consideration possible path and interior-point constraints. By using an iterative optimization process, solutions are obtained for the optimization variables (NLP variables) related to the controls and the initial values of states that are not a priori known.

Unlike the indirect shooting method, the direct shooting method does not require extraction and implementation of the analytical process of the first-order necessary conditions and co-states definition. This advantage nominates the direct shooting method as an easy-to-use candidate for solving OCPs. This feature further facilitates software realizations of this method using the following tools: Program to Optimize Simulated Trajectories (POST) [77] and Generalized Trajectory Simulation (GTS) [78]. The direct shooting method, however, suffers two major drawbacks; the first is the need for implicit integration to evaluate the gradient of NLP variables in each iteration – this imposes a high computational cost; and the second are the practical difficulties in solving a large NLP with particularly sensitivity to unknown initial conditions.

The direct multiple-shooting method is employed to alleviate the sensitivity issue of the direct shooting method to unknown initial conditions [71]. In this method, in a manner similar to that of indirect multiple-shooting method, the time interval $[t_0, t_f]$ is divided into several subintervals, and as a result a more detailed NLP is produced. In this regard, the NLP variables comprise the unknown coefficients in the control parametrization and the values of the states at the beginning of each time subinterval. While this formulation yields a much larger NLP compared to

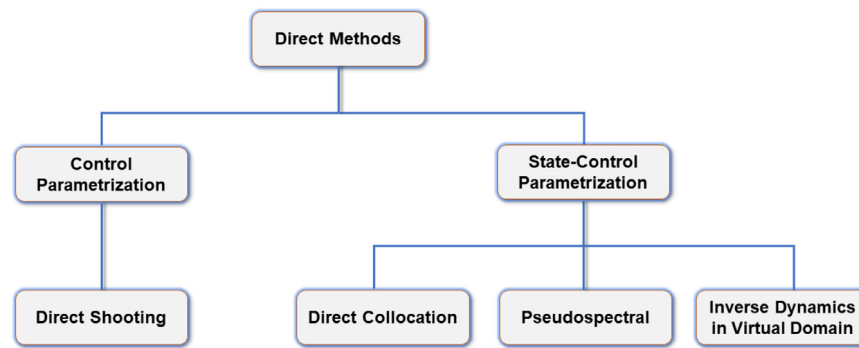


Fig. 6. Classification of Direct methods.

that of a simple direct shooting method, it has a direct impact on mitigating the sensitivity issue. This is because the integration is applied over smaller intervals as opposed to one large interval as in the direct shooting method.

5.2. Direct collocation methods

Direct collocation methods are state and control parametrization techniques in which states and controls are approximated and represented within by a discrete number of variables [79, 80]. By employing direct collocation methods, differential/integral constraints of a system are transformed into algebraic constraints such that an infinite continuous OCP is transcribed into a finite NLP. In this approach, the polynomial coefficients are treated as NLP variables and the proposed NLP can be solved using efficient and commonly used NLP solvers such as Sparse Nonlinear Optimizer (SNOPT) [81], Sparse Nonlinear Programming (SPRNL) [82], Nonlinear Interior-point Trust Region Optimization (KNITRO) [83], and Interior Point Optimizer (IPOPT) [84].

The direct collocation method is more commonly referred to as local collocation in the literature as it has local support along with numerical robustness to the initial guess but with a low algebraic convergence rate as opposed to global collocation methods with spectral accuracy. When local collocation methods are used to solve an OCP they result in a very large NLP problem.

To accommodate this difficulty in real-time trajectory optimization applications [71,80,85], differential flatness theory can be used to significantly reduce the NLP size that results from direct collocation methods [86].

The salient feature of differential flatness is to map a trajectory generation problem into a space, called flat space, that conserves the nonlinear dynamic characteristics of systems and allows representation of system's states and controls in terms of functions of flat outputs and their derivatives. By using this property, the constraints associated with the system dynamics are completely removed or alleviated to a great extent in the case of partially flat systems. As a result, the dimensions of the optimization problem are significantly reduced as only a few number of variables are participants, and consequently real-time computation becomes possible. As such, in recent years, more attention has been devoted to this theory for trajectory generation of nonlinear systems [87–104].

In real dynamic systems, particularly unmanned vehicles, determining flat outputs, is typically based on a trial-and-error process. The spatial coordinates are usually good candidates for flat outputs as the rest of the states and control variables can be computed from a vehicle's physical path. In effect, when using the concept of differential flatness there is no need to develop a specific algorithm to determine the flat outputs as relying on spatial coordinates will in most cases suffice.

5.3. Pseudospectral methods

Pseudospectral (PS) methods were initially employed for solving computational fluid dynamic (CFD) problems [105,106]. Since 1990, however, PS methods have been used to solve a wide range of optimal control problems [107,108]. Although, a plethora of PS methods have been used to produce trajectory optimization solutions for aerospace applications [109–113], these methods have seldom been employed for underwater trajectory optimization problems [114–117].

The fundamental of PS methods is that the states and controls can be approximated using a finite basis of an orthogonal polynomial, like the Chebyshev or the Lagrange polynomial, at a set of discretization points. These points, called collocation points, are normally selected as roots of the of Legendre polynomials (called Legendre–Gauss or LG nodes), roots of Chebyshev polynomials (called Chebyshev–Gauss or CG nodes), extrema of Legendre polynomials (called Legendre–Gauss–Lobatto or LGL nodes), extrema of Chebyshev polynomials (called Chebyshev–Gauss–Lobatto or CGL nodes), and roots of linear combinations of Legendre polynomials (called Legendre–Gauss Radau or LGR nodes). Unlike local collocation methods which use equally-spaced nodes, the PS methods take advantage of unequally-spaced distributions of orthogonal nodes (global collocation) in the time domain. This property provides higher accuracy of interpolation functions and rely on a fewer number of nodes, as opposed to local collocation methods.

The following approach is adopted for PS methods to transcribe an OCP into an equivalent NLP. The defect constraints, for approximating the dynamics of the system, are defined by taking derivatives of the interpolant polynomial and then setting them to be equal to the right-hand-side dynamic equations at all or a sub-set of collocation points. The path and boundary constraints are imposed on the intermediate and terminal collocation points, respectively. Finally, the integral cost function is approximated by means of a highly accurate Gaussian quadrature rule [71,85,111]. For mainstream PS methods such as Gauss, Legendre, Chebyshev and Radau, there exists a viable transformation between the Karush–Kuhn–Tucker (KKT) conditions of NLP and the continuous differential equations of co-states. Unlike indirect methods which have practical difficulties in terms of estimating co-states trajectories, when as in most cases a good initial guess of the co-states is not available, PS methods have the advantage that they are able to efficiently provide co-states information [71,85]. The co-states information enables the post-optimality process which is an investigation of the feasibility/optimality of solutions, to become possible.

Using orthogonal collocation points enables PS methods to show spectral convergence at an exponential rate for a group of smooth OCPs. However, this is not the case when dealing with

non-smooth OCPs, as PS methods exhibit extremely slow convergence rate (due to the high computational burden) even when employing high-degree basis functions, and as a consequence converging to feasible solutions may in some cases not be possible [118–120]. To accommodate these difficulties, and to simultaneously improve accuracy and computational efficiency of PS methods, the authors have developed adaptive mesh refinement approaches [121,122].

5.4. Inverse dynamics in the virtual domain method

The inverse dynamics in the virtual domain (IDVD) method is another branch of direct methods that exploits the concept of differential flatness to significantly reduce the dimension of optimization problem and thus enable fast prototyping of feasible trajectories [123,124]. The trajectory computations occur in the output space instead of the control space. The reference functions are normally combinations of any function such as orthogonal, monomial, and trigonometric. Then, by exploiting the inverse dynamics, state and control vectors are represented as functions of the output. The IDVD method performs optimization in the virtual domain as opposed to the time domain which is advantageous for decoupling space and time parametrizations. Compared to PS methods, the real-time version of the IDVD method does not exploit co-states estimation for generating optimal solutions and therefore trajectories generated are of a near-optimal form [125]. However, unlike other direct methods which use many optimization variables (large NLP size) and thus require extensive computational power, the IDVD method uses relatively few decision parameters (usually less than 10), enabling this method to be implemented in an online form within a closed-loop configuration. Additionally, the IDVD method is also easy to modify and code, offering suitable flexibility to operators to adjust it with respect to mission scenarios.

The effectiveness and computational tractability of the IDVD method have been verified in different realms of research areas. For instance, AUV rendezvous and path planning [104,126], collision-free trajectory planning of quadcopters [127], experimental implementation of the planar manoeuvre of a spacecraft docking with a rotating target [128], real-time trajectory optimization of an unmanned combat aerial vehicle performing agile air-to-surface attack [129], terminal guidance of autonomous parafoils [130], and minimum-time aircraft manoeuvres [131]. It has been mentioned in the literature that the computational speed of the IDVD is more than an order of magnitude faster than for example PS methods, in exchange for a small loss of optimality [119,121]. Thus, the IDVD method would seem to be a suitable technique for continuously updating trajectories online during a mission execution in order to mitigate unforeseen disturbances and unmodelled dynamics, as well as dynamically react to the pop-up threats.

5.5. Closed-loop vs open-loop solutions

More often than not, solutions generated by direct methods are considered as open-loop solutions since they just depend on time but not directly on state variables. To build up a state-dependent guidance law, the concept of online trajectory generation (optimization) should be used. The ultimate goal of online trajectory generation is to provide a set of trajectories for a vehicle during the mission not a priori, unlike offline applications, but based on the situational awareness of the operating environment and on updates of the vehicle's mission objectives. In this regard, the vehicle does not follow a pre-determined trajectory or path but instead continuously computes and tracks an optimized trajectory taking into consideration mission updates. In these

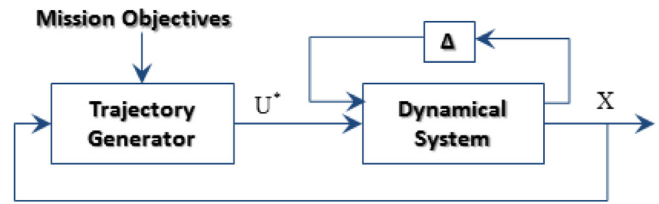


Fig. 7. Closed-loop configuration for online trajectory generation.

circumstances, the optimized trajectory is generated taking into consideration the mathematical representations of the vehicle's model and pre-determined boundary conditions, as well as the impact of uncertainties.

Several instances have been reported in the literature concerning the development of online trajectory generation systems for unmanned aerial and underwater vehicles [132–135]. In these studies, however, the refinement process is undertaken at the path planning level and not in the trajectory generation. In other words, the dynamics of vehicle are not involved in re-planning. Examples of online trajectory generation with a closed-loop implementation, include spacecraft docking as proposed and experimentally validated in [128], and a missile interceptor problem formulated and solved using a combinatory online trajectory optimization scheme in the closed-loop form [136].

A recent and systematic approach to generating optimized trajectories based on the instantaneous conditions of a vehicle involves implementing the trajectory generation process within the control loop [89,136]. In this loop, shown in Fig. 7, the trajectory generation block plays the role of a nonlinear controller which generates optimal control commands (U^*) based on feedback of the vehicle's instantaneous states (X). In this figure, delta (Δ) represents unmodelled dynamics and uncertainties. Ideally, the optimized trajectory should be generated at the same fast rate as the state updates, however, this is impossible in reality given the delay of the nonlinear controller computation time. If the trajectory generation computation time can be significantly reduced, then the optimized trajectory can be achieved based on the instantaneous conditions of vehicle. For realization of the concept of online trajectory generation, two major components should be taken into account

- the trajectory generation computational time
- the closed-loop trajectory generation process

In fact, trajectory generation within a closed-loop configuration is considered as a new control approach, in which controls are generated based on trajectory generation. This approach is similar to what is referred to as model predictive control [137–141].

Today, this approach is more typically known as a receding horizon control (RHC) since the optimal control sequences are generated for future time horizons. The repetitive nature of RHC leads to a state-dependent control law in which the physical limitations on the states and controls are incorporated as hard or soft constraints. In this approach, frequent changes of cost functions and constraints are possible, as the OCP is solved in a sequence of short time intervals [140]. The closed-loop stability in RHC has been discussed in several documented research in which various Lyapunov control functions are employed to stabilize the RHC [140,142].

The early use of RHC has been successfully demonstrated for controlling industrial processes such as chemical processes because of their relatively slow dynamics [143]. During that period, the limited performance of computing platforms initially hampered the implementation of RHC for nonlinear systems with

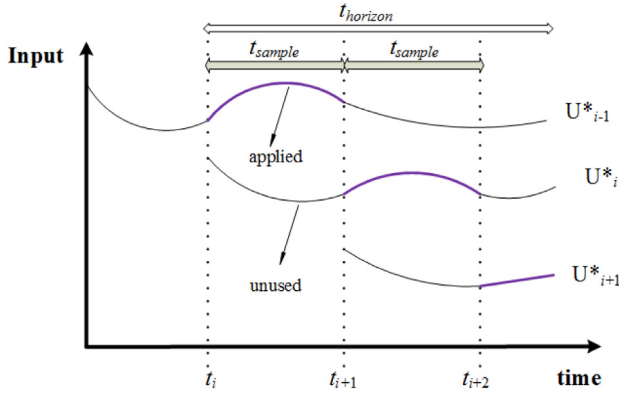


Fig. 8. Online trajectory generation based on the RHC scheme.

fast dynamics [144]. Nowadays, with the advent of affordable and powerful computational tools and also through better understanding of the RHC stability, this approach is once again attracting the attention of the control community. The use of RHC for aerospace applications has been documented in several research publications [136,145,146]. An example of timing schemes for implementing RHC and constructing the control loops based on the trajectory generation (optimization) is shown in Fig. 8 [136,144]. As seen, the control commands (U_{i-1}^* , U_i^* , U_{i+1}^*) are computed for a $t_{horizon}$ time interval, but only a t_{sample} time interval of control commands is utilized.

6. Towards the development of a universal docking guidance framework

As identified in Section 3, it is important for an AUV to have the capacity to robustly perform a range of optimal, safe docking manoeuvres under a variety of operating conditions. This section describes the desired guidance framework based on the IDVD method presented in Section 5. Such a framework comprises transcription of the docking problem into a high-fidelity two-point-boundary-value problem (TPBVP) and employs one of the most prominent direct methods to numerically solve the TPBVP. The consequence of this approach is to form a trajectory generator engine capable of generating trajectories that would enable an AUV to efficiently and reliably dock in a variety of operating conditions. This trajectory generator should

- combine both homing and docking stages together to overcome the range limitation of point-to-point guidance systems
- allow both 2D and 3D docking manoeuvres as opposed to the conventional docking guidance mostly designed for 2D operations
- involve both dynamics and kinematics of the AUV as opposed to the conventional docking guidance approaches
- incorporate the impact of current disturbances, particularly cross-current disturbances
- generate feasible and tractable trajectories for on-board implementation on the vehicle
- enable the vehicle to perform docking operations in a cluttered environment with obstacles and/or NFZs
- satisfy the tolerance accuracies for a safe and successful operation in the terminal phase of docking operations to ensure smooth arrival of the vehicle into the DS entrance
- facilitate minimum-energy, minimum-time, or simultaneous minimum time-energy docking operations

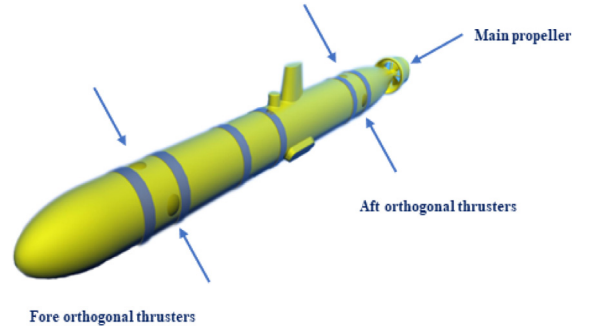


Fig. 9. Flinders AUV.

- assure robustness to within a reasonable bound of uncertainty associated with variations in the AUV pose, DS pose, and current disturbance
- allow a closed-loop online implementation required to handle unknown or dynamic environments or unforeseen changes and uncertainties

The overall performance of a prototype docking guidance framework and particularly the capabilities and main features of the direct methods employed as the trajectory generator engines, have been investigated through a series of representative docking scenarios in operating environments comprised of realistic currents and NFZs with respect to a priori known and unknown poses of the DS [147–150]. In these studies, conducted using a high-fidelity simulation platform, the configuration assumed the provision of a USBL communications beacon on the DS and a USBL transponder array on the AUV. To examine the performance of the guidance system, a representative scenario of an AUV docking with a DS towed behind a ship was defined. Without loss of generality, it was assumed that the AUV executing the docking mission receives a USBL update about the DS pose every t_{update} . The DS pose information is corrupted by both sensor and environmental noises. As the AUV approaches the DS, the impact of uncertainty reduces, and the DS pose information becomes more accurate. To make the problem more challenging a cluttered environment with six NFZs areas was considered.

The trajectory generation is based on a relatively high degree-of-freedom (DoF) model. For example, a torpedo-shaped AUV fitted with orthogonal fore and aft thrusters as illustrated in Fig. 9 would be modelled as a 4-DoF model

$$\begin{aligned}\dot{x} &= u \cos(\psi) + c_x \\ \dot{y} &= u \sin(\psi) + c_y \\ \dot{z} &= w \\ \dot{\psi} &= r\end{aligned}\quad (1)$$

In these equations, x , y , and z are the coordinates of the AUV's centre of gravity in the North-East-Down (NED) coordinate frame; u and w are the surge and heave components of the velocity vector in the body frame; ψ is the yaw angle; c_x and c_y are the components of the current velocity in NED, and r is the yaw rate. As opposed to the most of other existing approaches, the kinematic equations (1) would be complemented with dynamic equations to assure the candidate trajectory feasibility.

$$\begin{aligned}m\dot{u} - (X_u + X_{u|u}|u|)u &= T_u \\ m\dot{w} - (Z_w + Z_{w|w}|w|)w &= T_w \\ I_z\dot{r} - (N_r + N_{r|r}|r|)r &= T_r\end{aligned}\quad (2)$$

In Eq. (2), X_u , Z_w , and N_r are the linear drag terms; $X_{u|u}$, $Z_{w|w}$, and $N_{r|r}$ are the quadratic drag terms; m represents the AUV mass; I_z is the vehicle's inertia around the z axis; and T_u , T_w ,

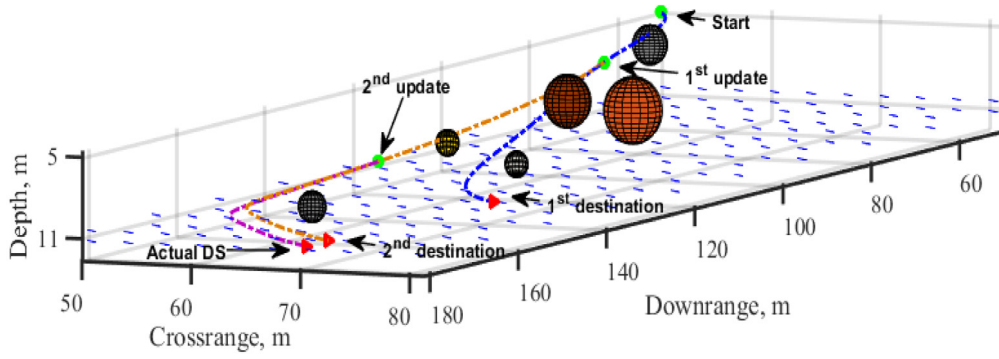


Fig. 10. 3D collision-free path re-optimized based on a better knowledge of the DS location.

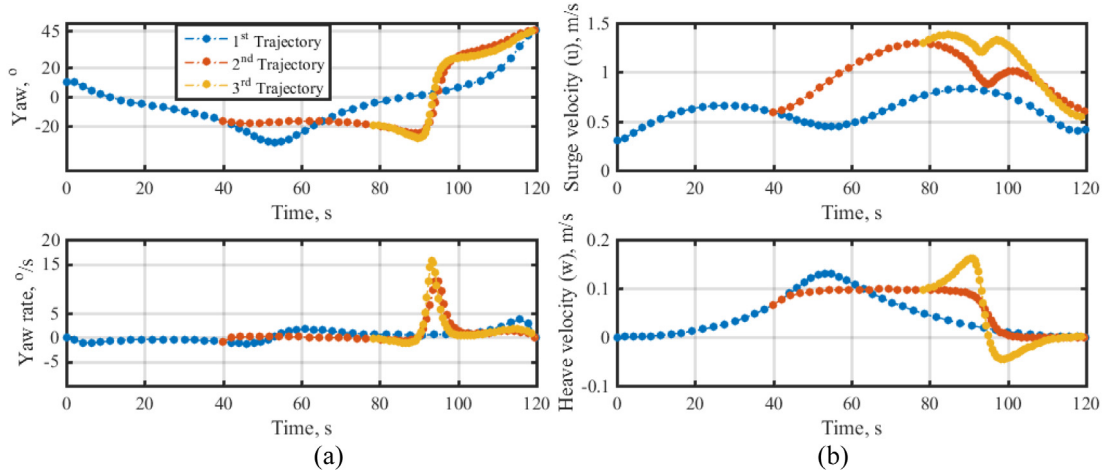


Fig. 11. Refinement of yaw and yaw rate (a); Refinement of surge and heave velocities (b).

and T_r are the control inputs in surge, heave and yaw directions, respectively.

To take into account the practical constraints associated with thrusters, the controls and yaw rate are bounded.

$$|T_u| \leq T_u^{\max}, \quad |T_w| \leq T_w^{\max}, \quad |T_r| \leq T_r^{\max}, \quad |r| \leq r^{\max} \quad (3)$$

To illustrate the universal docking guidance framework, the following presents a simple example of executing one specific mission by the Flinders AUV (Fig. 9) characterized by

$$\begin{aligned} m &= 30.4 \text{ kg} & I_z &= 3.45 \text{ kg/m}^2 \\ X_u &= -13.5 \text{ kg/s} & X_{u|u|} &= -1.62 \text{ kg/m} \\ Z_w &= -66.6 \text{ kg/s} & Z_{w|w|} &= -131 \text{ kg/m} \\ N_r &= -6.87 \text{ N.m.s/rad} & N_{r|r|} &= -94 \text{ kg.m}^2/\text{rad}^2 \\ T_u^{\max} &= T_w^{\max} = 20 \text{ N} & T_r^{\max} &= 15 \text{ Nm} & r^{\max} &= 15^\circ/\text{s} \end{aligned} \quad (4)$$

For this mission, $c_x = c_y = 0.25 \text{ m/s}$ and the initial and final conditions are defined as

$$\begin{aligned} X_0 &= [50 \text{ m}, 50 \text{ m}, 5 \text{ m}, 10^\circ, 0.3 \text{ m/s}, 0 \text{ m/s}, 0^\circ/\text{s}]^T \\ X_f &= [180(1 + \delta(D_r))\text{m}, 70(1 + \delta(D_r))\text{m}, 11(1 + \delta(D_r))\text{m}, \\ &\quad 45^\circ, u_f \text{ m/s}, 0 \text{ m/s}, 0^\circ/\text{s}]^T \end{aligned} \quad (5)$$

where the first three elements of the final state vector are dependent on the range from the true DS position, D_r , with normally distributed uncertainty $\delta(D_r) = N(0, 4.1 \times 10^{-6} D_r^2)$. The final surge u_f is set free to provide more flexibility for the vehicle to use its full range of manoeuvring capability. As a result, for each USBL update, i.e., every t_{update} , the AUV reference trajectory needs to be recomputed to account for the updated $X_f(D_r(t))$. The new trajectory generation uses the current AUV states and controls

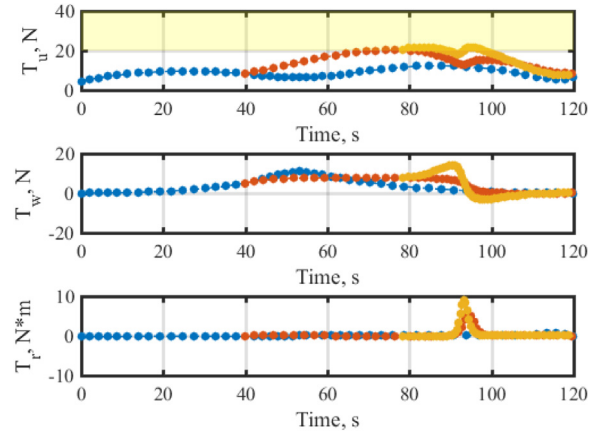


Fig. 12. Time histories of the control inputs for online docking trajectory generation.

as the new initial states; i.e., $X_0 = X(t)$ and $U_0 = U(t)$. In this scenario, the concept of RHC is explicitly used in such a way that it is suitable for the slow dynamic nature of the AUV docking operation. As such, t_{horizon} is set to be equal to t_{update} , and t_{sample} is set to be equal to the termination time of the optimization run. In addition, for this scenario, the docking procedure is required to be performed with minimum control effort (energy minimization) within a fixed operation time $t_f = 120 \text{ s}$. In this scenario, $N = 50$ computational nodes are used for the IDVD initialization.

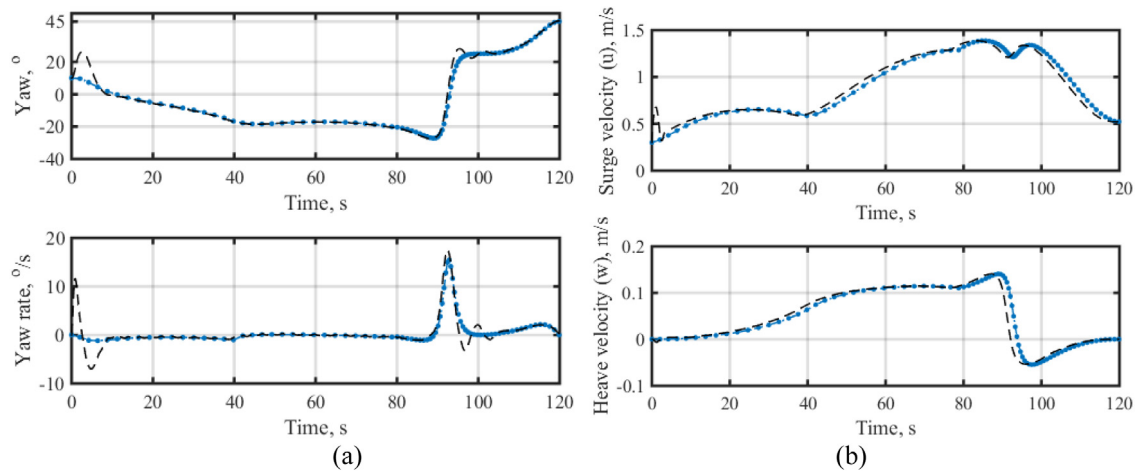


Fig. 13. SITL-based evolution of yaw and yaw rate (a); surge and heave velocities (b).

Figs. 10–14 illustrate the overall performance of the universal docking guidance framework in this test scenario. While executing this scenario, the AUV receives two USBL updates, which in turn trigger the reference trajectory to be updated twice. Fig. 10 presents the generated path in 3D, revealing NFZs and current fields. The three solid circles in Fig. 10 (including the start point) indicate the position along the trajectory at which the DS position is updated while the three triangles show the corresponding perception of the DS position at the corresponding time update. Thus, for example, the guidance system generates the first reference trajectory from the AUV initial position, indicated by the first solid circle and denoted as Start, to the best-known DS position indicated by a green triangle and denoted as 1st destination. With this reference trajectory, the AUV continues its motion to this destination. Forty seconds later, at the AUV position depicted as 1st update, the vehicle receives a new ping from the USBL forcing the guidance system to refine the reference trajectory starting from the current state and leading to the 2nd destination point. The AUV keeps tracking the second reference trajectory until the next USBL ping is received, resulting in the 2nd update. A third reference trajectory is thus generated with respect to the updated estimate of the DS position denoted as the Actual DS in Fig. 10. As seen in Fig. 10, each of the three reference trajectories forces the AUV to manoeuvre around NFZs (represented by spheres in Fig. 10) and this is where the IDVD-approach capability in generating spatial non-singular arc solutions in real-time pays off.

Figs. 11–12 show the time histories of surge and heave velocities, yaw angle and yaw rate, and controls obtained by the guidance system. Fig. 10(a) clearly shows that the AUV's final yaw and yaw rate permit correct alignment with the DS centre-line. Fig. 10(b) shows the capacity of the guidance system to adjust the surge and heave velocities (as opposed to the fixed speed profile in the classic docking guidance systems) while assuring smooth entry into the DS. The control time histories shown in Fig. 11 demonstrate that all controls are within their limits (within pre-set tolerances).

Figs. 13 and 14 compare the corresponding time histories of surge and heave velocities, yaw angle and yaw rate, and controls obtained by the guidance system (indicated by the blue-dotted lines) and their equivalent trajectories (indicated by the black-dashed lines) obtained using the Flinders software-in-the-loop (SITL) Platform, [136]. Figs. 10–14 prove that the developed IDVD-based guidance framework is able to generate a smooth trajectory guaranteeing a safe arrival of an AUV into a DS while meeting all the requirements put forward at the beginning of this section, including fast online generation of traceable trajectories incorporating the AUV's dynamics and kinematics, current disturbances,

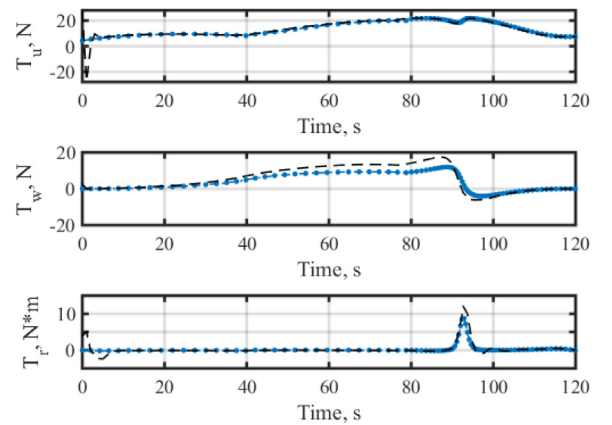


Fig. 14. SIL-based evolution of controls.

NFZs and DS pose uncertainties. In addition, for this particular scenario, the described guidance system allowed saving of about 23% of the AUV battery life for the entire docking operation as compared with a guidance system in which the docking manoeuvre is performed at the control bounds ($T_u(t) = T_u^{max}$). Accordingly, the guidance framework contributes to the cause of improving AUV autonomy by enabling longer mission durations while assuring reliable and cost-efficient docking operations.

7. Conclusion

The paper presented a thorough technical review of existing methodologies and equipment employed by autonomous underwater vehicles to execute an autonomous underwater docking. It also highlighted the challenges in attaining a safe repeatable docking in the complex and variable ocean environments. To address the question of what enabling factors are required to facilitate a robust fully autonomous docking capability, the emphasis is put on investigating the existing docking guidance systems. As a result of a review, the requirements for a general-purpose guidance framework are formulated. It is argued that such a framework should rely on a direct method of calculus of variations allowing satisfying most of the requirements as illustrated by a simple representative scenario of AUV docking.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Further reading

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