Cayley Contest

(Grade 10)

Thursday, February 23, 2012
(in North America and South America)

Friday, February 24, 2012
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

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Time: 60 minutes   ©2011 University of Waterloo
Calculators are permitted
Instructions

1. Do not open the Contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name and city/town in the box in the upper left corner.

5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**

6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.

7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

8. Diagrams are *not* drawn to scale. They are intended as aids only.
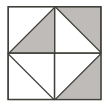
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.
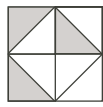
Part A: Each correct answer is worth 5.
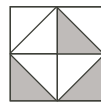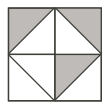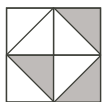
1. The value of $\dfrac{5-2}{2+1}$ is

   (A) 3   **(B)** 1   **(C)** -1 **(D)** -5 **(E)** 5

2. The average of 1, 3 and x is 3. What is the value of x?

   (A) 4   **(B)** 5   **(C)** 2   **(D)** 3   **(E)** 1

3. Which of the following is obtained by rotating the figure to the right clockwise by 90°?

   (A)   **(B)**   **(C)**

   (D)   **(E)**
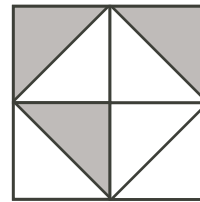
4. The value of $(-1)^3 + (-1)^2 + (-1)$ is

   (A) 2   **(B)** 1   **(C)** -3 **(D)** -1 **(E)** -2

5. If $\sqrt{100-x} = 9$, then x equals

   **(A)** 9   **(B)** 91   **(C)** $\sqrt{19}$ **(D)** 97   **(E)** 19

6. A basket contains 12 apples, 15 bananas and no other fruit. If 3 more bananas are added to the basket, what fraction of the fruit in the basket will be bananas?

   (A) $\frac{2}{5}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{4}{9}$ **(E)** $\frac{5}{9}$

7. The circle graph shows the results of asking 150 students to choose pizza, Thai food, or Greek food. How many students chose Greek food?

   (A) 78   **(B)** 32   **(C)** 48

   (D) 58   **(E)** 63

   Choice of Food

   Pizza 20%

   Thai 38%

   Greek
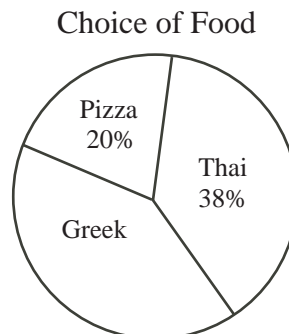
8. The product $\left(1 - \dfrac{1}{3}\right)\left(1 - \dfrac{1}{4}\right)\left(1 - \dfrac{1}{5}\right)$ is equal to

   (A) $\frac{2}{5}$ **(B)** $\frac{1}{60}$ **(C)** 1   **(D)** 3   **(E)** $\frac{59}{60}$

9. A class of 30 students was asked what they did on their winter holiday.
    20 students said that they went skating.
    9 students said that they went skiing.
    Exactly 5 students said that they went skating and went skiing.
    How many students did not go skating and did not go skiing?

    (A) 1   **(B)** 6   **(C)** 11   **(D)** 19   **(E)** 4

10. A solid rectangular prism has dimensions 4 by 2 by 2.
    A 1 by 1 by 1 cube is cut out of the corner creating the
    new solid shown. What is the surface area of the new
    solid?

    (A) 34   **(B)** 37   **(C)** 40

    (D) 15   **(E)** 39
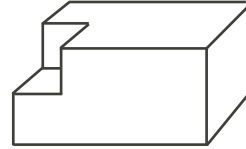

Part B: Each correct answer is worth 6.

11. Matilda has a summer job delivering newspapers.
    She earns $6.00 an hour plus $0.25 per newspaper delivered.
    Matilda delivers 30 newspapers per hour.
    How much money will she earn during a 3 hour shift?

    (A) $40.50   **(B)** $18.75   **(C)** $13.50   **(D)** $25.50   **(E)** $28.50

12. The point (p, q) is on the line $y = \frac{2}{5}x$, as shown. Also,
    the area of the rectangle shown is 90. What is the value
    of p?

   **(A)** 12   **(B)** 9   **(C)** 10

    (D) 15   **(E)** 30
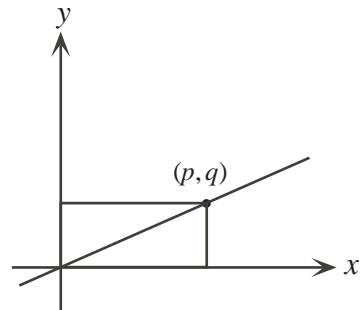
13. There is one odd integer N between 400 and 600 that is divisible by both 5 and 11.
    The sum of the digits of N is

    (A) 11   **(B)** 8   **(C)** 10   **(D)** 16   **(E)** 18

14. In the diagram, $\triangle PQR$ and $\triangle STU$ overlap
    so that $RTQU$ forms a straight line
    segment. What is the value of x?

    (A) 10   **(B)** 20   **(C)** 30

    (D) 40   **(E)** 50
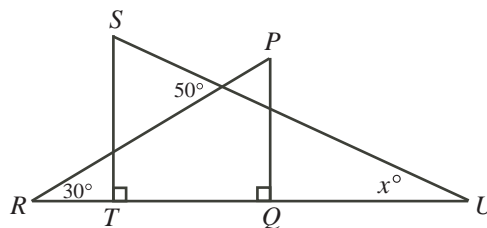
15. In the diagram, each of the two circles has centre O. Also, $OP : PQ = 1 : 2$. If the radius of the larger circle is 9, what is the area of the shaded region?

(A) 12π (B) 36π (C) 54π

(D) 72π (E) 81π
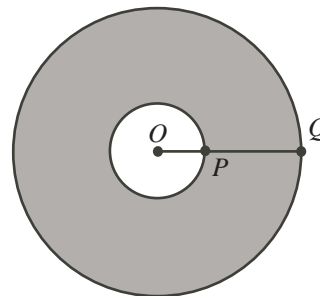
16. The equation $y = ax^2 + bx + c$ was used to create the table of values below:

x	0	1
y	8	9

What is the value of $a + b$?

(A) -2 (B) 1   (C) 3   (D) 5   (E) -5

17. A string has been cut into 4 pieces, all of different lengths. The length of each piece is 2 times the length of the next smaller piece. What fraction of the original string is the longest piece?

(A) $\frac{8}{15}$ (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{6}{13}$ (E) $\frac{1}{4}$

18. Six consecutive integers are written on a blackboard. When one of them is erased the sum of the remaining five integers is 2012. What is the sum of the digits of the integer that was erased?

(A) 5   (B) 6   (C) 7   (D) 9   (E) 11

19. In the star shown, the sum of the four integers along each straight line is to be the same. Five numbers have been entered. The five missing numbers are 19, 21, 23, 25, and 27. Which number is represented by q?

(A) 25   (B) 21   (C) 23

(D) 27   (E) 19
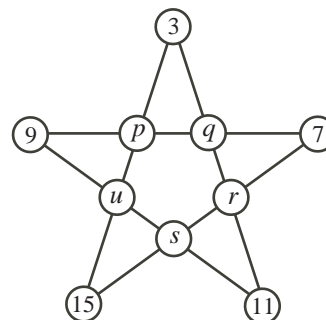
20. If N is the smallest positive integer whose digits have a product of 2700, then the sum of the digits of N is

(A) 23   (B) 24   (C) 25   (D) 26   (E) 27

Part C: Each correct answer is worth 8.

21. If x and y are positive integers with $x > y$ and $x + xy = 391$, what is the value of $x + y$?

(A) 38   (B) 39   (C) 40   (D) 41   (E) 42

22. Five monkeys are seated around a table. Their seats are labelled P, Q, R, S, and T, in clockwise order, as shown. The five monkeys are randomly numbered Monkey 1, Monkey 2, Monkey 3, Monkey 4, and Monkey 5. Monkey 1 remains in its seat. The remaining four monkeys then sit themselves in the remaining seats so that they are seated in clockwise order as Monkey 1, Monkey 2, Monkey 3, Monkey 4, and Monkey 5. What is the probability that the Monkey originally in seat R moves to seat P?
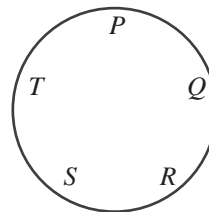
(A) $\frac{1}{20}$ (B) $\frac{1}{10}$ (C) $\frac{3}{20}$

(D) $\frac{1}{5}$ (E) $\frac{1}{4}$

23. In the diagram, points P, Q and R lie on a circle with centre O and radius 12, and point S lies on OR. If $\angle POR = 135°$, the area of trapezoid $OPQS$ is closest to
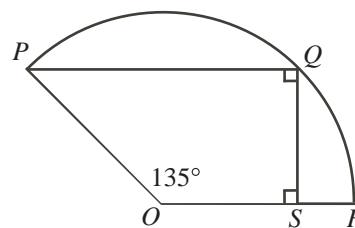
(A) 216   (B) 144   (C) 108

(D) 112.5   (E) 114.6

24. Six friends will exchange books in their book club. Each friend has one book to give to a friend, and will receive one book from a different friend. (No two friends trade books with each other.) In how many ways can the books be exchanged?

(A) 200   (B) 120   (C) 140   (D) 240   (E) 160

25. The digits of the positive integer n include no 9s, exactly four 8s, exactly three 7s, exactly two 6s, and some other digits. If the sum of the digits of n is 104 and the sum of the digits of $2n$ is 100, then the number of times that the digit 5 occurs in n is

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

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