Xiao Sun

POS6933 Bayesian Statistics and Data

Professor Badredine Arfi

Final project

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Abstract

This project replicates and extends Baldassarri and Park's research on American public opinion dynamics in economic issues, civil rights, moral issues, and foreign policy/security. They argue that issue partisanship can explain economic and civil rights issues, but partisan secular trends can be better for explaining moral issues. My project agrees with the author's findings and finds that the extended model especially works well in the domain of moral issues.

Keywords: public opinion, issue partisanship, partisan secular trend, replication, extension

Introduction

My final project first replicates the main results in Delia Baldassarri and Barum Park's paper (2020), "Was there a culture war? Partisan polarization and secular trends in US public opinion," and then improves the authors' main models. The authors employ two mechanisms: issue partisanship and secular trends, to explain American public opinion change in four domains: economic issues, civil rights, moral issues, and foreign policy (security). They argue that the "social process that brought about opinion change in the moral domain over the last four decades is substantively different from what has occurred in other issue domains" (Baldassarri and Park, 2020, 810). The process of issue partisanship (sorting political preferences along partisan lines) can account for public opinion dynamics in the economic and civil rights domains. But in moral issues, the significant change is a partisan secular trend, in which both Democrats and Republicans are adopting more progressive views. In particular, Democrats adopt progressive views faster than Replicants.

This paper is organized into four sections. I introduce the data and methodology in the first part. Then, I show the replication results and discuss the difficulties in the second. In the third part, I explain how I extend the models and solve primary problems; meanwhile, I briefly interpret the results. Last, I compare different time trend models for each domain to see which model is the best.

Data and Methodology

The authors use 78 issues data, which appeared at least three times in American National Election Study (ANES) and General Social Survey (GSS) data from 1972 to 2016. And these issues are classified into four domains: moral, economic, civil rights, and security/foreign policy domains. The economic domain includes issues on federal spending, health insurance, job provision, and the size of government. The discrimination against African Americans and other minorities is included in issues of civil rights. Instances of security and foreign policy are defense spending and urban unrest.

It is worth noting that the data has a hierarchical structure. The data with time is nested within a specific issue. Regarding the hierarchical data structure and the proportion main outcome, they adopt a multi-level Beta regression model to estimate American opinion trends. And their main outcome of interest is the proportion of liberal responses on each issue in a specific year. The liberal response is below the midpoint of each issue scale, which the authors have defined in their codebook. In short, they employ multi-level beta regression models with a logit link function. These models are estimated using a Bayesian approach, where they assign weakly informative priors to all parameters. After that, they fit separate models to each domain.

They let y_{it} be the proportion of liberal responses at year t on issue i. And they assume that $y_{it|}\ \mu_{it}, v \sim \text{Beta}(\mu_{it}, v)$, where the beta distribution is parameterized through the mean, $\mu_{it,}$ and the "precision" parameter v. And they model the main outcome as a function of time, partisanship, and their interactions. The mean is modeled as follows:

$$\mu_{it} = \text{logit}^{-1}[\gamma_{0,it} + \gamma_{1,it} \text{*IND}_t + \gamma_{2,it} \text{*DEM}_t + \gamma_{3,it} \text{*TIME}_t + \gamma_{4,it} \text{*(TIME}_t\text{* IND}_t) + \gamma_{5,it} \text{*(TIME}_t\text{* DEM}_t) + \ldots]$$

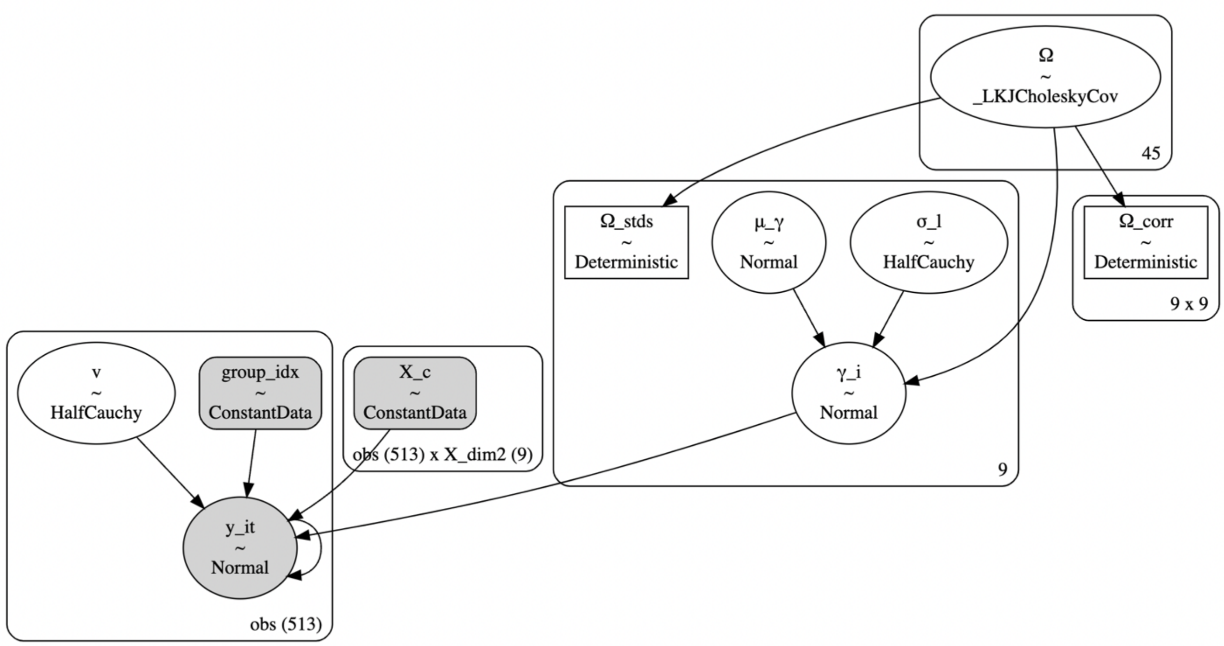
The authors set Republicans as the reference and IND and DEM as dummy variables for the partisanship. The primary variable of interest in the analysis is the interaction term between Democrats and time. A positive interaction term indicates that the partisan groups have been growing further apart. In other words, Democrats are becoming more liberal on specific issues at a faster speed than Replicants, although Republicans are also becoming more liberal rather than conservative over time. And a negative coefficient on time predictor means Republicans have become more conservative (Baldassarri and Park 2020).

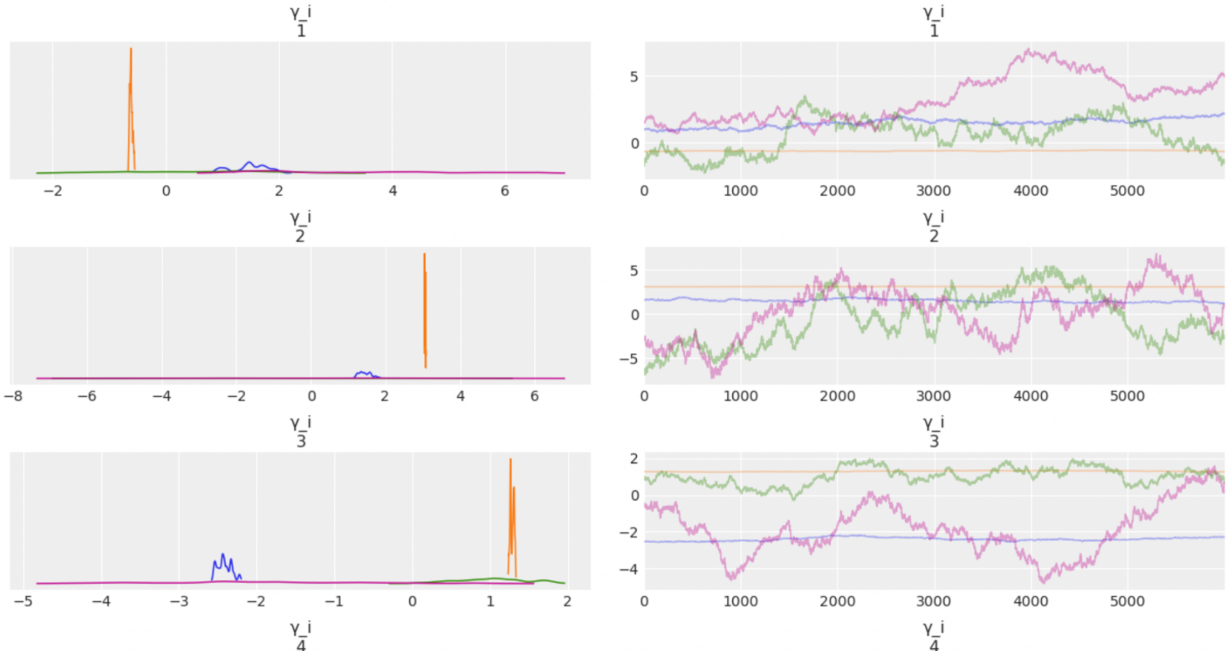
Replication Challenges and Results

I have followed almost all the necessary steps the authors did in the R. But in Python, I could not use the same functions as in R and so failed to generate the same results. For example, I could not use MvNormal to specify the covariance between predictors and consequently was unable to employ Beta regression to estimate the likelihood in model building. These two functions seemingly are not developed well in PYMC, which prevents me from executing the code chunk and continuing the model extension. Having consulted with Professor Arfi, I changed both MvNormal and Beta to Normal distribution in order to run the model and proceed with the following steps. Thus, I end up using multi-level normal regression instead of multi-level Beta regression.

Besides these two function problems, I have followed all the other steps, which means I used the same priors and built the same correlations between parameters. But the replication results are not good even if I increased the draws, tunes, and acceptance range. For instance, the authors only did 3,000 draws and used the first 2,000 iterations for tuning. But I used 6,000 draws and 3,000 iterations to tune the model and increased the target acceptance to 0.98. However, this sampling was still unsuccessful given the bad convergences (756 divergences) as shown in the trace plot and big r-hats in the summary table. The sampling process was also unreasonably time-consuming given the small to medium sample size. I think this is because Cholesky containing many parameters is included in the model as a hyper prior. Also, the 94% high-density interval (HDI) of all the predictors is very large, which offers little valuable information for interpretation. Their ranges are between big negative and positive values. These all indicated that the models in Python have something wrong. I have to change the model in my extension project.

As I have mentioned that I did not generate the same results, but I replicated all the steps as much as I could. Please see the following diagram, which specifies the relationships among parameters and the dimensions of the data.





	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
γ_i[0]	2.399	1.527	-0.783	5.346	0.652	0.488	6.328	14.120	2.119
γ_i[1]	1.162	1.765	-1.500	4.877	0.794	0.598	5.816	14.108	1.881
γ_i[2]	1.023	2.493	-4.911	4.025	0.779	0.567	11.102	27.374	1.544
γ_i[3]	-0.504	1.823	-2.880	1.900	0.829	0.626	6.415	40.384	1.735
γ_i[4]	0.674	2.699	-4.379	5.282	0.693	0.500	20.939	23.431	1.360
γ_i[5]	-1.278	2.512	-4.246	4.731	0.804	0.586	10.316	26.937	1.531
γ_i[6]	0.994	1.573	-1.950	3.509	0.693	0.521	5.887	31.422	1.876
γ_i[7]	0.109	1.842	-2.291	2.487	0.840	0.634	6.218	40.106	1.777
γ_i[8]	-0.736	1.811	-4.433	1.899	0.819	0.617	5.769	16.039	1.903

Model Extension and Comparison

Since the replication results are not satisfying, I have improved the models in three ways. (1) I used simpler uninformative priors to remove unnecessary priors like Cholesky. (2) I extended the data by incorporating the most recent available American National Election Studies (ANES) data and the General Social Survey (GSS) data [1] to see if the results would be different. (3) I examined both linear and non-linear time trends, and I added the interaction terms of time trend and DEM or IND in the model since the authors primarily focused on the non-linear (quadratic) time trend model.

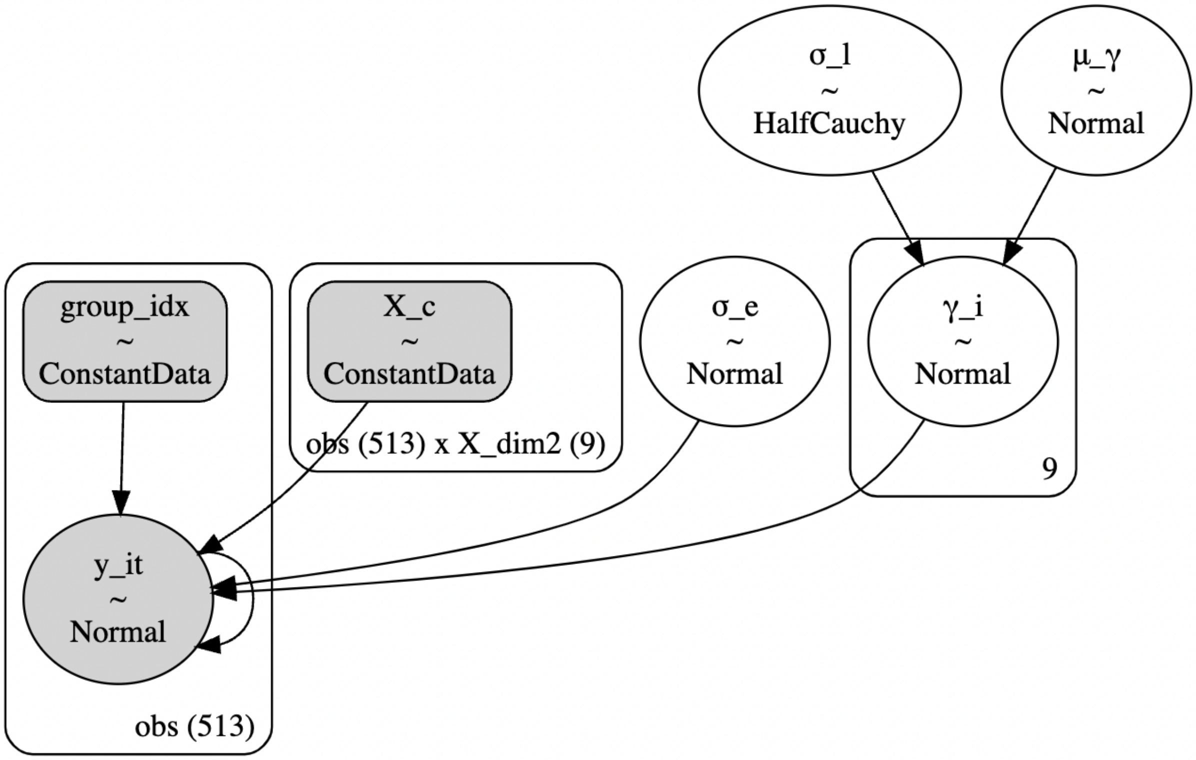
After having made these changes, I compared my models' results with the original (quadratic time trend) model's results. I also compared which time trend model would be better, but some results support linear, and some support the quadratic time model. Despite the disagreement, one thing I could confirm is that the cubic time trend model does not work well. The last point I have to mention is that I added 2018 or 2020 data to specific issues, like health insurance, foreign aid, etc. it should be noted that the surveys do not cover every issue the authors include in their datasets. That means the newly added data is incomplete, which may have bias. However, I hope the Bayesian approach could remove this biased influence.

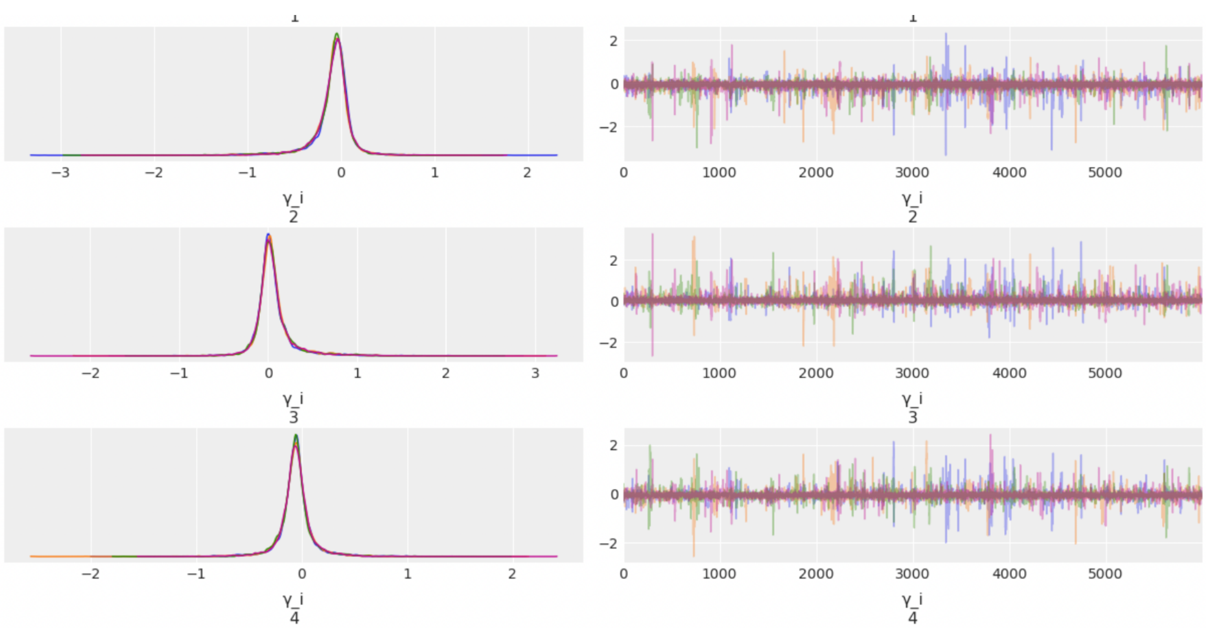
Next, I take the moral issue linear time trend model as an example to show how I changed priors, how many convergencies emerged from sampling, and what r_hats look like. Please see the results of other models in my Jupyter Notebook. It is important to note I have dealt with two problems in the model building and extension process. The primary difficulty is dealing with multi-level data. I have to group data by different issues, given that the data is nested within each specific issue. There are four domains, and each domain has 15 to 40 particular issues. So, I created a group index for each domain to categorize the responses in different issues and then put

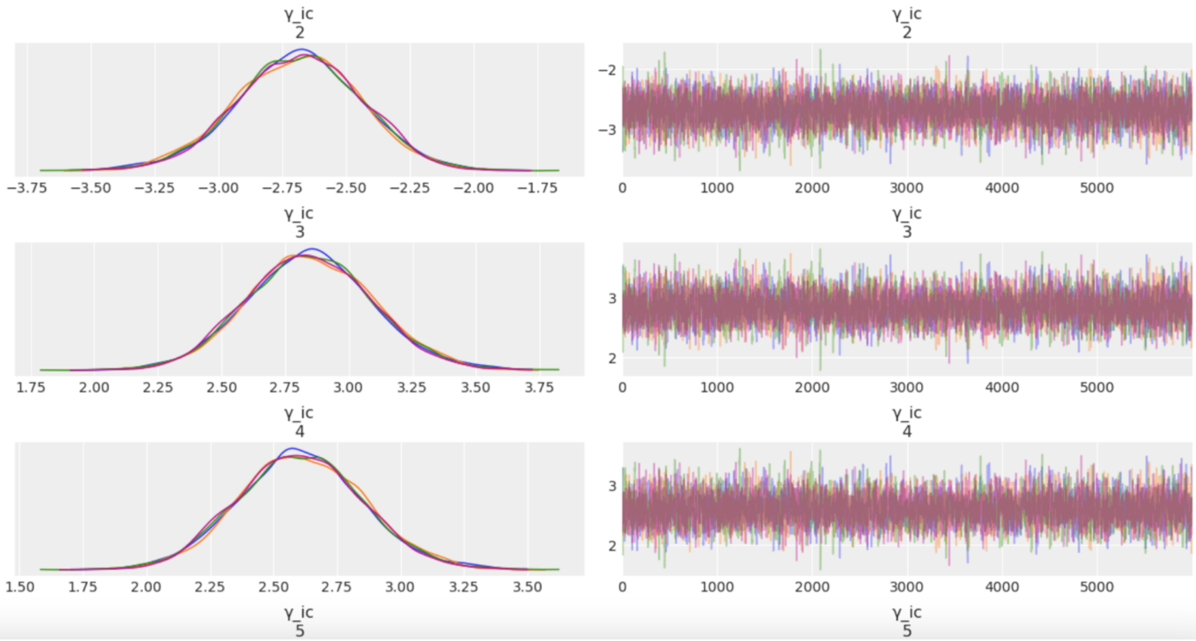
[1] Get the GSS data, https://gss.norc.org/Get-The-Data; Download ANES data, https://electionstudies.org/data-center/2020-time-series-study/

data grouped by "group_index" in models. The second is about setting the dimension and shape of the data in model building. It should be noted that the number of predictors used in your model is the number of shapes. Since I added different time trends, such as linear, quadratic, and cubic time trends, to the model, I had to specify and change the shape when I set priors. Sometimes it is confusing to set up shape and dimensions. But drawing the graph makes it easier to identify the structure.

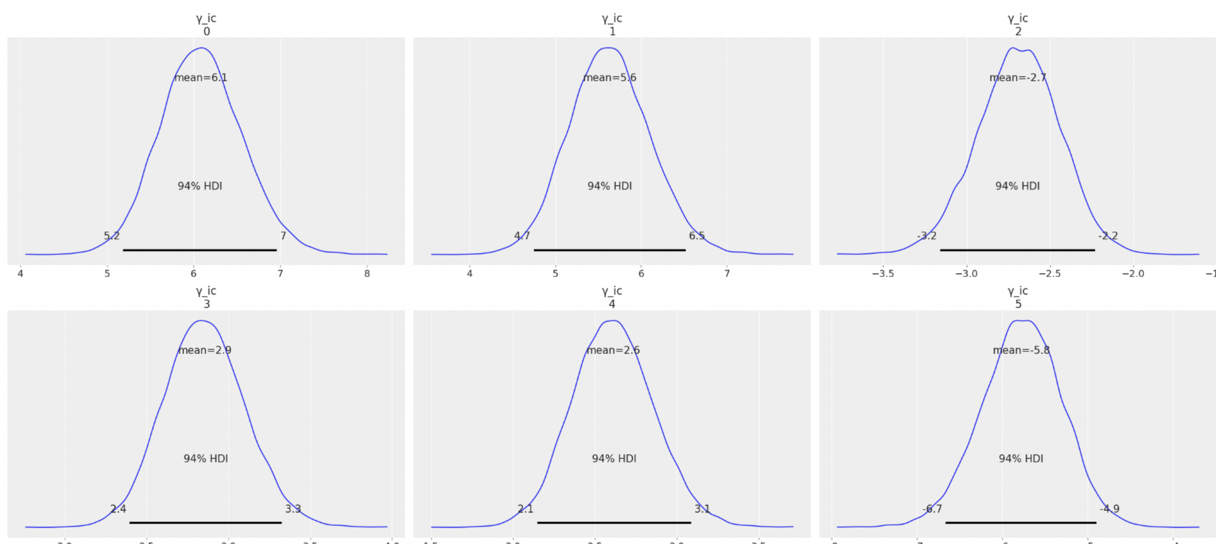
Last, I have run linear, quadratic, and cubic time trend models in four sperate domain models. That means I created 12 models. For the purpose of simplicity, I only show the economic issue model with quadratic time trend here for illustration purposes. γ_i is the prior for predictors, and I further set μ_γ and σ_l for γ_i. I also give a prior σ_e for the error term. For a good sampling, I set 6,000 draws, 3,000 tunes, 0.98 target accept range. There are not many convergencies between the four lines. And all the r-hats for predictors are less than 1.03. In this respect, the new model is satisfying.


σ_l
~
HalfCauchy
μ_γ
~
Normal
group_idx
~
ConstantData
X_c
~
ConstantData
obs (513) x X_dim2 (9)
σ_e
~
Normal
γ_i
~
Normal
9
y_it
~
Normal
obs (513)


γ_i
2
γ_i
3
γ_i
4



	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
γ_ic[0]	6.073	0.468	5.211	6.969	0.007	0.005	4161.879	5858.377	1.001
γ_ic[1]	5.620	0.468	4.751	6.510	0.007	0.005	4154.443	6098.932	1.001
γ_ic[2]	-2.685	0.245	-3.153	-2.234	0.004	0.003	4130.878	6043.639	1.001
γ_ic[3]	2.849	0.246	2.401	3.328	0.004	0.003	4145.194	6174.307	1.001
γ_ic[4]	2.607	0.247	2.153	3.081	0.004	0.003	4151.373	6317.387	1.001
γ_ic[5]	-5.765	0.467	-6.657	-4.900	0.007	0.005	4151.714	6053.409	1.001



Since my later comparison shows linear time trend model is also good, I provide the summary tables of the four domains' linear time trend models. I take the moral issue liner model as an example to interpret the coefficients. The most important two predictors are γ3 time and γ4 the interaction term of Democrat and time. The true value of γ3 time is within 0.2401 and 3.328, and the mean is 2.849, which means that Republicans have been becoming more progressive on moral issues. As for γ4 the interaction of Democrat and time, the true value is between 2.153 and 3.081, and the mean is 2.607. This positive interaction indicates that Democrats are increasingly progressive on moral issues over time and faster than Republicans.

Moral issues

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
γ_ic[0]	6.073	0.468	5.211	6.969	0.007	0.005	4161.879	5858.377	1.001
γ_ic[1]	5.620	0.468	4.751	6.510	0.007	0.005	4154.443	6098.932	1.001
γ_ic[2]	-2.685	0.245	-3.153	-2.234	0.004	0.003	4130.878	6043.639	1.001
γ_ic[3]	2.849	0.246	2.401	3.328	0.004	0.003	4145.194	6174.307	1.001
γ_ic[4]	2.607	0.247	2.153	3.081	0.004	0.003	4151.373	6317.387	1.001
γ_ic[5]	-5.765	0.467	-6.657	-4.900	0.007	0.005	4151.714	6053.409	1.001

Economic issues

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
γ_i[0]	0.371	0.431	-0.352	1.169	0.005	0.005	9290.764	6881.887	1.001
γ_i[1]	-0.326	0.406	-1.079	0.363	0.005	0.005	9819.059	6182.471	1.001
γ_i[2]	0.062	0.426	-0.665	0.836	0.006	0.006	7904.812	5032.599	1.001
γ_i[3]	0.093	0.437	-0.613	0.914	0.006	0.006	7766.871	5317.569	1.000
γ_i[4]	0.003	0.427	-0.759	0.741	0.006	0.006	7895.476	5044.911	1.001
γ_i[5]	0.024	0.405	-0.692	0.760	0.005	0.005	9722.949	6208.609	1.001

Civil rights

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
γ_ic[0]	-0.371	0.579	-1.491	0.685	0.009	0.008	4318.893	4015.776	1.001
γ_ic[1]	0.268	0.579	-0.842	1.343	0.009	0.009	4260.964	3927.243	1.001
γ_ic[2]	0.341	0.270	-0.163	0.846	0.004	0.003	4330.643	3899.458	1.001
γ_ic[3]	-0.728	0.276	-1.243	-0.214	0.004	0.003	4473.428	4255.497	1.001
γ_ic[4]	0.813	0.310	0.229	1.387	0.005	0.004	5088.370	4847.096	1.001
γ_ic[5]	-0.233	0.577	-1.272	0.901	0.009	0.009	4302.569	4020.714	1.001

Foreign policy/security

	mean	sd	hdi_3%	hdi_97%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
γ_ic[0]	13.250	6.792	1.580	26.929	0.136	0.103	2984.814	3096.220	1.002
γ_ic[1]	-4.957	6.257	-17.307	6.396	0.137	0.115	2576.277	2187.513	1.005
γ_ic[2]	2.247	5.950	-8.598	14.242	0.150	0.113	1993.690	1819.126	1.002
γ_ic[3]	7.401	6.206	-3.665	19.869	0.157	0.134	2010.906	1791.414	1.003
γ_ic[4]	-1.545	5.951	-12.971	9.864	0.150	0.116	1996.176	1824.364	1.002
γ_ic[5]	5.710	6.256	-5.835	17.878	0.137	0.114	2578.096	2193.250	1.005

Comparison between models

I use PSIS-LOOIC and WAIC to see how good these models' fit is. These statistics are approximations of out-of-sample deviance. A lower value means a better predictive fit. I use "arviz.waic" and "arvizc.loo" in Python to calculate each of the model's LOOIC and WAIC. Among all time trend models, linear time trend models in four domains have the lowest LOOIC and WAIC values. Then, I use "arviz.compare" to see which model is better. This "arviz.compare" is also based on loo or waic cross-validation.[2] But the results tell me that the quadratic time trend model is the best for each domain model. In this regard, comparing the fitness of the models is very tricky, given the fact that when we use different calculations or functions, we may get different results.

Regarding the simplicity and interpretation, I think the linear time trend model is better than the non-linear time trend model. For one thing, it has fewer predictors; for the other, its HDI makes more sense than the other two. In particular, the linear model works very well for explaining American public opinion change on moral issues. This model tells us that Americans have become more progressive on moral matters. This also fits our expectations in reality.

Economic issue

[2] https://arviz-devs.github.io/arviz/api/generated/arviz.compare.html. LOO is leave-one-out (PSIS-LOO loo) cross-validation and WAIC is the widely applicable information criterion.

	rank	loo	p_loo	d_loo	weight	se	dse	warning	loo_scale
Quadratic	0	94.869652	9.506008	0.000000	9.514199e-01	12.148748	0.000000	False	log
Cubic	1	90.967749	5.357646	3.901904	1.408966e-14	12.252251	2.514728	False	log
Linear	2	74.651214	3.982896	20.218438	4.858008e-02	11.758079	6.387043	False	log

```
az.waic(eco_trace1)
```

```
Computed from 24000 by 513 log-likelihood matrix

          Estimate       SE
elpd_waic    74.65    11.76
p_waic        3.98        -
```

```
az.waic(eco_trace2)
```

```
Computed from 24000 by 513 log-likelihood matrix

          Estimate       SE
elpd_waic    90.97    12.25
p_waic        5.35        -
```

```
az.waic(eco_trace3)
```

```
Computed from 24000 by 513 log-likelihood matrix

          Estimate       SE
elpd_waic    94.88    12.15
p_waic        9.49        -
```

Civil rights

	rank	loo	p_loo	d_loo	weight	se	dse	warning	loo_scale
Quadratic	0	141.634536	8.848332	0.000000	1.000000e+00	16.247942	0.000000	False	log
Cubic	1	133.266208	6.685274	8.368328	1.907197e-12	16.056846	3.348991	False	log
Linear	2	114.895008	5.247535	26.739529	0.000000e+00	15.822735	6.020853	False	log

```
az.waic(cv_trace2)
```

```
Computed from 24000 by 549 log-likelihood matrix

          Estimate       SE
elpd_waic   133.27    16.06
p_waic        6.68        -
```

```
az.waic(cv_trace1)
```

```
Computed from 24000 by 549 log-likelihood matrix

          Estimate       SE
elpd_waic   114.90    15.82
p_waic        5.25        -
```

```
az.waic(cv_trace3)
```

```
Computed from 24000 by 549 log-likelihood matrix

          Estimate       SE
elpd_waic   141.64    16.25
p_waic        8.84        -
```

Moral issues

	rank	loo	p_loo	d_loo	weight	se	dse	warning	loo_scale
Quadratic	0	700.294719	8.891878	0.000000	8.318527e-01	29.156402	0.000000	False	log
Cubic	1	649.674504	7.305995	50.620215	1.681473e-01	29.908374	12.701415	False	log
Linear	2	325.806524	7.573784	374.488195	5.153877e-12	26.800523	25.155973	False	log

```
az.waic(mo_trace1)
```

```
Computed from 24000 by 2049 log-likelihood matrix

          Estimate       SE
elpd_waic   325.81    26.80
p_waic        7.57        -
```

```
az.waic(mo_trace2)
```

```
Computed from 24000 by 2049 log-likelihood matrix

          Estimate       SE
elpd_waic   649.67    29.91
p_waic        7.31        -
```

```
az.waic(mo_trace3)
```

```
Computed from 24000 by 2049 log-likelihood matrix

          Estimate       SE
elpd_waic   700.30    29.16
p_waic        8.89        -
```

Foreign policy/security

	rank	loo	p_loo	d_loo	weight	se	dse	warning	loo_scale
Quadratic	0	104.225361	5.499665	0.000000	1.0	8.883345	0.000000	False	log
Cubic	1	104.039504	5.414480	0.185857	0.0	8.831225	0.217788	False	log
Linear	2	74.075559	4.342724	30.149801	0.0	8.632467	7.701042	False	log

```
az.waic(mo_trace1)
```

```
Computed from 24000 by 2049 log-likelihood matrix

          Estimate       SE
elpd_waic   325.81    26.80
p_waic        7.57        -
```

```
az.waic(mo_trace2)
```

```
Computed from 24000 by 2049 log-likelihood matrix

          Estimate       SE
elpd_waic   649.67    29.91
p_waic        7.31        -
```

```
az.waic(mo_trace3)
```

```
Computed from 24000 by 2049 log-likelihood matrix

          Estimate       SE
elpd_waic   700.30    29.16
p_waic        8.89        -
```

Conclusion

This paper offers very interesting findings that public opinion changes on economic and civil rights issues can be explained by issue partisanship, but opinion dynamics on moral issues are better explained by partisan secular trend. The authors adopt Bayesian approach to reveal that Americans have been becoming more progressive on moral issues, such as abortion, gay rights, legalization of marijuana, sexual behavior, and gender roles. By replicating and extending their research, I successfully gained the most important interpretation of American public opinion change on these issues although I have been unable to replicate every step they did in R. To know and improve the reliability of the results, I simplified the authors' models, extended the data, and compared the linear time trend model with the non-linear time trend models in the extension part. From my results, I find the moral issue linear time trend model is more interpretable and simpler than the other models in other issue domains. In short, my project confirms the authors' findings. Thus, their research on American public opinion dynamics holds the water.

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