Linear Programming: Introduction

Daniel Kane

Department of Computer Science and Engineering
University of California, San Diego

Advanced Algorithms and Complexity
Data Structures and Algorithms

Learning Objectives

- See an example of the type of problem solved by linear programming.

Factory

You are running a widget factory and trying to optimize your production procedures to save money.

Machines vs. Workers

Can use combination of machines and workers.

- Have only 100 machines.
- Unlimited workers.
- Each machine requires 2 workers to operate.

Production

- Each machine makes 600 widgets a day.
- Each worker makes 200 widgets a day.

Limited Demand

Total demand for only $100,000$ widgets a day.

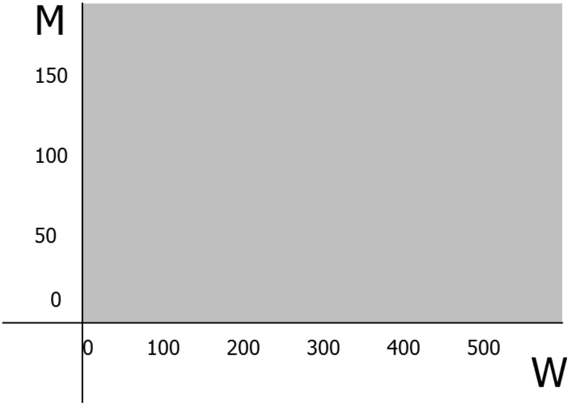
Algebra

Let W be the number of workers and M the number of machines.
Constraints:

- $W \geq 0.$
- $100 \geq M \geq 0.$
- $W \geq 2M.$
- $100,000 \geq 200(W - 2M) + 600M.$

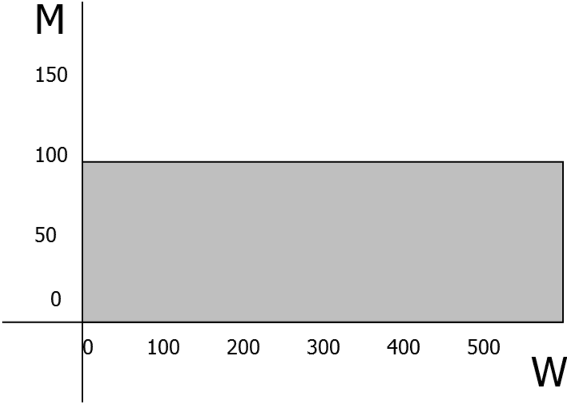
Graph

$M, W \geq 0$



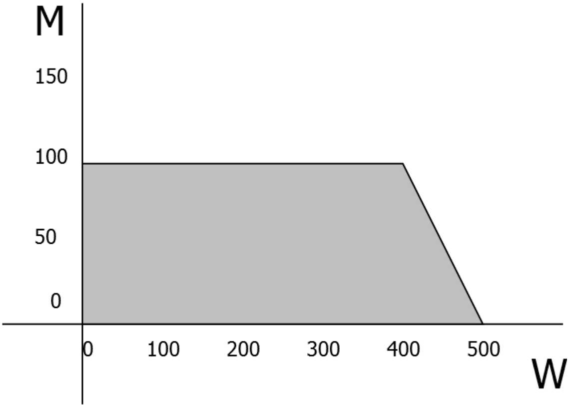
Graph

$M \leq 100$



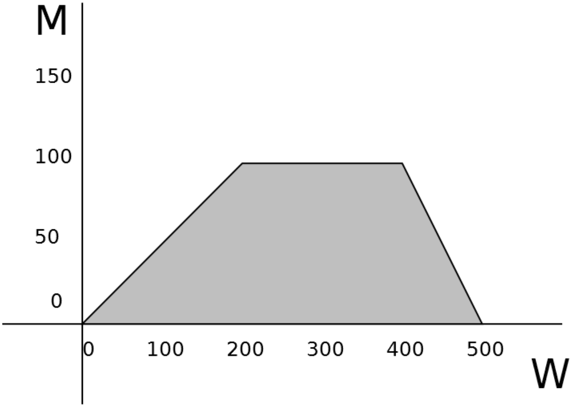
Graph

$$M + W \leq 500$$



Graph

Diagram of possible configurations:



Profits

Profits are determined as follows:

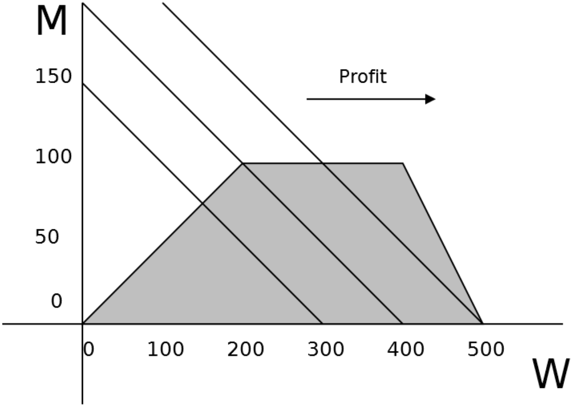
- Each widget earns you \$1.
- Each worker costs you \$100/day.

Total profits (in dollars per day):

$$200(W-2M)+600M-100W = 100W+200M.$$

Graph

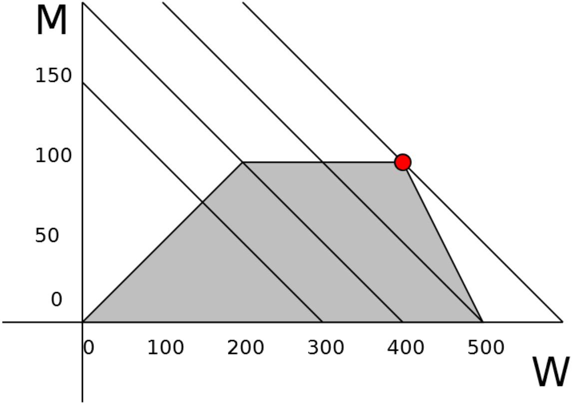
Profit mapped on graph:



Optimum

Best: $M = 100, W = 400$ [NB: A corner]

Profit $= \$60,000/\text{day}$.



Proof of Optimality

$$\begin{array}{rrcrcl}
100 \cdot [& 001 \cdot M & + & 000 \cdot W & \leq & 100] \\
+0.5 \cdot [& 200 \cdot M & + & 200 \cdot W & \leq & 100,000] \\
\hline
 & 200 \cdot M & + & 100 \cdot W & \leq & 60,000.
\end{array}$$

Summary

Maximized:

$$200M + 100W$$

subject to constraints:

$$\begin{aligned}
0M + 1W &\geq 0 \\
1M + 0W &\geq 0 \\
-1M + 0W &\geq -100 \\
-2M + 1W &\geq 0 \\
-1M - 1W &\geq -500
\end{aligned}$$