

# Flows in Networks: Slow Example

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Advanced Algorithms and Complexity  
Data Structures and Algorithms

# Learning Objectives

- Understand why the Ford-Fulkerson algorithm is sometimes slow.
- Provide an example of a potential slow execution of this algorithm.

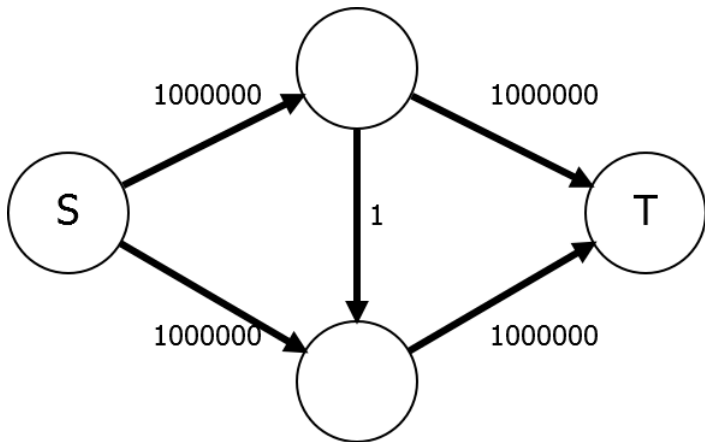
# Last Time

Ford-Fulkerson Algorithm for Maxflow:

- Runtime  $O(|E||f|)$ .
- Potentially bad if  $|f|$  is large.
- Does this problem ever actually happen?

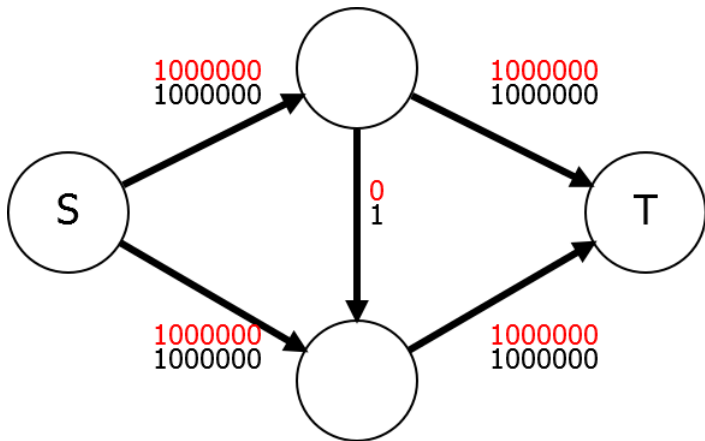
# Example

Consider the following example:

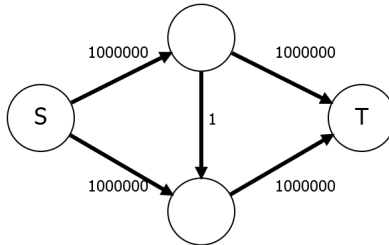
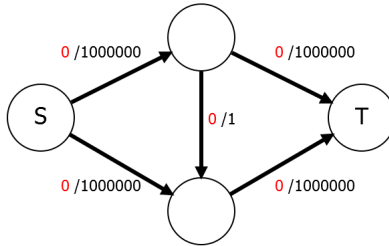


# Flow

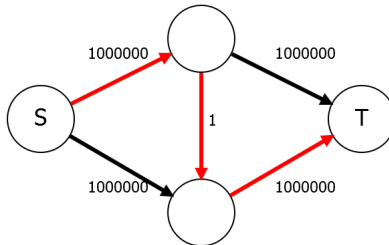
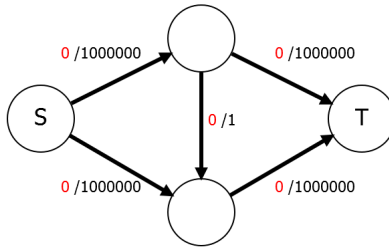
Has following maxflow ( $|f| = 2000000$ ):



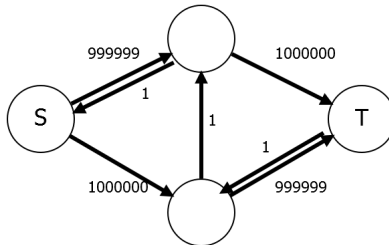
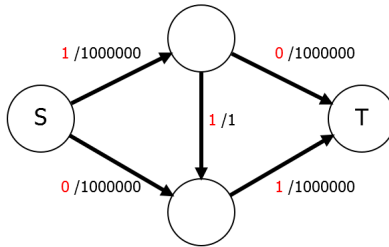
# Possible Execution



# Possible Execution

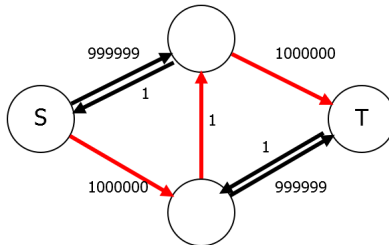
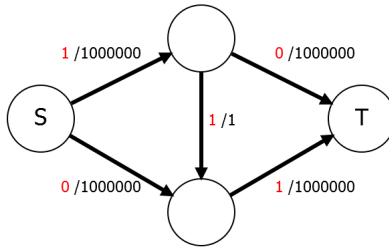


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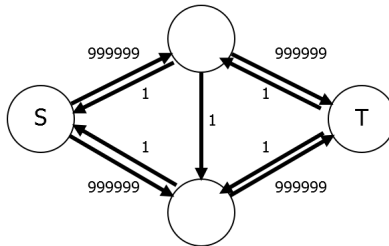
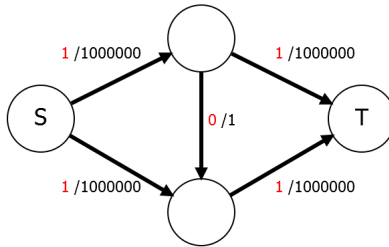




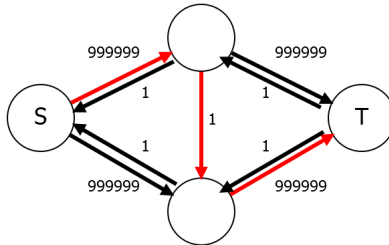
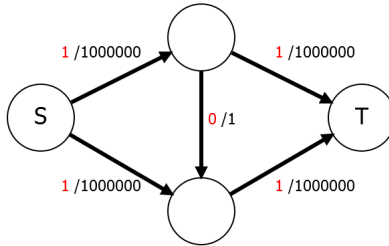
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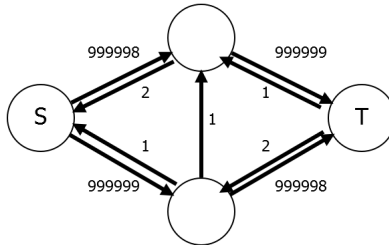
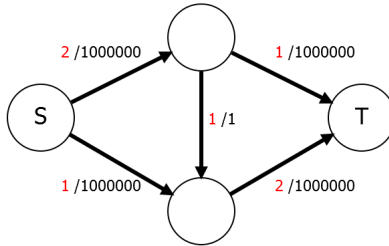
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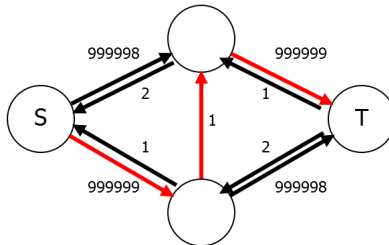
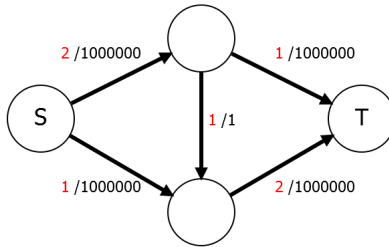
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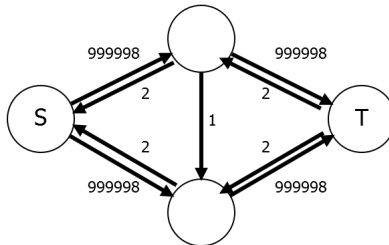
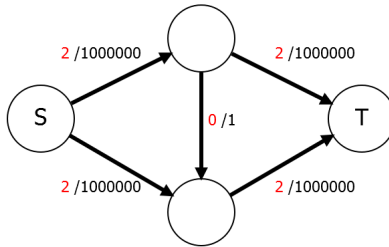
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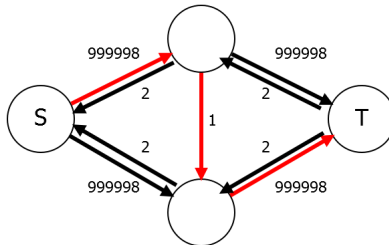
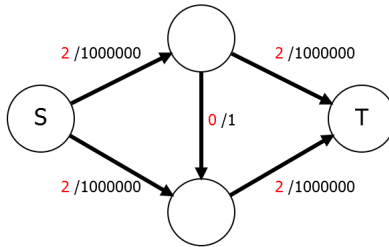
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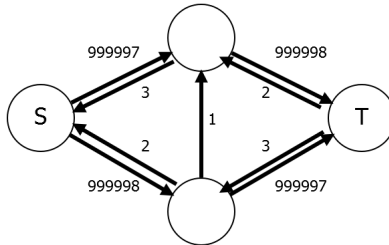
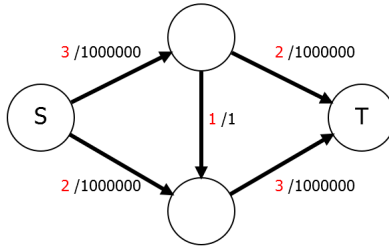
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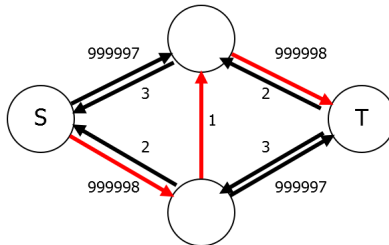
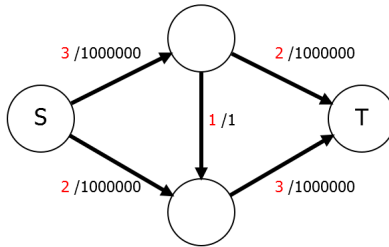


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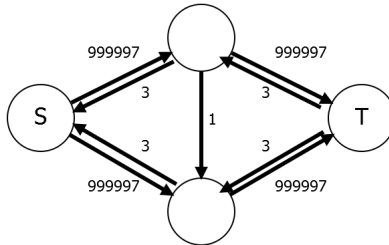
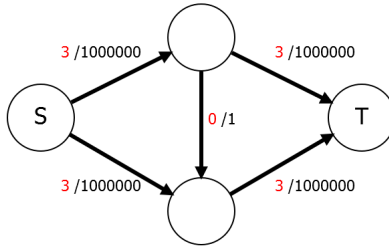




# Possible Execution



# Possible Execution



# Problem

How many iterations of Ford-Fulkerson will be required to compute a maximum flow, if it keeps choosing augmenting paths in this way?

# Time

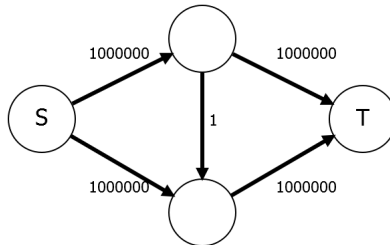
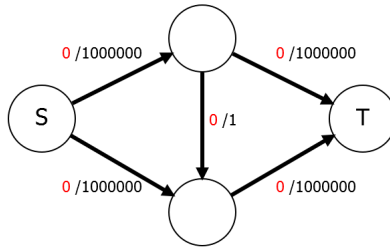
- Each step adds only one unit of flow (limited by middle edge).
- Need 2000000 total units.
- Ford-Fulkerson requires 2000000 iterations.

# Time

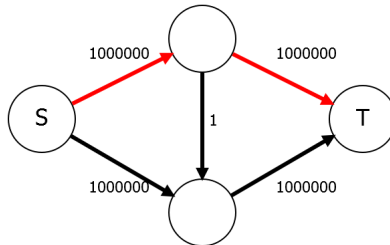
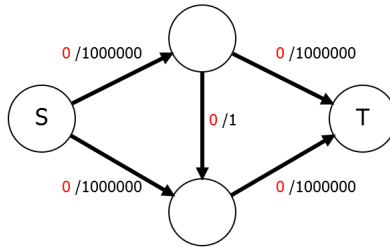
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- Need 2000000 total units.
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Doesn't need to be this bad.

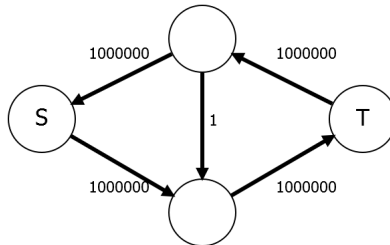
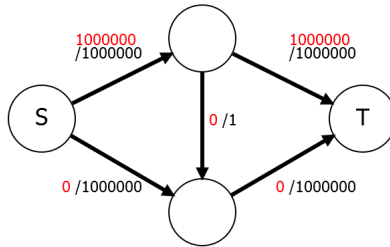
# Another Possible Execution



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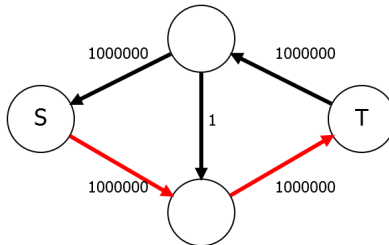
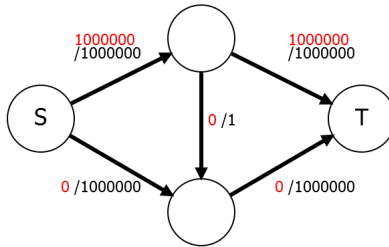


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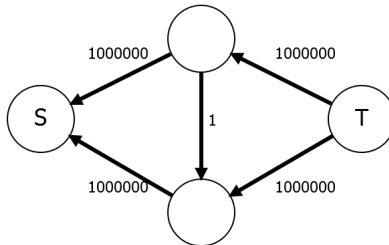
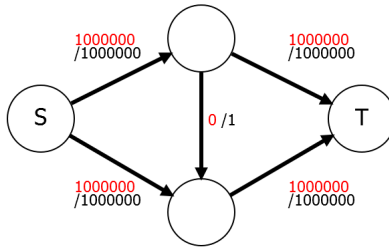




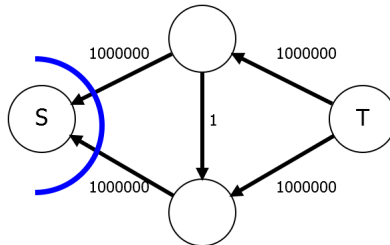
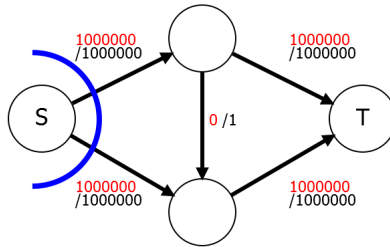
# Another Possible Execution



# Another Possible Execution



# Another Possible Execution



# Next Time

We'll find a way to ensure that we never have this type of problem.