<u>Clock Report</u>

MAE 03

Professor Qi Huihui

Rayyan Khalid

B04

Executive Summary

The pendulum clock is the first project undertaken by the students enrolled in MAE 03, Introduction to Engineering graphics and design class. During this project, we made a clock pendulum that consists of several main parts, including the pulley, the pendulum, the escapement wheel, and steel nut(s). The primary source of power to the clock is the gravity. The whole mechanism is that the hanging steel nut(s) is/are pulled down by the gravity, providing energy to the escapement wheel through the pulley, which then spins in a clockwise direction. That spinning escapement wheel ticks the pendulum which then starts to oscillate.

A pendulum's natural frequency determines the time it takes for one revolution of the escapement wheel. I used two different methods to find the pendulum's natural frequency: point mass analysis and the rigid body analysis. For the point mass analysis, a center point of the pendulum was calculated where the whole mass of the pendulum was assumed to concentrate. I find it important to mention here that in this analysis we assumed zero rotational inertia. Whereas, in the rigid body analysis, the whole body of the pendulum as well as the rotational inertia was taken into consideration. I mainly used the CAD design in Fusion 360 to obtain the necessary data about the pendulum, such as its mass, density, volume etc. It is important to note here that some assumptions were made during these analysis: friction was taken to be negligible; the amplitude of the oscillation is relatively small; the pendulum swings freely without consideration for the escapement wheel.

The rigid body analysis was observed to give more precise results compared to the point mass analysis. The reason for the more accurate analysis by rigid body analysis is that it takes into account the whole body of the pendulum and also the rotational moment of inertia of the pendulum clock. Whereas, the assumption of mass concentrating at a single point in the point mass analysis is an oversimplification of the real-life. The percentage error

produced by the point mass analysis was about 34.7% whereas with the rigid body analysis it was reduced down to 5.79% which proves my previous comment on the accuracy of the rigid body analysis.

Theoretical Analysis

Point Mass Analysis:

The basic Equations of Point Mass Pendulum were used in Point Mass Analysis in order to determine the pendulum's natural frequency. First, the pendulum's center of mass without the bolts was calculated, and then the pendulum's center of mass with the bolts was calculated. Each step had to be validated using the intermediate verification by comparing it with the physical pendulum, which was balanced and measured with one's finger.

Equation

$$w = \sqrt{\frac{g}{L_{com}}} \text{ (radians / seconds)} \qquad (1)$$

where,

w = natural frequency

g = gravity

L_{com} = Effective length of pendulum (distance from the pivot to the center of mass)

To calculated the center of mass of a pendulum without bolts, I used Fusion 360 to examine the properties of the CAD model of the pendulum to determine the center of mass without bolts. Due to our almost symmetrical pendulum, the center of mass was expected to be very close to zero for the X and Z coordinates. In the next step, I used the "Measure" tool under the "Inspect" menu to get the exact area of the sketch of the pendulum so it wouldn't be confused with the surface area of the pendulum (A). Based on the data we collected, we found that the Y-coordinate of the center of mass (La) was 2.905 cm, which is very similar to the value of 3.05 cm in the physical pendulum.

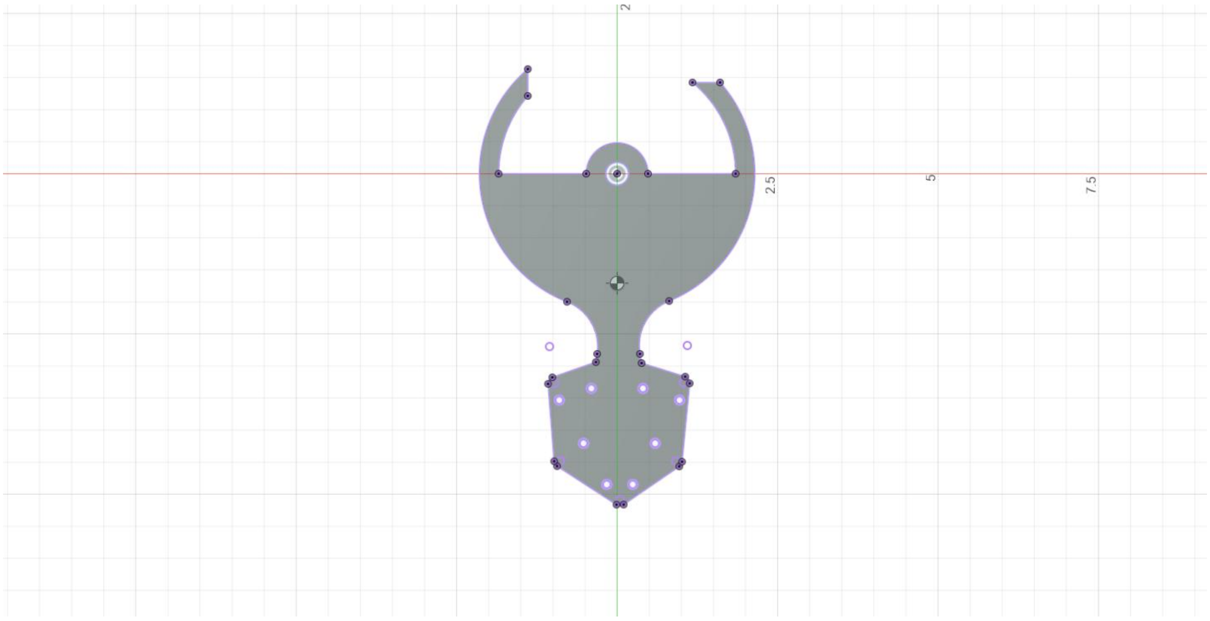

Figure 1: The circle in the middle shows the center of mass of the pendulum.

Then I calculated the center of mass with bolts. To do that, I used the design of my pendulum in fusion 360 to determine the length from the pivot point to the bolt holes. The lengths were as shown below in figure 2.

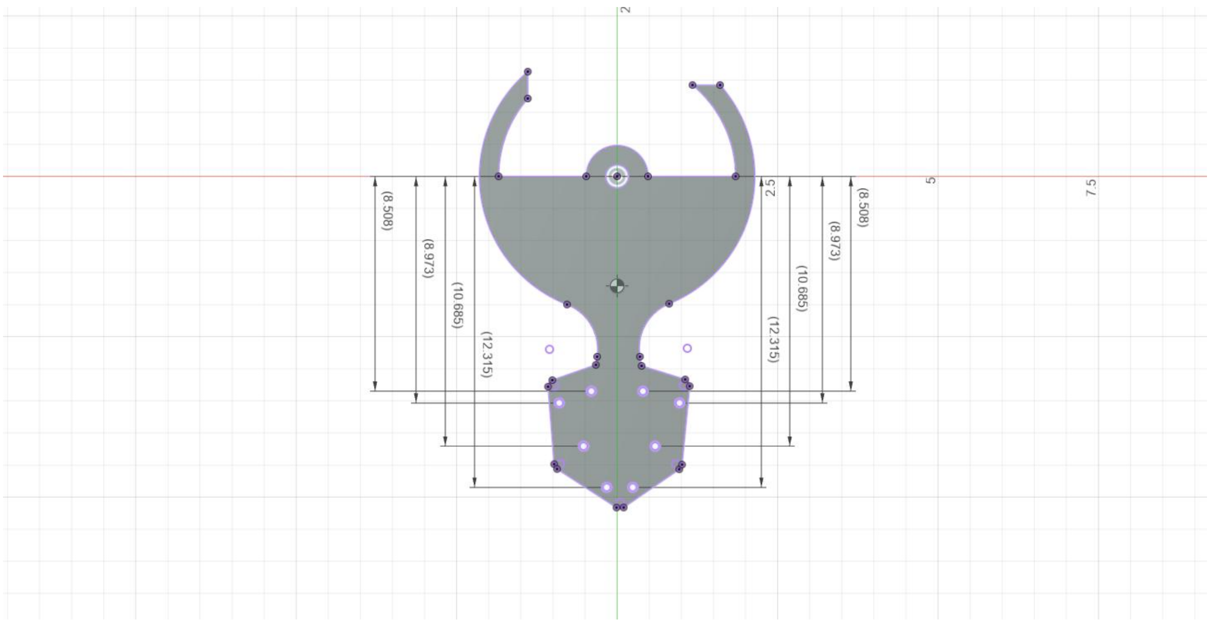

Figure 2: Distances of bolts from the pivot point in cm.

Then the total mass of the pendulum with nuts and bolts was calculated using the equation below.

Total Mass of Pendulum with Nuts & Bolts

$$Mt = M_{Calc} + n * M_b \qquad (2)$$

where,

M_{Calc} = Mass of the acrylic

n = number of bolts with two nuts = 8

M_b = Mass of one bolt and two nuts = 4.0 g

The calculated total mass of pendulum with nuts and bolts came out to be 94.13 g. Next, I determined the length of the point of center of mass from the pivot using the equation 3.

Length to Center of Mass

$$L_{com} = \frac{M_{calc} * L_a + M_b * (L_{b1} + L_{b2} + \cdots + L_{bn})}{M_t} \qquad (3)$$

where,

M_{Calc} = Calculated mass of acrylic

L_a = Distance between center and the pivot of the pendulum

M_b = Mass of one bolt and two nuts

$L_{b1}, L_{b2}, \ldots, L_{bn}$ = Vertical distance between nut bolts and pivot

Table 1: Lengths to Center of Mass

Calculate Center of Mass of Pendulum with Bolts			
Length to Center of Mass of Bolt 1	L_bolt1	0.12315	m
Length to Center of Mass of Bolt 2	L_bolt2	0.10685	m
Length to Center of Mass of Bolt 3	L_bolt3	0.08973	m
Length to Center of Mass of Bolt 4	L_bolt4	0.08508	m
Length to Center of Mass of Bolt 5	L_bolt5	0.08508	m
Length to Center of Mass of Bolt 6	L_bolt6	0.08973	m
Length to Center of Mass of Bolt 7	L_bolt7	0.10685	m
Length to Center of Mass of Bolt 8	L_bolt8	0.12315	m
Length to Center of Mass **in Meters**	Lcom_meter	2.905617025	**m**

I used equation 3 with this data provided in table 1 to calculate the length of center of mass from the pivot, which turned out to be 0.029 meters.

Then I used equation 1 to find the natural frequency in radians/seconds, using the L_{com} value calculated above, to get the natural frequency of 1.83 rad/sec. After, I converted the natural frequency into hertz(Hz) by using equation 4.

Converting natural frequency from rad/sec to Hz

$$f = w\,(\frac{rad}{sec}) * (\frac{cycles}{2*\pi*rad}) \qquad (4)$$

Period of Oscillation

$$T = \frac{1}{f} \qquad (5)$$

Total Time of One Revolution

$$Total\ Time = \frac{sec}{cycle} * \frac{cycles}{revolution} * revolution \qquad (6)$$

Using equation 5 and 6, I then calculated the total time it took my escapement wheel to complete one revolution. It turned out to be 47.89 seconds. This value was way off than the actual because of ignoring rotational inertia of the pendulum. So, to get a better picture of the pendulum clock, I performed a rigid body analysis.

Rigid Body Analysis:

The main equation used in the rigid body analysis was equation 7 below. All the work was done to obtain the values of the variables in this equation, i.e. the rotational inertia of the pendulum.

Natural Frequency in Radians

$$w = \sqrt{\frac{m_t*g*L_{Com}}{I}}\ (\frac{radians}{seconds}) \qquad (7)$$

where,

m_t = Total Mass of Pendulum with nuts and bolts

g = gravity

L_{Com} = Effective Length of the Pendulum

I = Rotation Inertia of the Pendulum

To find the rotational moment of inertia of the pendulum, I used the design of the pendulum in fusion 360 to measure the actual length between the pivot point and the bolts.

Rotational Moment of Inertia

$$I = \int_m r^2 \, dm \qquad (8)$$

The actual length was collected the same way as for the point mass analysis, using the dimension tool in the fusion 360.

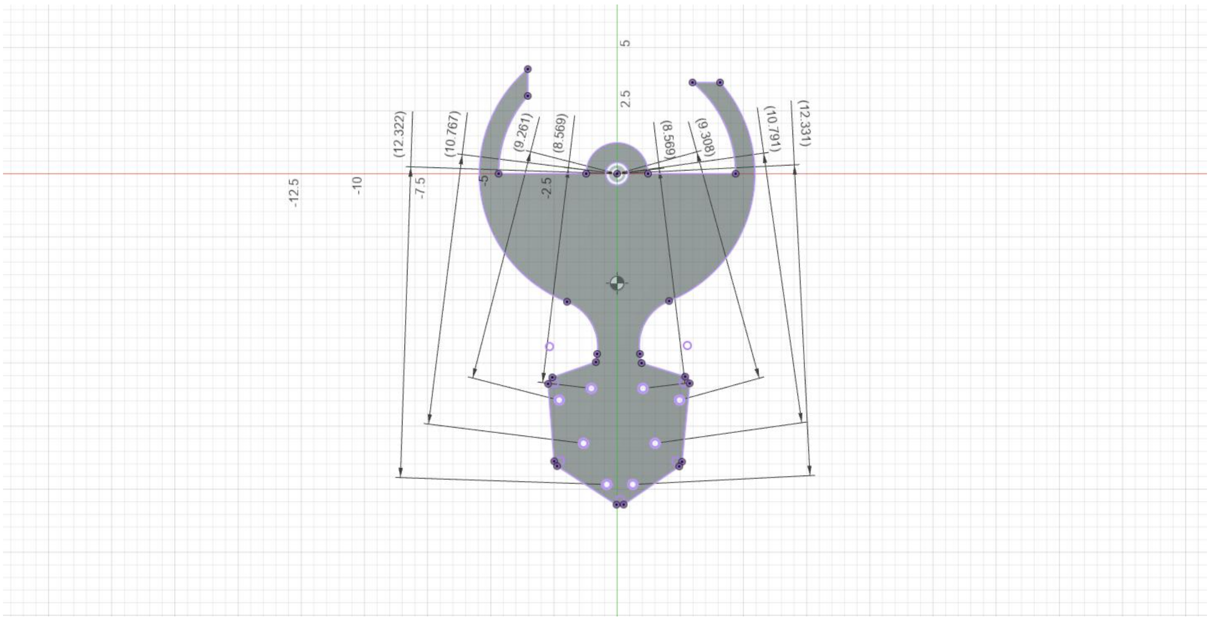

Figure 3: Actual length between bolts and pivot point

I multiplied the length of each bolt with the mass of the nuts, and calculated the bolt's moment of inertia. In addition to assuming uniform weights and densities in the bolts and nuts, I assumed uniform densities in the acrylics. The table below includes the moment inertia of the pendulum.

Table 2: Moments of inertia of bolts

Moment of Inertia of Pendulum	I_a	0.2630815 g.m^2
Moment of Inertia of Bolt 1	I_bolt1	0.0293711044 g.m^2
Moment of Inertia of Bolt 2	I_bolt2	0.0293711044 g.m^2
Moment of Inertia of Bolt 3	I_bolt3	0.0343064484 g.m^2
Moment of Inertia of Bolt 4	I_bolt4	0.0346555456 g.m^2
Moment of Inertia of Bolt 5	I_bolt5	0.0463713156 g.m^2
Moment of Inertia of Bolt 6	I_bolt6	0.0465782724 g.m^2
Moment of Inertia of Bolt 7	I_bolt7	0.0607326736 g.m^2
Moment of Inertia of Bolt 8	I_bolt8	0.0608214244 g.m^2
Total Moment of Inertia	I_total	0.6052893888 g.m^2

The total moment of inertia of the pendulum with bolts and nuts was then calculated by summing up all the moments of inertia, which gave me the value of 0.605 g.m^2. Then, using the equation 7, I plugged in the value of rotational inertia into the equation and calculated the natural frequency in rad/sec. I got 66.54 rad/sec. Converting this value in Hertz by using equation 4, and then using equation 5 and equation 6 to find the time of one complete revolution of the escapement wheel, I got 11.32 seconds as the time for one complete revolution. It was observed that the values given by rigid body analysis were more precise as the rotational moment of inertia was taken into account for this analysis.

Experimental Result

The analysis gave me a very powerful answer to the problem when I performed a simple task. It took me two weeks to fabricate and assemble the Clock Pendulum. Once it has reached one revolution of the escapement wheel, I simply used a stopwatch to record it. I recorded multiple times. In order to compare the data for Point Mass Analysis and Rigid Body Analysis, I averaged the timing after a full set of revolutions. As humans, our physical abilities would not allow us to keep up with the exact timing of the one revolution of the escapement wheel, so I took an average of all the measurements. With this calculation, I found the average time of the pendulum clock to be 10.7 seconds. To analyze which of the analysis of the clock was more precise, I calculated the percentage error in the timing of the clock using the equation 9 below.

Percent Error

$$\% \, Error = \frac{Time_{measured} - Time_{calculated}}{Time_{measured}} * 100\% \qquad (9)$$

where,

$Time_{measured}$ is the real-time values

$Time_{calculated}$ is found by analysis

The percentage error measured by this method was 34.7% for the point mass analysis and 5.79% for the rigid body analysis.

Discussion

I was able to estimate the clock pendulum timing based on the theoretical calculations. So far, it is the easiest and best method to calculate the timing of one revolution of the escapement wheel, although it is not perfectly able to demonstrate the natural frequency. According to this research, the percentage error in clock timing using point mass analysis was very large in comparison to the percentage error in clock timing using rigid body analysis. L_{com} estimate has a low Percent Error but the timing fails to meet desirable and real-life data standards. According to our hypothesis, it happened because we were not taking into account the pendulum's rotational moment of inertia. It is for this reason that Rigid Body Analysis is superior to Point Mass Analysis.

As a result of these two analyses, I was able to demonstrate that for a pendulum that isn't symmetrical around all axis, rigid body analysis should be used because the mass of the pendulum is distributed over all its body. If it is a simple shaped pendulum, in which mass is centered at the center of the object, we could use the Point Mass Analysis to estimate the timing correctly without much error.

The Rigid Body Analysis performed better for this Pendulum Clock because it takes into account the distributed mass in the Pendulum instead of the Point Mass Analysis. It is evident from the lower error percentage of the rigid body analysis as compared to the point mass analysis. It follows that the rotational moment of inertia is not negligible in this particular pendulum.

Appendix

Pendulum Timing Analysis

Name: Rayyan Khalid
Section: B04

Variable Description	Variable Name	Values/Equations	Units	Comments
Acrylic Pendulum Specifications				
Area	A	82.361	cm^2	
Thickness	t	0.635	cm	
Volume	Vol	52.299	cm^3	
Density	p	1.188	gm/cm^3	
Calculated Mass of Acrylic	M_Calc	62.131212	gm	
Actual Mass of Acrylic	M_Act		gm	
Length to Center of Mass of Acrylic	La	4.35	cm	
Percent Error in Acrylic Mass Calculation	M_Error	#DIV/0!	%	
Calculate Total Mass of Pendulum				
Mass of One Bolt with Two Nuts	Mb	4	gm	
Number of Bolts with Two Nuts	Nb	8		
Total Mass of Pendulum with Nuts and Bolts	Mt	94.131212		calculated
Calculate Center of Mass of Pendulum with Bolts				
Length to Center of Mass of Bolt 1	L_bolt1	0.12315	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 2	L_bolt2	0.10685	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 3	L_bolt3	0.08973	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 4	L_bolt4	0.08508	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 5	L_bolt5	0.08508	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 6	L_bolt6	0.08973	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 7	L_bolt7	0.10685	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 8	L_bolt8	0.12315	m	distance from pivot point to bolt
Length to Center of Mass **in Meters**	Lcom_meter	2.905617025	**meters**	make sure you convert meters
Estimated Center of Mass of Pendulum with Nuts and Bolts	Lcom_est			
Percent Error in Pendulum Nuts and Bolts Lcom Estimate	Lcom_error			
Calculate Natural Frequency and Timing using Point Mass Assumption				
Gravitational Constant	g	9.8	m/s^2	
Natural Frequency in radians/sec	nat_freq_rad_sec	1.836512345	rad/sec	
Natural Frequency in Hz	nat_freq_hz	0.2922900178	Hz	
Period of Oscillation	period	3.4212595	sec/cycle	
Number of Teeth on Escapement Wheel	nteeth	14		
Calculated Time of One Revolution of Escapement Wheel	time_calc	47.89763299	sec	
Measured Time of One Revolution of Escapement Wheel	time_meas			
Percent Error in Clock Timing	time_error	#DIV/0!		
Calculate Natural Frequency and Timing using Rigid Body Assumption				
Moment of Inertia of Pendulum	I_a	0.2630815	g.m^2	
Moment of Inertia of Bolt 1	I_bolt1	0.0293711044	g.m^2	
Moment of Inertia of Bolt 2	I_bolt2	0.0293711044	g.m^2	
Moment of Inertia of Bolt 3	I_bolt3	0.0343064484	g.m^2	
Moment of Inertia of Bolt 4	I_bolt4	0.0346555456	g.m^2	
Moment of Inertia of Bolt 5	I_bolt5	0.0463713156	g.m^2	
Moment of Inertia of Bolt 6	I_bolt6	0.0465782724	g.m^2	
Moment of Inertia of Bolt 7	I_bolt7	0.0607326736	g.m^2	
Moment of Inertia of Bolt 8	I_bolt8	0.0608214244	g.m^2	
Total Moment of Inertia	I_total	0.6052893888	g.m^2	
Natural Frequency in radians/sec	rb_nat_freq_rad_sec	66.54532013	rad/sec	
Natural Frequency in Hz	rb_nat_freq_hz	10.59101664	Hz	
Period of Oscillation	rb_period	0.09441964205	sec	
Calculated Time of One Revolution of Escapement Wheel	rb_time_calc	11.321874989	sec	
Percent Error in Clock Timing	rb_time_error			

Pendulum Timing Analysis

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Section: B04

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Area	A	82.361	cm^2	
Thickness	t	0.635	cm	
Volume	Vol	52.299	cm^3	
Density	p	1.188	gm/cm^3	
Calculated Mass of Acrylic	M_Calc	=Vol*p	gm	
Actual Mass of Acrylic	M_Act		gm	
Length to Center of Mass of Acrylic	La	4.35	cm	
Percent Error in Acrylic Mass Calculation	M_Error	=(M_Act-M_Calc)/M_Act * 100	%	
Calculate Total Mass of Pendulum				
Mass of One Bolt with Two Nuts	Mb	4	gm	
Number of Bolts with Two Nuts	Nb	8		
Total Mass of Pendulum with Nuts and Bolts	Mt	=M_Calc+Nb*Mb		calculated
Calculate Center of Mass of Pendulum with Bolts				
Length to Center of Mass of Bolt 1	L_bolt1	0.12315	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 2	L_bolt2	0.10685	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 3	L_bolt3	0.08973	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 4	L_bolt4	0.08508	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 5	L_bolt5	0.08508	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 6	L_bolt6	0.08973	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 7	L_bolt7	0.10685	m	distance from pivot point to bolt
Length to Center of Mass of Bolt 8	L_bolt8	0.12315	m	distance from pivot point to bolt
Length to Center of Mass **in Meters**	Lcom_meter	=(M_Calc*La+Mb*(SUM(C22:C29)))/I	**meters**	make sure you convert meters
Estimated Center of Mass of Pendulum with Nuts and Bolts	Lcom_est			
Percent Error in Pendulum Nuts and Bolts Lcom Estimate	Lcom_error			
Calculate Natural Frequency and Timing using Point Mass Assumption				
Gravitational Constant	g	9.8	m/s^2	
Natural Frequency in radians/sec	nat_freq_rad_sec	=(g/Lcom_meter)^(1/2)	rad/sec	
Natural Frequency in Hz	nat_freq_hz	=nat_freq_rad_sec/(2*PI())	Hz	
Period of Oscillation	period	=1/nat_freq_hz	sec/cycle	
Number of Teeth on Escapement Wheel	nteeth	14		
Calculated Time of One Revolution of Escapement Wheel	time_calc	=period*nteeth*1	sec	
Measured Time of One Revolution of Escapement Wheel	time_meas			
Percent Error in Clock Timing	time_error	=((time_calc-time_meas)/time_meas)		
Calculate Natural Frequency and Timing using Rigid Body Assumption				
Moment of Inertia of Pendulum	I_a	0.2630815	g.m^2	
Moment of Inertia of Bolt 1	I_bolt1	=Mb*0.08569^2	g.m^2	
Moment of Inertia of Bolt 2	I_bolt2	=Mb*0.08569^2	g.m^2	
Moment of Inertia of Bolt 3	I_bolt3	=Mb*0.09261^2	g.m^2	
Moment of Inertia of Bolt 4	I_bolt4	=Mb*0.09308^2	g.m^2	
Moment of Inertia of Bolt 5	I_bolt5	=Mb*0.10767^2	g.m^2	
Moment of Inertia of Bolt 6	I_bolt6	=Mb*0.10791^2	g.m^2	
Moment of Inertia of Bolt 7	I_bolt7	=Mb*0.12322^2	g.m^2	
Moment of Inertia of Bolt 8	I_bolt8	=Mb*0.12331^2	g.m^2	
Total Moment of Inertia	I_total	=SUM(C45:C53)	g.m^2	
Natural Frequency in radians/sec	rb_nat_freq_rad_sec	=((Mt*g*Lcom_meter)/I_total)^(1/2)	rad/sec	
Natural Frequency in Hz	rb_nat_freq_hz	=rb_nat_freq_rad_sec/(2*PI())	Hz	
Period of Oscillation	rb_period	=1/rb_nat_freq_hz	sec	
Calculated Time of One Revolution of Escapement Wheel	rb_time_calc	=rb_period*nteeth*1	sec	
Percent Error in Clock Timing	rb_time_error			

Work Cited

MAE 03, Tutorial 6

MAE 03, Tutorial 7