Controls Report:
Rectilinear Position Control

MAE 171A
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I. ABSTRACT

In this study, we explored and characterized concepts of control systems, specifically PID control, by conducting experiments with a mass-spring system with varying degrees of freedom in order to determine parameters (mass, spring, and damping constants). We investigated the behaviour and control of a rectilinear position control system consisting of a two degrees of freedom mechanical system, which included coupled carts, springs, and dampers. The objective was to plan and design a closed-loop PID control system to achieve step displacement with minimal overshoot (< 25%) and a fast settling time (< 2% error to final value). The system dynamics were characterized by open-loop step response experiments which allowed us to obtain estimated values for mass, damping, and stiffness parameters. We carried out a validation of these parameters using an linear time-invariant (LTI) model, whose results were utilised for PID controller design. We evaluated the performance of these controllers through step response tests and compared them against simulation predictions. Results demonstrated that a tuned PID control can minimize overshoot and settling time while maintaining stability. The experiment demonstrates the practical application of control theory in a real-world dynamic system.

II. INTRODUCTION

The study of mechanical systems and their dynamic behaviour is fundamental to engineering applications ranging from vehicle suspensions to robotic automation. Understanding how these systems respond to external forces, damping, and control inputs enables engineers to design stable and efficient mechanisms [1]. This experiment focuses on a 2 DOF rectilinear system consisting of two carts connected by springs and dampers. The primary objective is to analyze the system's response under different conditions and implement control strategies to achieve precise motion with minimal overshoot and settling time.

The experiment was conducted in three phases. First, key system parameters, including mass, damping, and stiffness, were determined by observing the system's free and forced responses. Next, the system was modeled using a linear time-invariant (LTI) approach, allowing for numerical simulations to predict system behaviour. Finally, Proportional-Integral-Derivative (PID) controllers were designed and tested to optimize system response. The PID controller was fine-tuned to minimize oscillations and achieve fast stabilization.

This report details the experimental setup, methodology, and findings, comparing real-world results with theoretical predictions. Through this study, valuable insights were gained into system identification, control implementation, and the challenges of real-world dynamic system regulation in applications such as robotics, aerospace, industrial automation, and more [1].

III. PROCEDURES

III.1 Calculating System Parameters

Calculating the physical characteristics of our setup (masses, the spring constants and damping coefficients) [2]. This involved three experiments:

1. Dual Free Mass Experiment (2 degrees of freedom): In our first setup, we allowed both masses connected by springs to move freely. This approach helped us understand the impact of the first spring when the system stabilized, as the second mass moved without constraint, simulating a scenario where only one spring influenced the system dynamics significantly after initial movements. We calculated k_1 from this experiment using the formula $k = F/x$, where F is the force and x is the displacement, both of which were obtained from the simulation setup.
2. Single Free Mass 1 Experiment (1 degree of freedom): In the second experiment, we fixed the second mass and only allowed the first mass to move. This configuration stretched the first spring while compressing the second, providing us with data on how the springs individually and collectively contribute to the system's behaviour. So from this experiment, we could calculate m1, d1 and k^{hat} using the formulas in [A1].
3. Single Free Mass 2 Experiment (1 degree of freedom): For the third test, we fixed the first mass and manually applied a force to the second mass, engaging only the second spring and damper. From the results of the first two experiments, we could calculate the individual spring constant of the second spring k2 by subtracting the first spring's constant from the combined stiffness observed when both springs were engaged. We could also calculate m2 and d2 through this experiment using the formulas in appendix [A3].

In each scenario, we conducted the tests five times to ensure accuracy and used standard formulas to compute the average values of the parameters and minimize errors.

III.2 PID Design: ZIEGLER - NICHOLS Tuning Method

The Ziegler-Nichols tuning method was used to design the PID controller for the rectilinear position control system. This method provides a systematic approach to determining the proportional (Kp), integral (Ki), and derivative (Kd) gains based on system response characteristics. The tuning process began by analyzing the open-loop system response, where a step input was applied without any control action to observe the system's natural behaviour. It was ensured that the system exhibited a stable oscillatory response before proceeding with the tuning process [3].

To determine the appropriate controller gains, the Ziegler-Nichols Ultimate Sensitivity Method was employed. Initially, the system was controlled using proportional-only (P) control, setting Ki and Kd to zero. The proportional gain (Kp) was then gradually increased until the system exhibited sustained oscillations (attained at Kp = 2). The ultimate gain (Ku) was recorded as the value of Kp at which continuous oscillations were observed, and the ultimate period (Pu) was measured as the time between consecutive peaks in the oscillatory response (Pu = 0.1067) [4].

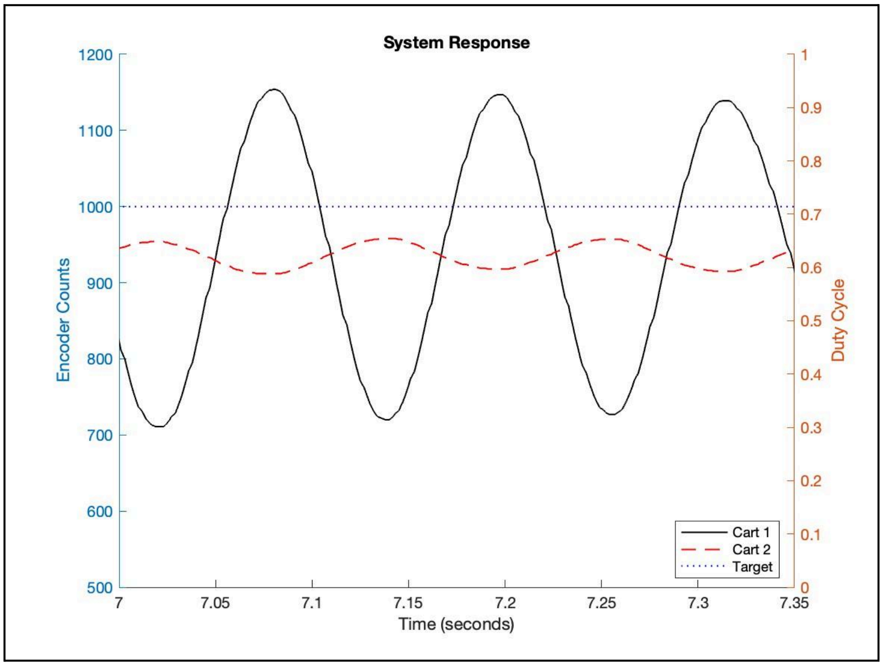


Figure 1: Zoomed view t = (7,7.35) [s] System Response for Ziegler-Nichols Ultimate Sensitivity Tuning Method, displaying evenly spaced oscillations.

Once Ku and Pu were determined, the standard Ziegler-Nichols tuning formulas were applied to compute the PID controller parameters using Fig.1 (full view in [A5]). According to the method, the full PID controller, which was selected for this experiment, was assigned the gains Kp = 0.45Ku, Ki = 1.2Ku/Pu, and Kd =0.075KuPu [4]. These values were then implemented in the control algorithm within the system's software.

Table 1: Kp, Ki, Kd Values for each PID Controller Using the Ziegler-Nichols Method

Controller Type	Kp	Ki	Kd
P	0.5Ku	0	0
PI	0.45Ku	0.54Ku/Pu	0
PD	0.8Ku	0	0.1KuPu
PID	0.45Ku	1.2Ku/Pu	0.075KuPu

After computing the initial PID gains, the controller was implemented and tested by applying a step input and analyzing the system's response. Key performance metrics such as overshoot (ensuring it remained below 25%), settling time (minimizing time to reach within 2% of final value), and steady-state error (ensuring near-zero error) were monitored. Ziegler-Nichols method provided a good starting point with all our constraints being met on the first attempt. The final controller design was validated by comparing the experimental closed-loop response with simulated LTI system results, demonstrating the practical application of PID tuning techniques in optimizing system performance while ensuring robustness in real-time control.

IV. RESULTS

After following the first section of the outlined procedures and applying the equations given to us in lecture [A1][A2][A3], we determined the key parameters of mass, damping, and stiffness constants for each cart. With these values, we could create plots to test the accuracy of our data in preparation for designing and tuning the PID controller. The mass, damping, and stiffness coefficients we determined are outlined in Table 1 below, averaging repeated experiments for consistent parameter values.

Cart	Mass	Damping	Stiffness Constant
Cart 1	1.616e-6	6.959e-6	1.9969e-4
Cart 2	2.905e-6	4.1673-6	5.2359e-4

To validate these parameters, we conducted step response tests and compared the results with those predicted by our simulations using the code provided on Canvas [A4] to form plots displaying the step response of the model, comparing the simulated step response to the measured step response. For instance, the step response of the 2 DOF model for cart 1 in Figure 2 shows the simulated and measured responses. This comparison highlighted the accuracy of our system modeling, as the measured response closely followed the simulated trajectory, confirming the correctness of the estimated stiffness, spring, and damping values.

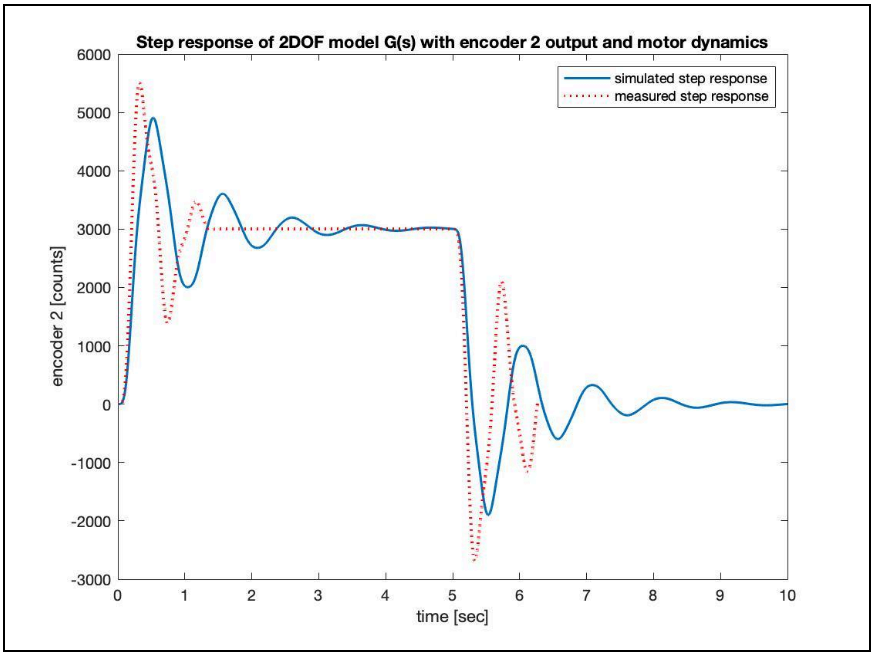


Figure 2: *Simulated step response (blue) and measured step response (red dots) of a 2 Degrees of Freedom (DOF) Model with Encoder*

The slight discrepancies between the simulated and actual responses could be attributed to simplifications in the model, such as assuming linear behaviour and ignoring minor nonlinearities present in the actual setup. These differences are valuable for refining our model and understanding the limitations of our experimental setup.

PID Controller Performance

After validating the system parameters, we implemented the PID controller using the Ziegler-Nichols tuning method. The controller was designed to minimize overshoot and ensure rapid settling to the desired position. The performance of the PID controller was assessed by applying step inputs and observing the system's response.

The System Response plot in [A6] features the performance of Cart 1 (red line) and Cart 2 (black line) over time (the zoomed in version in Fig. 3). The target encoder counts (dotted line) represent the desired position of the carts.

The PID controller effectively managed the system’s response, keeping overshoot below 25% and achieving a settling time within 2% of the target position quickly. This performance was consistent across multiple trials, demonstrating the robustness of our control strategy.

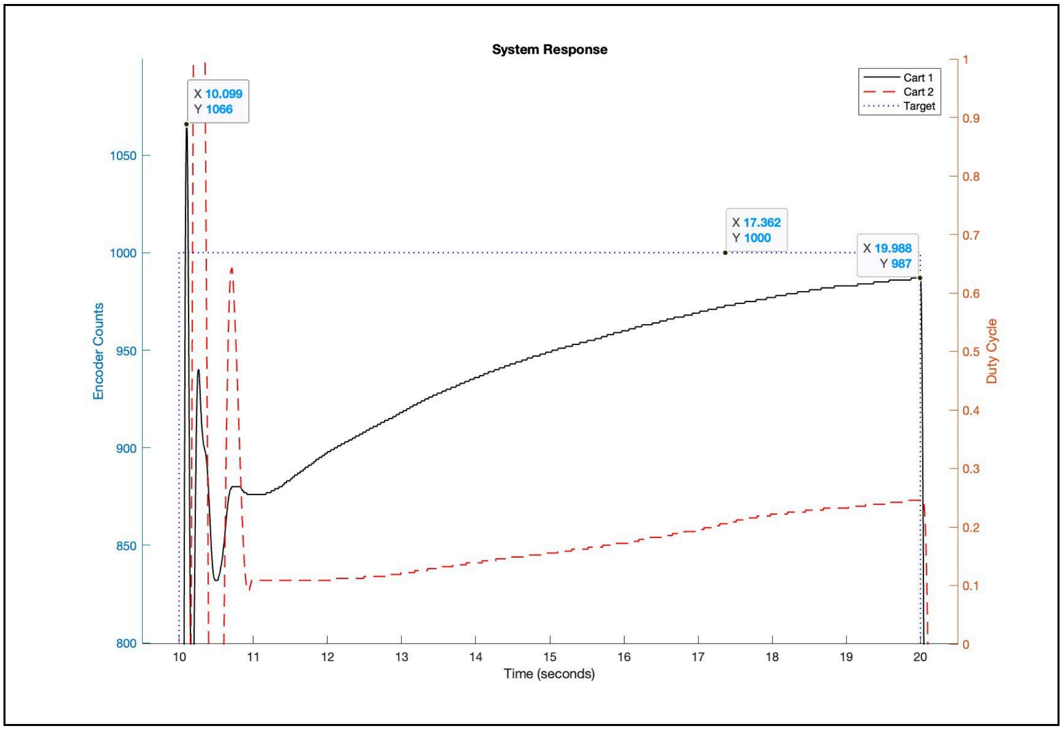


Figure 3: *Zoomed view t = (10,20) [s] PID controlled carts system response with time interval of 10-20 seconds, Cart 1 (Black), Cart 2 (Red Stripes) and the Target (Blue dots) encoder counts.*

Cart 1 shows rapid escalation towards the target, overshooting slightly before stabilizing. The sharp peaks early in the response indicate aggressive initial corrections by the PID controller, which quickly dampens to fine-tune the cart's position. At approximately 17 seconds, Cart 1 achieves a stable state very close to the target position. This quick stabilization showcases the effectiveness of the PID tuning, adhering to our design specifications of keeping over-shoot under 25% and settling within 2% of the target position in a short duration.

The PID controller settings, derived from the Ziegler-Nichols tuning approach, are detailed in Table 2 below.

Controller Type	Kp	Ki	Kd
PID	0.9	0.293	0.016

The visualizations and plots from the experiment illustrated how changes in system parameters and controller gain constants uniquely impact the system's response. Ensuring that the obtained parameters were precise was crucial in order to set up the physical system correctly. Then, tuning the gain values with well-known theorems and methods allowed for designing a PID that fit the required tolerances even small mathematical adjustments had large effects on real-life performance.

V. DISCUSSION

The experiments conducted for this rectilinear position control system experiment provided a comprehensive evaluation of the mass-spring system's dynamic properties and effectiveness of PID control in achieving desired operational criteria. Our approach involved initially determining the system's mass, damping, and stiffness values to model the system through controlled experimental setups.

With the parameters established, we implemented a PID controller using the Zeigler-Nichols tuning method. This classic approach allowed us to refine the controller settings based on the system's response to initial testing, focusing on minimizing overshoot and optimizing settling times.

The effectiveness of the PID controller was evident in the system's response to step inputs. Based on the data points on Figure 3, the percent overshoot (%OS) and steady state error (Ess) is as follows:

$$\%OS = \left|\frac{1066-1000}{1000}\right| \times 100\%$$
$$\%OS = 6.6\,\%$$

$$\%Ess = \left|\frac{987-1000}{1000}\right| \times 100\%$$
$$\%Ess = 1.31\%$$

As seen above, the designed controller adeptly managed to keep overshoot below 25% and ensured rapid settling to the target position within 2% error margin, aligning with our design goals. By calculating the percent overshoot (%OS) and steady-state error (%Ess), we quantified the controller's precision and efficiency. The %OS and %Ess were calculated based on the deviations observed from the target encoder counts, with results indicating an overshoot of 6.6% and a steady-state error of 1.31%. These performance metrics underscore the PID controller's capability to stabilize the system swiftly and maintain control over its dynamic response, even in the presence of potential disturbances and modeling inaccuracies.

CONCLUSION

Our exploration into PID control of a rectilinear position control system provided substantial insights and demonstrated the application of theoretical control concepts in a tangible setup. Throughout our experiment, we focused on understanding the system's dynamic and refining a PID controller to meet specific performance goals, such as minimal overshoot and quick settling times.

The successful implementation of the PID controller highlighted its effectiveness in achieving precise control over the system's movement. By adhering to the Ziegler-Nichols tuning method, we were able to configure the controller to respond adeptly to changes, maintaining stability and accuracy, in the face of dynamic challenges. The experiments validated our theoretical models, showing only minor deviations which were essential for further refinement of our control strategies.

Through testing and adjustment, the controller consistently met our expectations of our approach. The error analysis further affirmed the controller's capability to manage system dynamics efficiently, providing a reliable basis for future enhancements and applications in more complex systems. Our findings contribute valuable knowledge to the field of control systems, supporting ongoing efforts to optimize mechanical systems for better performance and reliability in various engineering contexts.

REFERENCES

[1] TechTarget. *Control system*. Retrieved from https://www.techtarget.com/whatis/definition/control-system

[2] Kleissl, J., De Callafon, R. (2025). *Position control*. MAE 171A.

[3] University of Utah, Department of Electrical and Computer Engineering. (2022). *ECE 3510 – Week 10 lecture notes*. Retrieved from https://my.ece.utah.edu/~ece3510/ECE3510_Week10_F22.pdf

[4] Ctrl Alt FTC. *Tuning methods of a PID controller*. Retrieved from https://www.ctrlaltftc.com/the-pid-controller/tuning-methods-of-a-pid-controller

Individual Contributions:

Alice: Wrote parts of the results section, wrote parts of the discussion section, created citations and references, proofreading.

Tanmay: Wrote Abstract, Introduction, parts of results, PID calculations, proofread.

Ravi: Wrote PID controller error analysis,part of PID calculation, plotted and titled all figures/tables, formatted appendix, proofread.

Rayyan: Wrote first half of procedures, Introduction, parts of the results section, parts of the discussion section, and wrote the conclusion.

APPENDIX

Background theory: step response of a 1DOF system

RESULT:

Consider a 1DOF system with single mass m, damping d and stiffness k and let us define

with $\omega_n := \sqrt{\frac{k}{m}}$ (resonance) and $\beta := \frac{1}{2}\frac{d}{\sqrt{mk}}$ (damping ratio)

then a step input $u(t) = U,\ t \geq 0$ of size U on the 1DOF system results in the output response

$$y(t) = \frac{U}{k}\left[1 - \mathrm{e}^{-\beta\omega_n t}\sin(\omega_d t + \phi)\right]$$

where

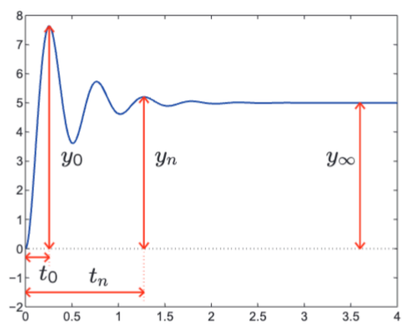
$$\omega_d = \omega_n\sqrt{1-\beta^2} \quad \text{damped resonance frequency in rad/s}$$

$$\phi = \tan^{-1}\frac{\sqrt{1-\beta^2}}{\beta} \quad \text{phase shift of response in rad}$$

[A1]: Formulas obtained from lecture of resonance and damping ratio obtained for a 1 DOF system

Outline of Lab Work: estimation of model parameters

With the times t_0, t_n and the values y_0, y_n and y_∞ from step response:



Allows us to estimate:

$$\hat{\omega}_d = 2\pi\frac{n}{t_n - t_0} \quad \text{(damped resonance frequency)}$$

$$\beta\hat{\omega}_n = \frac{1}{t_n - t_0}\ln\left(\frac{y_0 - y_\infty}{y_n - y_\infty}\right) \quad \text{(exponential decay term)}$$

where $n =$ number of oscillations between t_n and t_0.

[A2]: Controls lecture notes specifying y_0, y_n, and y_{inf} in a step response graph.

Outline of Lab Work: estimation of model parameters

With the computed

$$\widehat{\omega}_n = \sqrt{\widehat{\omega}_d^2 + (\beta\widehat{\omega}_n)^2} \quad \text{(undamped resonance frequency)}$$

$$\widehat{\beta} = \frac{\beta\widehat{\omega}_n}{\widehat{\omega}_n} \quad \text{(damping ratio)}$$

we can now find the estimates

$$\widehat{k} = \frac{U}{y_\infty} \quad \text{(stiffness constant)}$$

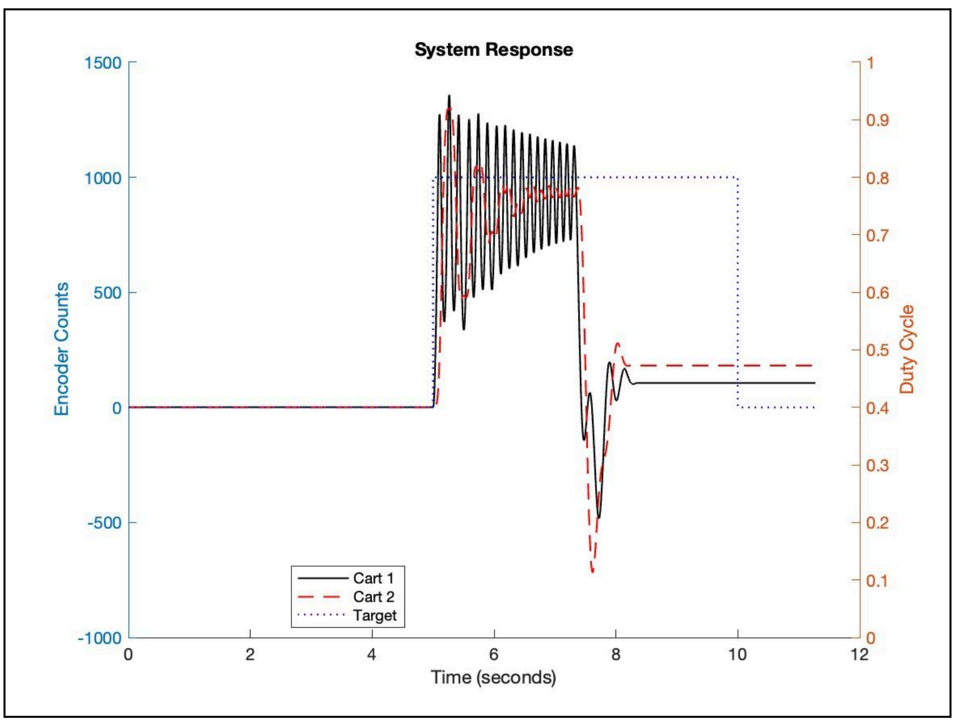
$$\widehat{m} = \widehat{k} \cdot \frac{1}{\widehat{\omega}_n^2} \quad \text{(mass/inertia)}$$

$$\widehat{d} = \widehat{k} \cdot \frac{2\widehat{\beta}}{\widehat{\omega}_n} \quad \text{(damping constant)}$$

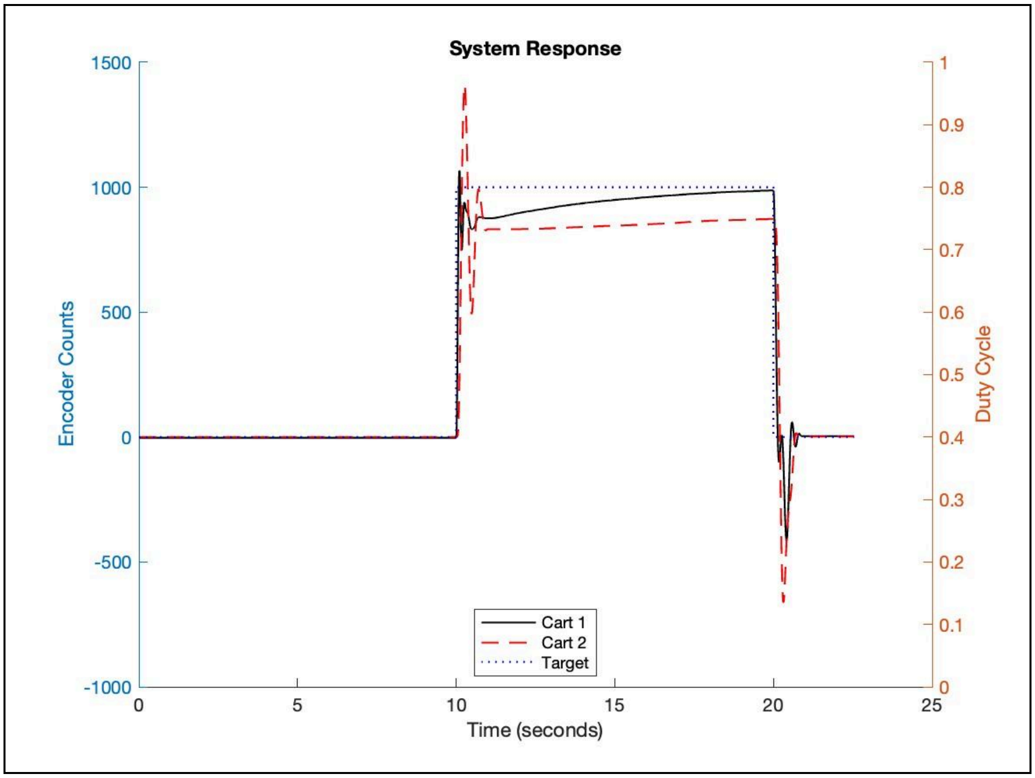
[A3]: Controls lecture notes specifying formulas for damping ratio, stiffness constant, mass/inertia and damping constant.

https://drive.google.com/drive/u/1/folders/1N3z8n0JceIOZHUxohEzzWKhaSu712k0B

[A4]: Link to Team Google Drive, including all MATLAB codes and lab results



[A5]: Full view of *system response for Ziegler-Nichols Ultimate Sensitivity Tuning Method*



[A6]: Full view of *PID controlled carts system response with Cart 1 (black), Cart 2 (red stripes) and the Target (blue dots) encoder counts.*