4.2 – Linear Programming Problems: Minimization – The Dual Problem

Suppose you are given a minimization problem:

Minimize
$$C = -2x - 3y$$

subject to $5x + 4y \le 32$
 $x + 2y \le 10$
 $x \ge 0, y \ge 0$

Notice that the constraints appear to be set up in a standard maximization problem. BUT... the problem is a minimization problem.

To remedy this, instead of minimizing C = -2x - 3y, we maximize -C = 2x + 3y. Set up the problem again, but as a maximization

Maximize
$$-C = P = 2x + 3y$$

subject to $5x + 4y \le 32$
 $x + 2y \le 10$
 $x \ge 0, y \ge 0$

Finally, place the data into a simplex tableau and finish the problem.

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Standard Minimization Problem

A standard minimization problem (where inequalities involve \geq) can be transformed into a standard maximization problem (where inequalities involve \leq) by the following method:

- 1. Write the table of data for the original problem
- 2. Put the objective function at the bottom (without the minus signs)
- 3. Now reverse the rows and columns to obtain a new data table
- 4. Make the inequalities involve \leq
- 5. Use u, v, ... for the standard variables.

The original problem is called the PRIMAL The associated problem is called the DUAL

Example:

Minimize
$$C = 10x + 11y$$

 $20x + 10y \ge 300$
Subject to $15x + 15y \ge 300$
 $10x + 20y \ge 250$

Write down the primal information:

\boldsymbol{x}	y	Constant
20	10	300
15	15	300
10	20	250
10	11	

Interchange the columns and rows and use variables u, v, w (one for each equation)

и	v	w	Constant
20	15	10	10
10	15	20	11
300	300	250	

This can be represented by the problem:

Maximize
$$P = 300u + 300v + 250w$$

$$20u + 15v + 10w \le 10$$

Subject to
$$10u + 15v + 20w \le 11$$

$$u \ge 0, v \ge 0, w \ge 0$$

This is a standard maximization problem.

Create the initial simplex tableau adding slack variables x and y.

и	v	W	$\boldsymbol{\mathcal{X}}$	У	P	Const
20	15	10	1	0	0	10
10	15	20	О	1	Ο	11
-300	-300	-250	0	0	1	0

What's the point?

The maximum of the original (PRIMAL) problem is the minimum of the DUAL problem, and vice versa.

Now	let's	solve	it:

Const

Const

Const

Const

Const

и	ν	W	х	У	Р	Const

и	ν	w	$\boldsymbol{\mathcal{X}}$	y	P	Const

и	ν	W	X	У	P	Const

и	ν	W	х	У	P	Const

Example:

Minimize
$$C = 21x + 14y$$

$$2x + 3y \ge 12$$

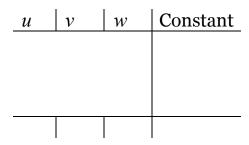
Subject to
$$3x + y \ge 6$$

$$x + 3y \ge 9$$

Primal Table

y	Constant
	<i>y</i>

Dual Table



Initial Simplex Tableau

и	v	W	X	У	P	Const

и	ν	w	$\boldsymbol{\mathcal{X}}$	У	P	Const

и	ν	W	х	У	Р	Const

u	ν	W	х	У	P	Const

и	v	\mathcal{W}	$\boldsymbol{\mathcal{X}}$	У	P	Const

и	ν	W	х	У	P	Const

u	ν	w	$\boldsymbol{\mathcal{X}}$	\mathcal{Y}	P	Const