4.2 – Linear Programming Problems: Minimization – The Dual Problem

Suppose you are given a minimization problem:

$$
\begin{aligned}
\text{Minimize} \quad & C = -2x - 3y \\
\text{subject to} \quad & 5x + 4y \le 32 \\
& x + 2y \le 10 \\
& x \ge 0, y \ge 0
\end{aligned}
$$

Notice that the constraints appear to be set up in a standard maximization problem. BUT… the problem is a minimization problem.

To remedy this, instead of minimizing $C = -2x - 3y$, we maximize $-C = 2x + 3y$. Set up the problem again, but as a maximization

$$
\begin{aligned}
\text{Maximize} \quad & -C = P = 2x + 3y \\
\text{subject to} \quad & 5x + 4y \le 32 \\
& x + 2y \le 10 \\
& x \ge 0, y \ge 0
\end{aligned}
$$

Finally, place the data into a simplex tableau and finish the problem.

	Const

	Const

	Const

	Const

	Const

	Const

	Const

Standard Minimization Problem

A standard minimization problem (where inequalities involve $\geq$) can be transformed into a standard maximization problem (where inequalities involve $\leq$) by the following method:

1. Write the table of data for the original problem

2. Put the objective function at the bottom (without the minus signs)

3. Now reverse the rows and columns to obtain a new data table

4. Make the inequalities involve $\leq$

5. Use $u, v, \ldots$ for the standard variables.

The original problem is called the PRIMAL
The associated problem is called the DUAL

Example:

Minimize $C = 10x + 11y$

Subject to
$$\begin{aligned} 20x + 10y &\geq 300 \\ 15x + 15y &\geq 300 \\ 10x + 20y &\geq 250 \end{aligned}$$

Write down the primal information:

x	y	Constant
20	10	300
15	15	300
10	20	250
10	11	

Interchange the columns and rows and use variables u, v, w (one for each equation)

u	v	w	Constant
20	15	10	10
10	15	20	11
300	300	250	

This can be represented by the problem:

Maximize $P = 300u + 300v + 250w$

$$20u + 15v + 10w \le 10$$

Subject to $10u + 15v + 20w \le 11$

$$u \ge 0, v \ge 0, w \ge 0$$

This is a standard maximization problem.

Create the initial simplex tableau adding slack variables x and y.

u	v	w	x	y	P	Const
20	15	10	1	0	0	10
10	15	20	0	1	0	11
-300	-300	-250	0	0	1	0

What's the point?

The maximum of the original (PRIMAL) problem is the minimum of the DUAL problem, and vice versa.

Now let's solve it:

	Const

	Const

	Const

	Const

	Const

u	v	w	x	y	P	Const

u	v	w	x	y	P	Const

u	v	w	x	y	P	Const

u	v	w	x	y	P	Const

Example:

Minimize $C = 21x + 14y$

Subject to
$$2x + 3y \geq 12$$
$$3x + y \geq 6$$
$$x + 3y \geq 9$$

Primal Table

x	y	Constant

Dual Table

u	v	w	Constant

Initial Simplex Tableau

u	v	w	x	y	P	Const

u	v	w	x	y	P	Const

u	v	w	x	y	P	Const

u	v	w	x	y	P	Const

u	v	w	x	y	P	Const

u	v	w	x	y	P	Const

u	v	w	x	y	P	Const