Characteristics That Impact Convergence Time of Recombination Markov Chain on Dual Graphs

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1 Result

1.1 Mixing Time of Plans with Three Districts and Nine Vertices

We mainly examine graphs with 9 vertices and plot different factors against the mixing for graph. Note that the y-axis in each graph stands for the time to convergence. Each unit represents one recombination.

1.2 Edges of Meta Graph

We first explore the relationship between the number of edges of meta graph and the convergence time. We want to see the strength of correlation between these two factors. The R^2-value is not very large while the p-value is almost zero which suggests that our result is statistically significant.

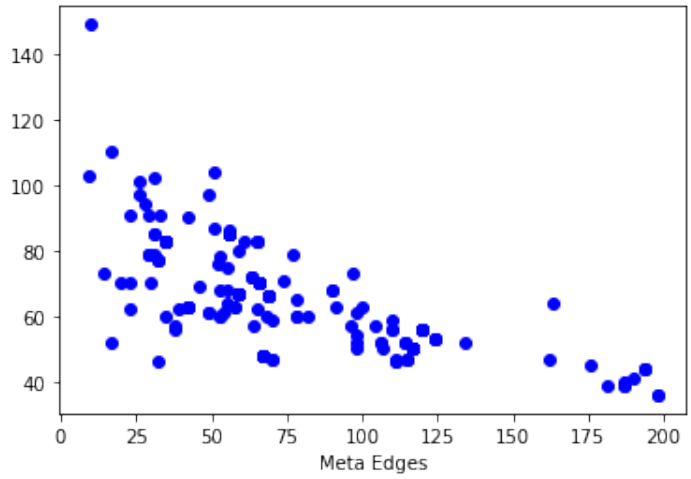
OLS Regression Results

Dep. Variable:	Time	R-squared (uncentered):	0.591
Model:	OLS	Adj. R-squared (uncentered):	0.588
Method:	Least Squares	F-statistic:	287.0
Date:	Thu, 25 Mar 2021	Prob (F-statistic):	1.91e-40
Time:	21:00:37	Log-Likelihood:	-1037.8
No. Observations:	200	AIC:	2078.
Df Residuals:	199	BIC:	2081.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Meta Edges	0.6068	0.036	16.941	0.000	0.536	0.677

Omnibus:	10.830	Durbin-Watson:	1.589
Prob(Omnibus):	0.004	Jarque-Bera (JB):	11.117
Skew:	-0.514	Prob(JB):	0.00386
Kurtosis:	3.528	Cond. No.	1.00

Notes:
[1] R² is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.



In the same vein, given that density is calculated in proportion to number of edges, it is only reasonable that we obtain similar results when we run this variable against convergence time.

OLS Regression Results

Dep. Variable:	Time	R-squared (uncentered):	0.956
Model:	OLS	Adj. R-squared (uncentered):	0.956
Method:	Least Squares	F-statistic:	4344.
Date:	Thu, 25 Mar 2021	Prob (F-statistic):	3.88e-137
Time:	21:00:37	Log-Likelihood:	-814.31
No. Observations:	200	AIC:	1631.
Df Residuals:	199	BIC:	1634.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Meta Density	245.2576	3.721	65.911	0.000	237.920	252.595

Omnibus:	58.303	Durbin-Watson:	2.091
Prob(Omnibus):	0.000	Jarque-Bera (JB):	284.863
Skew:	-1.008	Prob(JB):	1.39e-62
Kurtosis:	8.488	Cond. No.	1.00

Notes:
[1] R^2 is computed without centering (uncentered) since the model does not contain a constant.
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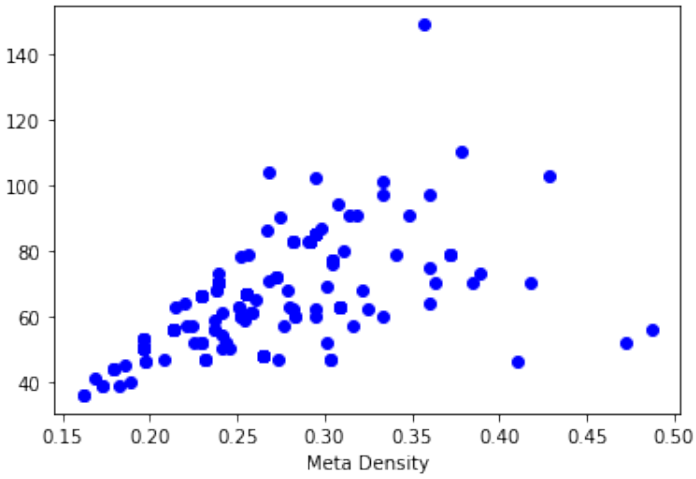


1.3 Edges of Dual Graph

We then begin to focus our attention on the planar graphs. We first run our regression on the number of edges in the dual graph against the mixing time. Note that we still see a very strong correlation between these two variables.

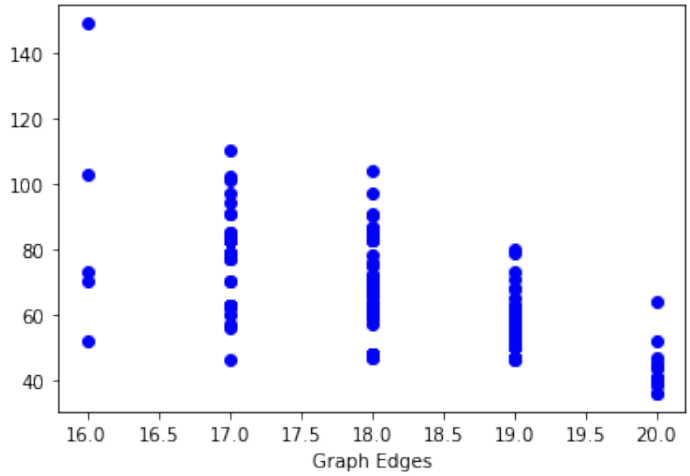
OLS Regression Results

Dep. Variable:	Time	R-squared (uncentered):	0.927
Model:	OLS	Adj. R-squared (uncentered):	0.927
Method:	Least Squares	F-statistic:	2541.
Date:	Thu, 25 Mar 2021	Prob (F-statistic):	2.85e-115
Time:	21:00:37	Log-Likelihood:	-864.89
No. Observations:	200	AIC:	1732.
Df Residuals:	199	BIC:	1735.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Graph Edges	3.5931	0.071	50.403	0.000	3.453	3.734

Omnibus:	30.087	Durbin-Watson:	2.011
Prob(Omnibus):	0.000	Jarque-Bera (JB):	56.352
Skew:	0.754	Prob(JB):	5.80e-13
Kurtosis:	5.118	Cond. No.	1.00

Notes:
[1] R^2 is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.



Like our analysis on the meta graph, because graph density is derived from the number of vertices (which is fixed) and edges. we arrive at a similar result.

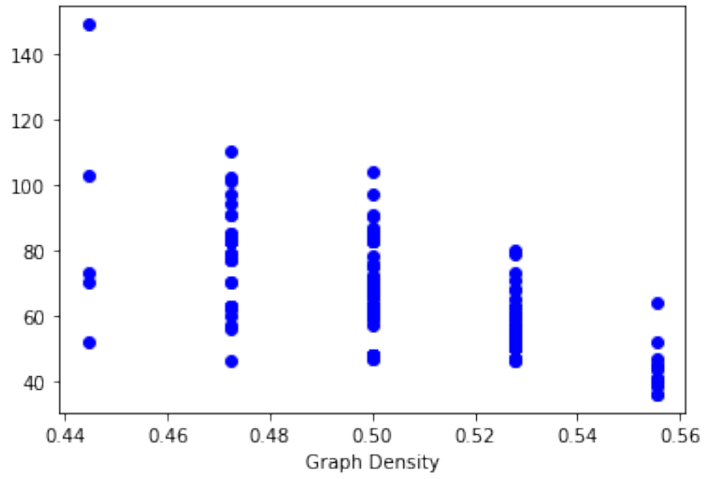
OLS Regression Results

Dep. Variable:	Time	R-squared (uncentered):	0.927
Model:	OLS	Adj. R-squared (uncentered):	0.927
Method:	Least Squares	F-statistic:	2541.
Date:	Thu, 25 Mar 2021	Prob (F-statistic):	2.85e-115
Time:	21:00:37	Log-Likelihood:	-864.89
No. Observations:	200	AIC:	1732.
Df Residuals:	199	BIC:	1735.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Graph Density	129.3509	2.566	50.403	0.000	124.290	134.412

Omnibus:	30.087	Durbin-Watson:	2.011
Prob(Omnibus):	0.000	Jarque-Bera (JB):	56.352
Skew:	0.754	Prob(JB):	5.80e-13
Kurtosis:	5.118	Cond. No.	1.00

Notes:
[1] R^2 is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.



1.4 Number of Spanning Trees

Next, we examine the number of spanning trees which is defined as a tree that include all vertices of our graph. We want to see if this variable has any correlation with convergence time. // To my

surprise, even though the number of spanning seems to capture more information about the structure of the graph in theory, this variable actually has a weaker correlation with convergence time than other factors that we have tested with a lower R^2-value than previous variables.

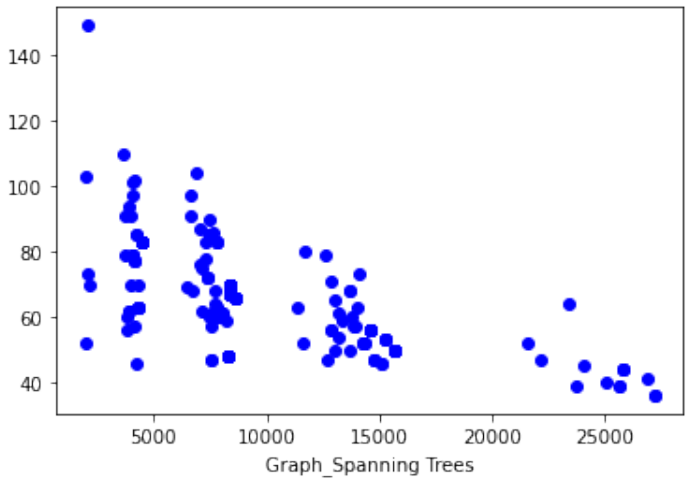
OLS Regression Results

Dep. Variable:	Time	R-squared (uncentered):	0.588
Model:	OLS	Adj. R-squared (uncentered):	0.586
Method:	Least Squares	F-statistic:	284.5
Date:	Thu, 25 Mar 2021	Prob (F-statistic):	3.21e-40
Time:	21:00:37	Log-Likelihood:	-1038.3
No. Observations:	200	AIC:	2079.
Df Residuals:	199	BIC:	2082.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Graph_Spanning Trees	0.0046	0.000	16.866	0.000	0.004	0.005

Omnibus:	16.316	Durbin-Watson:	1.553
Prob(Omnibus):	0.000	Jarque-Bera (JB):	18.202
Skew:	-0.647	Prob(JB):	0.000112
Kurtosis:	3.716	Cond. No.	1.00

Notes:
[1] R^2 is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.



1.5 Number of Triangles

Finally, we want to explore the number of triangles in our dual graph. It appears that the regression also has a high R^2-value and a low p-value.

OLS Regression Results

Dep. Variable:	Time	R-squared (uncentered):	0.911
Model:	OLS	Adj. R-squared (uncentered):	0.911
Method:	Least Squares	F-statistic:	2039.
Date:	Thu, 25 Mar 2021	Prob (F-statistic):	1.56e-106
Time:	21:00:37	Log-Likelihood:	-885.10
No. Observations:	200	AIC:	1772.
Df Residuals:	199	BIC:	1776.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Number of Triangles	2.0530	0.045	45.158	0.000	1.963	2.143

Omnibus:	24.311	Durbin-Watson:	1.947
Prob(Omnibus):	0.000	Jarque-Bera (JB):	42.657
Skew:	0.640	Prob(JB):	5.46e-10
Kurtosis:	4.865	Cond. No.	1.00

Notes:
[1] R² is computed without centering (uncentered) since the model does not contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.