2017 IEEE 56th Annual Conference on Decision and Control (CDC)
December 12-15, 2017, Melbourne, Australia

Electric Vehicle Charging Control in Residential Distribution Network: A Decentralized Event-Driven Realization

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Abstract— **Uncontrolled or improperly controlled electric vehicle (EV) charging can negatively impact electric distribution networks. In this paper, we develop a decentralized event-driven EV charging control scheme to achieve "valley-filling" (i.e., flattening demand profile during overnight charging), meanwhile meeting heterogeneous individual charging requirements and satisfying distribution network constraints. This control scheme is capable of handling EVs' random arrivals and departures. The formulated problem is an optimization problem with a non-separable objective function and strongly coupled inequality constraints. We describe a novel shrunken-primal-dual subgradient (SPDS) algorithm to support the decentralized control scheme, and verify its efficacy and convergence with a simple distribution network model.**

I. Introduction

Electric vehicles (EVs) have been proved to be effective in increasing energy conversion efficiency and reducing GHG emissions by relieving the usage of fossil-based fuel [1]. Beyond the GHG-reduction nature, the potential of controlling the charging process of EVs for the provision of grid services is envisioned in [2]. Extensive studies have been conducted on the modeling and control problems of EVs for "valley-filling" (i.e., flattening demand profile during overnight charging) [3], [4], load balancing [5], and frequency regulation [6], [7], to name a few. The potential of EV charging control for facilitating renewable energy integration and providing additional services are discussed in [8], [9]. Though showing promising application potentials, results in the aforementioned literature are all based on the assumption of no network constraints.

Apart from the benefits, the charging process of a large population of EVs would negatively affect the distribution network if it is not properly controlled. Lopes *et al.* [10] elaborated on the challenges of EVs' integration into the traditional mid- or low-voltage distribution network, including nodal voltage drop, transformer overloading, network congestion, and increased power loss. Authors in [11] verified the negative impacts by non-network-aware controlled charging. Other literature reviewing the impacts includes [12], [13]. Thus, controlling EV charging in the context of distribution network constraints is urged.

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This work was supported by the Natural Sciences and Engineering Research Council of Canada, the California Energy Commission Award EPIC 15-013, the National Science Foundation CyberSEES No. 1539585, and the State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources (Grant No. 17006).

Recently, more attention has been paid to studying EV charging control considering nodal voltage drop [14], [15]. Although these results can help alleviate the impacts, they were solely designed for meeting the constraints, thus not being able to be generalized for providing grid services. Currently, only few works have addressed this type of problem. Considering the network impacts, Richardson *et al.* [16] optimized the EV charging profiles to minimize the total power consumption. Luo and Chan [17] studied a real-time control to minimize power losses. Among all types of grid services, studies in a large amount of literature indicate that the best way EVs could serve the grid is to fill the overnight load valley [4], [18]. In our paper, we target at valley-filling meanwhile satisfying both local charging needs and distribution network constraints.

In EV charging control algorithm design, most literature takes a centralized approach [13]–[17], [19], which does not scale well as the number of EVs grows. Our objective is to design a decentralized optimal controller that can be embedded in local chargers and does not require a communication network among chargers. Designing such a control framework is notably challenging, because firstly the valley-filling objective is a coupled and non-separable function, and secondly the nodal voltages strongly couple the individual charging powers in form of a linear inequality. To our knowledge only a limited number of researchers have addressed the distributed/decentralized realization of such a problem. Chang *et al.* [20] proposed a consensus-based primal-dual perturbation algorithm for a distributed realization. Koshal *et al.* [21] developed a regularized primal-dual subgradient (PDS) algorithm for a decentralized realization, however errors exist due to the regularization of the Lagrangian function. Therefore, a decentralized algorithm that can solve the aforementioned optimization problem without regularization or convergence errors is deemed crucial.

Another important issue in charging control lies in the uncertainties. In the valley-filling problem, drivers may 1) arrive home and plug in EVs after the valley-filling has already started; and 2) break the commitment by plugging out EVs before the designated or committed charging deadlines. Having these two uncertainties present, it is impractical to keep the schedule calculated before valley-filling starts till the end. Therefore, developing a control scheme that can handle EVs' random arrivals and departures is urged.

The main contribution of this paper is two-fold. Firstly, this is the first paper studying network-aware EV charging control for the provision of valley-filling under mobility uncertainties. Secondly, a novel decentralized shrunken-primal-dual subgradient (SPDS) algorithm is described to solve optimization problems consisting of non-separable objective functions and strongly coupled network constraints without regularizing the Lagrangian function.

II. EV Charging and Distribution Network

A. EV charging model

The dynamics of the state-of-charge (SOC) of the ith EV can be represented by a first-order discrete-time system as

$$SOC_i(k+1) = SOC_i(k) + \eta_i \Delta t \frac{\bar{P}_i}{\bar{E}_i} u_i(k), \tag{1}$$

where η_i is the charging efficiency, Δt is the sampling time interval, $\bar{P}_i$ is the maximum charging power, $\bar{E}_i$ is the battery capacity, and $u_i(k) \in [0,1]$ is the charging rate.

Let $x_i(k)$ denote the energy remained to be charged to the ith EV for achieving the total required energy $E_{i,r}$, we have the charging dynamics written as

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k), \tag{2}$$

where $A_i = \mathbf{I}_1$ and $B_i = -\eta_i \Delta t \bar{P}_i$. In this paper, EVs' plug-in time, charging deadlines, initial SOCs, charging efficiencies, battery capacities, and maximum charging powers are heterogeneous. Augmenting all EVs at time k, we have

$$x(k+1) = x(k) + \sum_{i=1}^{n} B_{i,c} u_i(k), \tag{3}$$

where

$$x(k) = [x_1(k) \ x_2(k) \ \cdots \ x_n(k)]^\mathsf{T} \in \mathbb{R}^n,$$
$$u(k) = [u_1(k) \ u_2(k) \ \cdots \ u_n(k)]^\mathsf{T} \in \mathbb{R}^n,$$

and $B_{i,c} \in \mathbb{R}^n$ is the ith column of $B = \bigoplus_{i=1}^n B_i$. $\bigoplus$ denotes direct sum hereinafter.

Let $\tilde{k}$ and $\tilde{k}+K$ denote the start and end time of valley-filling, respectively; let k_i and $k_i + K_i$ denote the plug-in time and the designated (committed) charging deadline of the ith EV, respectively. Consider the case where all EVs are committed to the charging deadlines, then by augmenting system (3) along the valley-filling period $[\tilde{k}, \tilde{k}+K]$, we have

$$\begin{aligned} \mathcal{X}(\tilde{k}) &= \left[x(\tilde{k}+1|\tilde{k})^\mathsf{T} \ \cdots \ x(\tilde{k}+K|\tilde{k})^\mathsf{T}\right]^\mathsf{T} \\ &= \mathcal{M} x(\tilde{k}) + \sum_{i=1}^{n} \mathcal{B}_i \mathcal{U}_i(\tilde{k}) \in \mathbb{R}^{nK}, \end{aligned} \tag{4}$$

where

$$\mathcal{M} = [\mathbf{I}_n \ \mathbf{I}_n \ \cdots \ \mathbf{I}_n]^\mathsf{T} \in \mathbb{R}^{nK \times n},$$

$$\mathcal{B}_i = \begin{bmatrix} B_{i,c} & & \\ \vdots & \ddots & \\ B_{i,c} & \cdots & B_{i,c} \end{bmatrix} \in \mathbb{R}^{nK \times K},$$

and

$$\mathcal{U}_i(\tilde{k}) = \left[u_i(\tilde{k}|\tilde{k})^\mathsf{T} \ u_i(\tilde{k}+1|\tilde{k})^\mathsf{T} \ \cdots \ u_i(\tilde{k}+1|\tilde{k})^\mathsf{T}\right]^\mathsf{T} \in \mathbb{R}^K. \tag{5}$$

Note that control signals for the ith EV should be strictly constrained to 0 before the time $\max\{k_i, \tilde{k}\}$ and after the time $\min\{k_i + K_i, \tilde{k}+K\}$. Detailed charging control constraints will be elaborated on in Section III.

B. Distribution network model

In this paper, we consider a radial distribution network as shown in Fig. 1. This network is the single-phase IEEE-13 Node Test Feeder by discarding the transformer between Nodes 4 and 5, discarding the switch between Nodes 6 and 7, and assuming no capacitors. Let $\mathbb{H} = \{\imath | \imath = 1, \ldots, h\}$ denote

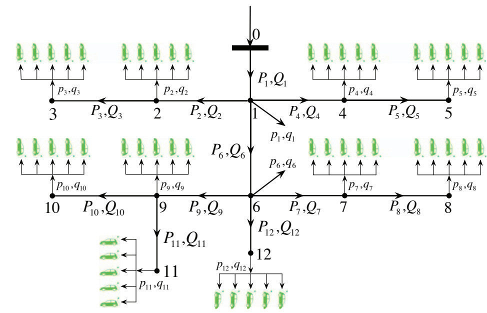

Fig. 1. A radial distribution network connected with EVs.

the set of nodes and let $\mathbb{E}$ denote the set of all downstream line segments. Node 0 is the feeder head, decoupling interactions in the downstream distribution system from the rest of the grid and maintaining its own nodal voltage magnitude $|V_0|$.

At time k, let $|V_\imath(k)|$ denote the voltage magnitude at Node $\imath$; let $p_\imath(k)$ and $q_\imath(k)$ denote the consumed real and reactive power at Node $\imath$; and with a slight abuse of notations let $r_{\imath\jmath}$ and $x_{\imath\jmath}$ denote the resistance and reactance of the line segment $(\imath, \jmath)$. According to [15], [22], [23], by omitting the line losses and higher-order terms, the LinDistFlow model of this distribution network can be written as

$$\boldsymbol{V}(k) = \boldsymbol{V}_0 - 2\boldsymbol{R}p(k) - 2\boldsymbol{X}q(k), \tag{6}$$

where

$$\begin{aligned} \boldsymbol{V}(k) &= \left[|V_1(k)|^2 \ \ |V_2(k)|^2 \ \ \cdots \ \ |V_h(k)|^2\right]^\mathsf{T} \in \mathbb{R}^h \\ \boldsymbol{V}_0 &= \left[|V_0|^2 \ \ |V_0|^2 \ \ \cdots \ \ |V_0|^2\right]^\mathsf{T} \in \mathbb{R}^h \\ p(k) &= [p_1(k) \ p_2(k) \ \cdots \ p_h(k)]^\mathsf{T} \in \mathbb{R}^h \\ q(k) &= [q_1(k) \ q_2(k) \ \cdots \ q_h(k)]^\mathsf{T} \in \mathbb{R}^h \end{aligned} \tag{7}$$

and

$$\boldsymbol{R} \in \mathbb{R}^{h \times h}, \ \boldsymbol{R}_{\imath\jmath} = \sum_{(\hat{\imath},\hat{\jmath}) \in \mathbb{E}_\imath \cap \mathbb{E}_\jmath} r_{\hat{\imath}\hat{\jmath}},$$

$$\boldsymbol{X} \in \mathbb{R}^{h \times h}, \ \boldsymbol{X}_{\imath\jmath} = \sum_{(\hat{\imath},\hat{\jmath}) \in \mathbb{E}_\imath \cap \mathbb{E}_\jmath} x_{\hat{\imath}\hat{\jmath}},$$

where $\mathbb{E}_\imath$ and $\mathbb{E}_\jmath$ are the sets containing downstream line segments connecting Node 0 and Node $\imath$ and connecting Node 0 and Node $\jmath$, respectively [15], [23].

At Node $\imath$, the consumed real and reactive powers are represented as

$$\begin{aligned} p_\imath(k) &= p_{\imath,b}(k) + p_{\imath,EV}(k), \\ q_\imath(k) &= q_{\imath,b}(k) + q_{\imath,EV}(k), \end{aligned} \tag{8}$$

where $p_{\imath,b}(k)$ and $q_{\imath,b}(k)$ denote the real and reactive baseline power, respectively, and $p_{\imath,EV}(k)$ and $q_{\imath,EV}(k)$ denote the real and reactive EV charging power, respectively.

Since (6) is linear, letting $\boldsymbol{V}_b(k)$ denote the squared voltage drop caused by the baseline load, we have

$$\boldsymbol{V}(k) = \boldsymbol{V}_0 - \boldsymbol{V}_b(k) - 2\boldsymbol{R}p_{EV}(k) - 2\boldsymbol{X}q_{EV}(k), \quad (9)$$

where $p_{EV}(k) = \left[p_{1,EV}(k)^\mathsf{T}\ p_{2,EV}(k)^\mathsf{T}\ \cdots\ p_{h,EV}(k)^\mathsf{T}\right]^\mathsf{T}$ and $q_{EV}(k) = \left[q_{1,EV}(k)^\mathsf{T}\ q_{2,EV}(k)^\mathsf{T}\ \cdots\ q_{h,EV}(k)^\mathsf{T}\right]^\mathsf{T}$. We further assume EVs only consume real power, resulting in $q_{\imath,EV}(k) = 0,\ \forall\ \imath \in \mathbb{H}$. Thus, (9) is rewritten as

$$\boldsymbol{V}(k) = \boldsymbol{V}_0 - \boldsymbol{V}_b(k) - 2\boldsymbol{R}p_{EV}(k). \quad (10)$$

Suppose $n_\imath$ EVs are connected at Node $\imath$, we have

$$p_{\imath,EV}(k) = \sum_{\hat{\imath}=1}^{n_\imath} \bar{P}_{\imath,\hat{\imath}} u_{\imath,\hat{\imath}}(k),\ \imath = 1, \ldots, h, \quad (11)$$

where $u_{\imath,\hat{\imath}}(k)$ denotes the charging rate of the EV connected at charger $\hat{\imath}$ of node $\imath$, and $\bar{P}_{\imath,\hat{\imath}}$ is the associated maximum charging power. It's worth noting that $\sum_{\imath=1}^{h} n_\imath = n$.

By following ascending orders of $\imath$ and $\hat{\imath}$, we replace the subscripts $\imath, \hat{\imath}$ by $i = 1, \ldots, n$, thus having (10) written as

$$\boldsymbol{V}(k) = \boldsymbol{V}_0 - \boldsymbol{V}_b(k) - 2\boldsymbol{R}G\bar{P}u(k), \quad (12)$$

where

$$G = \bigoplus_{\imath=1}^{h} G_\imath \in \mathbb{R}^{h\times n},\ \bar{P} = \bigoplus_{i=1}^{n} \bar{P}_i \in \mathbb{R}^{n\times n},$$

and $G_\imath = \mathbf{1}_{n_\imath}^\mathsf{T}$ is the charging power aggregation vector.

Let $D \in \mathbb{R}^{h\times n}$ denote $-2\boldsymbol{R}G\bar{P}$, $y_d(k)$ denote $\boldsymbol{V}_0 - \boldsymbol{V}_b(k)$, and $y(k)$ denote $\boldsymbol{V}(k)$, we have

$$y(k) = y_d(k) + Du(k), \quad (13)$$

where $y(k) = [y_1(k)\ \cdots\ y_h(k)]^\mathsf{T} \in \mathbb{R}^h$ and $y_d(k) = [y_{d1}(k)\ \cdots\ y_{dh}(k)]^\mathsf{T} \in \mathbb{R}^h$. Augmenting the system output $y(k)$ from $\tilde{k}$ to $\tilde{k} + K - 1$, we have

$$\mathcal{Y}_{\tilde{k}} = \mathcal{Y}_{d\tilde{k}} + \sum_{i=1}^{n} \mathcal{D}_i \mathcal{U}_i(\tilde{k}), \quad (14)$$

where

$$\mathcal{Y}_{\tilde{k}} = \left[y(\tilde{k}|\tilde{k})^\mathsf{T}\ y(\tilde{k}+1|\tilde{k})^\mathsf{T}\ \cdots y(\tilde{k}+K-1|\tilde{k})^\mathsf{T}\right]^\mathsf{T},$$
$$\mathcal{Y}_{d\tilde{k}} = \left[y_d(\tilde{k}|\tilde{k})^\mathsf{T}\ y_d(\tilde{k}+1|\tilde{k})^\mathsf{T}\ \cdots y_d(\tilde{k}+K-1|\tilde{k})^\mathsf{T}\right]^\mathsf{T},$$
$$\mathcal{D}_i = \bigoplus_{\kappa=1}^{K} D_i \in \mathbb{R}^{hK\times K},\ D = [D_1\ D_2\ \cdots\ D_n].$$

III. Controller Design

In this section, we start from a deterministic control problem, followed by the novel decentralized algorithm, and then present the event-driven control scheme.

A. Deterministic control problem

In the deterministic case, all EVs are plugged in before $\tilde{k}$ and all designated charging deadlines are later than $\tilde{k} + K$. Let $P_b(\tilde{k}) \in \mathbb{R}^K$ denote the estimated aggregated value of all non-adjustable loads during the time $[\tilde{k}, \tilde{k} + K - 1]$. In the rest of this paper, without the loss of generality, we drop the time stamp $\tilde{k}$ for simplicity.

The objective function of valley-filling can be written as

$$\mathcal{F}(\mathcal{U}) = \frac{1}{2}\left\|P_b + \sum_{i=1}^{n} f_i(\mathcal{U}_i)\right\|_2^2 + \frac{\rho}{2}\|\mathcal{U}\|_2^2 \quad (15)$$

where $f_i(\mathcal{U}_i) = \bar{P}_i \mathcal{U}_i$ and $\mathcal{U} = [\mathcal{U}_1^\mathsf{T} \cdots \mathcal{U}_n^\mathsf{T}]^\mathsf{T}$. The term $\frac{\rho}{2}\|\mathcal{U}\|_2^2$ is added to protect battery state of health.

Solving EV charging sequences is subject to local and network constraints. The local constraint can be written as

$$\mathcal{U}_i \in \mathbb{U}_i, \quad (16)$$

where

$$\mathbb{U}_i := \{\mathcal{U}_i | \mathbf{0} \le \mathcal{U}_i \le \mathbf{1}, x_i(k) + \mathcal{B}_{i,l}\mathcal{U}_i = 0\},$$
$$\mathcal{B}_{i,l} = [B_{i,c}\ B_{i,c}\ \cdots\ B_{i,c}] \in \mathbb{R}^{n\times K}.$$

Herein, $\mathbf{0} \le \mathcal{U}_i \le \mathbf{1}$ constrains the charging power within $[0, \bar{P}_i]$; $x_i(k) + \mathcal{B}_{i,l}\mathcal{U}_i$ denotes the energy remained to be charged at the end of valley-filling and it must be zero.

To maintain power quality, the network constraint aims to keep all nodal voltage magnitudes within $[\underline{\nu}|V_0|, \overline{\nu}|V_0|]$. Since energy generation and reactive power supply are discarded, the network constraint can be represented as

$$\mathcal{Y}_b - \sum_{i=1}^{n} \mathcal{D}_i \mathcal{U}_i \le \mathbf{0}. \quad (17)$$

where $\mathcal{Y}_b = \underline{\nu}^2 \boldsymbol{V}_0 - \mathcal{Y}_{d\tilde{k}}$.

To summarize, the deterministic optimal charging control sequences of all connected EVs can be obtained by solving

$$\begin{aligned} &\min_{\mathcal{U}}\ \mathcal{F}(\mathcal{U}) \\ &\text{s.t. } \mathcal{U}_i \in \mathbb{U}_i,\ \forall\ i = 1, 2, \ldots, n, \\ &\qquad \mathcal{Y}_b - \sum_{i=1}^{n} \mathcal{D}_i \mathcal{U}_i \le \mathbf{0}. \end{aligned} \quad (18)$$

B. Shrunken-primal-dual subgradient (SPDS) algorithm

The Lagrange dual problem of (18) is

$$\max_{\lambda \in \mathbb{R}_+^{hK}} \left\{\min_{\mathcal{U}\in\mathbb{U}} \mathcal{L}(\mathcal{U}, \lambda)\right\}, \quad (19)$$

where $\mathbb{U} = \mathbb{U}_1 \times \mathbb{U}_2 \times \cdots \times \mathbb{U}_n$, $\lambda \in \mathbb{R}_+^{hK}$ is the dual variable associated with the inequality constraint, and $\mathcal{L} : \mathbb{R}^{nK} \times \mathbb{R}^{hK} \to \mathbb{R}^1$ is the Lagrangian given by

$$\mathcal{L}(\mathcal{U}, \lambda) = \mathcal{F}(\mathcal{U}) + \lambda^\mathsf{T} d(\mathcal{U}). \quad (20)$$

According to the Saddle-Point Theorem [24] and by using the decomposable structure of $\mathbb{U}$, the primal-dual subgradient (PDS) algorithm [25] suggests that the ℓth updating iteration for a centralized realization can be performed as

$$\mathcal{U}^{(\ell+1)} = \Pi_{\mathbb{U}}\left(\mathcal{U}^{(\ell)} - \alpha_\ell \nabla_{\mathcal{U}} \mathcal{L}(\mathcal{U}^{(\ell)}, \lambda^{(\ell)})\right), \quad (21a)$$

$$\lambda^{(\ell+1)} = \Pi_{\mathbb{R}_+^{hK}}\left(\lambda^{(\ell)} + \beta_\ell \nabla_\lambda \mathcal{L}(\mathcal{U}^{(\ell)}, \lambda^{(\ell)})\right), \quad (21b)$$

where $\alpha_\ell > 0$ and $\beta_\ell > 0$ are the iteration step sizes.

Let $\mathcal{U}_{-i}$ denote the collection of all $\mathcal{U}_j$, $j \neq i$. Assume problem (18) satisfies Slather condition with $\bar{\mathcal{U}}$ the Slater point, therefore the strong duality holds. Then, by the first-order optimality conditions and the decomposable structure of $\mathbb{U}$, $(\mathcal{U}^*, \lambda^*)$ is a solution to (18) if and only if each $\mathcal{U}_i^*$ is a zero of the parameterized natural map $\mathbf{F}^{\text{nat}}_{\mathbb{U}_i}\left(\mathcal{U}_i; \mathcal{U}^*_{-i}, \lambda^*\right) = 0$ and λ^* is a zero of $\mathbf{F}^{\text{nat}}_{\mathbb{R}^{hK}_+}\left(\lambda; \mathcal{U}^*\right) = 0$ [21], [26], where

$$\mathbf{F}^{\text{nat}}_{\mathbb{U}_i}\left(\mathcal{U}_i; \mathcal{U}^*_{-i}, \lambda^*\right) \triangleq \mathcal{U}_i - \Pi_{\mathbb{U}_i}\left(\mathcal{U}_i - \nabla_{\mathcal{U}_i}\mathcal{L}\left(\mathcal{U}_i; \mathcal{U}^*_{-i}, \lambda^*\right)\right),$$
$$\mathbf{F}^{\text{nat}}_{\mathbb{R}^{hK}_+}\left(\lambda; \mathcal{U}^*\right) \triangleq \lambda - \Pi_{\mathbb{R}^{hK}_+}\left(\lambda + \nabla_\lambda \mathcal{L}\left(\mathcal{U}^*, \lambda\right)\right).$$

Built upon the parametrized natural maps, the regularized PDS (RPDS) algorithm [21] allows each individual agent to solve for its local problem iteratively. However, RPDS is built on a regularized Lagrangian, introducing unnecessary regularization errors. Therefore, to solve (18) in a decentralized fashion with guaranteed convergence and without regularization or approximation errors, we propose a shrunken-primal-dual subgradient (SPDS) algorithm as follows. At the ℓth iteration, primal and dual variables update by following

$$\mathcal{U}_i^{(\ell+1)} = \Pi_{\mathbb{U}_i}\left(\frac{1}{\tau_{\mathcal{U}}}\Pi_{\mathbb{U}_i}\left(\tau_{\mathcal{U}}\mathcal{U}_i^{(\ell)} - \alpha\nabla_{\mathcal{U}_i}\mathcal{L}(\mathcal{U}^{(\ell)}, \lambda^{(\ell)})\right)\right), \tag{22a}$$

$$\lambda^{(\ell+1)} = \Pi_{\mathbb{D}}\left(\frac{1}{\tau_\lambda}\Pi_{\mathbb{D}}\left(\tau_\lambda\lambda^{(\ell)} + \beta\nabla_\lambda\mathcal{L}(\mathcal{U}^{(\ell)}, \lambda^{(\ell)})\right)\right), \tag{22b}$$

where

$$\mathbb{D} := \left\{\lambda | \lambda \geq \mathbf{0},\ \|\lambda\|_2 \leq d_\lambda\right\}, \tag{23}$$

and $0 < \tau_{\mathcal{U}}, \tau_\lambda < 1$.

The SPDS features a two-tier projection where primal and dual variables are shrunken then expanded. Take (22a) for example, at tier-1 projection, solution from the previous iteration $\mathcal{U}_i^{(\ell)}$ is shrunken by $\tau_{\mathcal{U}}$, moved towards the descent direction of the Lagrangian, and then projected into $\mathbb{U}_i$. Because the shrinkage introduces conservativeness, result from tier-1 projection is expanded by $1/\tau_{\mathcal{U}}$ and then projected back into $\mathbb{U}_i$ at the tier-2 projection. The shrinkage-expansion and the two-tier projection in (22) guarantee the convergence without any regularizations. In the SPDS,

$$d_\lambda \triangleq \frac{\mathcal{F}(\bar{\mathcal{U}}) - \tilde{l}}{\gamma} + \sigma, \tag{24}$$

where $\sigma > 0$, $\gamma = \min_{j=1,\ldots,hK}\left\{-d_j(\bar{\mathcal{U}})\right\}$, $d_j(\mathcal{U})$ is the jth entry of $d(\mathcal{U})$, and

$$\tilde{l} = \min_{\mathcal{U}_i \in \mathbb{U}_i, i=1,\ldots,n} \mathcal{L}(\mathcal{U}_i, \ldots, \mathcal{U}_n, \tilde{\lambda}),\ \forall \tilde{\lambda} \in \mathbb{R}^{hK}_+. \tag{25}$$

It has been shown in [20], [27], [28] that, under Slater condition, the primal solution $\mathcal{U}_i^*$ exists and therefore strong duality holds. Further, the dual optimal set $\mathbb{D}$ is not empty and bounded by d_λ. In the rest of this paper, we assume that the bound d_λ can be obtained a priori.

C. Event-driven control problem

The deterministic control problem defined in Section III-A is conservative because it does not consider drivers' random behaviors. On the one hand, plug-in time and charging deadlines must be randomly distributed around $\tilde{k}$ and $\tilde{k}+K$. On the other hand, drivers may plug out before the designated charging deadlines. To handle these random behaviors, we define two events that will trigger the whole system to recompute control sequences. Specifically, at time $\kappa > \tilde{k}$,

- EVENT 1 is triggered if one or more EVs plug in;
- EVENT 2 is triggered if one of more connected EVs plug out before their designated charging deadlines.

Assume that the ith EV does not plug out unexpectedly before its designated charging deadline. Then by comparing $[\tilde{k}, \tilde{k}+K]$ and $[k_i, k_i + K_i]$, the ith EV can be classified into four groups. Suppose the system is triggered by either event at time κ, the optimal control sequences can be obtained by re-solving (18) at time κ, however the local constraint $\mathbb{U}_i$ needs amendments to adapt to the events. Let $\tilde{u}_i(k),\ k = \tilde{k}, \cdots, \kappa - 1$ denote the control sequence that have already been applied before κ. The charging sequence of the ith EV in the recomputation at κ should satisfy $\mathcal{U}_i \in \mathbb{U}_i$, where

- GROUP 1:
$$\begin{aligned}\mathbb{U}_i := \Big\{\mathcal{U}_i | & u_i(k) = \tilde{u}_i(k),\ k = \tilde{k}, \cdots, \kappa - 1,\\ & u_i(k) = 0,\ k = k_i, \cdots, \tilde{k} - 1,\\ & 0 \leq u_i(k) \leq 1,\ k = \tilde{k}, \cdots, k_i + K_i - 1,\\ & u_i(k) = 0,\ k = k_i + K_i, \cdots, \tilde{k} + K,\\ & x_i(k_i) + [B_i\ \cdots\ B_i]\mathcal{U}_i = 0\big\}.\end{aligned}$$
- GROUP 2:
$$\begin{aligned}\mathbb{U}_i := \Big\{\mathcal{U}_i | & u_i(k) = \tilde{u}_i(k),\ k = \tilde{k}, \cdots, \kappa - 1,\\ & u_i(k) = 0,\ k = k_i, \cdots, \tilde{k} - 1,\\ & 0 \leq u_i(k) \leq 1,\ k = \tilde{k}, \cdots, \tilde{k} + K - 1,\\ & x_i(k_i) + [B_i\ \cdots\ B_i]\mathcal{U}_i\\ & \leq (k_i + K_i - \tilde{k} - K - 1)\eta\bar{P}_i\Big\}.\end{aligned}$$
- GROUP 3:
$$\begin{aligned}\mathbb{U}_i := \Big\{\mathcal{U}_i | & u_i(k) = \tilde{u}_i(k),\ k = \tilde{k}, \cdots, \kappa - 1,\\ & u_i(k) = 0,\ k = \tilde{k}, \cdots, k_i - 1,\\ & 0 \leq u_i(k) \leq 1,\ k = k_i, \cdots, k_i + K_i - 1,\\ & u_i(k) = 0,\ k = k_i + K_i, \cdots, \tilde{k} + K,\\ & x_i(k_i) + [B_i\ \cdots\ B_i]\mathcal{U}_i = 0\big\}.\end{aligned}$$
- GROUP 4:
$$\begin{aligned}\mathbb{U}_i := \Big\{\mathcal{U}_i | & u_i(k) = \tilde{u}_i(k),\ k = \tilde{k}, \cdots, \kappa - 1,\\ & u_i(k) = 0,\ k = \tilde{k}, \cdots, k_i - 1,\\ & 0 \leq u_i(k) \leq 1,\ k = k_i, \cdots, \tilde{k} + K - 1,\\ & x_i(k_i) + [B_i\ \cdots\ B_i]\mathcal{U}_i\\ & \leq (k_i + K_i - \tilde{k} - K - 1)\eta\bar{P}_i\Big\}.\end{aligned}$$

Note that, if the ith EV belongs to GROUP 3 or GROUP 4, its charger keeps generating an all-zero control sequence until the EV is plugged in. If the ith EV is unplugged unexpectedly at $k = \kappa$, its charger keeps generating $[\tilde{u}_i(\tilde{k}) \cdots \tilde{u}_i(\kappa - 1)\ 0 \cdots 0]$ after κ. The set $\mathbb{U}_i$ is convex, ensuring that the ith EV will reach the desired SOC by its designated deadline given that it is not plugged out unexpectedly.

To summarize, the event-driven decentralized control based on SPDS for valley-filling is shown in **Algorithm 1**.

Algorithm 1 SPDS Algorithm for Valley-Filling

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procedure
    if k = k̃ or EVENTs are triggered at κ then
        ℓ = 0; Chargers initialize U_i^(0); Operator initializes λ^(0); Tolerance τ_ε; Initial error ε = 10^9; Maximum iteration ℓ_max.
        while ε > τ_ε and ℓ ≤ ℓ_max do
            All EV chargers transmit their own U_i^(ℓ) to the operator.
            Operator transmits the dual variable λ^ℓ to all EV chargers.
            All EV-connected chargers perform (22a).
            All idle chargers generate all-zero control sequences.
            Unexpectedly plugged-out chargers generate constant control sequences, i.e., [ũ_i(k̃) ··· ũ_i(κ − 1) 0 ··· 0].
            System operator performs (22b).
            ρ = ‖U^(ℓ+1) − U^(ℓ)‖_2.
            ℓ = ℓ + 1.
        end while
    end if
end procedure
```

IV. Simulation Results

The simulation is conducted with the distribution network shown in Fig. 1. Each node except Nodes 1 and 6 has 5 EV chargers. Step sizes $\alpha = 2 \times 10^{-9}$ and $\beta = 1$; dual bound $d_\gamma = 2 \times 10^6$; maximum iteration number $\ell_{max} = 30$; tolerance $\tau_\epsilon = 1 \times 10^{-4}$; shrinking parameters $\tau_{\mathcal{U}} = \tau_\lambda = 0.974$; initial values of $\mathcal{U}_i^{(0)}(k)$ and $\lambda^{(0)}$ are both all-zero vectors; lower bound $\underline{\nu} = 0.95$ p.u. The baseline load data is collected from Southern California Edison and scaled. The valley-filling service period is from 19:00 to 8:00 next day.

A. Deterministic charging control

In this case, all 50 EVs are plugged in before 19:00 and none of them plug out before 8:00. Controlled by **Algorithm 1**, Fig. 2 shows the baseline load and the controlled total load in 30 iterations. Considering the result at the 30th iteration,

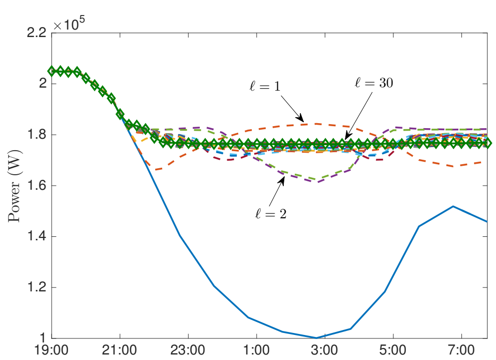

Fig. 2. Baseline load and total loads in 30 iterations.

all EVs are commanded to start charging and filling the valley and keeps the total load profile flat after 21:00.

Nodal voltage magnitudes of the baseline load (solid lines) and the controlled total loads at the 1st and 30th iterations (dashed lines and diamond marked lines) are shown in Fig. 3. At the 30th iteration, all nodal voltage magnitudes are in line

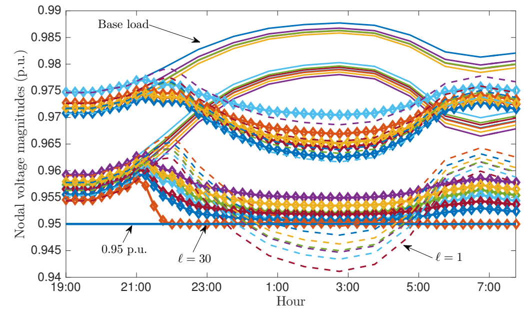

Fig. 3. Nodal voltage magnitudes of baseline load and total load.

with or above 0.95 p.u., indicating that network constraints are satisfied under our proposed control scheme.

B. Event-driven charging control

In this case, 14 EVs are randomly selected to plug in during 19:00–1:00; EVs' designated charging deadlines are randomly distributed around 8:00; 10 EVs are randomly selected to unexpectedly leave during 23:00–6:30 before their designated charging deadlines. Fig. 4 shows the valley-filling performance under the proposed **Algorithm 1**. It can be

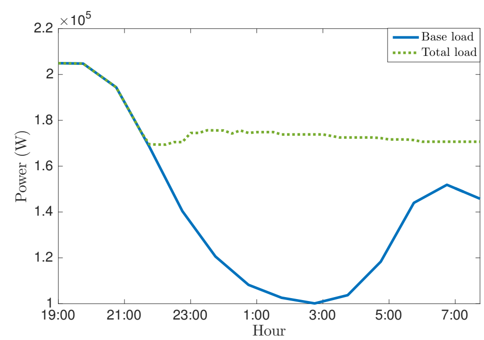

Fig. 4. Valley-filling performance when EVs randomly join and leave.

seen that, due to less number of connected EVs, EVs start filling the valley at 22:00, which is one hour later than the deterministic case. During the whole process, the total load profile is being slightly adjusted to accommodate two events.

Fig. 5 shows the time stamps of two events and the charging profiles of all EVs. Since EVs start charging and filling the valley at 22:00, any event happens after that directly impacts the total load profile. Between 23:00 and 00:00 several EVs successively plug in, we can clearly observe the increase of the total load during that time in Fig. 4. From 00:00 to 1:00, two events are triggered alternatively, driving the total load up and down. After 1:00, only EVENT 2 happens, resulting in a decrease of the total load.

Nodal voltage magnitudes in the event-driven case are shown in Fig. 6. All nodal voltage levels are no less than 0.95 p.u., thus distribution network constraints are satisfied.

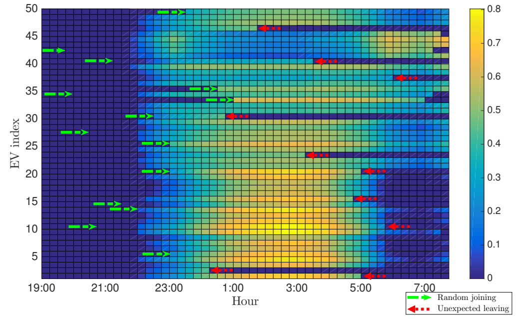

Fig. 5. Time stamps of events and charging profiles of all EVs.

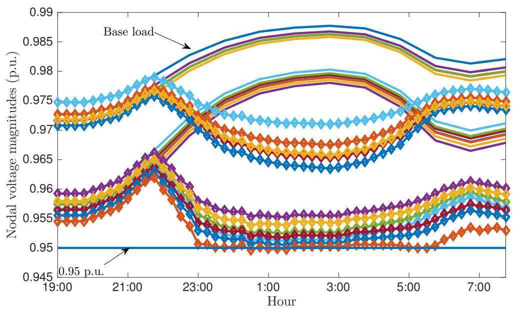

Fig. 6. Nodal voltage magnitudes of the baseline load and the controlled total load in the event-driven case.

During 23:00–5:30, voltage magnitude at Node 11 stays at 0.95 p.u.; After 5:30, all nodal voltage magnitudes are higher than the corresponding part in the deterministic case because a certain number of EVs have already plugged out.

V. Conclusions

This paper developed a network-aware decentralized EV charging control framework to achieve valley-filling. A novel SPDS algorithm, which can effectively solve the valley-filling problem as well as general optimization problems consisting of non-separable objective functions and strongly coupled inequality constraints, was described. Built upon the SPDS, an event-driven control scheme was developed to accommodate EV drivers' random behaviors. In both deterministic and event-driven cases, the proposed control scheme successfully achieved valley-filling, fulfilled charging requirements, and satisfied distribution network constraints.

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