Final project file

Contents

1. Abstract

Quantum scrambling is the dispersal of local information into many-body quantum correlations distributed throughout an entire system. This concept accompanies the dynamics of thermalization in closed quantum systems, and has recently emerged as a powerful tool for characterizing chaos in black holes.

However, the direct experimental measurement of quantum scrambling is difficult, since it is hard to distinguish its effect from ordinary decoherence and noise.

Landsman et al. “Verified quantum information scrambling” Nature, 567(7746), 2019 Conducted an experiment using trapped ions where scrambling can be distinguished from decoherence using a teleportation protocol. We would like to investigate the robustness of this experiment against changing the initial state. In particular, we will focus on a class of states known as the thermofield double states (see Wu et. al, “Variational thermal quantum simulation via thermofield double states”, 123(22), 2019) which interpolates between the bell pairs used in Landsman et al. and a product of two vacuum state of an Ising Hamiltonian for which the teleportation protocol is expected to fail.

2. Theoretical Background

2.1. Quantum Scrambling

Quantum scrambling is the dispersion of local information into many-body quantum correlations distributed throughout the entire system, due to thermalization qualities of the system. This is a phenomenon we see mainly in strongly-interacting quantum systems.

Similar behavior has been associated with black hole horizons in the context of gravitational holography (AdS/CFT correspondence).

One main definition used vastly today is in terms of OTOC's (Out of Time Order Correlations). These are correlation functions of the structure $\langle O_X O_Y(t) O_X O_Y(t)\rangle$, where O_X is a Hermitian operator that acts on a subsystem at time 0, and O_Y is a Hermitian operator that operates on a different subsystem (of the same system) at time t. The intuition behind this correlator is the attempt to measure the influence of one observable at time 0 on another observable at time t. This reminds the test for ill-conditioned systems (not by chance) [3].

The OTOC correlation functions have proven to be powerful theoretical tool for diagnosing chaos in quantum systems. However, their experimental measurement remain an essential challenge. This is due to the resemblance of the effect of quantum scrambling, ordinary decoherence and noise on the OTOC, which is its decay to zero.

In [1], a teleportation protocol is proposed, which measures un-ambiguously the scrambling qualities of a quantum system. In this scheme, a scrambling operator is built, and through the state fidelity of the qubit which is supposed to bare the teleported state, and an OTOC surveillance (it's decay), the scrambling qualities are determined.

We wish to analyze the scrambling qualities of a quantum system through its teleportation qualities by altering the correlation between the coupled qubits in the circuit suggested in [1], and by thus test the robustness of the system suggested in [1], [3].
In particular we are interested in finding the dependency between the scrambling qualities of a system, the initial state of the qubit we wish to teleport and the temperature of the Thermofield-double (TFD) state in which we initialize the remaining qubits in.

2.2. Thermofield-double states

Given two copies of any quantum mechanical system, one may want to prepare them in the thermofield double state for the purpose of studying thermal physics or black holes.

The Thermofield double state of a system is the following unique pure state:

$$|TFD\rangle = \frac{1}{\sqrt{Z}}\sum_{n} e^{-\frac{\beta E_n}{2}}|n\rangle_L \otimes |n\rangle_R \tag{1}$$

Where $|n\rangle_L, |n\rangle_R$ are the energy states of the individual systems. This is an entangled pure state of the full system with the property that each of the two copies is in the thermal density matrix with temperature β^{-1}.

In our system, we adjust the correlation between specific couples of qubits, by altering the temperature $T = \frac{1}{\beta}$ where for $T \to \infty$ $(\beta \to 0)$ results in 3 EPR pairs (fully correlated) between the appropriate qubits ((1, 4), (2, 3), (5, 6)), and $T \to 0$ $(\beta \to \infty)$ results in an un-correlated state, in which the two sub-systems act separately, with no effect on one another. This in turn changes the **teleportation qualities** of the circuit, which is what we wish to analyze.

2.3. Quantum teleportation

As explained in [4], quantum teleportation is a protocol in which the state of a qubit is transferred to another qubit (the first qubit ‘loses’ his state – as the no-cloning principle implies).

Say that Alice wants to teleport her qubit state to Bob. In order to do that, the teleportation protocol suggested by Qiskit implies the following: Alice and bob must be given each one qubit from 2 coupled qubits in an EPR state, say $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, **prior** to the teleportation itself. Then, by performing a bell measurement and sending Bob her results, Alice lets Bob know which of the X and Z gates (or both) he should apply to his qubit. He receives two classical bits of data, and performs the following action:

$00 \rightarrow Do\ nothing.$

$01 \rightarrow Apply\ X\ gate.$

$10 \rightarrow Apply\ Z\ gate.$

$11 \rightarrow Apply\ ZX\ gate.$

Once he has applied the appropriate gates on his qubit, it is in the wanted state, which is the state of the first (un-coupled) qubit in Alice’s possetion.

The following circuit performs exactly that:

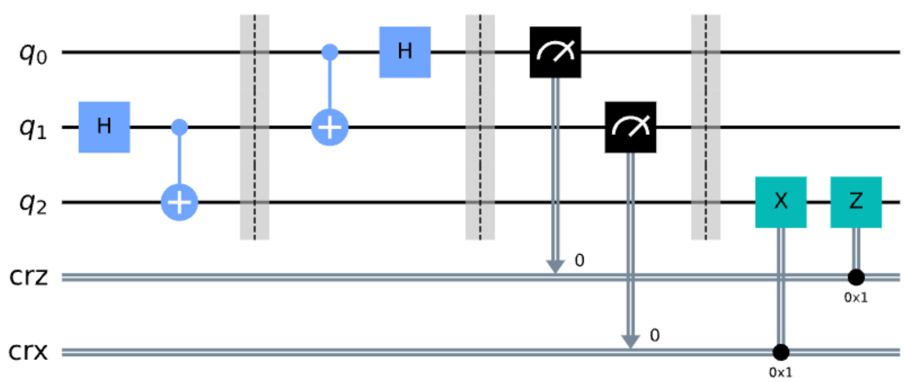

Figure 2.3.1: Quantum teleportation circuit

The first two quantum gates (until the first barrier) are the coupling (i.e., produce an EPR pair of state $|\Phi^+\rangle$) part of the system, which play the role of an outsider giving Alice and Bob each one qubit of the coupled pair.

The second section (between barrier 1 and 2) is the Bell-measurement (i.e., measurement in the Bell basis).

The third section is the measurements of Alice's qubits $(q0, q1)$ and 'sending' to Bob.

The last section is the section in which Bob applies his gates **according to the classical data in the classical bits** (that's why they are called **conditional gates**).

For the interested, the full proof is at [4].

In our project, we perform teleportation via <u>**scrambling**</u>, meaning that the strong interactions of the system, dictated by the scrambling unitary $\widehat{U_S}$, are the main factor we utilize to perform the teleportation.

One way of constructing such a scrambling operator $\widehat{U_S}$, is by building it so that it delocalizes all single-qubit operators into 3-qubit operators [1]. For the case of the unitary used for our analysis, we may prove this quality by proving that it satisfies the following equations:

$$U^\dagger(X \otimes I \otimes I)U = X \otimes Z \otimes Z \quad (2)$$

$$U^\dagger(I \otimes X \otimes I)U = Z \otimes X \otimes Z \quad (3)$$

$$U^\dagger(I \otimes I \otimes X)U = Z \otimes Z \otimes X \quad (4)$$

$$U^\dagger(Y \otimes I \otimes I)U = Y \otimes X \otimes X \quad (5)$$

$$U^\dagger(I \otimes Y \otimes I)U = X \otimes Y \otimes X \quad (6)$$

$$U^\dagger(I \otimes I \otimes Y)U = X \otimes X \otimes Y \quad (7)$$

$$U^\dagger(Z \otimes I \otimes I)U = Z \otimes Y \otimes Y \quad (8)$$

$$U^{\dagger}(I \otimes Z \otimes I)U = Y \otimes Z \otimes Y \quad (9)$$

$$U^{\dagger}(I \otimes I \otimes Z)U = Y \otimes Y \otimes Z \quad (10)$$

2.4. Execution frameworks

Main simulator: Aer simulator.

Qualities: The main simulator of the Aer backend, which is designated to mimic the operation of a real quantum computer, with **no** noise (other simulators can also simulate noise).

** This simulator can also save the state of the system at any time, a functionality we utilize in this experiment.

Differences from an actual quantum computer:

Sending a quantum circuit to a quantum simulator is different from sending it to a quantum computer in several ways:

- Quantum simulators support some non-unitary operations, which are not supported by quantum computers. Moreover, on qiskit's quantum simulators we can apply gates after measurements, which is forbidden in IBM's quantum computers.
- The quantum simulators we use are 'noiseless', meaning that the results we get are the expectation values of the results of a circuit sent to a quantum computer. This is an advantage for the building of the circuit and for learning motivations, since we can isolate the effects of the circuit components, with no noise stemmed results.

3. The experimental system

3.1 General

In our experiment, we built a circuit consisting of 7 qubits and 7 classical bits.
We initialize the state of the first qubit (the state to be teleported) in several different states (fixed or random) and the remaining 6 qubits in the TFD state of different temperatures, using the Ising Hamiltonian as our base system Hamiltonian:

$$H = -\sum_i \sigma_z^{[i]} \otimes \sigma_z^{[i+1]} - g\sum_i \sigma_x^{[i]} - h\sum_i \sigma_z^{[i]} \tag{11}$$

We choose the parameter values $g = -1.05, h = 0.5$ which gives us a Hamiltonian far from any integrability limits [5].
We then **scramble** the system information, and **measure** the states of the qubits in the Bell basis.
Finally, we analyze the teleportation qualities of the system, using statistical information returned from the execution of the circuit (many times).

3.2 Circuit

We built our circuit following the main characteristics of the circuit described in the main circuit of [1].

The circuit script is depicted in the attachments section. Nevertheless, we explain the main circuit flow and construction for clarity.

The flow of the circuit construct is as follows: We initialize qubit 0 in the state we wish to teleport (random or fixed) and initialize qubits 1-6 in the Thermofield-double (TFD) state, of some temperature T. We then **scramble** the system's information using a scrambling unitary $\widehat{U_S}$ which satisfies the constrains we depicted in equations (2)-(9).
After scrambling, we can perform a measurement in the Bell basis of each of the two pairs: (0, 5), (1, 4), (2, 3) which completes our teleportation protocol. We chose to measure the coupled qubits (2, 3).

We then test the scrambling qualities of this system, given the initial state of the first qubit, and TFD state in which qubits 1-6 were initialized with.

In the context of the classical teleportation protocol, we may think of the first qubit as Alice's qubit (to be teleported), and of qubit 6 (the last qubit) as Bob's second qubit (to receive the state of Alice's qubit). We may see our circuit as the following flow:

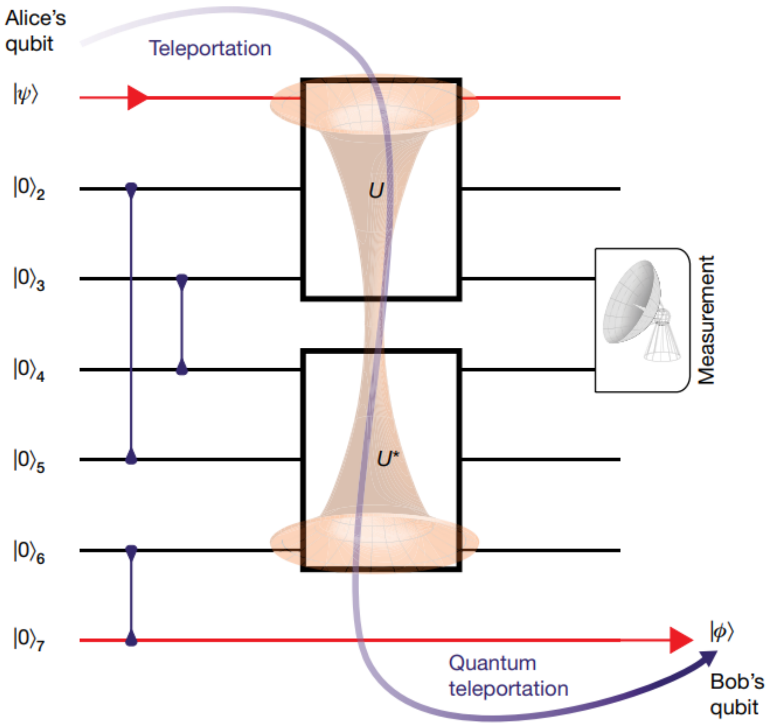

Figure 3.2.1: The Experimental quantum circuit [1].

Figure 3.2.1 displays the pairs of correlated qubits and the qubits which the scrambling operator and its conjugate operate upon[1].

We change the initial state of the first qubit, and the TFD state of the remaining 6 qubits by altering the temperature of the system, and analyze the teleportation statistics of the circuit. Specifically, we measure the number of counts of the state $|\Psi\rangle = |1000000\rangle$ which marks that the teleportation has failed, and the quantity P_{suc} which is the probability of successful teleportation, which is in our case the relative number of counts of the state $|\Psi\rangle = |0000000\rangle$[2] divided by the total counts of the experiment.

** Notice that when assigning the identity operator to U, the circuit becomes the classic quantum teleportation circuit for qubits 0, 5 and 6.

[1] Important note: In our circuit, qubits 3 and 5 are switched with one another compared to [1]. This is a matter of convenience when dealing with the TFD state.

[2] This state refers the state of the system after mapping the last qubit back to the σ_z basis, conditional to the successful teleportation.

4. Results

4.1 Constant input state:

We begin by showing the results of the scrambling circuit, where the initial state of qubit 0 (with the state to be teleported) is **constant**. We perform this experiment for all Pauli operators eigenstates: $|0\rangle_X, |1\rangle_X, |0\rangle_Y, |1\rangle_Y, |0\rangle_Z, |1\rangle_Z$, where:

$$|0\rangle_X = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{12}$$

$$|1\rangle_X = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{13}$$

$$|0\rangle_Y = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ -1 \end{pmatrix} \tag{14}$$

$$|1\rangle_Y = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix} \tag{15}$$

$$|0\rangle_Z = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{16}$$

$$|1\rangle_Z = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{17}$$

We conduct the experiment for β values in range $[0, 2.5]$ with steps of 0.01. This corresponds to temperatures from ∞ to 0.4 $[J]$[3].

For each temperature, we show the successful measurement probability, the unsuccessful teleportation counts and the state fidelity[4] with the state $|0000000\rangle$.

The successful measurement probability is the probability to measure the state $|0000000\rangle$ at the end of the circuit, meaning the amount of shots taken of the system in the state $|0000000\rangle$, divided by the total amount of shots taken, which is 20,000 in our case.

The unuccessful teleportation counts is the amount of times the system was measured and found to be in the state $|1000000\rangle$ at the end of the circuit, which marks that the teleportation has failed, since the target qubit is in state $|1\rangle$. This marks failed teleportation since as mentioned in previous sections, we apply the inverse gate to the initializing gate of

[3] We take $k_B = 1$, therefore the temperature T is in units of Energy $[J]$, and β is in units of $\left[\frac{1}{J}\right]$.

[4] The state fidelity is measured for β values in the range $[0, 1.25]$, with similar jumps (0.01) to the other experiments.

q0 (the 'teleportand') to q6 (the qubit that we want to receive the teleported state), which means if teleportation was successful, we should measure the state $|0\rangle$ for qubit 6 (q6) when the Bell measurement succeeded.

We wish to measure this fidelity, in order to understand better the teleportation qualities of the system. The **state fidelity** of the system state with the state $|0000000\rangle$ is the main measure of successful teleportation in the system. Intuitively, this quantity tells us how similar the system state and the state $|0000000\rangle$ are, where the state $|0000000\rangle$ marks **successful teleportation** of the circuit.

When we check this quantity, we eliminate the measuring of the qubits of the system, otherwise the fidelity function is un-measurable (due to the collapsing of the system state).

We show graphs of the fidelity between the system state, and the state $|0000000\rangle$, which marks **successful teleportation**.

Results for $|0\rangle_X$:

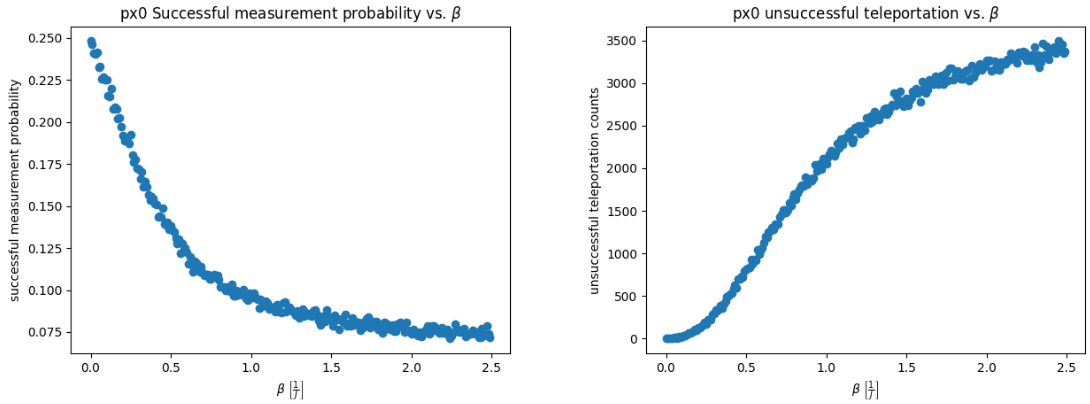

Figure 4.1: Successful teleportation probability and unsuccessful teleportation counts for $|0\rangle_X$.

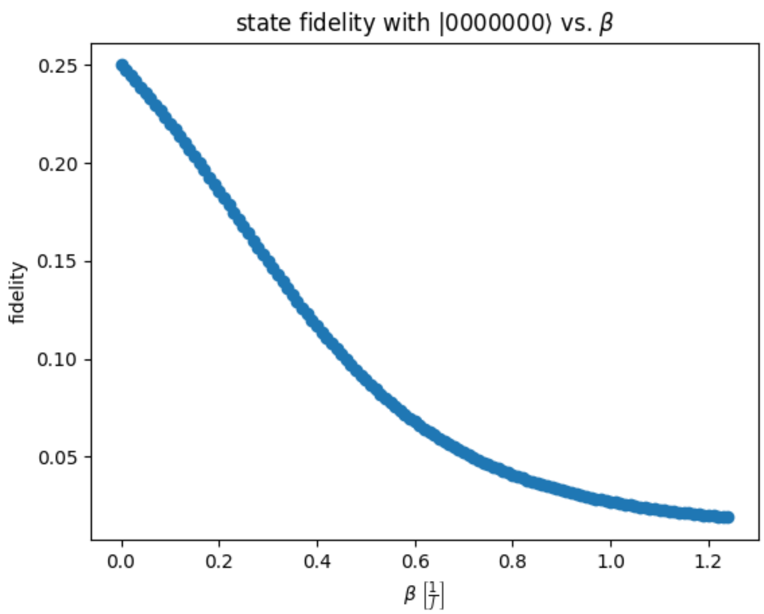

Figure 4.2: State fidelity with $|0000000\rangle$, for $|0\rangle_X$ as the initial state of q0.

We generate fits for the graphs near $\beta = 0$, and analyze the behavior of the circuit near the maximum-correlation regime.

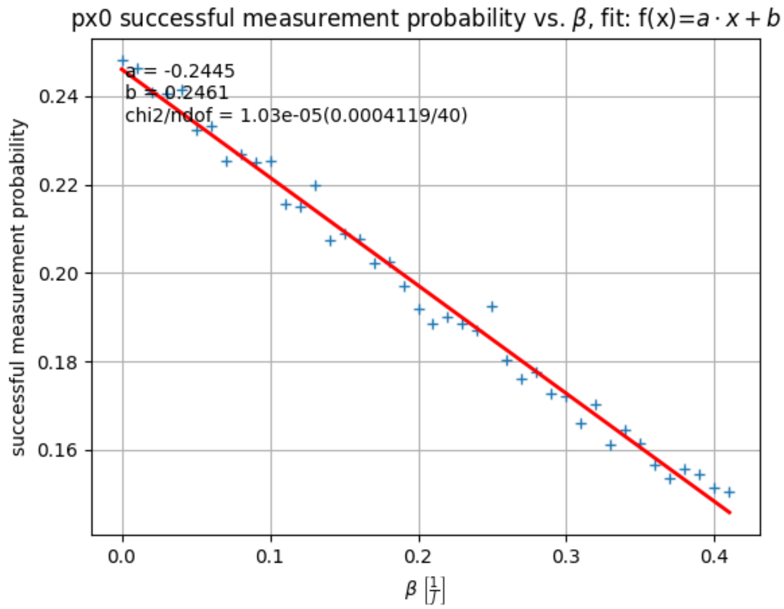

Figure 4.3: Linear fit for the successful measurement probability, where the initial state of q0 is $|0\rangle_X$.

A square fit for the unsuccessful teleportation vs. β:

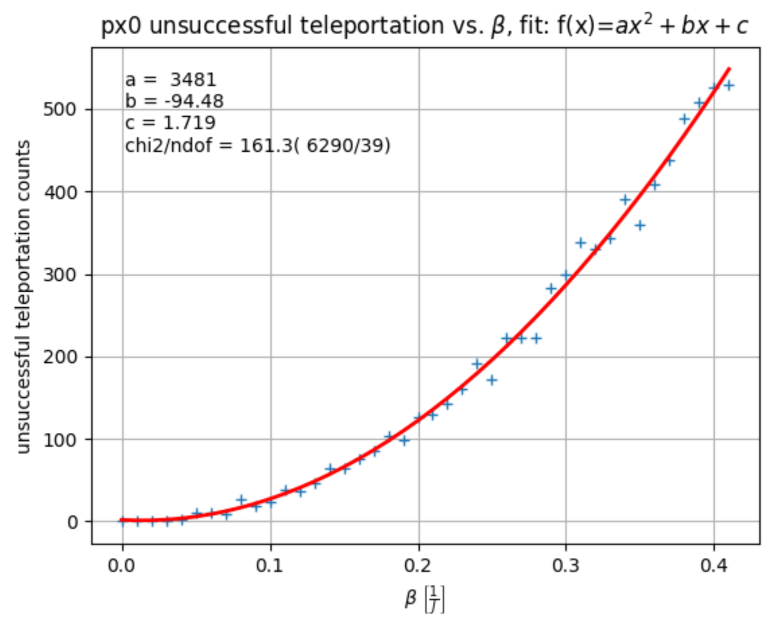

Figure 4.4: A square fit for the unsuccessful teleportation vs. β, where the initial state of q0 is $|0\rangle_X$.

A cubic fit for the state fidelity with the state $|0000000\rangle$:

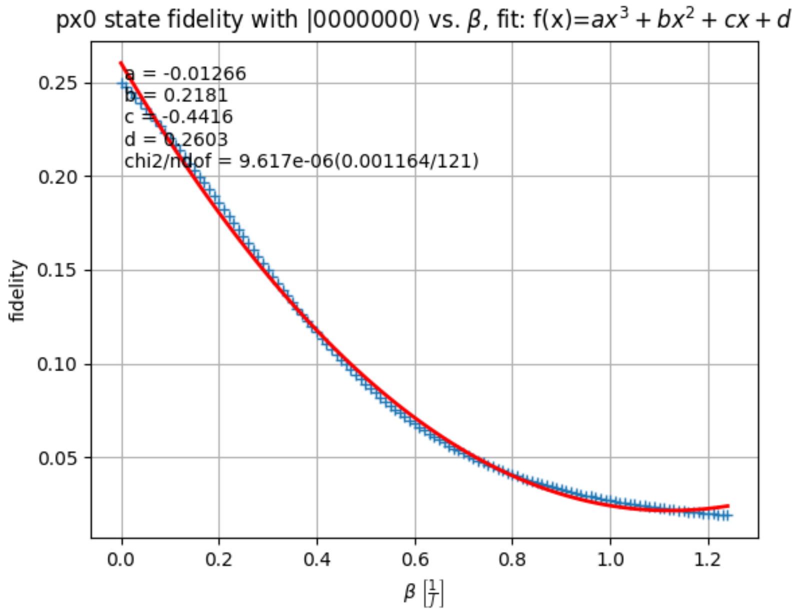

Figure 4.5: State fidelity of the system with $|0000000\rangle$, where the initial state of the system is $|0\rangle_X$.

Results for $|1\rangle_X$:

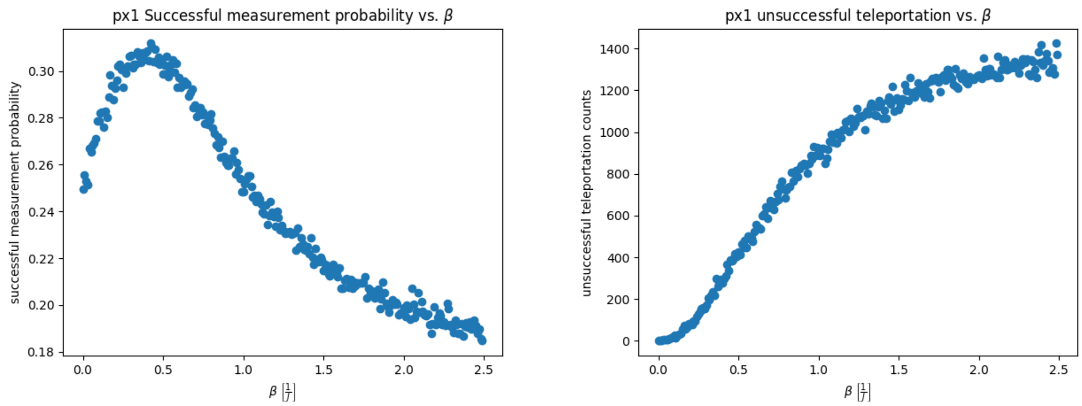

Figure 4.6: Successful teleportation probability and unsuccessful teleportation counts for $|1\rangle_X$.

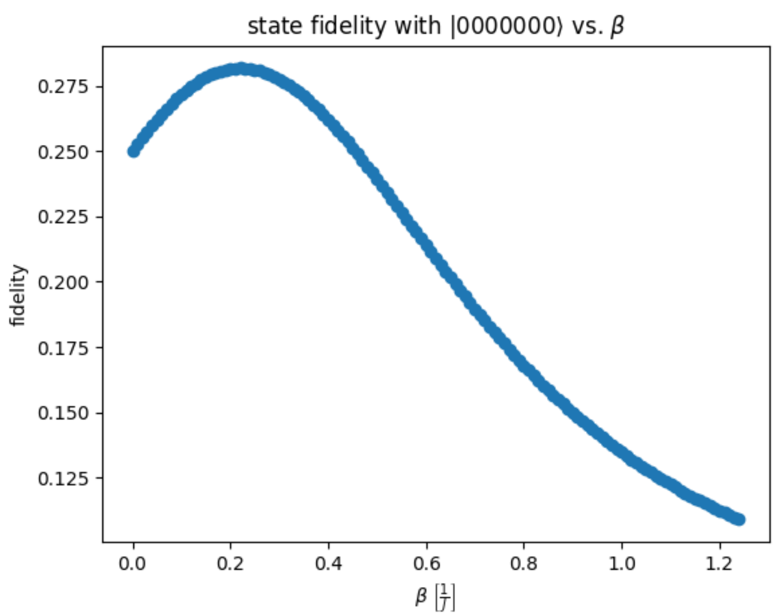

Figure 4.7: State fidelity with $|0000000\rangle$, for $|1\rangle_X$ as the initial state of q0.

For this case, we show a fit of the unsuccessful teleportation counts, in the regime of $\beta = 0$:

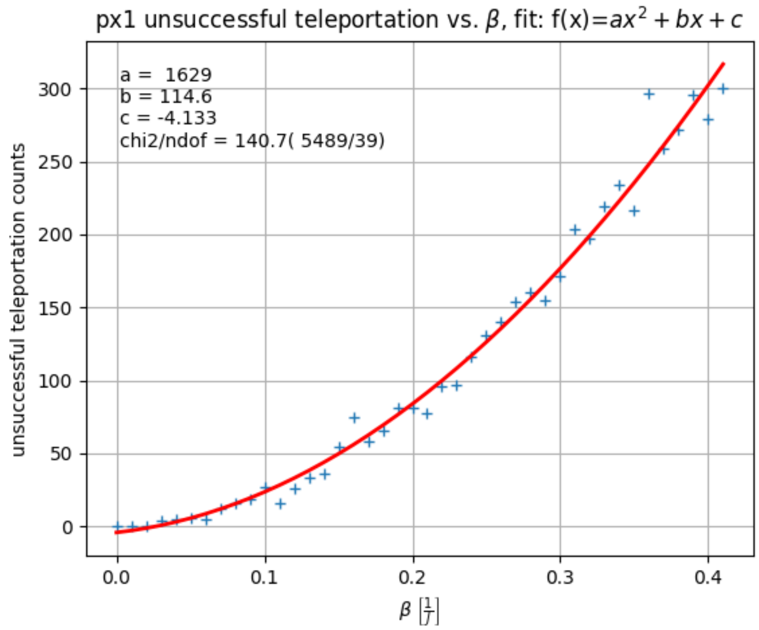

Figure 4.8: A square fit of the unsuccessful teleportation counts vs. β where the initial state of q0 is $|1\rangle_X$.

Results for $|0\rangle_Y$:

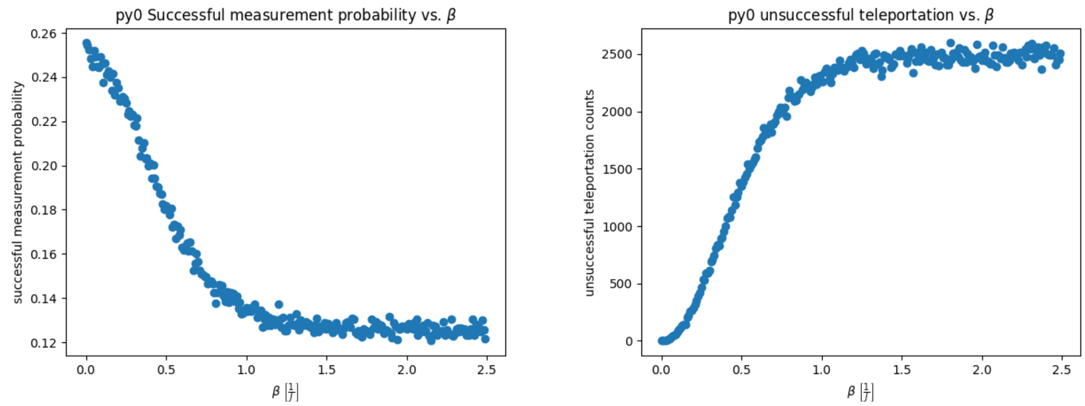

Figure 4.9: Successful teleportation probability and unsuccessful teleportation counts for $|0\rangle_Y$.

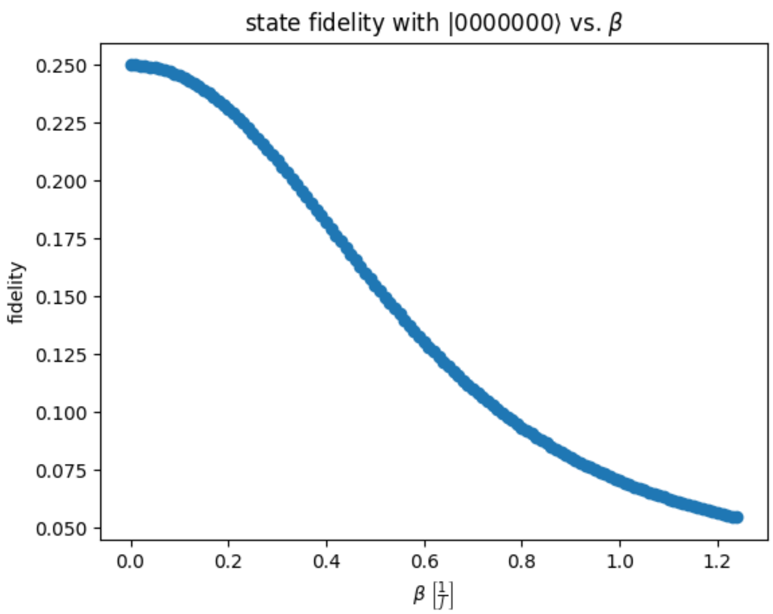

Figure 4.10: State fidelity with $|0000000\rangle$, for $|0\rangle_Y$ as the initial state of q0.

We show a square fit for the regime of $\beta = 0$ for the successful measurement probability data vs. β, a square fit in the same regime for the unsuccessful teleportation counts vs. β and a cubic fit of the state fidelity in the whole range of the experiment:

Fits for the above data:

First, a square fit for the successful measurement probability vs. β, in the regime of $\beta = 0$:

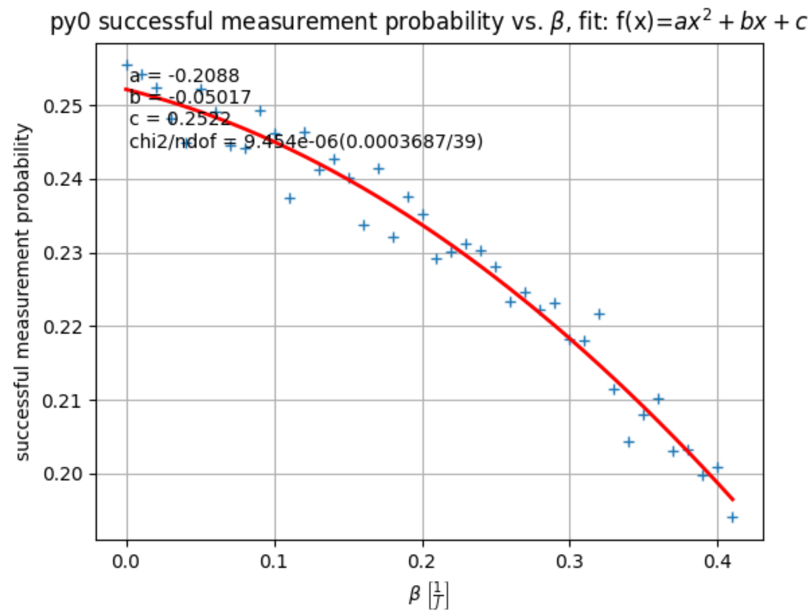

Figure 4.11: Square fit for the successful measurement probability vs. β, in the regime of $\beta = 0$, when the initial state of q0 is $|0\rangle_Y$.

Second, a square fit for the unsuccessful teleportation counts, in the same regime:

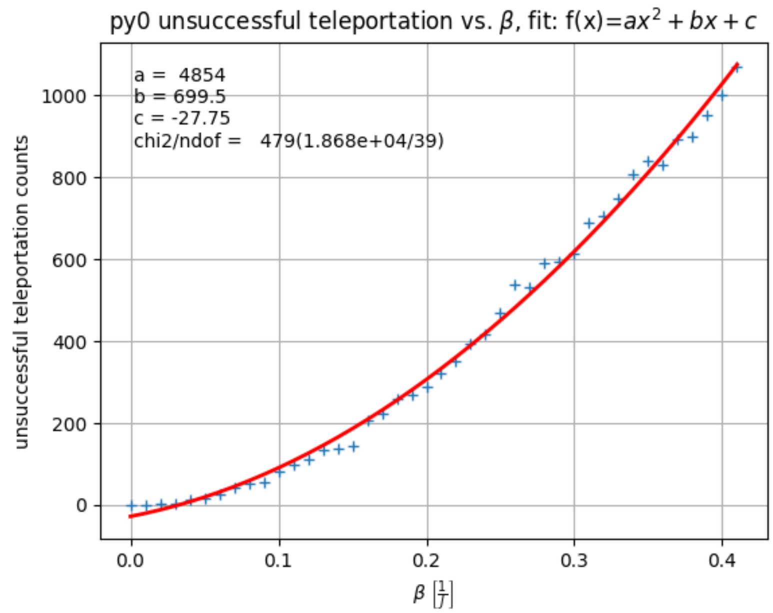

Figure 4.12: A square fit for the unsuccessful teleportation counts, near $\beta = 0$, when the initial state of q0 is $|0\rangle_Y$.

Finally, a cubic (polynomial of third degree) fit for the state fidelity, also in the above range:

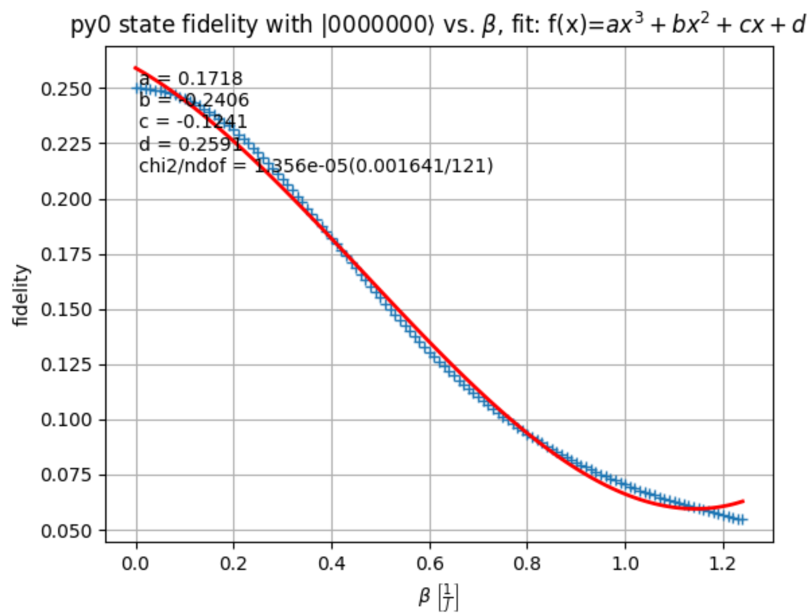

Figure 4.13: A cubit fit for the state fidelity of the system, when the initial state of q0 is $|0\rangle_Y$.

Results for $|1\rangle_Y$:

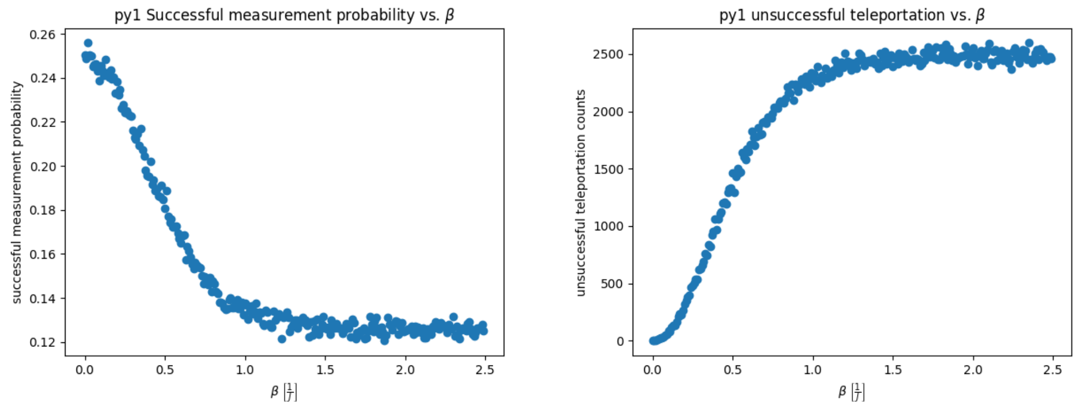

Figure 4.14: Successful teleportation probability and unsuccessful teleportation counts for $|1\rangle_Y$.

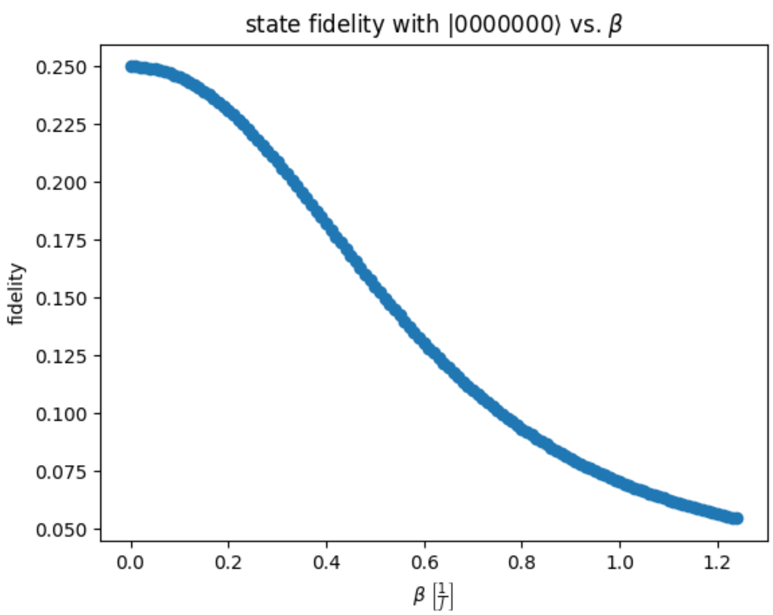

Figure 4.15: State fidelity with $|0000000\rangle$, for $|1\rangle_Y$ as the initial state of q0.

We generate fits for this case, similar to the case in which the first qubit's initial state was $|0\rangle_Y$:

First, a square fit for the successful measurement probability vs. β, in the regime of $\beta = 0$:

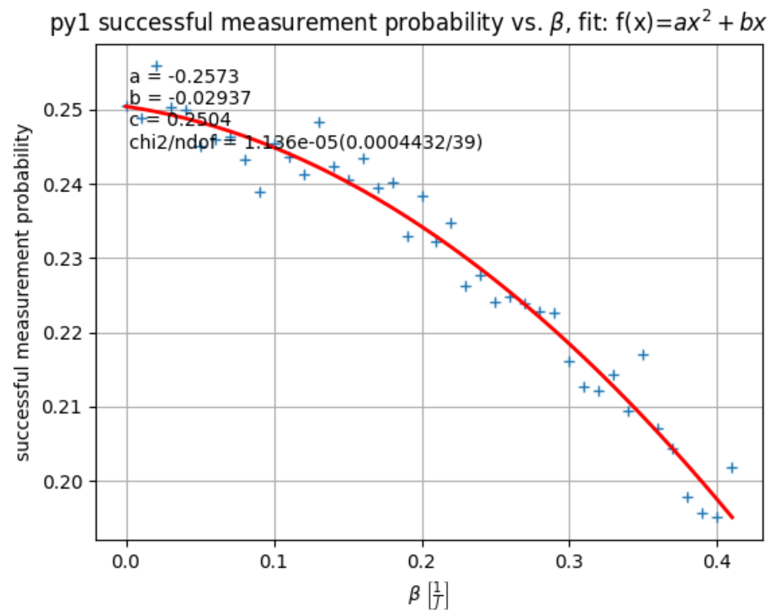

Figure 4.16: A square fit for the successful measurement probability vs. β, near $\beta = 0$, where the initial state of q0 is $|1\rangle_Y$.

Second, a square fit for the unsuccessful teleportation counts, in the same regime:

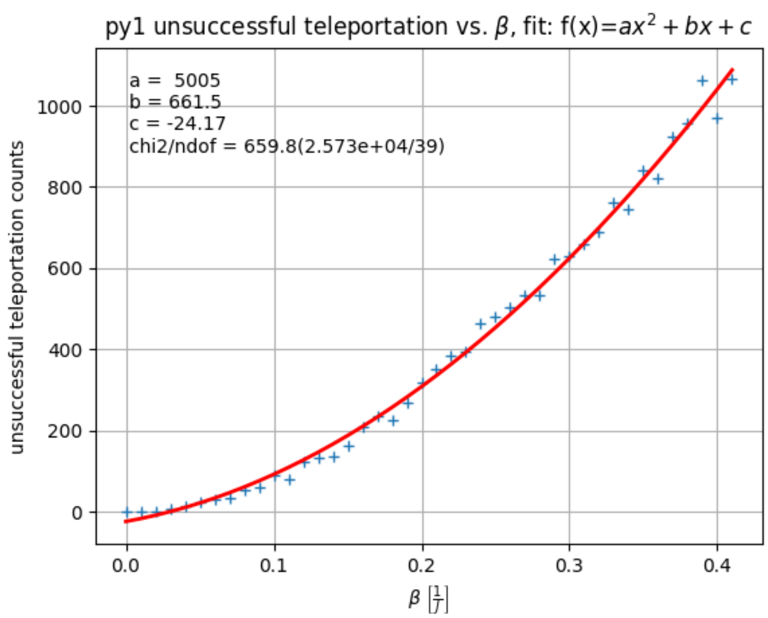

Figure 4.17: A square fit for the unsuccessful teleportation counts vs. β in the regime of $\beta = 0$, where the initial state of q0 is $|1\rangle_Y$.

Finally, a cubic (polynomial of third degree) fit for the state fidelity:

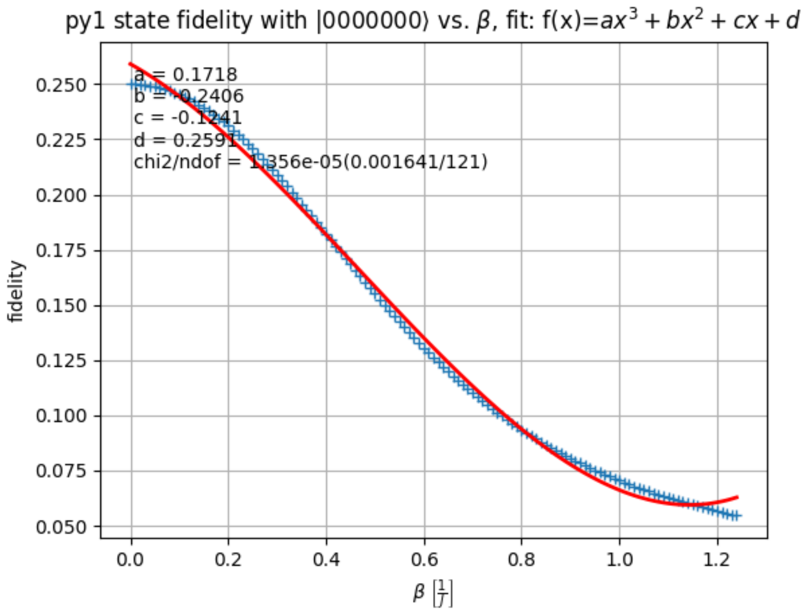

Figure 4.18: A cubic fit for the state fidelity of the system with $|0000000\rangle$, where the initial state of q0 is $|0\rangle_Y$.

Results for $|0\rangle_Z$:

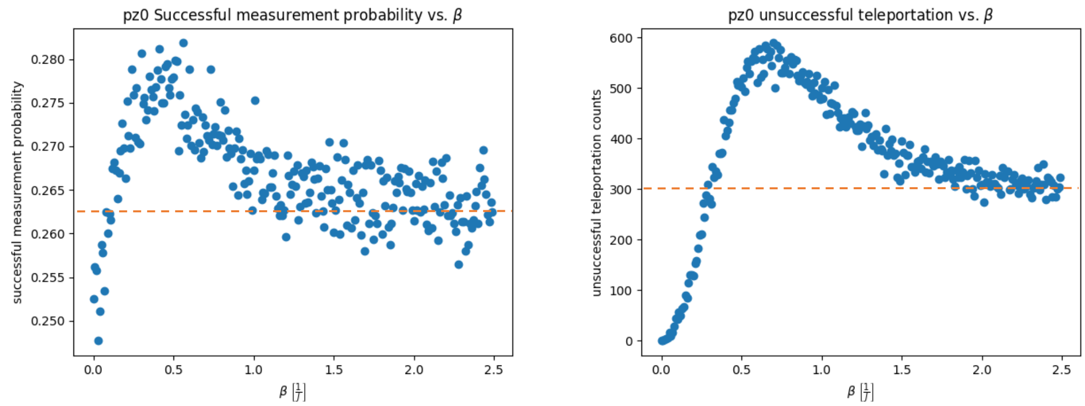

Figure 4.19: Successful teleportation probability and unsuccessful teleportation counts for $|0\rangle_Z$.

The orange stripped lines are the **asymptotes** of this graphs, achieved by generating a TFD state with minimum correlation[5] (no correlation), and running the experiment with it.

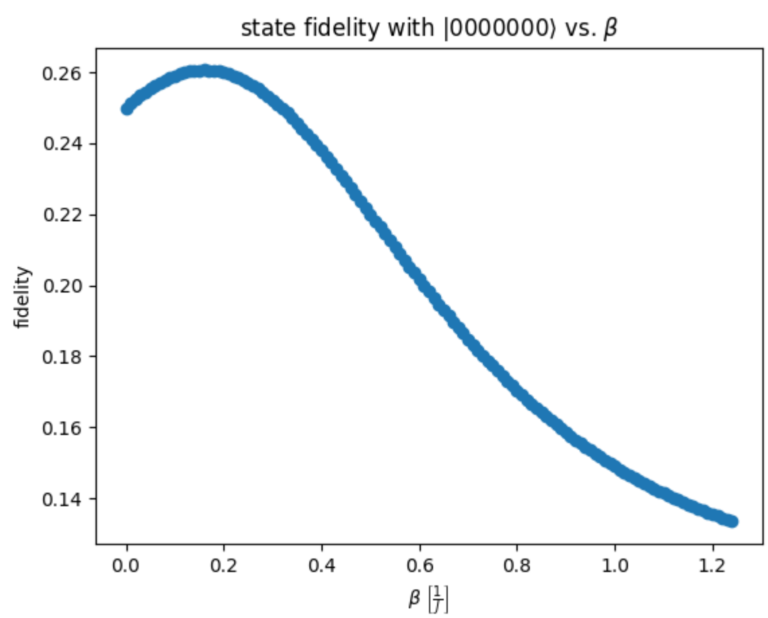

Figure 4.20: State fidelity with $|0000000\rangle$, for $|0\rangle_Z$ as the initial state of q0.

[5] This is practically done by generating a TFD state at temperature $T \to 0$.

Results for $|1\rangle_Z$:

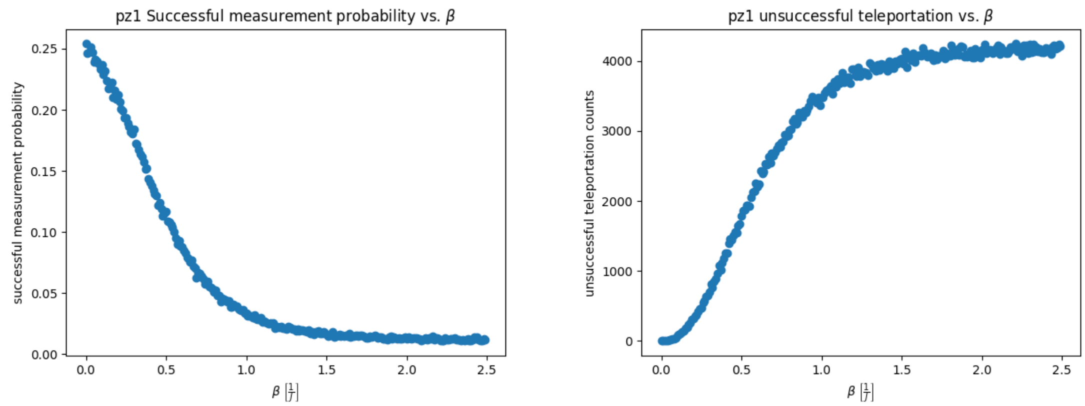

Figure 4.21: Successful teleportation probability and unsuccessful teleportation counts for $|1\rangle_Z$.

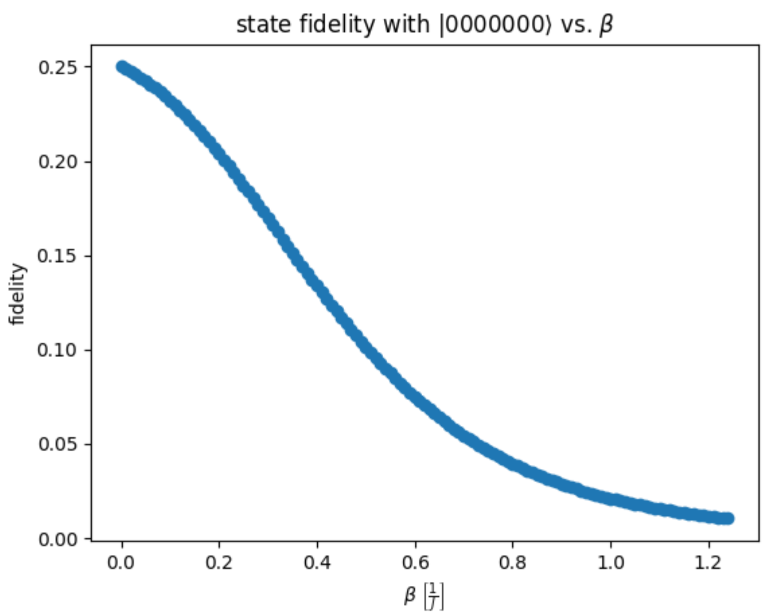

Figure 4.22: State fidelity with $|0000000\rangle$, for $|1\rangle_Z$ as the initial state of q0.

First, a square fit for the successful measurement probability vs. β, in the regime of $\beta = 0$:

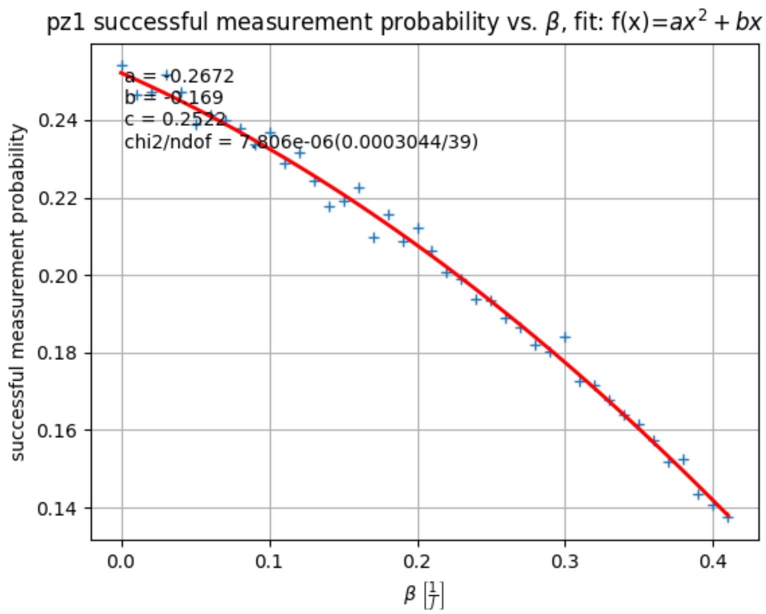

Figure 4.23: A square fit for the successful measurement's probability vs. β, where the initial state of q0 is $|1\rangle_Z$.

Second, a square fit for the unsuccessful teleportation counts, in the same regime:

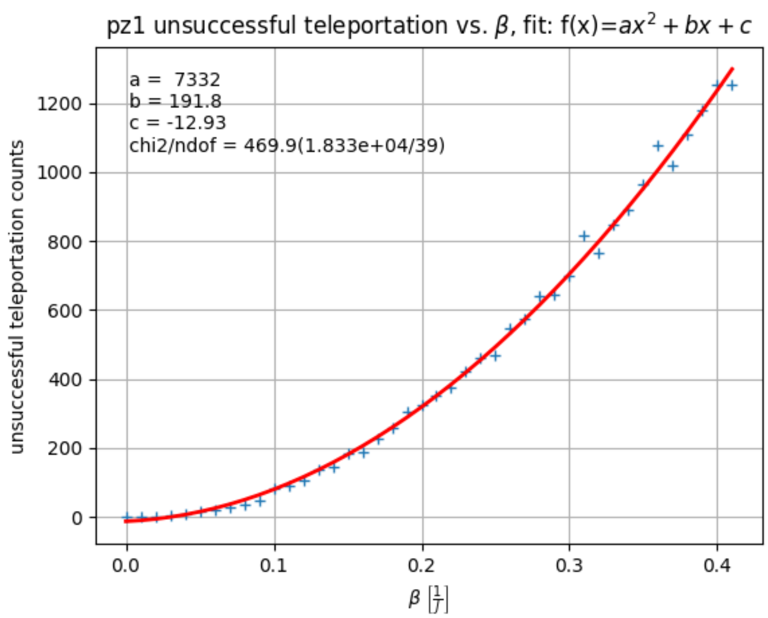

Figure 4.24: A square fit for the unsuccessful teleportation counts vs. β, where the initial state of q0 is $|1\rangle_Z$.

Finally, a cubic (polynomial of third degree) fit for the state fidelity, also in the above range:

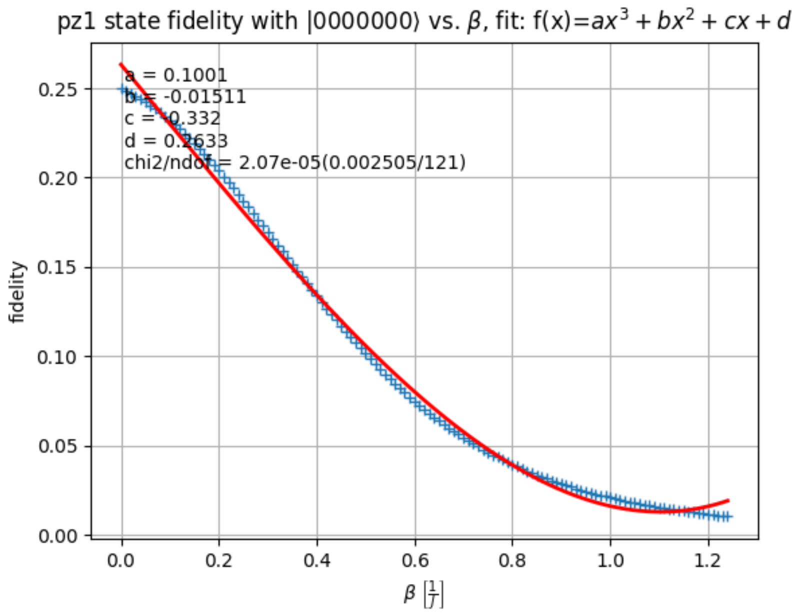

Figure 4.25: A cubic fit for the state fidelity with $|0000000\rangle$, where the initial state of q0 is $|1\rangle_Z$.

4.2 Random input state:

We then perform the experiment using a **random initial state** for the first qubit. This state is re-generated at each iteration of the circuit, meaning for each different TFD (i.e., different temperature), using qiskit's random state generator.

We conduct the experiment for β values in range $\beta \in [0, 1.5]$ with steps of $\delta\beta = 0.005$. This corresponds to temperatures from ∞ to 0.667 $[J]$.

Each experiment is made of 300 runs of the quantum circuit, 1 run for each temperature in the depicted range.

For each temperature, we show the mean values of the success probability and the unsuccessful teleportation counts out of 20,000 total shots, for 5, 10 and 20 experiments:

Success probability:

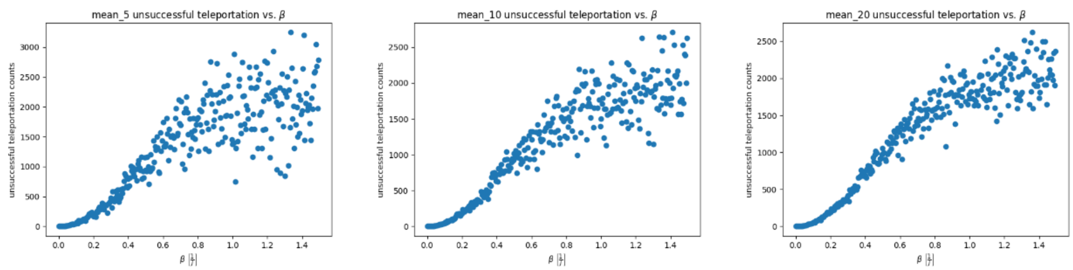

Figure 4.26: Mean values of unsuccessful teleportation for 5, 10 and 20 experiments (respectively from left to right), with a random initial state for q0.

Unsuccessful teleportation counts:

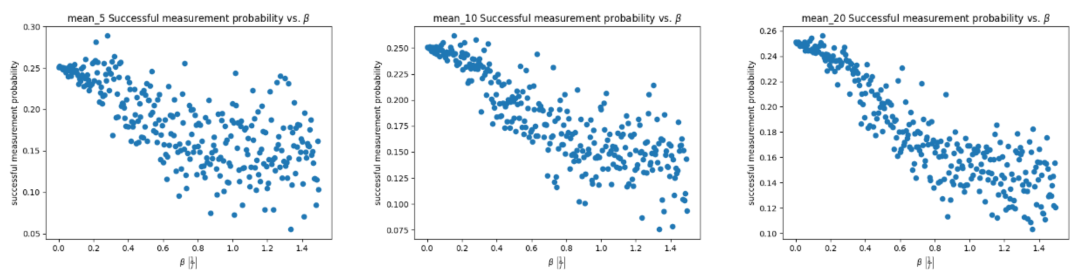

Figure 4.27: Mean values of the successful measurement probability for 5, 10 and 20 experiments (respectively from left to right), with a random initial state for q0.

The average fidelities of the system state with $|0000000\rangle$, where the initial state of the first qubit (q0) is random for 5, 10 and 20 experiments:

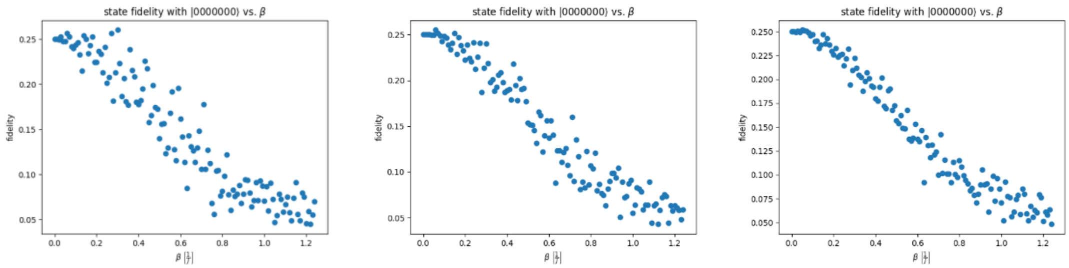

Figure 4.28: Average state fidelities for 5, 10 and 20 experiments (respectfully, from left to right) of the system state with $|0000000\rangle$, where the initial state of q0 is random. β is in the range $[0, 1.25]$, with steps 0f 0.01.

A square fit for the mean values of the unsuccessful teleportation counts from 20 experiments in which the initial state of q0 is random:

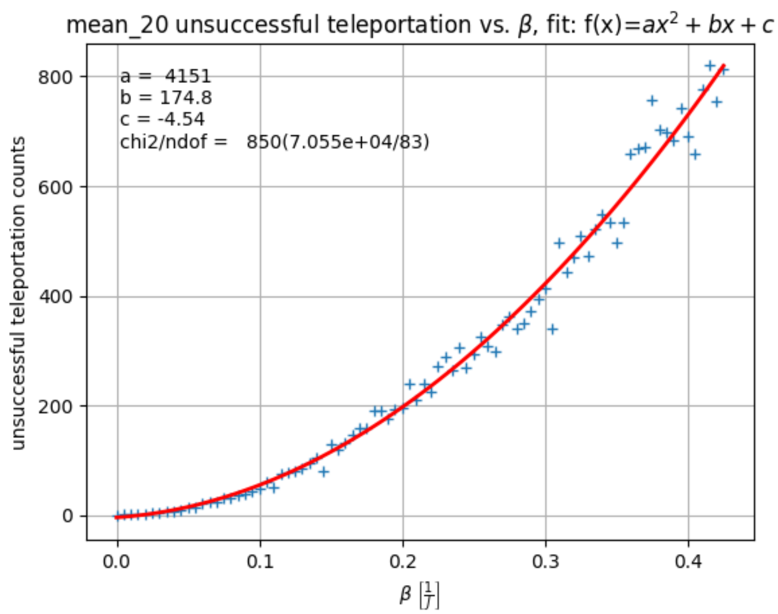

Figure 4.29: Unsuccessful teleportation vs. β, for the mean values of 20 experiments in which the initial state of q0 is random.

A square fit for the mean values of the successful measurement probability from 20 experiments in which the initial state is random, in the regime of the maximum correlation ($\beta = 0$):

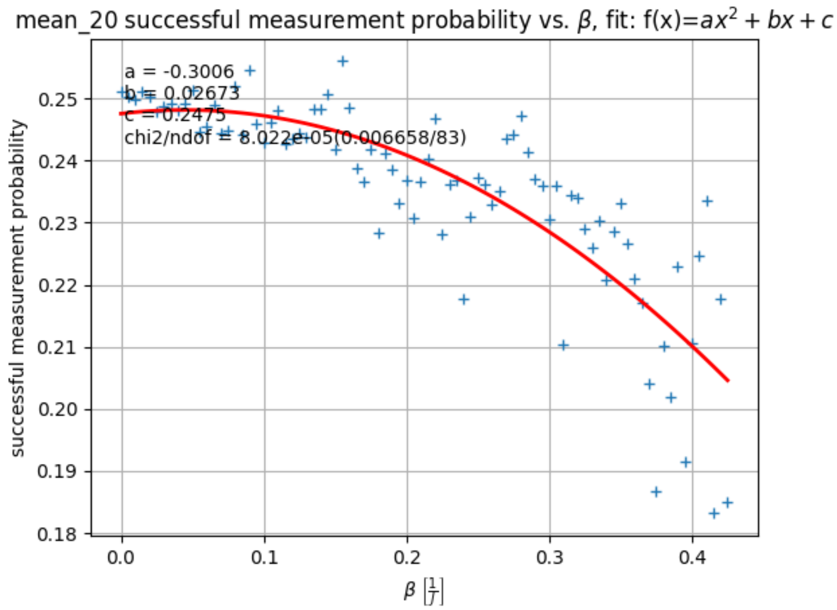

Figure 4.30: A square fit for the successful measurement probability vs. β, for the mean values of 20 experiments in which the initial state of q0 is random.

A cubic fit for the state-fidelity of the system with $|0000000\rangle$, for the average of 20 experiments in which the initial state of q0 is random:

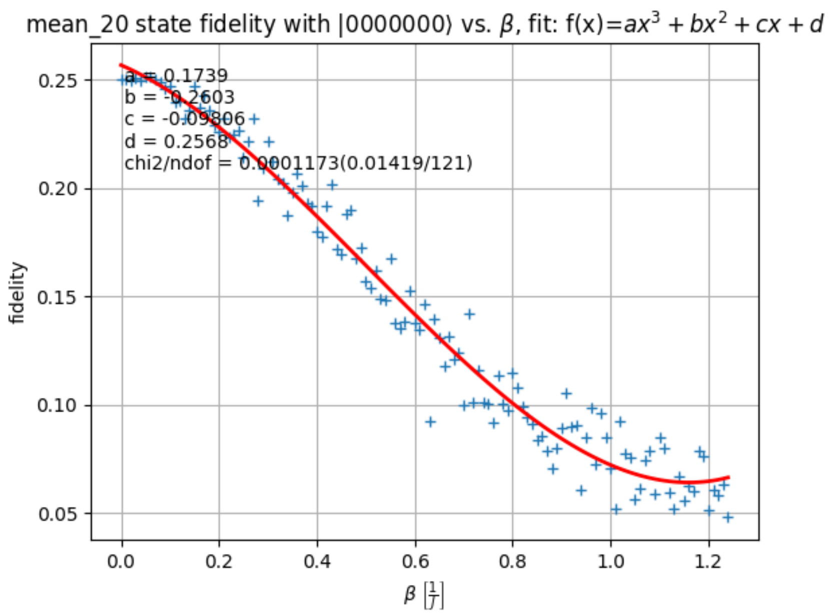

Figure 4.31: A cubic fit for the average state fidelity of 20 experiments, in which the initial state of the system is random.

5. Conclusions and outlook

5.1 Conclusions

First of all, our main conclusion from our research is that the results of the experiments indeed verify the robustness of the system suggested in [1], in the sense that the best teleportation qualities are achieved for the most correlation between the relevant qubits at the initial TFD state.

We see that in almost all cases the following properties are consistent:

- The amount of counts of the state $|1000000\rangle$ - which mark that the teleportation has failed - **increase monotonically** with respect to β (i.e., with $\frac{1}{T}$).

  We remind that $\beta = 0$ gives us maximum correlation between the relevant pairs in the TFD state (EPR pairs). This means that we the results indeed point out that the higher the correlation is between the relevant pairs of qubits, the lower the amount of counts of the state which marks that the teleportation has failed (i.e., the state $|1000000\rangle$).
- The probability of successful measurement (i.e., the probability of successful teleportation) **decreases monotonically** with respect to β (i.e., with $\frac{1}{T}$). This too, points out that the higher the correlation between the relevant qubits in the initial TFD state, the higher the probability for successful teleportation of the first qubit's state.
- The state-fidelity of the system state with $|0000000\rangle$ **decreases monotonically** with respect to β. This goes hand by hand with the prior points, yet it gives us another strong conformation that we are looking at things right.

Apart from the above encouraging insights, we gather the main conclusions of our work:

1. An interesting insight we conclude from the results of the experiments in which the initial state of q0 is a random state is that there exists an upper bound for the unsuccessful teleportation counts that depends on β and a lower bound of the successful measurement probability that too depends on β. We whish to find the function that serves as this bound for each case.

1.1 Unsuccessful teleportation counts:

After analysis, we reached the following bound: $f(x) = 5000 \cdot x^2 + 175 \cdot x + 9$.

We see that this function indeed serves as the wanted bound in the following plot:

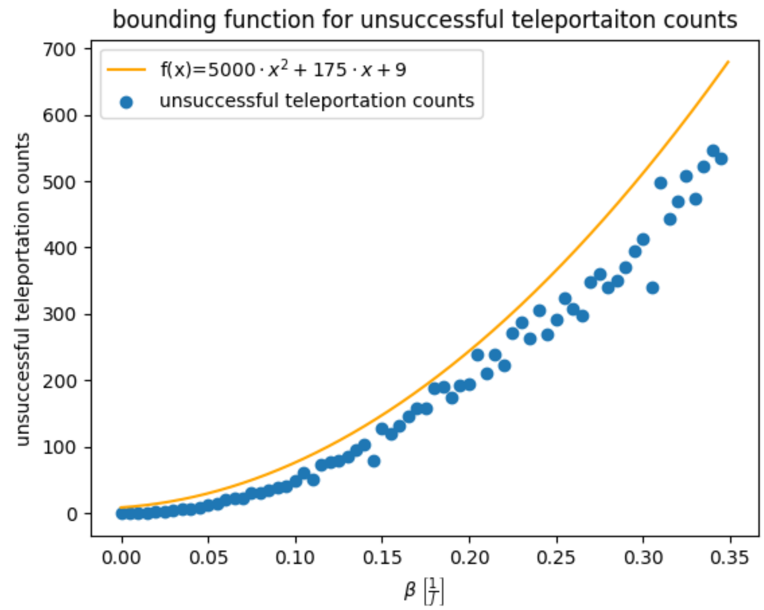

Figure 5.1: Upper bound of the unsuccessful teleportation counts.

1.2 Successful measurement probability:

After analysis, we reached the following bound: $f(x) = -0.52 \cdot x^2 + 0.025 \cdot x + 0.24$.

We see that this function indeed serves as the wanted bound in the following plot:

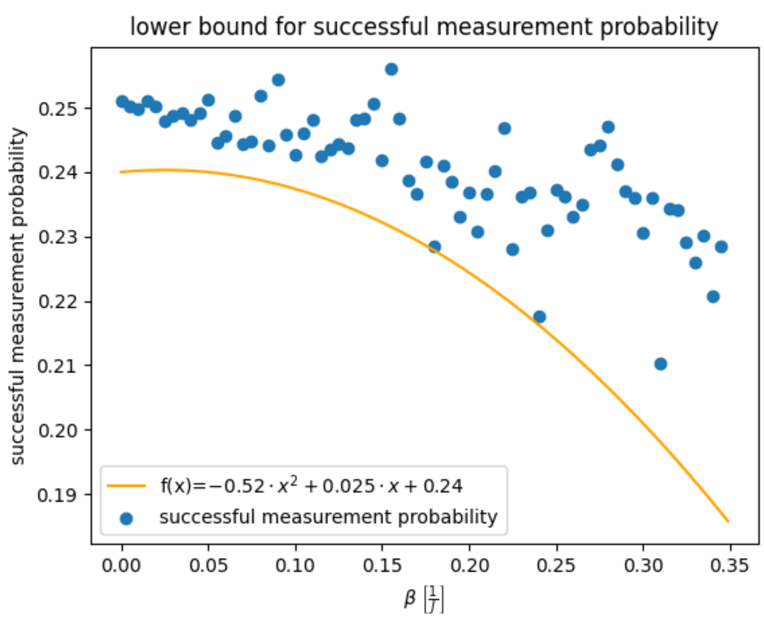

Figure 5.2: Lower bound of the successful measurement probability.

These bounds express the fact that when we change the correlations between the relevant pairs of the circuit by altering the temperature of the TFD state in which we initialize them to be, we don't ruin the teleportation qualities of the system at once, but we perform a rather gradual process which results in these bounds, which can be in turn very useful.

2. One case that gave us unexpected results (also the only case not to be consistent with the above insights) is the case in which the initial state of q0 is $|0\rangle_Z$. We got a surprising dataset for this experiment, which showed no decay in the successful measurement probability (not near the maximum-correlation zone and not asymptotically).
   Furthermore, the unsuccessful teleportation counts do not converge to a highest value, but peaks and then decays to a lower value, to which it indeed converges to (about 303 – pointed out by the orange stripped line).
   One possible explanation for this phenomenon, is that the generated system states, consisting of the initial state $|0\rangle_Z$ for q0, are somewhat close to a eigenstate of the Ising Hamiltonian that we use (same values for g and h). This possibly causes an illusion such as this, where it seems that the system is performing great teleportation for all temperatures, while it is at essence doing nothing.
   Later on, we perform the experiment using random initial states for q0, and take the average of the measured values. This leads to true teleportation results, given that the weight of this phenomenon is negligible.

5.2 Outlook

Although we have witnessed some interesting insights in this project, we do have some goals we would like to reach in the coming future. The main ones are:

1. We ran the circuits of this project on qiskit's Aer simulator only. The next step is to run it on a real trapped-ions quantum computer[6] and witness the footprints of the quantum computer on the experiment results (also, can conform or disprove our work, although the simulator is built to mimic the action of a real quantum computer).
2. We would like to check the results of this system averaged on several **Hamiltonians**, rather than initial states. Our intuition says that this should lead to similar results (and cancel out the circuit's behavior when the initial state of q0 is $|0\rangle_Z$), yet the Hamiltonian of the Ising system's integrability qualities varies when altering the values of h and g (see eq. (11)), therefore this is an interesting follow-up experiment to our opinion.
3. We would like to run the experiment with different sizes to test the robustness of this system. We (as [1]) tested the system for 7 qubits (scrambling operator for 3 qubits), we would like to test this mechanism using this experimental system for circuits with different numbers of qubits to earn agility and robustness.
4. Our findings in the fits we generated are interesting and intriguing, yet they are phenomenological for now. We would like to find (and by thus prove) the analytical solution for the measured quantities near the maximum correlation point ($\beta = 0$). This will allow us to understand whether our system is exact or incorrect, and correct it appropriately by further analysis of the results relative to the analytical predictions.
5. In our work, we build the TFD states using a python module that we run prior to the experiment on the quantum simulator. This TFD state is manually inserted to the quantum circuit and programmed to initialize the relevant qubits in the relevant states. This is effective and exact, yet we would like to perform **all** calculations on a single quantum computer without involving a classical computer computation. This means building a quantum circuit that generates the TFD state of a wanted system, given a

[6] One modern company that offers this kind of service is IonQ. One can buy an amount of windows on their trapped-ions quantum computer (that has well more then 7 qubits) and run this experiment for a very symbolical price.

temperature T. After doing so, we may run the whole experiment using one quantum circuit, which is obviously more comfortable and utilizes the quantum computer better.

6. In figures (4.26), (4.27) we see that around $\beta = 0.8 \left[\frac{1}{J}\right]$, there is a change in the general tendency of the data from a clear spread with a un-ambiguous trend to randomly spread data within some range.

   This is an interesting point which we would like to further investigate. In particular, we would like to characterize the factors that determine the values of this breaking point (breaking of trend).

7. In this work, we concentrated on the regime of $\beta = 0$ (for instance in the fits for the successful measurement probability and the unsuccessful teleportation counts). In future work we would like to expand our research to the farther regimes of the data from the maximum-correlation point.

6. Attachements

In order to avoid copying all project-related code (many pages), we opened a GitHub repository which contains all project files can be found in: GitHib repo.

** The data with which we generated all of the plots and fits of the project are not uploaded to the repository. If interested, you may email me at razmon@post.bgu.ac.il and I will happily transfer the data gathered throughout the project (also, a lot of data).

7. Bibliography

1. Landsman et al. “Verified quantum information scrambling” Nature, 567(7746).
2. Wu et al. “Variational quantum simulation via Thermofield double states”, 1811.11756v1.
3. Yoshida & Yao, “Disentangling scrambling and decoherence via quantum teleportation”, 1803.10772v1.
4. Qiskit – Quantum teleportation protocol.
5. Banuls, Cirac & Hastings. “Strong and weak thermalization on infinite non-integrable quantum systems”, 1007.3957v1.
6. Qiskit textbook.
7. Maldacena, Shenker & Stanford. “A bound on chaos”, 1503.01409v1, 2015.
8. Shenker & Stanford. “Black holes and the butterfly effect”, 1306.0622v3, 2014.

Old:

1. Qiskit – Quantum teleportation protocol.
2. Landsman et al. “Verified quantum information scrambling” Nature, 567(7746).
3. Wu et al. “Variational quantum simulation via Thermofield double states”, 1811.11756v1.
4. Banuls, Cirac & Hastings. “Strong and weak thermalization on infinite non-integrable quantum systems”, 1007.3957v1.
5. Yoshida & Yao, “Disentangling scrambling and decoherence via quantum teleportation”, 1803.10772v1.
6. Qiskit textbook.
7. Maldacena, Shenker & Stanford. “A bound on chaos”, 1503.01409v1, 2015.
8. Shenker & Stanford. “Black holes and the butterfly effect”, 1306.0622v3, 2014.

New one (change the numbers in the work…):

1. Landsman et al. “Verified quantum information scrambling” Nature, 567(7746).
2. Wu et al. “Variational quantum simulation via Thermofield double states”, 1811.11756v1.
3. Yoshida & Yao, “Disentangling scrambling and decoherence via quantum teleportation”, 1803.10772v1.
4. Qiskit – Quantum teleportation protocol.
5. Banuls, Cirac & Hastings. “Strong and weak thermalization on infinite non-integrable quantum systems”, 1007.3957v1.

** From here it’s the same.

6. Qiskit textbook.
7. Maldacena, Shenker & Stanford. “A bound on chaos”, 1503.01409v1, 2015.
8. Shenker & Stanford. “Black holes and the butterfly effect”, 1306.0622v3, 2014.