№1

- $\phi = 120\,mm \Rightarrow \phi = 0.12\,m \Rightarrow radius = 0.06\,m$
- $L = 2\,m$
- Maximum Torque $M = 300\,daNm \Rightarrow M = 3000\,Nm$
- Maximum speed of rotation $N = 4000\,RPM = 66.67\,RPS$
- $f_1 = \frac{\pi}{2}\sqrt{\frac{EI}{mL^3}}$
- $I = \pi r^3 t$ (area moment of inertia)

$J = \frac{\pi}{4}(r_o^4 - r_i^4)$

$+ \; I = \frac{J}{2}$ (for circle)

- Carbon/epoxy 60% $0°; 90°; +45° + -45°$
- $t = 0.125\,mm$
- $\rho_{carbon} = 1750\,Kg/m^3$; $\rho_{epoxy} = 1200\,Kg/m^3$

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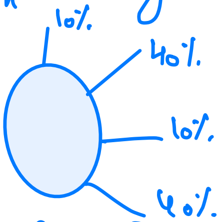

$\tau_{max} = \frac{M_T \times r}{J_p}$

$\tau_{max} = 327\,MPa$ (chart 5.3)

$\frac{N}{mm^2} = MPa$

with safety factor of 6 $\Rightarrow \tau_{allowable} = \frac{327}{6} = 54.5\,MPa$

$\Rightarrow \tau_{allowable} = \frac{3 \times 10^3 \times 0.06}{J_p = I} \Rightarrow 54.5 \times 10^6 = \frac{3000 \times 0.06}{\pi \times 0.06^3 \times t} \Rightarrow \boxed{t = 4.867 \times 10^{-3}\,m}$

$\Rightarrow \boxed{t = 2.434\,mm}$

$\Rightarrow$ № of plies $= \frac{2.434}{0.125} = 19.44 \approx \boxed{20}$

To check for f_1

we need E_x (chart 5.4)

$\Rightarrow E_x = 31.979\,GPa$

$I = \pi r^3 t = \pi \times (0.06)^3 \times (4.867 \times 10^{-3}) \Rightarrow \boxed{I = 3.303 \times 10^{-6}\,m^4}$

$m = \rho V$

But $\rho = \rho_f V_f + \rho_m V_m$

$= 1750(0.6) + (1200 \times 0.4) \Rightarrow \boxed{\rho = 1530\,Kg/m^3}$

$d = D - 2t$

$= 120 - 2(2.434) \Rightarrow d = 115.132\,mm$

$\boxed{r = 0.0576\,m}$

$\Rightarrow m = 1530 \times \frac{\pi}{4} \times (D^2 - d^2) L$

$\Rightarrow m = 1530 \times \pi (R^2 - r^2) L$

$= 1530 \times \pi \times (0.06^2 - 0.0576^2) \times 2$

$\Rightarrow \boxed{m = 2.713\,Kg}$

$\Rightarrow f_1 = \frac{\pi}{2}\sqrt{\frac{31979 \times 10^6 \times 3.303 \times 10^{-6}}{2.713 \times 2^3}} = 109.581\,RPS$

$= \boxed{6574.86\,RPM}$

$P_1 = 6574.86 > N = 4000\text{ RPM} \Rightarrow$ safe.

2) <u>steel</u>: $\tau_{max} = 300\text{ MPa}$
$\rho = 7800\text{ Kg/m}^3$

$$\Rightarrow \tau_{allowable} = \frac{300}{6} = 50\text{ MPa} \Rightarrow \tau = \frac{M \times r}{J}$$

$$\Rightarrow 50 \times 10^6 = \frac{3000 \times r}{\pi \times r^3 \times 2.434 \times 10^{-3}} \Rightarrow r = 0.088580\text{ m} \Rightarrow \boxed{r = 88.580\text{mm}}$$

$$m_{steel} = 7800 \times \pi \times (0.088580^2 - 0.086146^2) \times 2 \Rightarrow \boxed{m_{steel} = 20.843\text{kg}}$$

$$\Rightarrow \%m = \frac{20.843 - 2.713}{20.843} \times 100 \Rightarrow \boxed{86.984\%}$$

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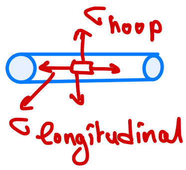
2) Thin walled pressure vessel $\Rightarrow$ stress is same along thickness

$\Rightarrow \frac{t}{r} < 0.1 \Rightarrow$ thin

open vessel $\Rightarrow \sigma_2 = 0 \Rightarrow \boxed{\sigma_x = 0}$

no shear $\Rightarrow$ normal highest 1 + lowest 2



$$\sigma_{hoop} = \sigma_1 = \sigma_y = \frac{P \cdot r}{t}$$

$$\sigma_{long} = \sigma_2 = \sigma_x = \frac{P r}{2t}$$

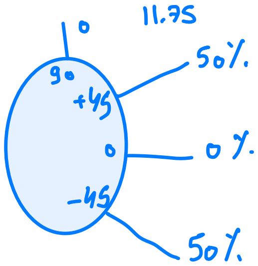
$P_0 = 10\text{ bars} = 1000\text{ KPa}$

FOS = 8

1) $\sigma_x = 0$ $\quad\quad \sigma_{y_{max}} = 94\text{ MPa}$

2) $\sigma_y = P_0 \frac{r}{t}$ $\quad \Rightarrow \sigma_{y_{allowable}} = \frac{94}{8} = 11.75\text{ MPa} \Rightarrow \sigma = \frac{P_0 \times r}{t} \Rightarrow t = \frac{P_0 \times r}{\sigma}$

$$\Rightarrow t = \frac{1}{11.75} \times 100 \Rightarrow \boxed{t = 8.511\text{ mm}}$$



3) <u>5.14</u> $E_x = 14130\text{ MPa}$
$E_y = E_x = 14130\text{ MPa}$
$\nu_{xy} = 0.57$
$\nu_{yx} = 0.57$

<u>5.15</u> $G_{xy} = 12760\text{ MPa}$

$\Rightarrow$ <u>law of behavior</u>:
$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{bmatrix} \frac{1}{14130} & \frac{-0.57}{14130} & 0 \\ \frac{-0.57}{14130} & \frac{1}{14130} & 0 \\ 0 & 0 & \frac{1}{12760} \end{bmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix}$$

4) $\sigma_x = 0$
$\sigma_y = 11.75$ (not threshold because there is FOS = 8)
$\tau_{xy} = 0$

$$\Rightarrow \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} -0.476 \\ 0.835 \\ 0 \end{pmatrix} \times 10^{-3}$$

$\Longrightarrow$ Now rotate by $\theta = 45°$

$$\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{pmatrix} = \begin{pmatrix}0.5 & 0.5 & 1\\ 0.5 & 0.5 & -1\\ -0.5 & 0.5 & 0\end{pmatrix}\begin{pmatrix}-0.476\\ +0.835\\ 0\end{pmatrix}\times 10^{-3} \Longrightarrow \varepsilon_2 = \left(\frac{0.835-0.476}{2}\right)\times 10^{-3} = 0.017\% < 0.1\%$$

$\Longrightarrow$ Take transverse of $1 \Rightarrow$ 2

N°3 Radius R

wall thickness e_0

P_0 ; glass/epoxy

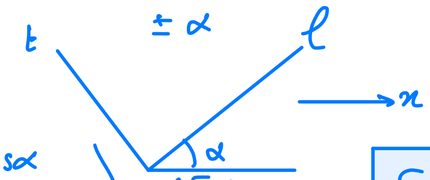
1) $\sigma_x = \sigma_2 = \dfrac{P_0 r}{2e_0}$

$\sigma_y = \sigma_1 = \dfrac{P_0 r}{e_0}$

theoretically $\Longrightarrow$ no shearing (cross_section)

2) isotensoid $\Longrightarrow$ in tension

fibers of orientation



a)

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{pmatrix} = \begin{pmatrix}\cos^2\alpha & \sin^2\alpha & 2\sin\alpha\cos\alpha\\ \sin^2\alpha & \cos^2\alpha & -2\sin\alpha\cos\alpha\\ -\sin\alpha\cos\alpha & \sin\alpha\cos\alpha & \cos^2\alpha-\sin^2\alpha\end{pmatrix}\begin{pmatrix}\sigma_L\\ 0\\ 0\end{pmatrix} \Longrightarrow \boxed{\begin{array}{l}\sigma_x = \sigma_L\cos^2\alpha \quad \text{--- } 1\\ \sigma_y = \sigma_L\sin^2\alpha \quad \text{--- } 2\end{array}}$$

b) $\sigma_x = \dfrac{P_0 R}{2t}$

$\sigma_y = \dfrac{P_0 R}{t}$

$$\frac{2}{1} \Longrightarrow \tan^2\alpha = \frac{\sigma_y}{\sigma_x} = \frac{\frac{P_0 R}{t}}{\frac{P_0 R}{2t}} = 2 \Longrightarrow \boxed{\tan\alpha = \sqrt{2}}$$

$\Longrightarrow \boxed{\alpha = 54{,}7°}$

$\Longrightarrow \sigma_L = \sigma_x + \sigma_y$ or $\sigma_x + \sigma_y = \sigma_L(\cos^2\alpha + \sin^2\alpha) \Longrightarrow \boxed{\sigma_L = \sigma_x + \sigma_y}$

$$\boxed{\sigma_L = \frac{3P_0 R}{2t}}$$

c) $R = 40\text{cm}$

$V_f = 80\%$

$S_{glass} = 3200\text{MPa}$

$P_0 = 200$ bars

$$\Longrightarrow \sigma_L = \frac{3P_0 r}{2t} = 3200$$

$\Longrightarrow \boxed{t = 3{,}75 \text{ mm}} \Longrightarrow \boxed{e_0 = \dfrac{t}{V_f} = 4{,}6875\text{mm}}$

No 4)

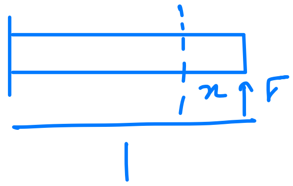
core: polystyrene
skin: aluminum

$e_p = 2.15mm$; $e_c = 80.2\ mm$
$E_p = 65200\ MPa$; $E_c = 21.5\ MPa$
$G_p = 24890\ MPa$; $G_c = 7.7\ MPa$

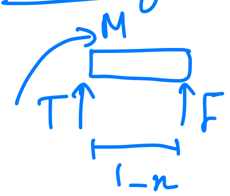
core support (is affected by shearing)
$\Longrightarrow$ Transverse shear

Castigliano:

no victious force
since study w.r.t F



take right side:



$\Longrightarrow T + F = 0 \Longrightarrow T = -F \Longrightarrow \boxed{\frac{\partial T}{\partial F} = -1}$

$M - (1-x)F = 0 \Longrightarrow M = F(1-x) \Longrightarrow \boxed{\frac{\partial M}{\partial F} = 1 - x}$

$$\frac{1}{\langle EI \rangle} = \frac{1}{E_p e_p \times (1) \times \frac{(e_c + e_p)^2}{2}} = \frac{1}{65200 \times 10^6 \times 2.15 \times 10^{-3} \times 0.1 \times \frac{(80.2 + 2.15)^2}{2} \times 10^{-6}}$$

width

$$\Longrightarrow \boxed{\frac{1}{\langle EI \rangle} = 2.104 \times 10^{-5} (N m^2)^{-1}}$$

$$\frac{K}{\langle GS \rangle} = \frac{1}{G_c (e_c + 2e_p) \times 0.1} = \frac{1}{7.7 \times 10^6 \times (80.2 + 4.3) \times 10^{-3} \times 0.1} \Longrightarrow \boxed{\frac{K}{\langle GS \rangle} = 1.537 \times 10^{-5} N^{-1}}$$

$$\delta = \frac{1}{\langle EI \rangle} \int M \frac{dM}{dF} dx + \frac{K}{\langle GS \rangle} \int T \frac{dT}{dF} dx$$

$$= 2.104 \times 10^{-5} \left[F \int_0^1 (1-x)(1-x) dx \right] + 1.537 \times 10^{-5} \left[F \int_0^1 dx \right]$$

$$\delta = 2.104 \times 10^{-5} \times F \left[\int_0^1 (1-x)^2 dx \right] + 1.537 \times 10^{-5} \times F \times x^1 - x^0$$

$$\delta = 0.02104 \int_0^1 \left[1 - 2x + x^2 \right] dx + 0.01537 \times 1$$

$$\delta = 0.02104 \left(x - x^2 + \frac{x^3}{3} \right]_0^1 + 0.01537$$

$$= 0.02104\ \ 1 - 1 + \frac{1}{3} + 0.01537 \Longrightarrow \boxed{\delta = 22.38 mm}$$

can't neglect shear in sandwich

Extra: Find max normal & shear stresses developed in beam.

M at $x=0$ & $M(x=L) \Rightarrow M_{max} = 1000$

$$\sigma = \frac{M_{max}}{0.1 \times e c_{ep}}$$

$$= \frac{1000}{0.1 \times 2.15 \times 80.2} \Rightarrow \boxed{\sigma = 57.99\ MPa}$$

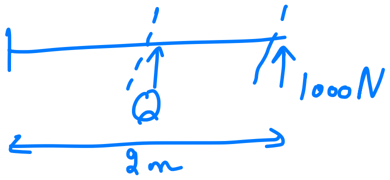
$$\tau_{max} = \frac{1000}{0.1 \times 80.2 \times 10^{-3}} \Rightarrow \boxed{\tau_{max} = 124.6\ MPa}$$

$K = 0.25$

$$P_{cr} = \frac{0.25\ \pi^2 (1.537 \times 10^{-5})^{-1}}{1 + \pi^2 (1.537 \times 10^{-5})^{-1} \times 0.25 \times (2.104 \times 10^{-5})}$$

$$\boxed{P_{cr} = 41.846\ KN}$$

Extra Problem:



$0 \le x \le 1$ left side

$T_1 = -Q - 1000 \Rightarrow \frac{\partial T_1}{\partial Q} = -1$

$M_1 = (1-x)Q + 1000(2-x)$

$\frac{\partial M_1}{\partial Q} = 1 - x$

$1 \le x \le 2$

$T_2 = -1000\ N \Rightarrow \frac{\partial T_2}{\partial Q} = 0$

$M_2 = 1000(2-x) \Rightarrow \frac{\partial M_2}{\partial Q} = 0$

$\boxed{Q = 0}$

$$\delta = \frac{1}{\langle EI \rangle} \int M \frac{dM}{dQ} dx + \frac{K}{\langle GS \rangle} \int T \frac{dT}{dQ} dx$$

$$\delta = \frac{1}{\langle EI \rangle} \int_0^1 1000(2-x)(1-x)dx + \int_1^2 1000(\cancel{2-x}) \times 0\, dx$$

$$+ \frac{K}{\langle GS \rangle} \int_0^1 +1000\, dx + 0$$

$$\delta = \frac{1}{\langle EI \rangle} \times 1000 \left[\int_0^1 2 - 3x + x^2\, dx \right] + \frac{K}{\langle GS \rangle} \times 1000$$

$$\delta = \frac{1}{\langle EI \rangle} \times 1000 \left[2x - \frac{3x}{2} + \frac{x^3}{3} \right]_0^1 + \frac{1000K}{\langle GS \rangle}$$

$$\delta = \frac{1}{\langle EI \rangle} \times 1000 \left[0.5 + \frac{1}{3} \right] + \frac{1000K}{\langle GS \rangle}$$

$$\Rightarrow \boxed{\delta = \qquad mm}$$

No 5 $F = 500N$

$$I = \frac{bh^3}{12} = \frac{0.1}{12} \times (5\times10^{-3})^3$$

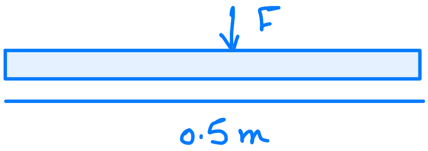
$$I = 1.042\times10^{-5}\, m^4$$

① $$A = \frac{PL^3}{48EI} = \frac{500\times0.5}{48\times65.2\times10^6\times1.042\times10^{-5}}$$

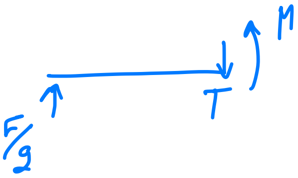
$$\Longrightarrow \boxed{A = 19.17mm}$$

② $$\frac{1}{(EI)} = \frac{1}{65.2\times10^6\times2.5\times10^3\times0.1\times\left(\frac{2.5+25}{2}\right)\times10^{-6}} = 1.622\times10^{-4}\,(Nm^2)^{-1}$$

$$\frac{K}{(GS)} = \frac{1}{20\times10^6\times(25+5)\times10^{-3}} = 1.67\times10^{-5}\,(N)^{-1}$$



$$0 \le x \le 0.25$$



$$T_1 = \frac{F}{2} \Rightarrow \frac{\partial T_1}{\partial F} = \frac{1}{2}$$

$$M_1 = \frac{F}{2}x \Rightarrow \frac{dM}{dF} = \frac{x}{2}$$

$$0.25 \le x \le 0.5$$

$$T_2 = \frac{F}{2} \Rightarrow \frac{\partial T_2}{\partial F} = \frac{1}{2}$$

$$M_2 = \frac{F}{2}\left(\frac{1}{2} - x\right) \Rightarrow \frac{dM_2}{dF} = \frac{1}{2}\left(\frac{1}{2} - x\right)$$

$$\delta = A' = 1.622\times10^{-4}\left[\frac{F}{4}\int_0^{0.25} x^2dx + \frac{F}{4}\int_{0.25}^{0.5}\left(\frac{1}{2}-x\right)^2dx\right]$$

$$+1.67\times10^{-5}\left[\frac{F}{4}\int_0^{0.25}dx + \frac{F}{4}\int_{0.25}^{0.5}dx\right]$$

$$\Longrightarrow \boxed{\delta = 1.251mm}$$

$$\frac{A}{A'} = 15.3$$

$$\frac{A}{l} = \frac{1.251}{500} = 0.0025 \approx \frac{1}{400} \Rightarrow \text{safe}$$

⑥ $t = 35\,mm$

$G = 18\,MPa$

$0°$  $I\ 0.125\,m$ $\quad V_f = 60\%$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} N/mm \; ; \; \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} N\,mm/mm \; ; \; \begin{bmatrix} Q_y \\ Q_z \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \end{bmatrix} N/mm$$

since 100% on $0° \Rightarrow$ 5.14 & 5.15 $\rightarrow E_1 = 45\,GPa$; $E_2 = 12\,GPa$; $\nu_{12} = 0.3$; $\nu_{21} = 0.08$;
$G_{12} = 4.5\,GPa$

$$Q_{11} = \frac{45}{1-(0.3\times 0.08)} = 46.107\,GPa \; ; \; Q_{12} = 0.3 \times 12.295 = 3.6885\,GPa$$

$$Q_{22} = \frac{12}{1-(0.3\times 0.08)} = 12.295\,GPa \; ; \; Q_{66} = 4.5\,GPa$$

$$[Q] = [\bar{Q}]_0$$

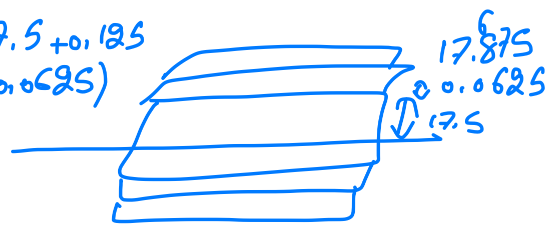
• Matrix $[A]$:

$$\left.\begin{aligned} A_{11} &= Q_{11} \times 0.125 \times 4 = 23.054 \\ A_{22} &= Q_{22} \times 0.125 \times 4 = 6147.5 \\ A_{12} &= Q_{12} \times 0.125 \times 4 = 1.844 \\ A_{66} &= Q_{66} \times 0.125 \times 4 = 2.250 \end{aligned}\right\} MN/m$$

$[B]$ & $[C] = 0$ (symmetry)

$$D_{ij} = \frac{h}{2}\left(L_{ij}^{(2)} - L_{ij}^{(1)}\right) \longrightarrow (17.5 + 0.0625) + (17.5 + 0.125 + 0.0625)$$

$$= h\,[L_{ij}]^{(2)}$$



isotropic core material $\Rightarrow [F] = \begin{bmatrix} 630 & 0 \\ 0 & 630 \end{bmatrix} N/mm$

Due to symmetry & uncoupling