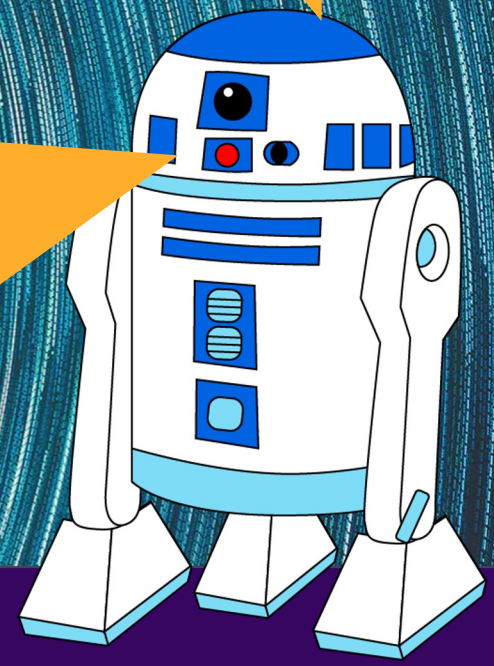
CIS 4210/5210:
ARTIFICIAL INTELLIGENCE

Informed Search

HW2 is due Wednesday by 11:59pm.

If you need an extension, you must request it 24 hours in advance to avoid a late penalty.



Review: Search problem definition

States: a set ***S***

An *initial state* $s_i \in S$

Actions: a set ***A***

$\forall$ ***s Actions(s)*** = *the set of actions that can be executed in* ***s****, that are applicable* in ***s***.

Transition Model: $\forall s \forall a \in$ ***Actions(s) Result(s, a)*** $\rightarrow s_r$

s_r *is called a successor of* ***s***

$\{s_i\} \cup$ ***Successors***(s_i)* = *state space*

Path cost (Performance Measure): Must be additive

e.g. sum of distances, number of actions executed, …

c(x,a,y) is the step cost, assumed ≥ 0

(where action ***a*** goes from state ***x*** to state ***y****)*

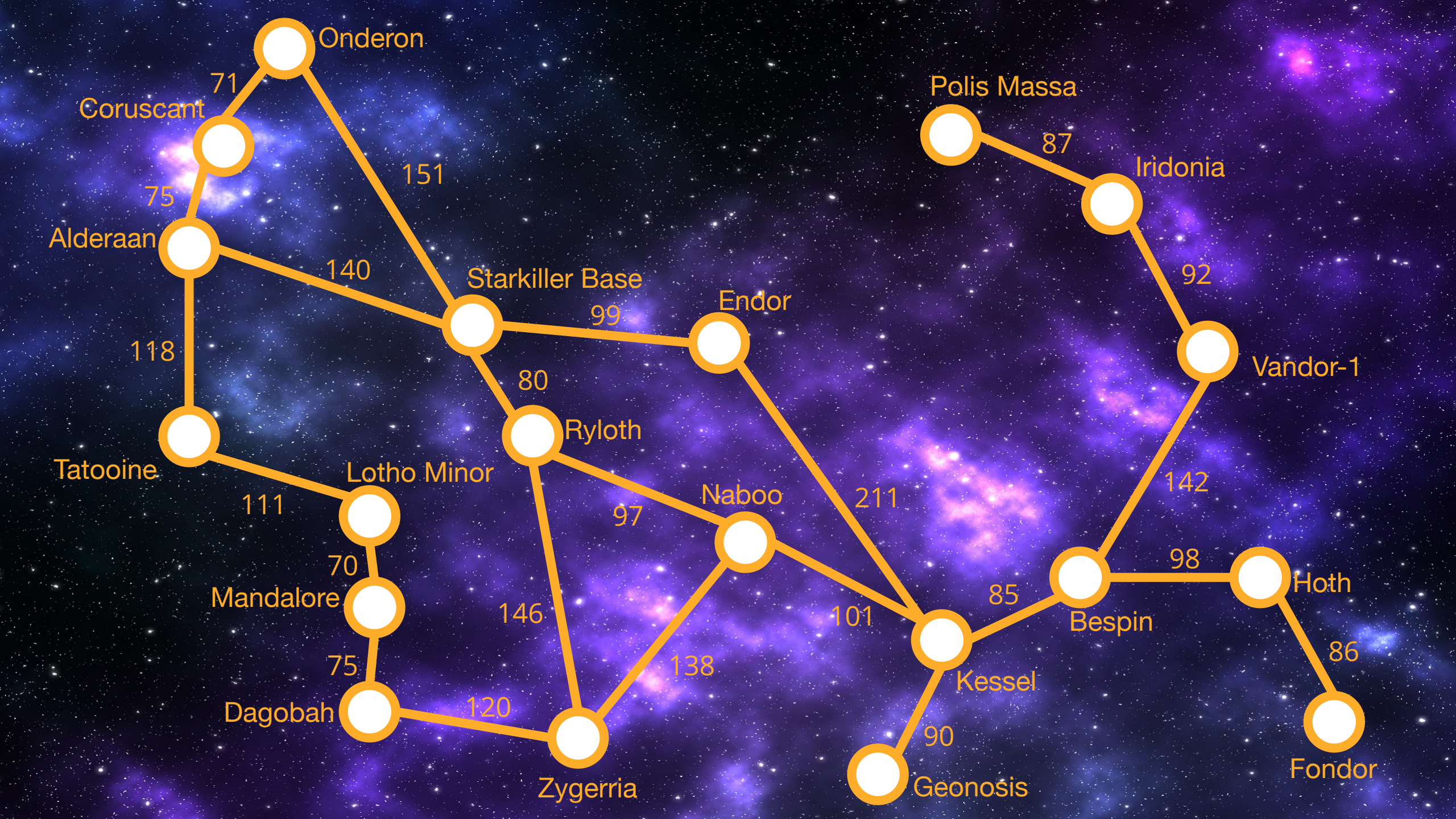
Goal test: ***Goal(s)***

Can be implicit, e.g. ***checkmate(s)***

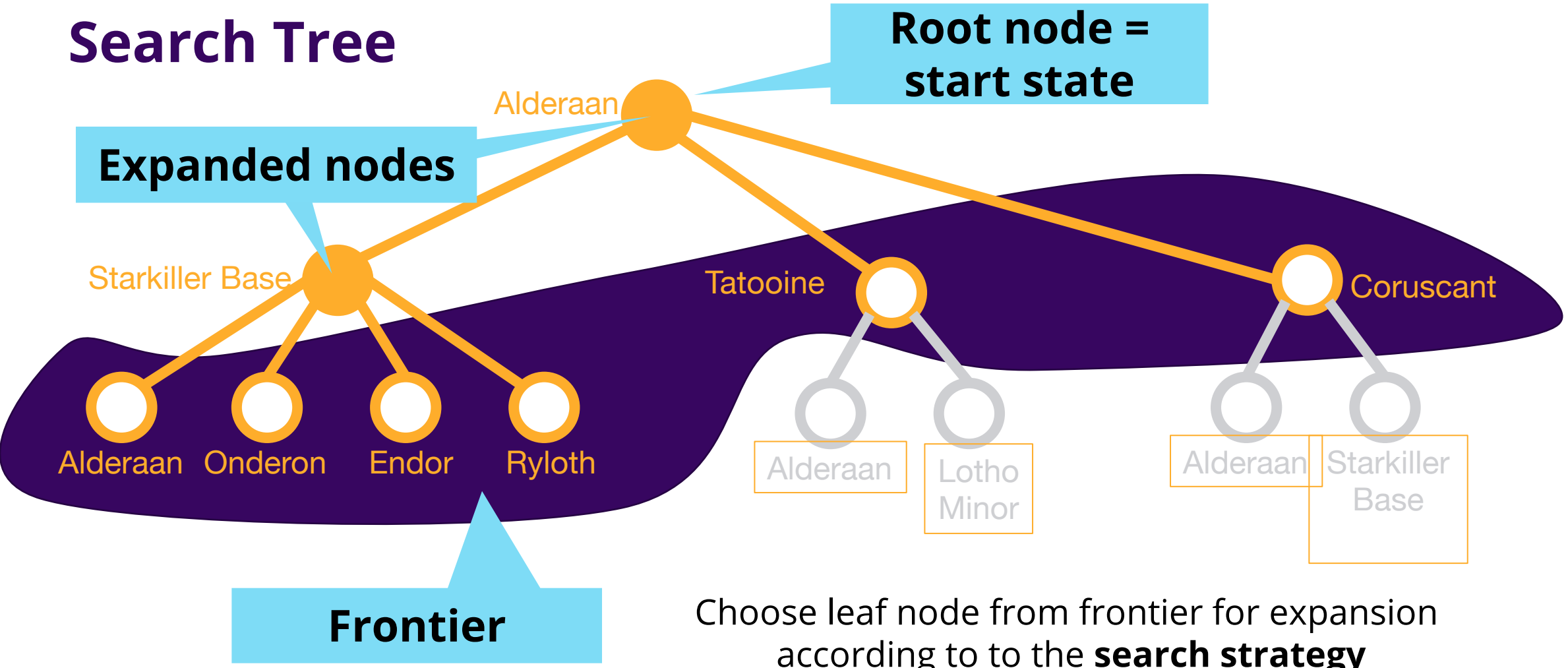
s is a *goal state* if ***Goal(s)*** is *true*

Review: Useful Concepts

- *State space*: the set of all states reachable from the initial state by *any* sequence of actions
  - *When several operators can apply to each state, this gets large very quickly*
  - *Might be a proper subset of the set of configurations*
- *Path*: a sequence of actions leading from one state s_j to another state s_k
- *Frontier:* those states that are available for *expanding* (for applying legal actions to)
- *Solution*: a path from the initial state s_i to a state s_g that satisfies the goal test


Onderon
71
Coruscant
151
75
Alderaan
140
Starkiller Base
99
Endor
118
80
Ryloth
Tatooine
Lotho Minor
111
Naboo
97
211
70
Mandalore
146
101
85
Bespin
98
Hoth
138
75
Kessel
86
Dagobah
120
90
Fondor
Zygerria
Geonosis
Polis Massa
87
Iridonia
92
Vandor-1
142

Search Tree



Choose leaf node from frontier for expansion according to to the **search strategy**

Determines the search process

Review: Search Strategies

Strategy = order of tree expansion

- Implemented by different **queue structures** (LIFO, FIFO, priority)

Dimensions for evaluation

- *Completeness-* always find the solution?
- *Optimality* - finds a least cost solution (lowest path cost) **first**?
- *Time complexity* - # of nodes generated *(worst case)*
- *Space complexity* - # of nodes simultaneously in memory *(worst case)*

Time/space complexity variables

- *b*, *maximum branching factor* of search tree
- *d*, *depth* of the shallowest goal node
- *m*, maximum length of any path in the state space (potentially ∞)

Animation of Graph BFS algorithm
set to music 'flight of bumble bee'

https://youtu.be/x-VTfcmrLEQ

Animation of Graph DFS algorithm
Depth First Search of Graph
set to music 'flight of bumble bee'

https://youtu.be/NUgMa5coCoE

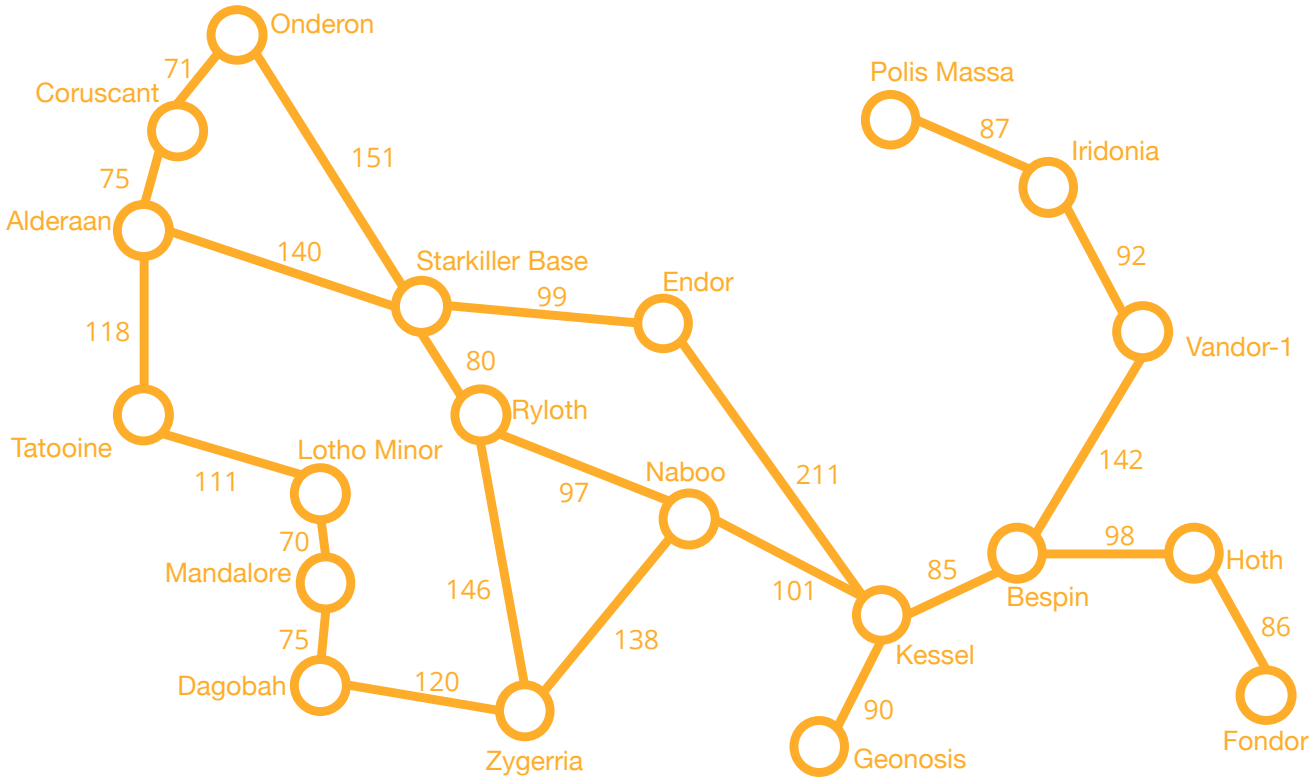
“Uniform Cost” Search

“In computer science, *uniform-cost* search (UCS) is a tree search algorithm used for traversing or searching a *weighted* tree, tree structure, or graph.” - Wikipedia

Motivation: Map Navigation Problems

All our search methods so far assume *step-cost = 1*

This is only true for some problems



g(N): the path cost function

- **Our assumption so far: All moves equal in cost**
  - Cost = # of nodes in path-1
  - *g(N)* = *depth(N)* in the search tree

- More general: Assigning a (potentially) unique cost to each step
  - N_0, N_1, N_2, N_3 = nodes visited on path *p* from N_0 to N_3
  - *C(i,j):* Cost of going from N_i to N_j
  - If N_0 the root of the search tree,

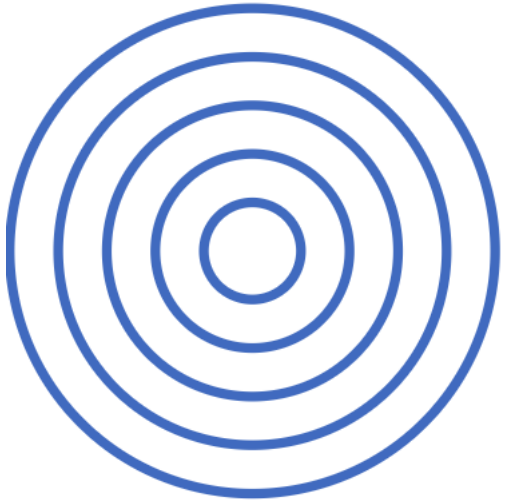
$$g(N3)=C(0,1)+C(1,2)+C(2,3)$$

Uniform-cost search (UCS)

- Extension of BF-search:
  - **Expand node with *lowest path cost***
- Implementation:
  - *frontier* = priority queue ordered by g(n)
- Subtle but significant difference from BFS:
  - Tests if a node is a goal state when it is selected for expansion, not when it is added to the frontier.
  - Updates a node on the frontier if a better path to the same state is found.
  - So always enqueues a node *before checking whether it is a goal.*

WHY???

Shape of Search



- **Breadth First Search** explores equally in all directions. Its frontier is implemented as a FIFO queue. This results in smooth contours or “plys”.

- **Uniform Cost Search** lets us prioritize which paths to explore. Instead of exploring all possible paths equally, it favors lower cost paths. Its frontier is a priority queue. This results in “cost contours”.

This may remind you of something...

Which algorithm from your algorithms class does Uniform Cost Search remind you of?

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Uniform Cost Search (UCS) vs. Dijkstra's algorithm

Foundational Concepts:

- Dijkstra's: Shortest path from initial to all vertices in a weighted graph.
- UCS: AI's pathfinding. Shortest path from initial state to a goal state.

Similarities:

- Use of **Priority Queue** to pick lowest cost node.
- **Greedy Nature**: Always choose optimal local solution.
- Both operate on a **weighted graph**.
- Guarantee **Optimality** for non-negative weights.

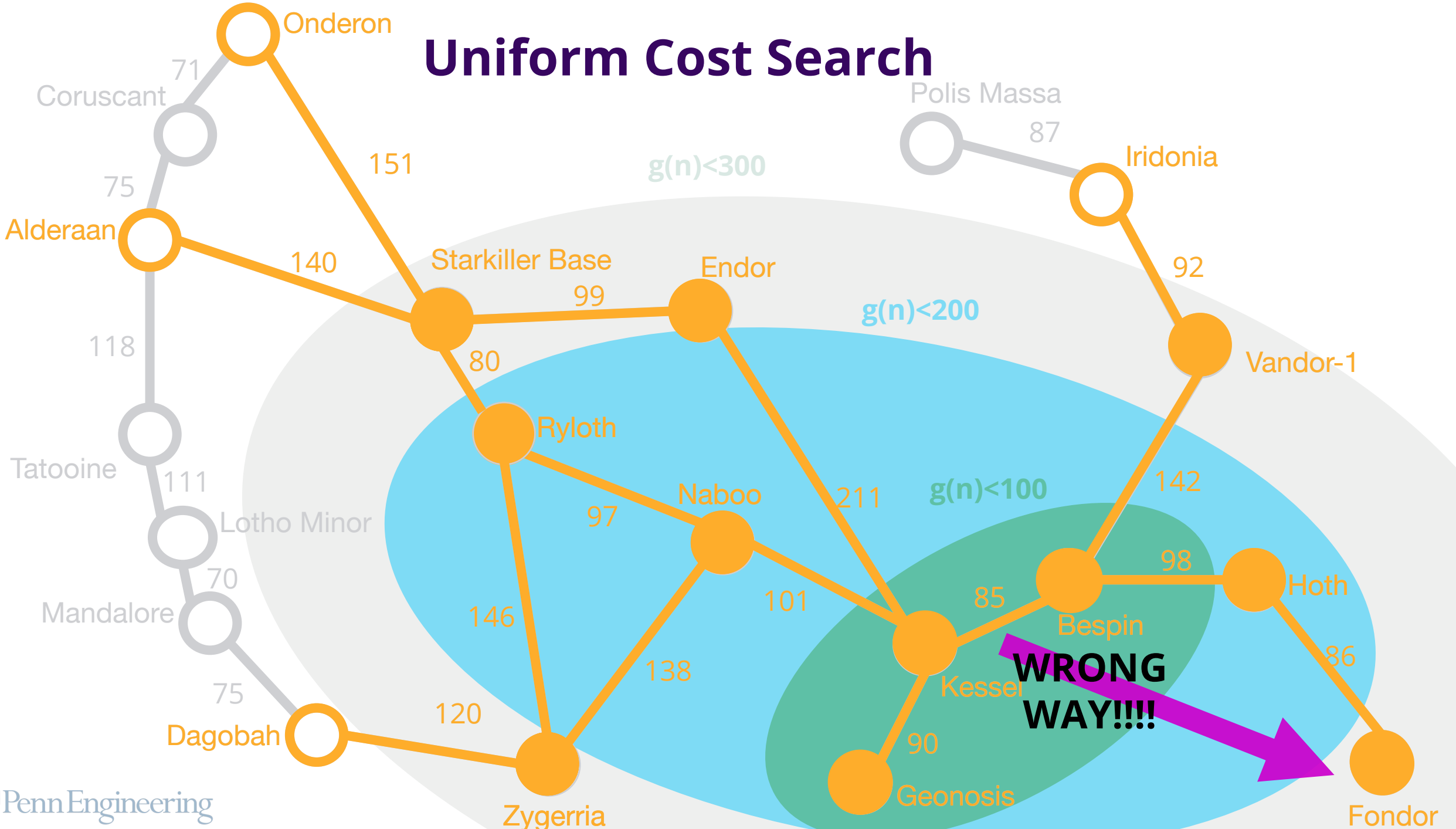
Differences:

- **Objective**: UCS targets specific goal node; Dijkstra's finds paths to all nodes.
- **Termination**: UCS stops at goal; Dijkstra's stops after all nodes have been reached.

Relation:

- UCS is a "targeted" Dijkstra’s: it seeks a path to a specific node, not all nodes.

Uniform Cost Search

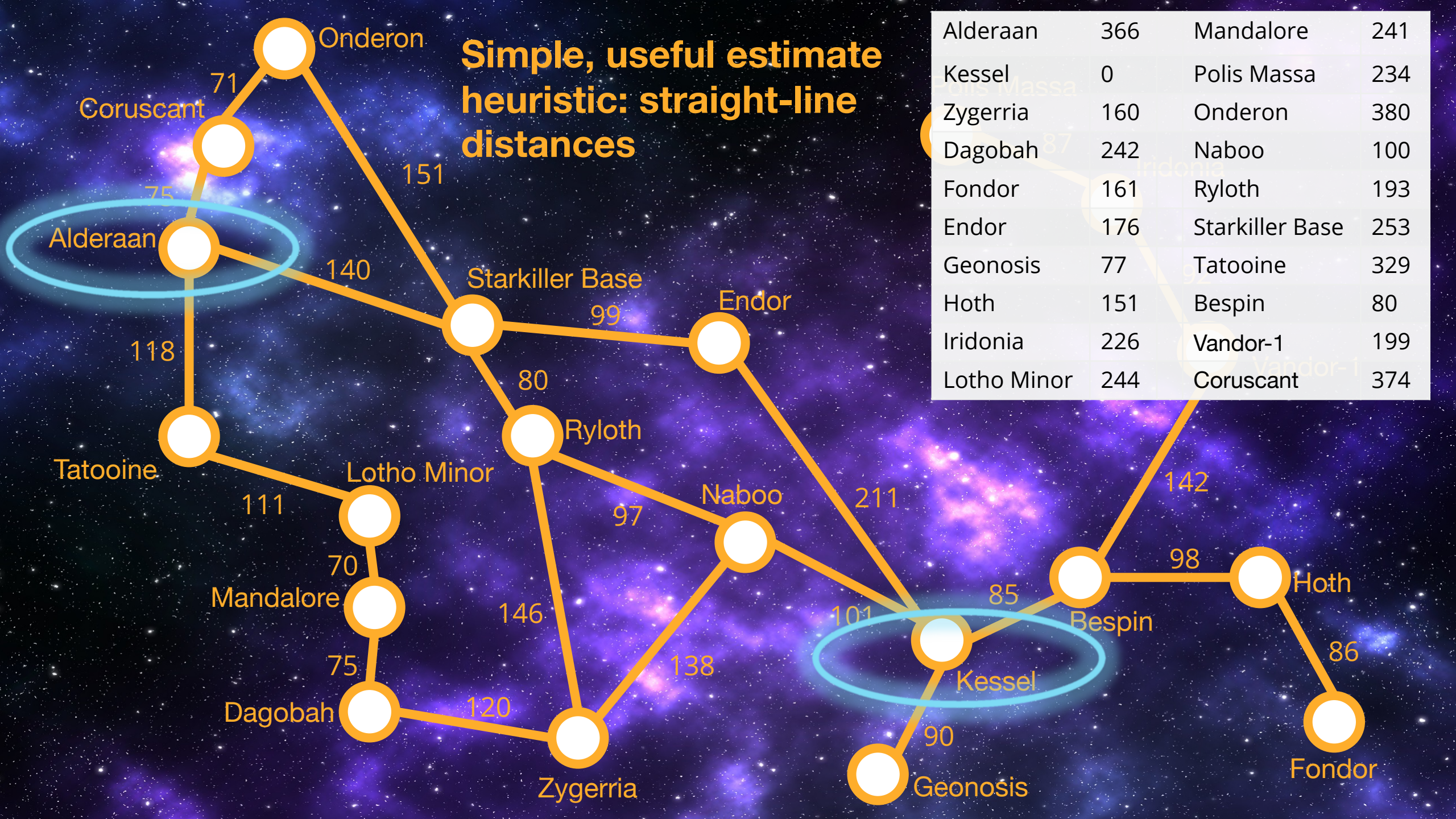


A better idea than UCS…

- Node expansion based on *an estimate* which *includes distance to the goal*
- General approach of **informed search**:
  - ***Best-first search***: node selected for expansion based on an ***evaluation function f(n)***
    - ✓ ***f(n)*** includes *estimate* of distance to goal ***(new idea!)***
  - Implementation: **Sort our frontier queue** by this **new *f(n)*.**

Instances of best-first search:

- **greedy search**
- ***A* search***



Alderaan	366	Mandalore	241
Kessel	0	Polis Massa	234
Zygerria	160	Onderon	380
Dagobah	242	Naboo	100
Fondor	161	Ryloth	193
Endor	176	Starkiller Base	253
Geonosis	77	Tatooine	329
Hoth	151	Bespin	80
Iridonia	226	Vandor-1	199
Lotho Minor	244	Coruscant	374

Heuristic (estimate) functions

Heureka! ---Archimedes

[dictionary]*"A rule of thumb, simplification, or educated guess that reduces or limits the search for solutions in domains that are difficult and poorly understood."*

Heuristic knowledge is useful, but not necessarily correct.

Heuristic algorithms use heuristic knowledge to solve a problem.

A *heuristic function h(n)* takes a state *n* and returns an *estimate* of the distance from *n* to the goal.

Greedy Best-First Search

First attempt at integrating heuristic knowledge

Best-first search

Basic idea:

Select node for expansion with minimal *evaluation function* ***f(n)***

- where ***f(n)*** is some function that includes ***estimate heuristic h(n)*** of the remaining distance to goal

Implement using priority queue

Exactly UCS with *f(n)* replacing *g(n)*

Greedy best-first search: *f(n) = h(n)*

Expands the node that *is estimated* to be closest to goal

Completely ignores *g(n):* the cost to get to *n*

In our Romanian map, $h(n) = h_{SLD}(n)$ = straight-line distance from *n* to Bucharest

In a grid, the heuristic distance can be calculated using the “Manhattan distance”:

```
def heuristic(a, b):
    # Manhattan distance on a square grid
    return abs(a.x - b.x) + abs(a.y - b.y)
```

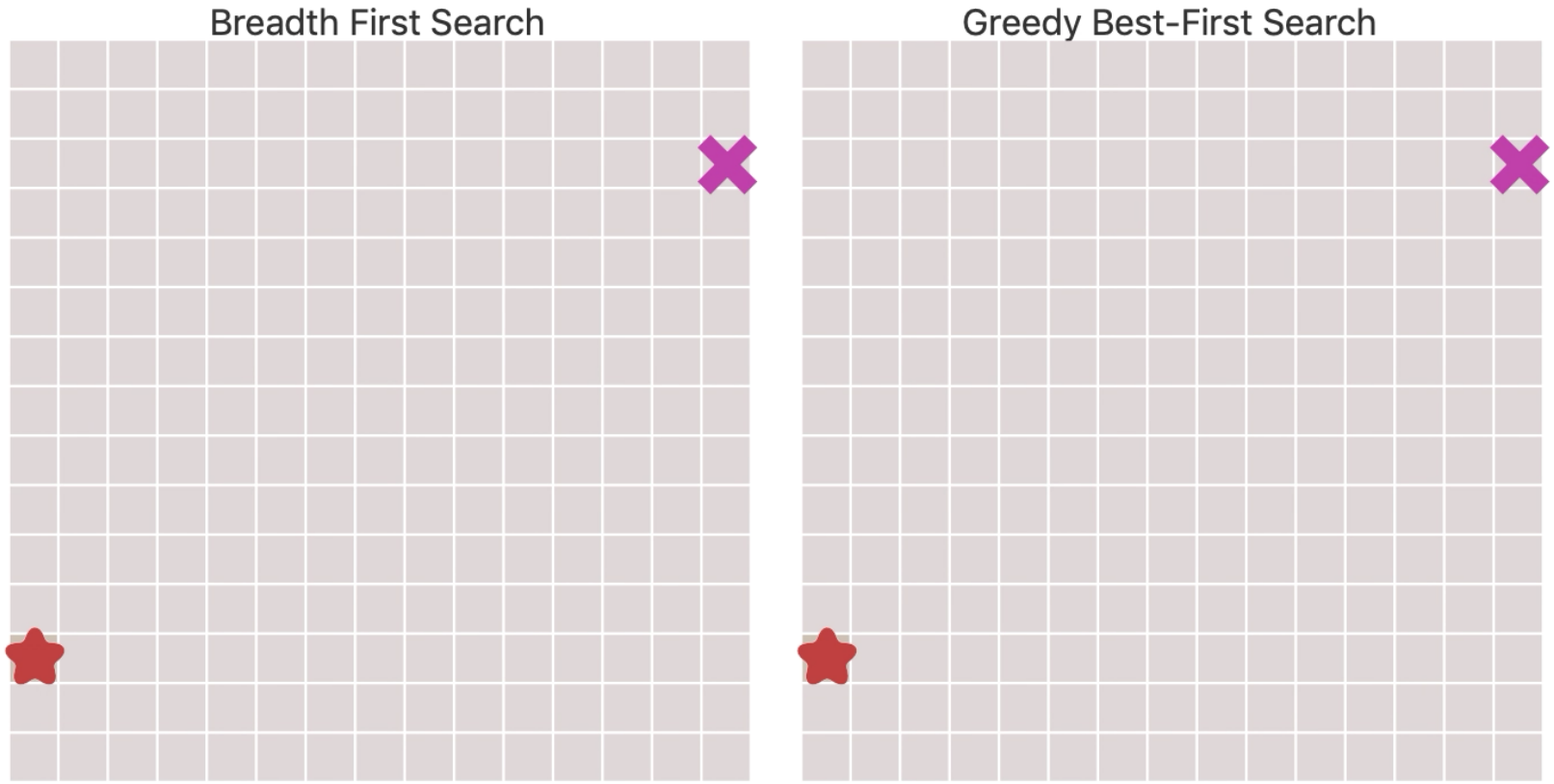
Greedy best-first search

```
frontier = PriorityQueue()
frontier.put(start, 0)
came_from = {}
came_from[start] = None

while not frontier.empty():
   current = frontier.get()

   if current == goal:
      break

   for next in graph.neighbors(current):
      if next not in came_from:
         priority = heuristic(goal, next)
         frontier.put(next, priority)
         came_from[next] = current
```

Code from Amit Patel
of Red Blob Games

This is not the solution to the HW assignment.

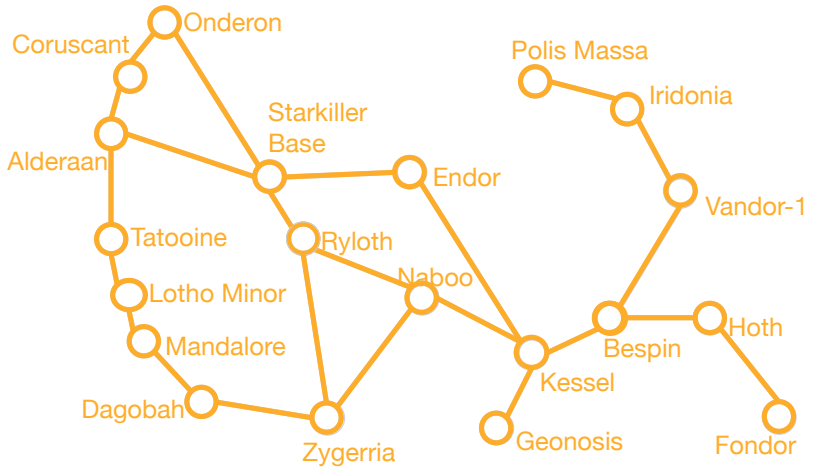
BFS v. Greedy Best-First Search



https://www.redblobgames.com/pathfinding/a-star/introduction.html

Greedy best-first search example

Initial State: **Alderaan**
Goal State: **Kessel**



Frontier queue:

Alderon 366

Planet	SLD		Planet	SLD
Alderaan	366		Lotho Minor	244
Bespin	80		Mandalore	241
Coruscant	374		Naboo	100
Dagobah	242		Onderon	380
Endor	176		Polis Massa	234
Fondor	161		Ryloth	193
Geonosis	77		Starkiller Base	253
Hoth	151		Tatooine	329
Iridonia	226		Vandor-1	199
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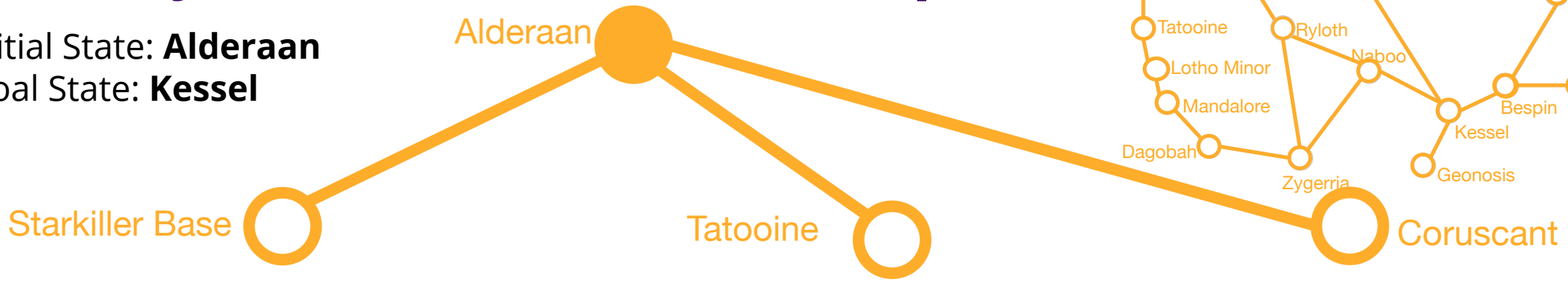
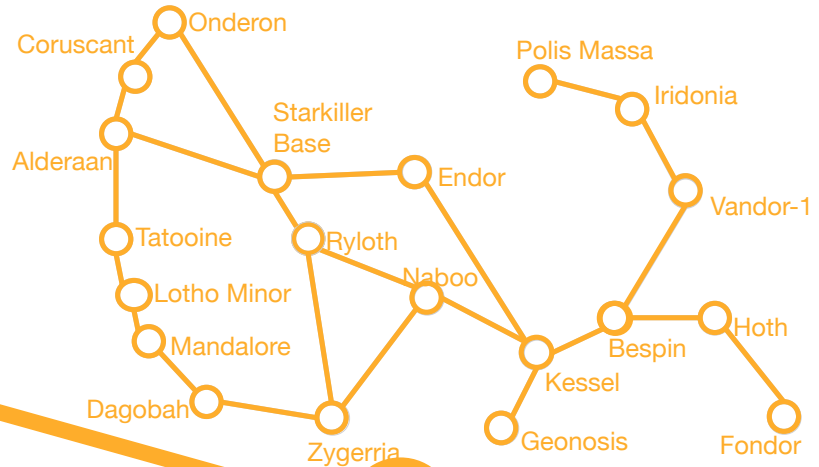
Greedy best-first search example

Initial State: **Alderaan**
Goal State: **Kessel**





Frontier queue:

Starkiller Base	**253**
Tatooine	**329**
Coruscant	**374**

Planet	SLD		Planet	SLD
Alderaan	366		Lotho Minor	244
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Greedy best-first search example

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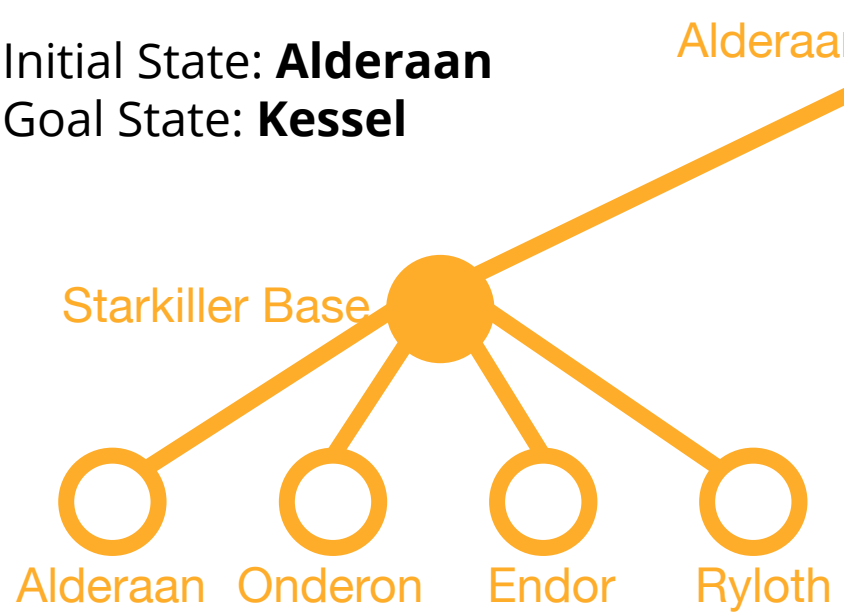

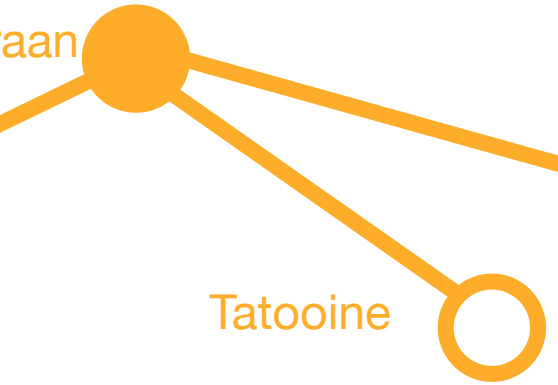

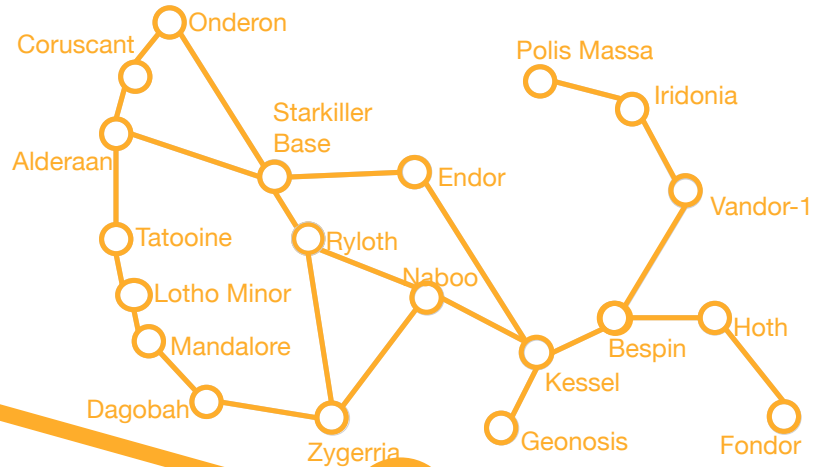

Are we doing **graph search** or **tree search**?

Frontier queue:

Tatooine	**329**
Coruscant	**374**
Alderaan	**366**
Onderon	**380**
Endor	**176**
Ryloth	**193**

Planet	SLD		Planet	SLD
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Graph search versus tree search

What is the difference between graph search and tree search?

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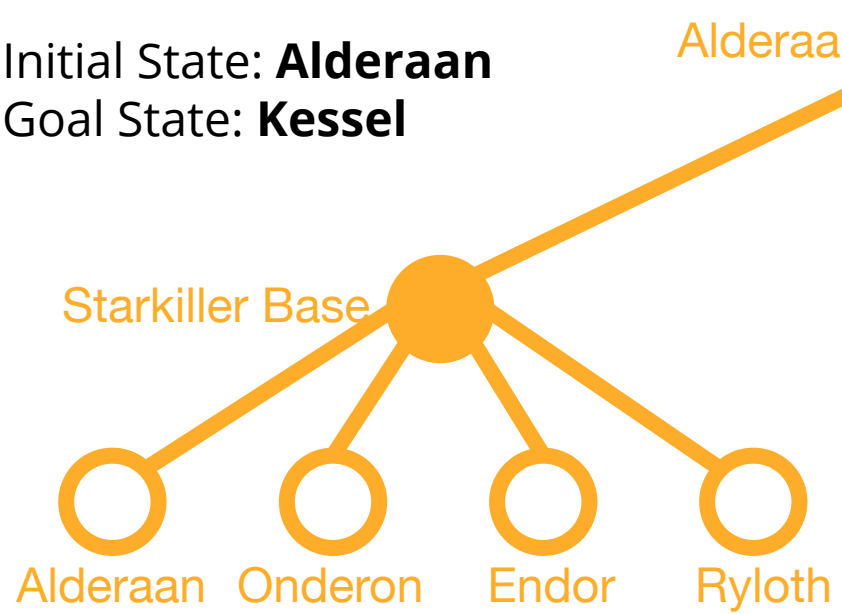
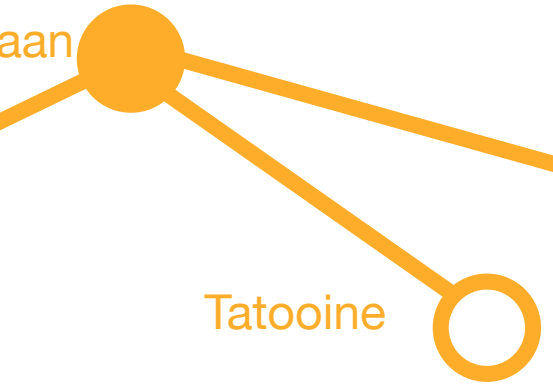
Greedy best-first search example

Initial State: **Alderaan**
Goal State: **Kessel**





Frontier queue:

Endor	**176**
Ryloth	**193**
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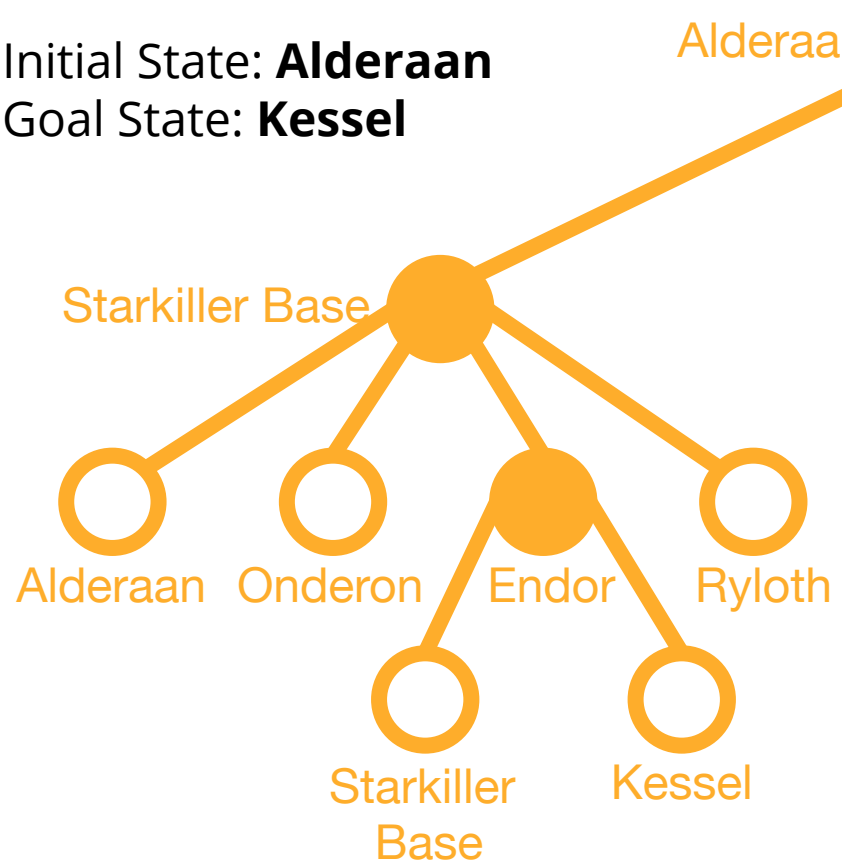
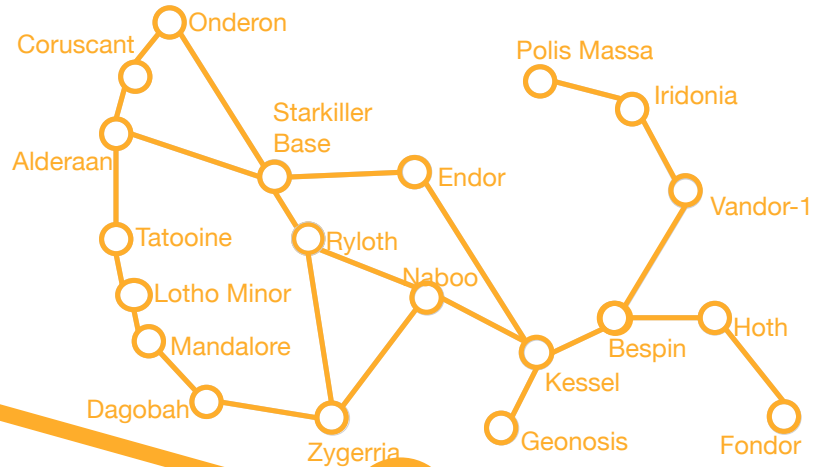
Greedy best-first search example

Initial State: **Alderaan**
Goal State: **Kessel**





Frontier queue:

Ryloth	**193**
Tatooine	**329**
Alderaan	**366**
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Starkiller Base	**253**
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Greedy best-first search example

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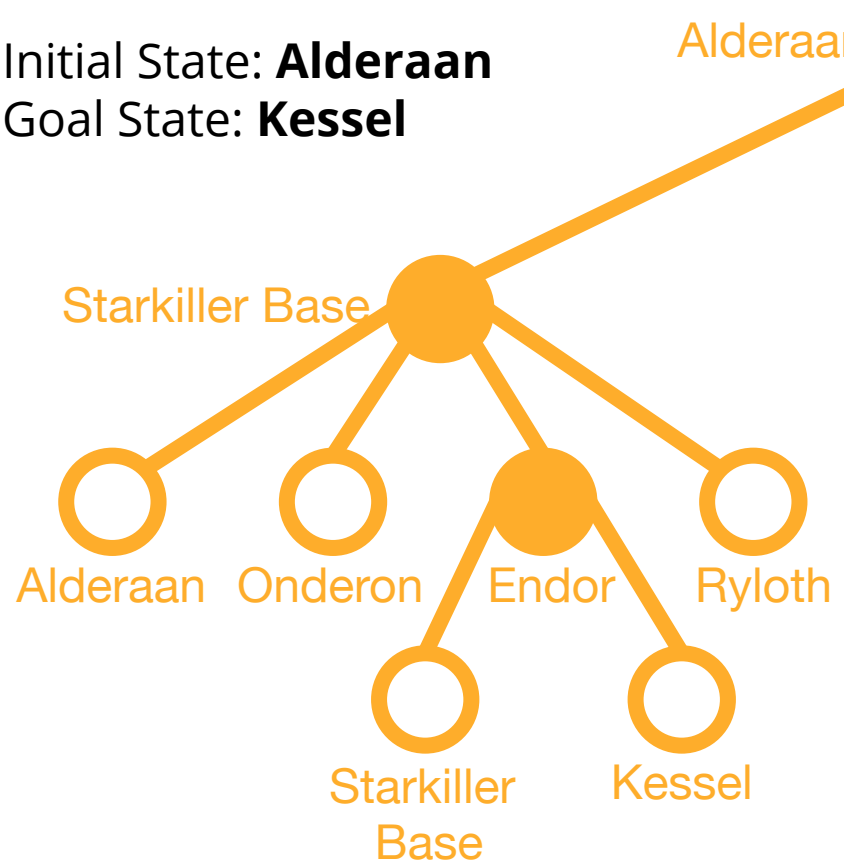

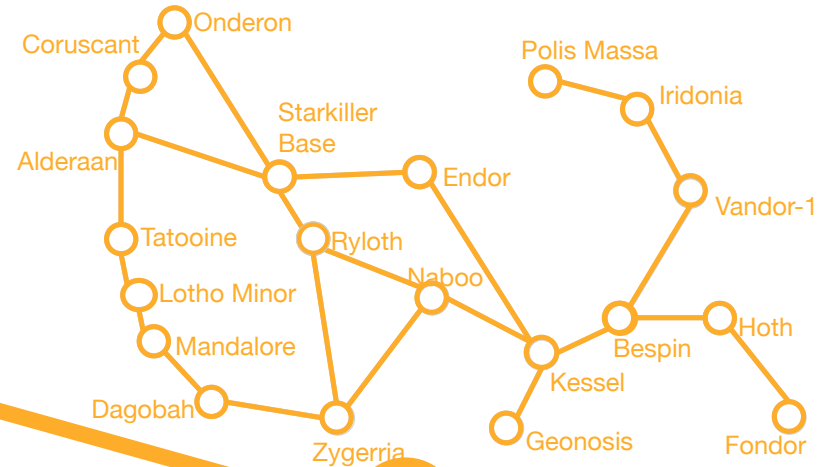

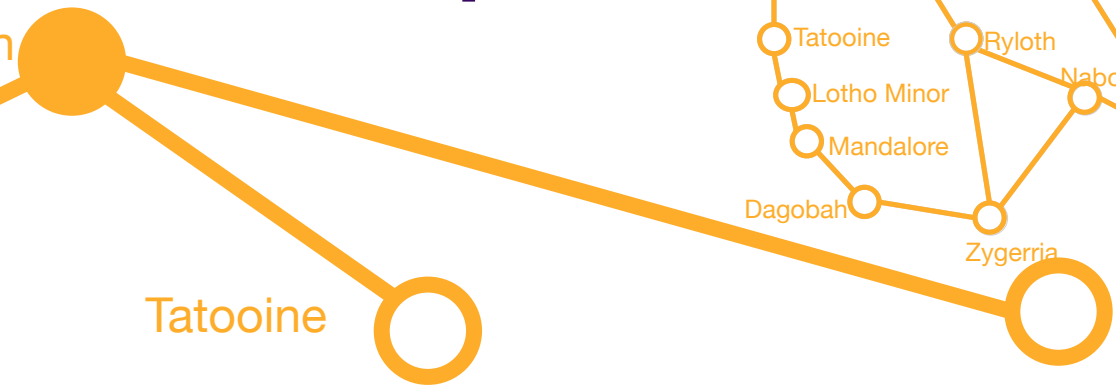

Frontier queue:

Kessel	**0**
Ryloth	**193**
Starkiller Base	**253**
Tatooine	**329**
Alderaan	**366**
Coruscant	**374**
Onderon	**380**

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Greedy best-first search example

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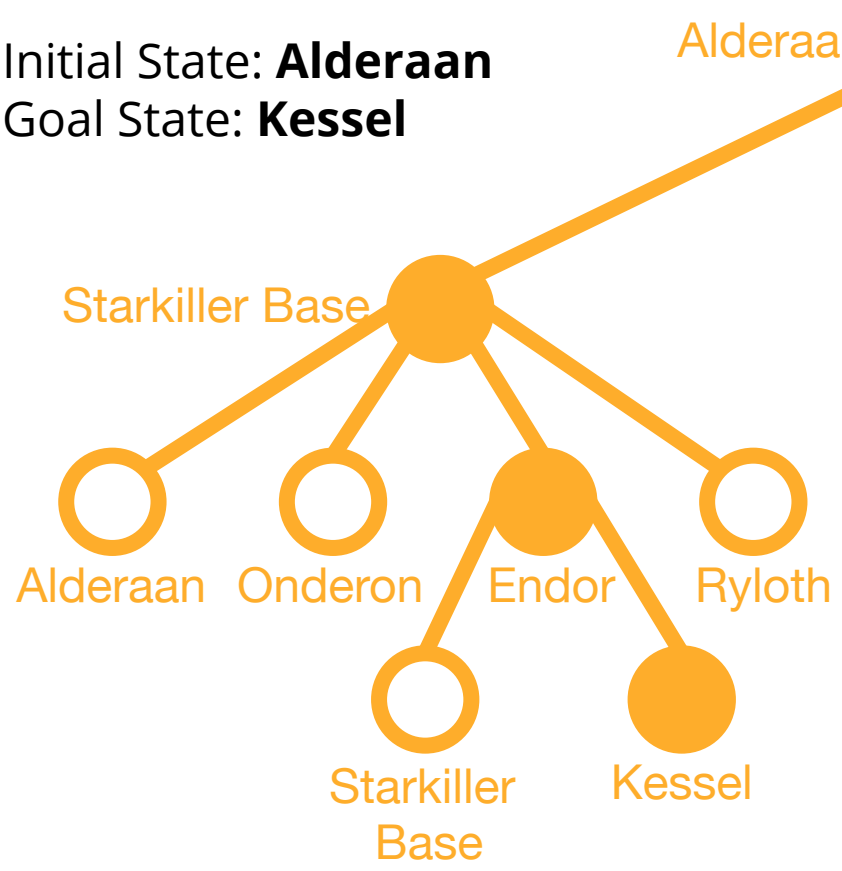

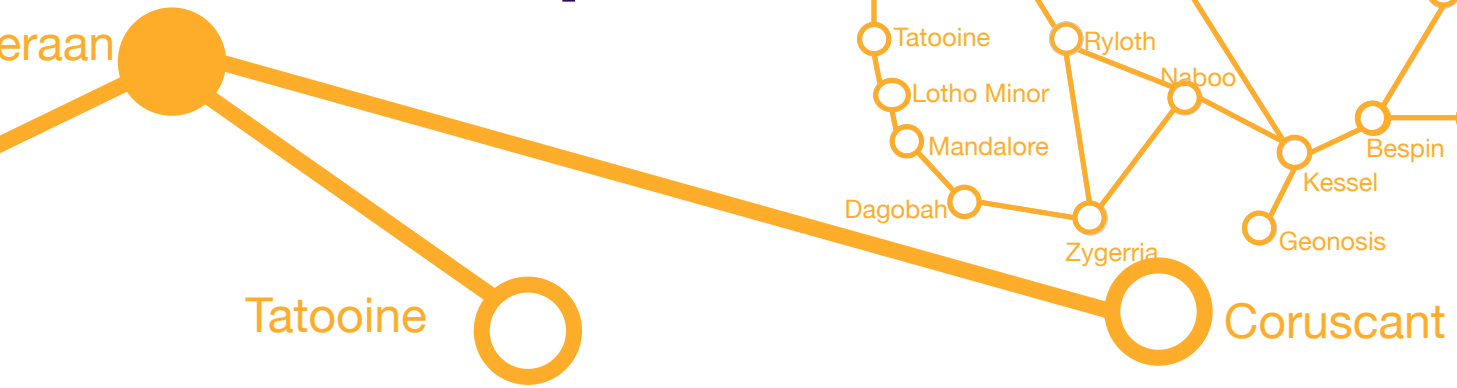

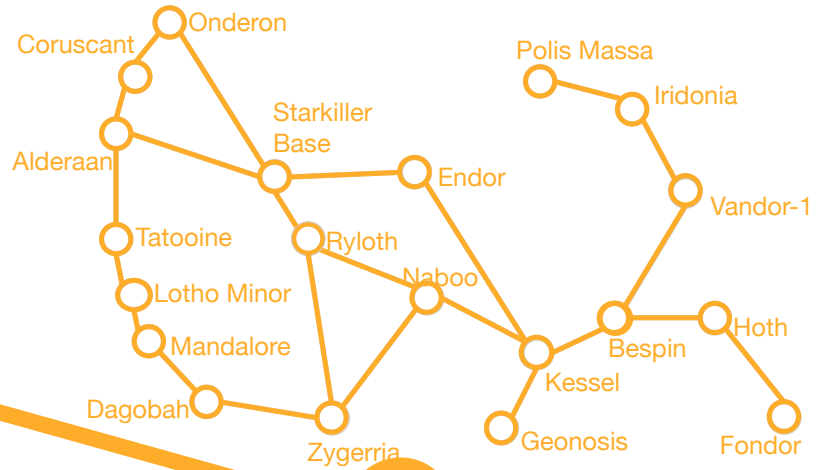

Goal reached !!

Frontier queue:

Ryloth	**193**
Starkiller Base	**253**
Tatooine	**329**
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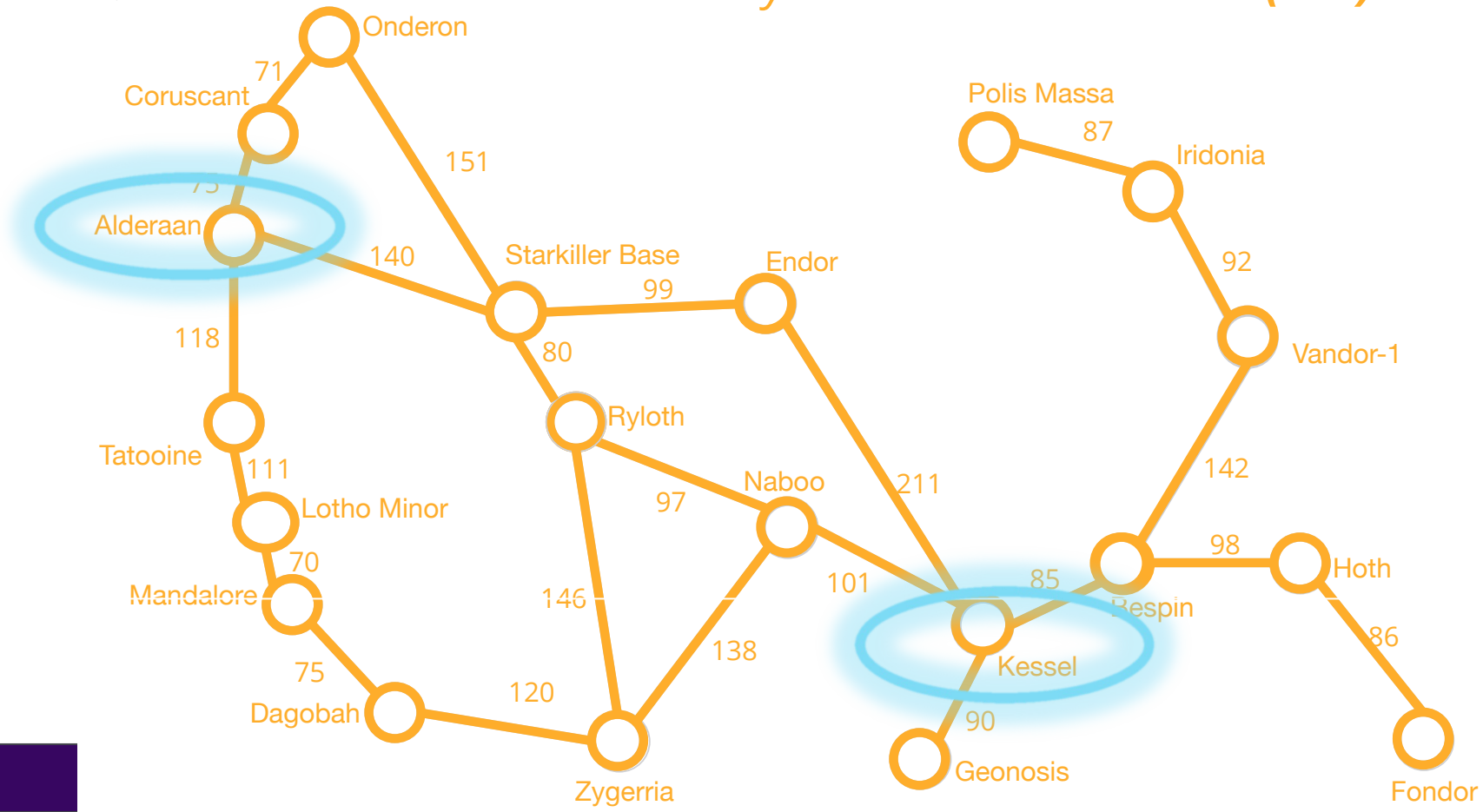
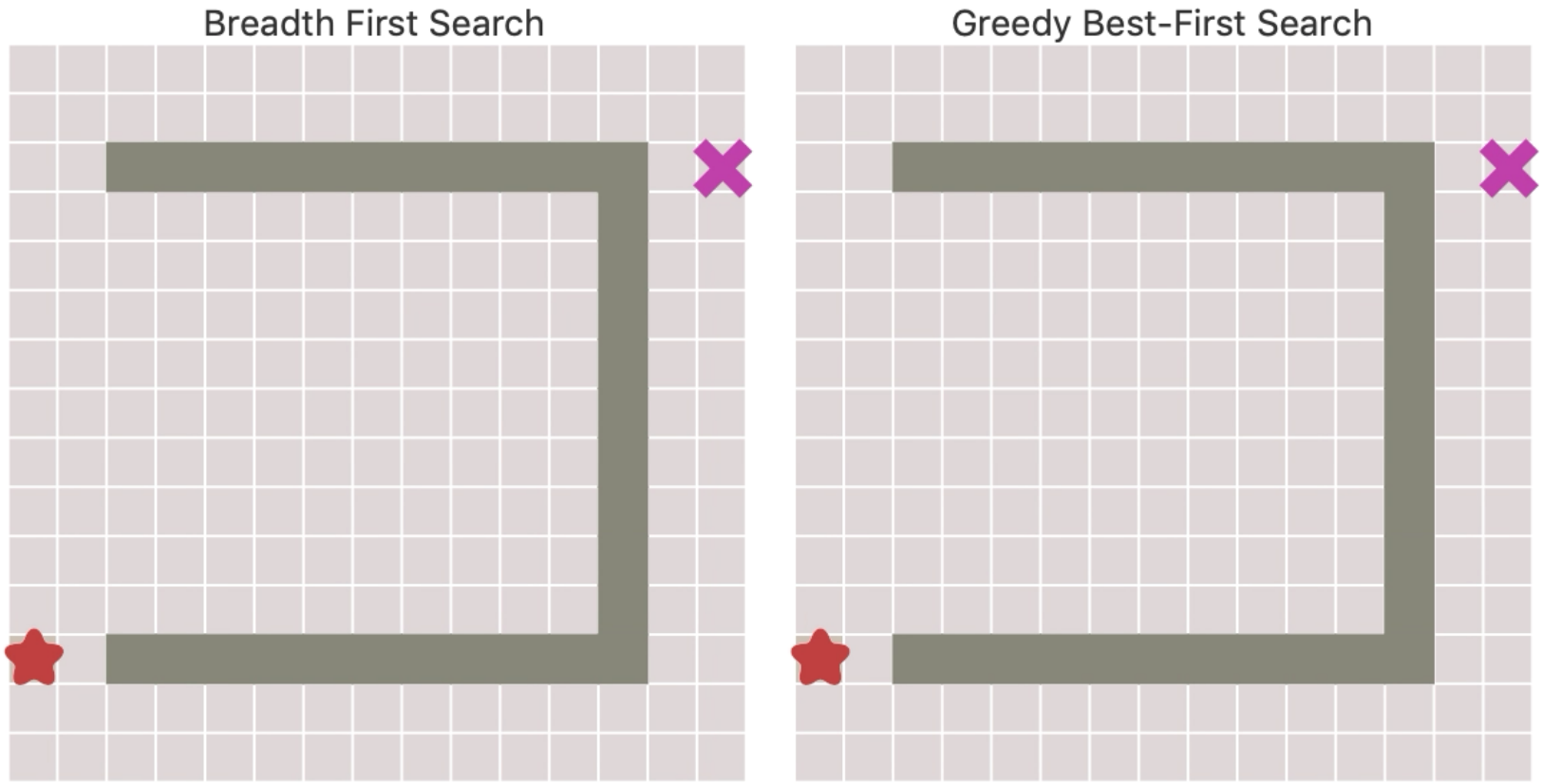
Properties of greedy best-first search

Optimal?

- No!
  - Found: *Alderaan → Starkiller Base → Endor → Kessel* ***(450)***
  - Shorter: *Alderaan → Starkiller Base → Ryloth → Naboo → Kessel* ***(418)***



BFS v. Greedy Best-First Search

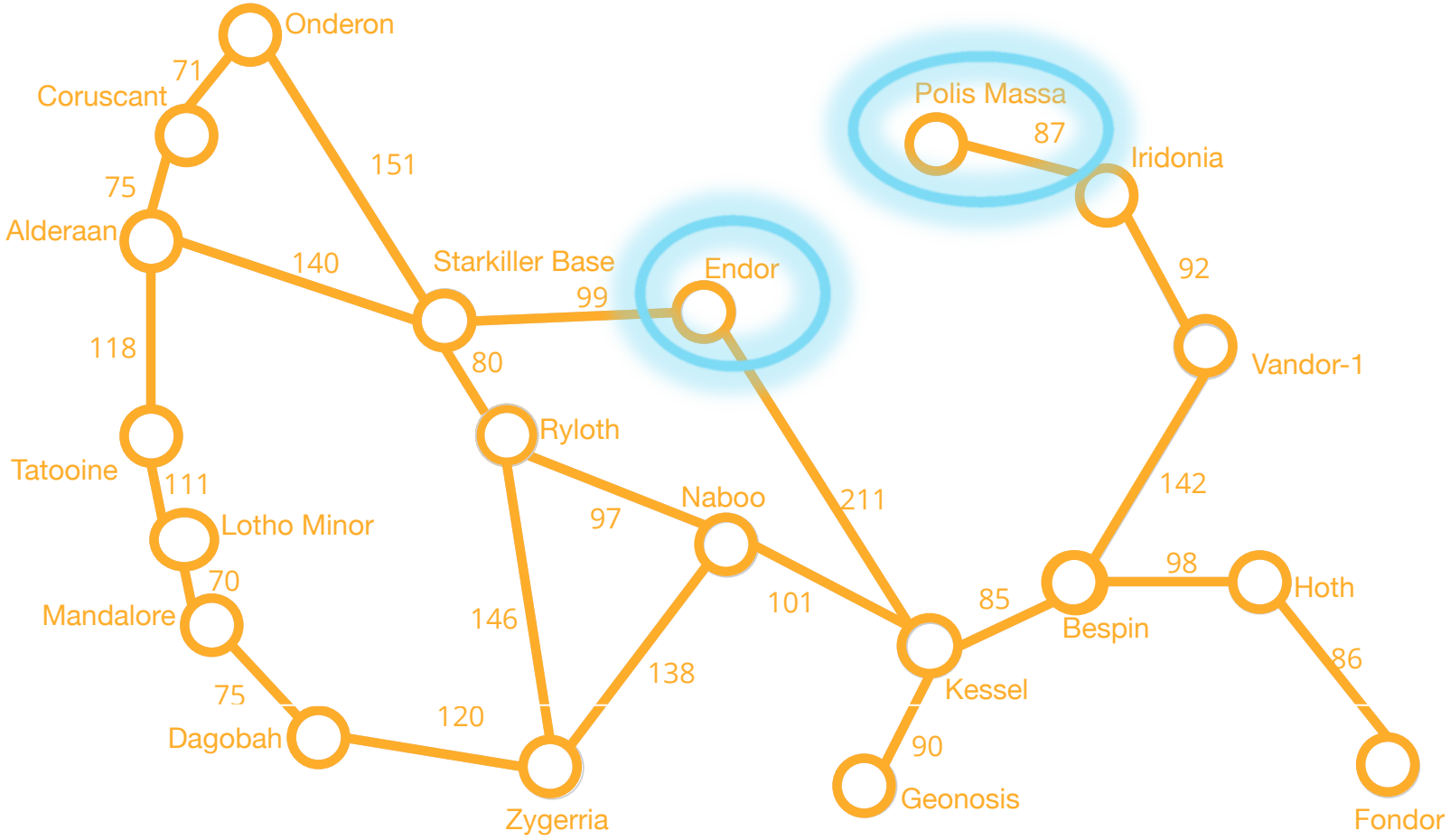


https://www.redblobgames.com/pathfinding/a-star/introduction.html

Properties of greedy best-first search

Complete?

- No – can get stuck in loops,
- e.g., Iridonia → Polis Massa→ Iridonia → Polis Massa →…

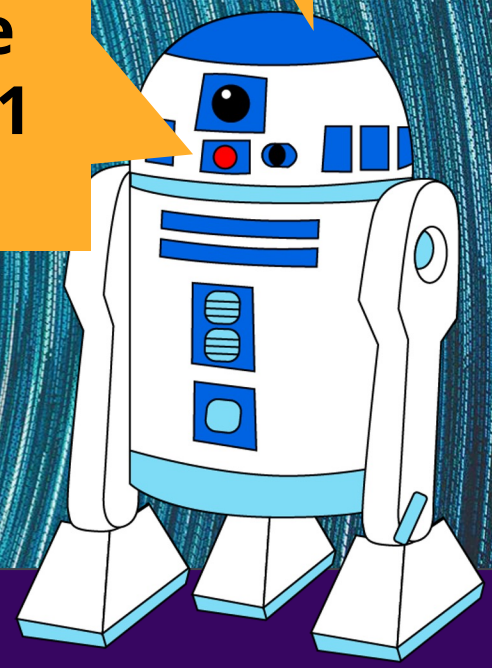


CIS 4210/5210:
ARTIFICIAL INTELLIGENCE

Informed Search

Quiz due on Sunday. HW3 is due on Wednesday.

If you need an extension, request is 24 hours in advance via Canvas (no need to email me). Usually, we grant a max extension of 1 or 2 days.



“That’s no moon…”

A* search

AIMA 3.5

“That’s a star…”

A* search

Best-known form of best-first search.

Key Idea: avoid expanding paths that are already expensive but expand most promising first.

Simple idea: *f(n)=g(n) + h(n)*

- ***g(n)*** the actual cost (so far) to *reach* the node
- ***h(n)*** estimated cost to *get from the node to the goal*
- ***f(n)*** estimated *total cost* of path through *n* to goal

Implementation: Frontier queue as priority queue by increasing *f(n)* *(as expected…)*

Key concept: Admissible heuristics

A heuristic *h(n)* is *admissible* if it *never overestimates* the cost to reach the goal;

i.e. it is *optimistic*

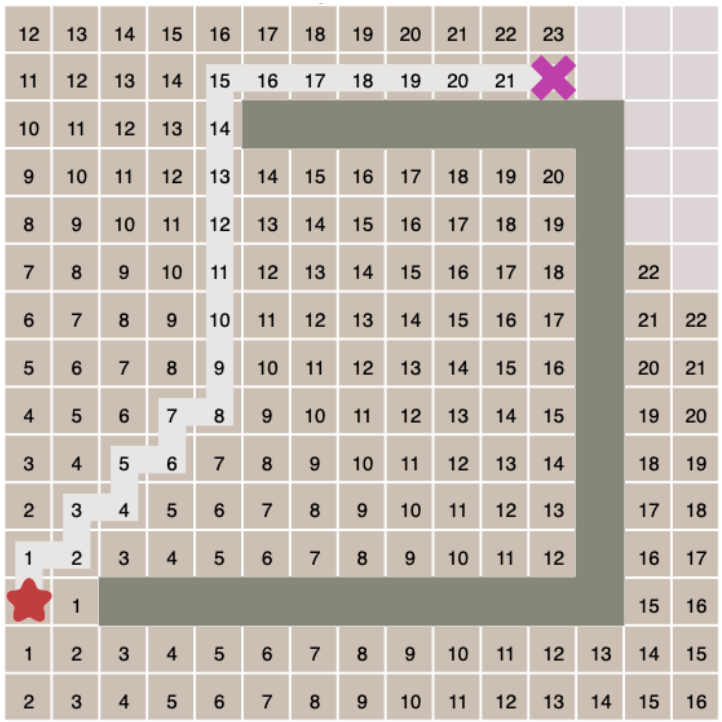
- Formally: $\forall n$, *n* a node:
  - $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from *n*
  - $h(n) \geq 0$ so $h(G)=0$ for any goal *G*.

Example: $h_{SLD}(n)$ never overestimates the actual road distance

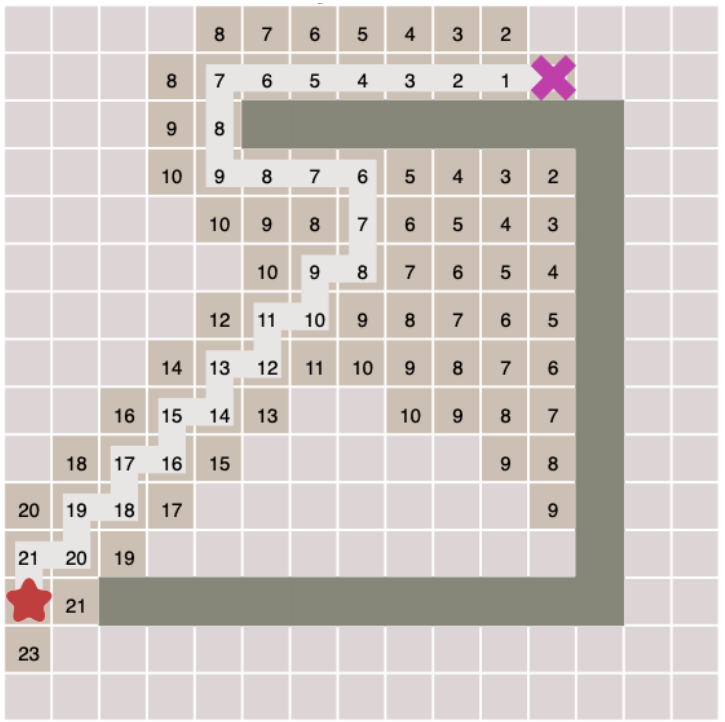
Theorem: If *h(n)* is *admissible*, A* using **Tree Search** is *optimal*

A* is optimal with admissible heuristic

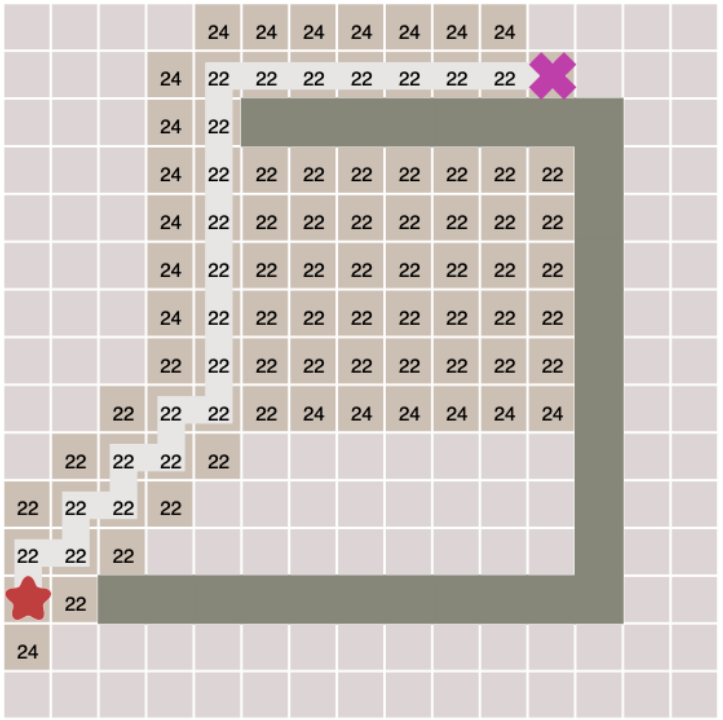
Uniform Cost Search



Greedy Best-First Search



A* Search



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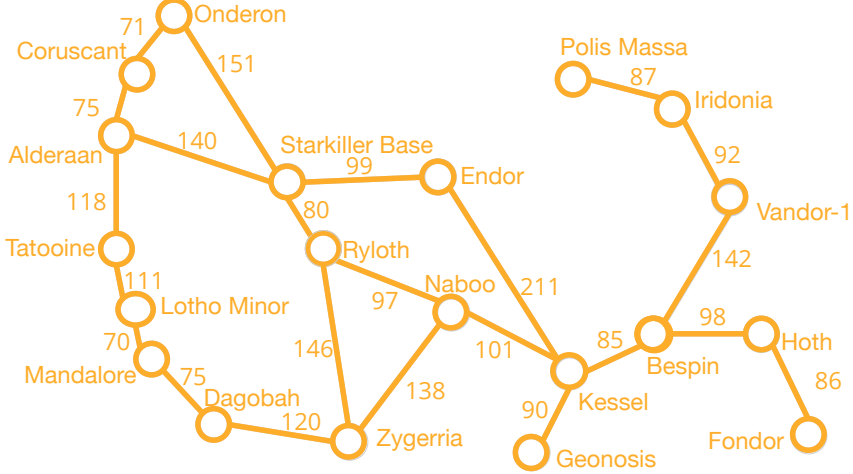
A* search example

Initial State: **Alderaan**
Goal State: **Kessel**



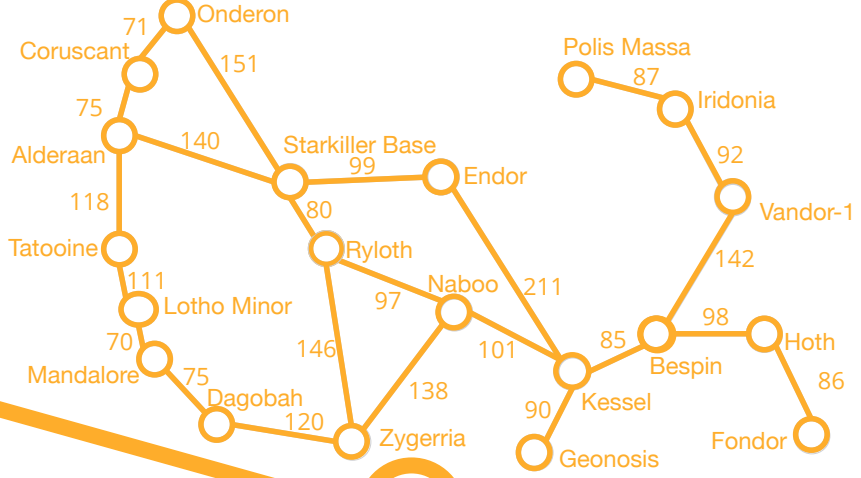
Frontier queue:

Alderon 366 = 0 + 366

Planet	SLD		Planet	SLD
Alderaan	366		Lotho Minor	244
Bespin	80		Mandalore	241
Coruscant	374		Naboo	100
Dagobah	242		Onderon	380
Endor	176		Polis Massa	234
Fondor	161		Ryloth	193
Geonosis	77		Starkiller Base	253
Hoth	151		Tatooine	329
Iridonia	226		Vandor-1	199
Kessel	0		Zygerria	160

A* search example

Initial State: **Alderaan**
Goal State: **Kessel**



$$f(n) = g(n) + h(n)$$

Frontier queue:

Starkiller Base	**393 = 140 + 253**
Tatooine	**447 = 118 + 329**
Coruscant	**449 = 75 + 374**

Planet	SLD		Planet	SLD
Alderaan	366		Lotho Minor	244
Bespin	80		Mandalore	241
Coruscant	374		Naboo	100
Dagobah	242		Onderon	380
Endor	176		Polis Massa	234
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A* search example

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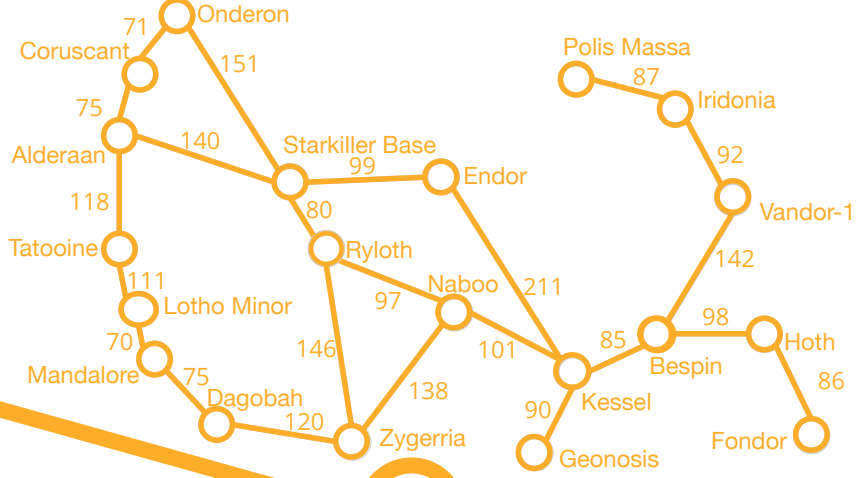

Sort queue by f(n).

f(n) = g(n) + h(n)

Frontier queue:

Starkiller Base **393 = 140 + 253**
Tatooine **447 = 118 + 329**
Coruscant **449 = 75 + 374**

Planet	SLD		Planet	SLD
Alderaan	366		Lotho Minor	244
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A* search example

Initial State: **Alderaan**
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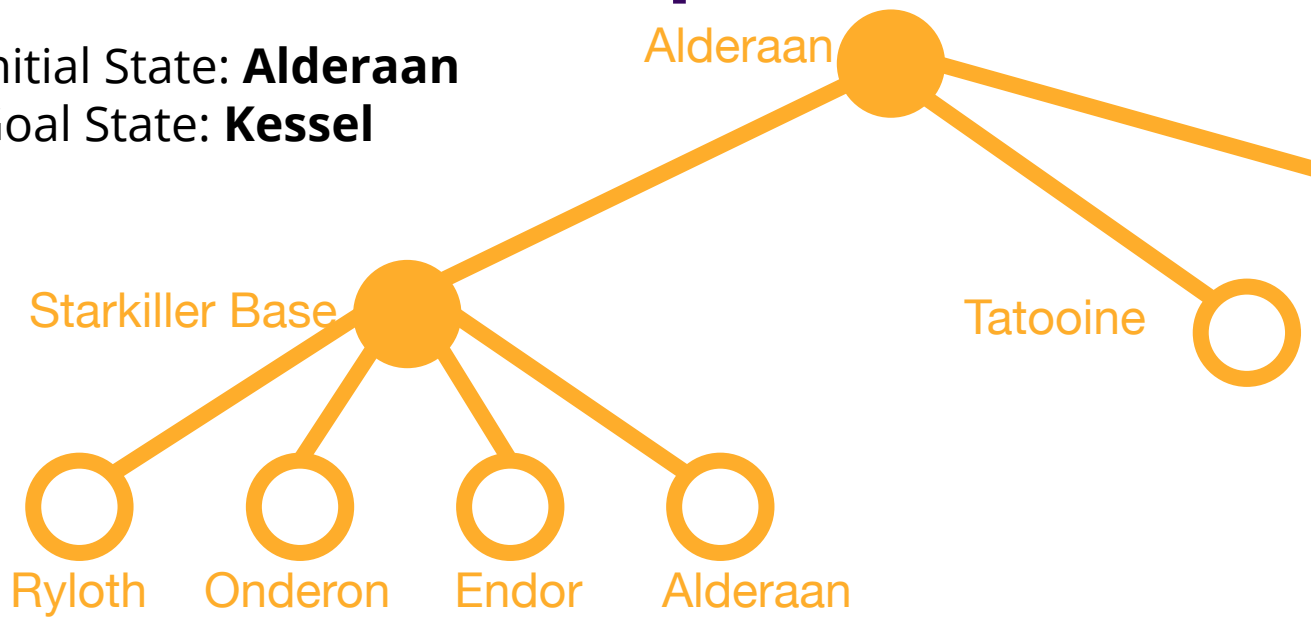
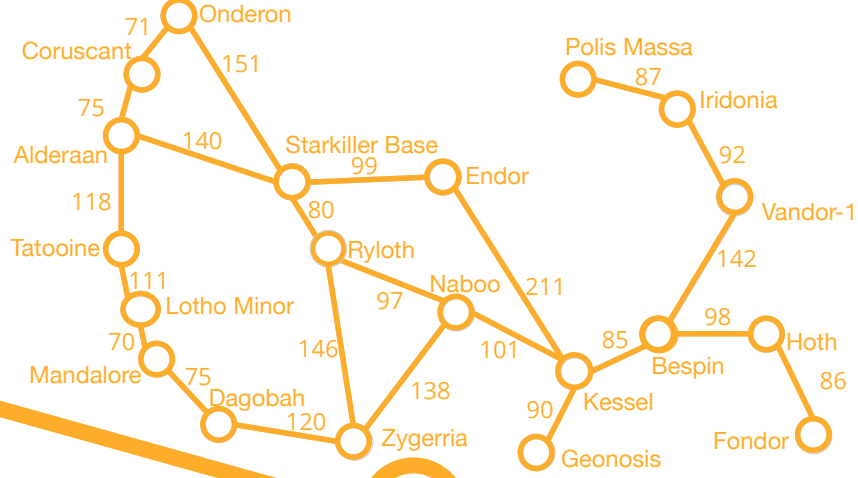




Are we doing **graph search** or **tree search**?

Frontier queue:

Tatooine	**447 = 118 + 329**
Coruscant	**449 = 75 + 374**
Alderaan	**646 = 280 + 366**
Onderon	**526 = 146 + 380**
Endor	**415 = 239 + 176**
Ryloth	**413 = 220 + 193**

Planet	SLD	Planet	SLD
Alderaan	366	Lotho Minor	244
Bespin	80	Mandalore	241
Coruscant	374	Naboo	100
Dagobah	242	Onderon	380
Endor	176	Polis Massa	234
Fondor	161	Ryloth	193
Geonosis	77	Starkiller Base	253
Hoth	151	Tatooine	329
Iridonia	226	Vandor-1	199
Kessel	0	Zygerria	160

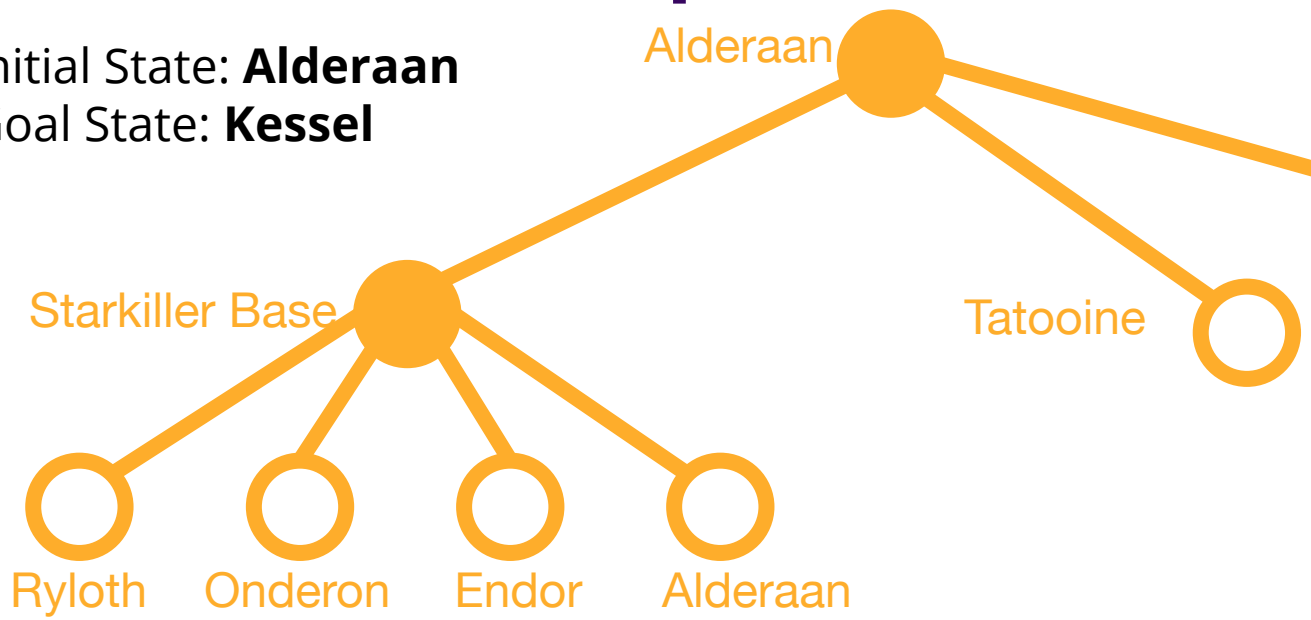
Graph search versus tree search

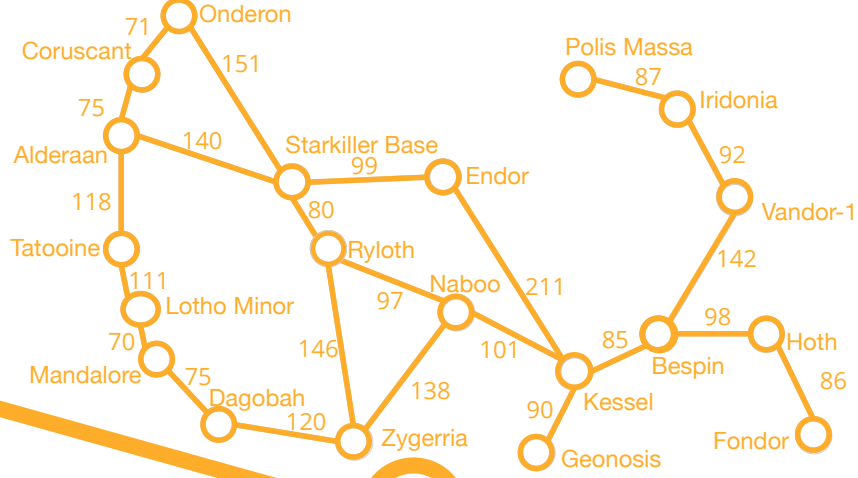
What is the difference between graph search and tree search?

PollEv.com/ccb

A* search example

Initial State: **Alderaan**
Goal State: **Kessel**





Frontier queue:

Ryloth	**413 = 220 + 193**
Endor	**415 = 239 + 176**
Tatooine	**447 = 118 + 329**
Coruscant	**449 = 75 + 374**
Onderon	**526 = 146 + 380**
Alderaan	**646 = 280 + 366**

Planet	SLD	Planet	SLD
Alderaan	366	Lotho Minor	244
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A* search example

Initial State: **Alderaan**
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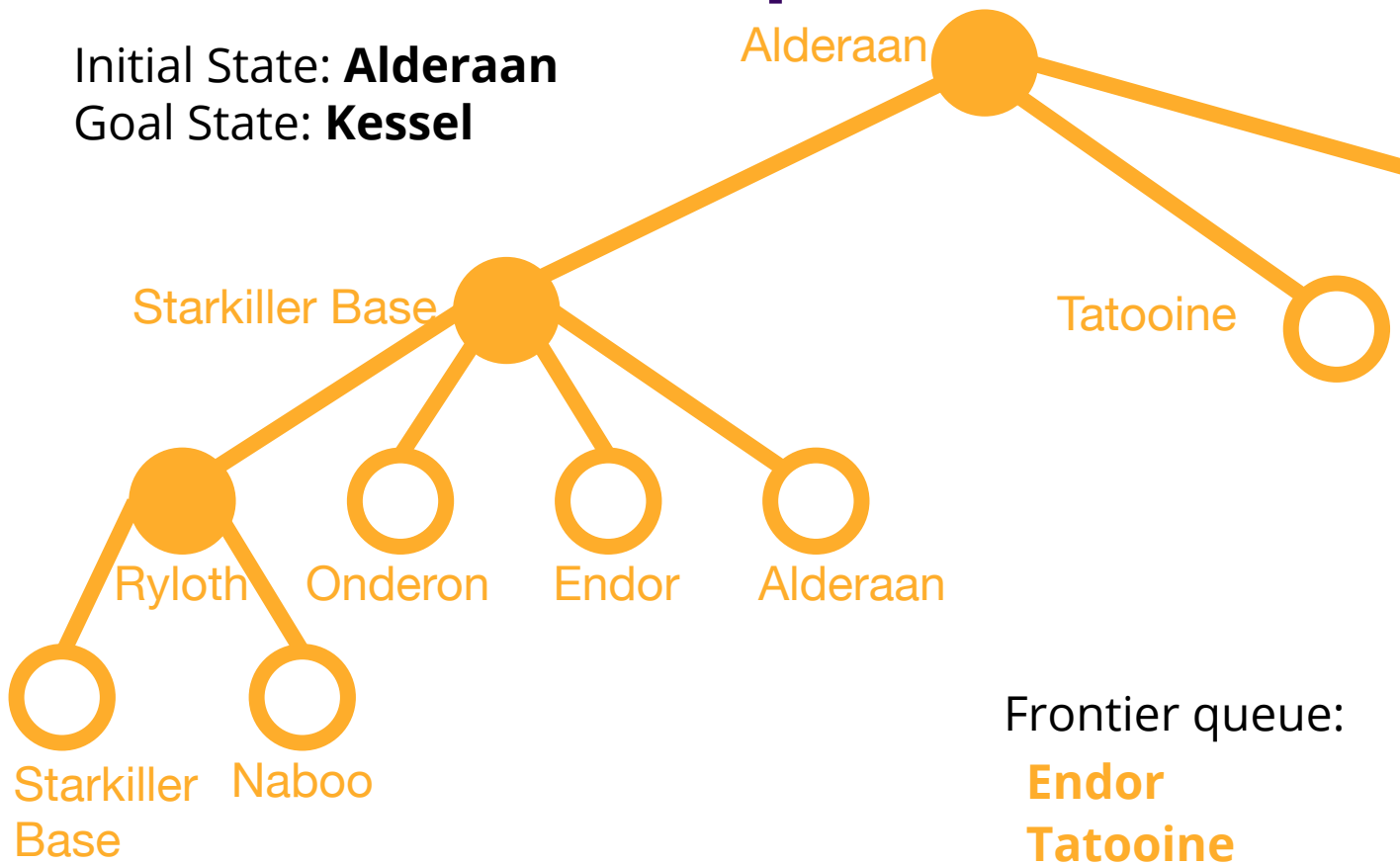

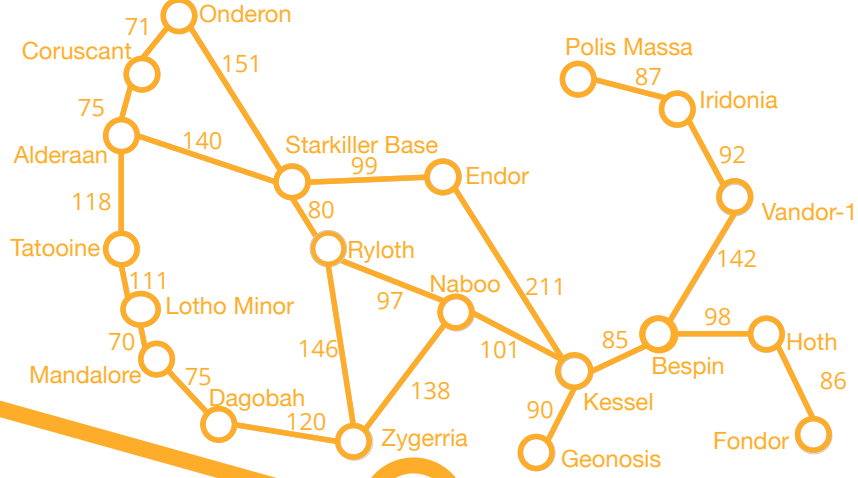

Frontier queue:

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Coruscant	**449 = 75 + 374**
Onderon	**526 = 146 + 380**
Alderaan	**646 = 280 + 366**
Starkiller Base	**532 = 300 + 253**
Naboo	**417 = 317 + 100**

Planet	SLD	Planet	SLD
Alderaan	366	Lotho Minor	244
Bespin	80	Mandalore	241
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A* search example

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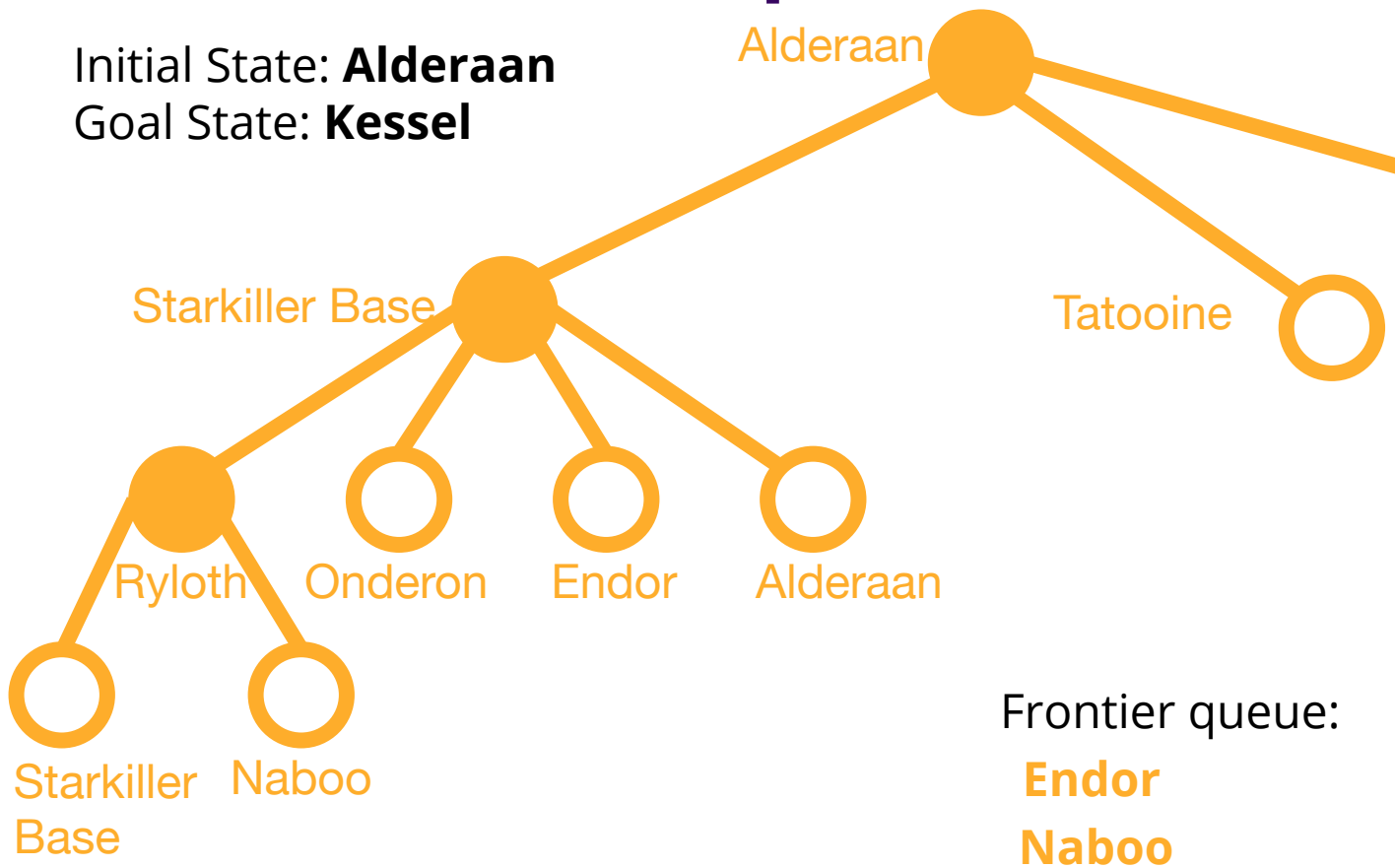
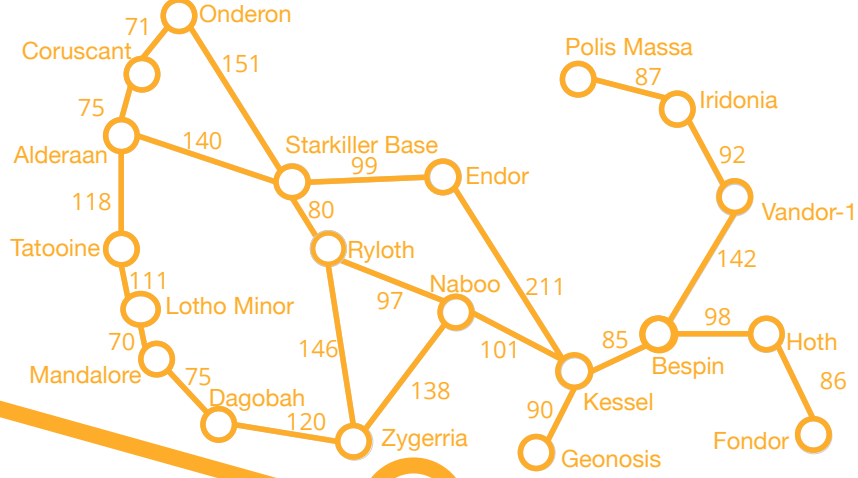
Goal State: **Kessel**





Frontier queue:

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Naboo	**417 = 317 + 100**
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Onderon	**526 = 146 + 380**
Starkiller Base	**532 = 300 + 253**
Alderaan	**646 = 280 + 366**

Planet	SLD		Planet	SLD
Alderaan	366		Lotho Minor	244
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A* search example

Initial State: **Alderaan**
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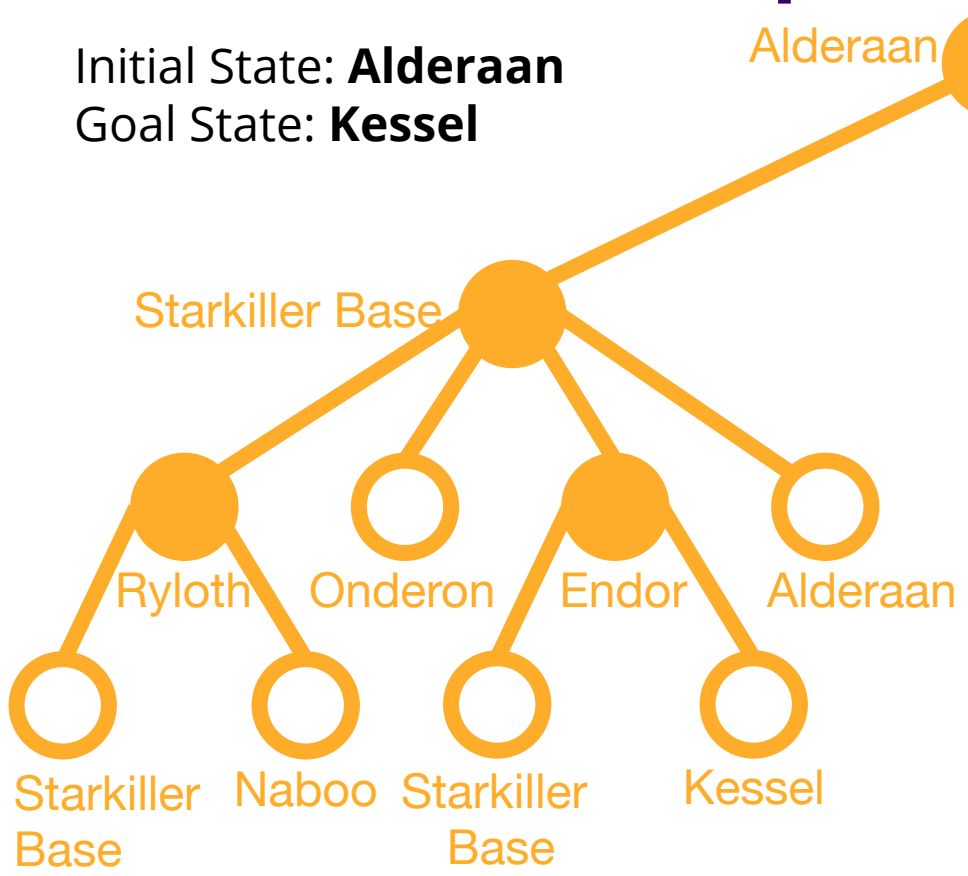

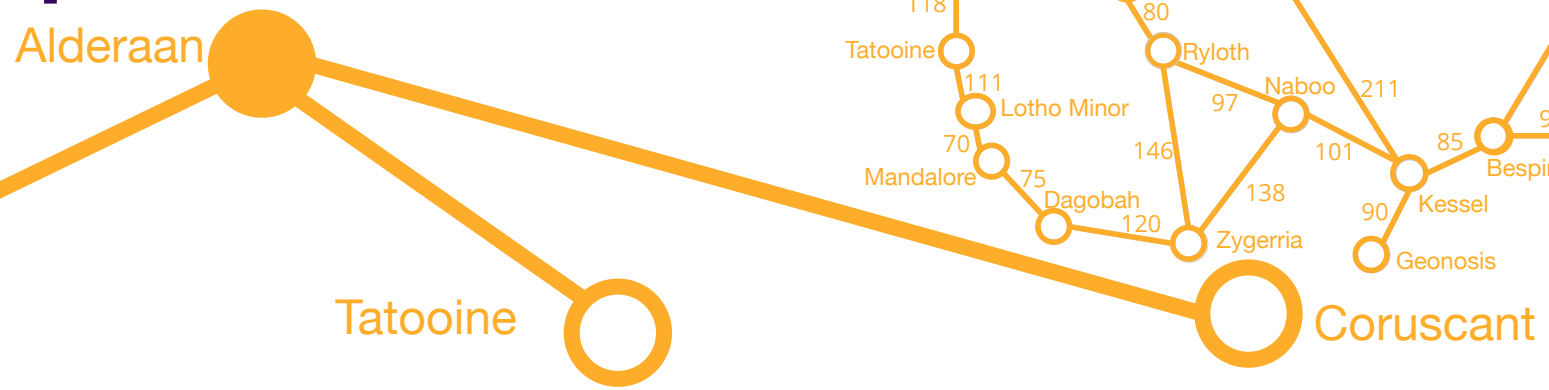

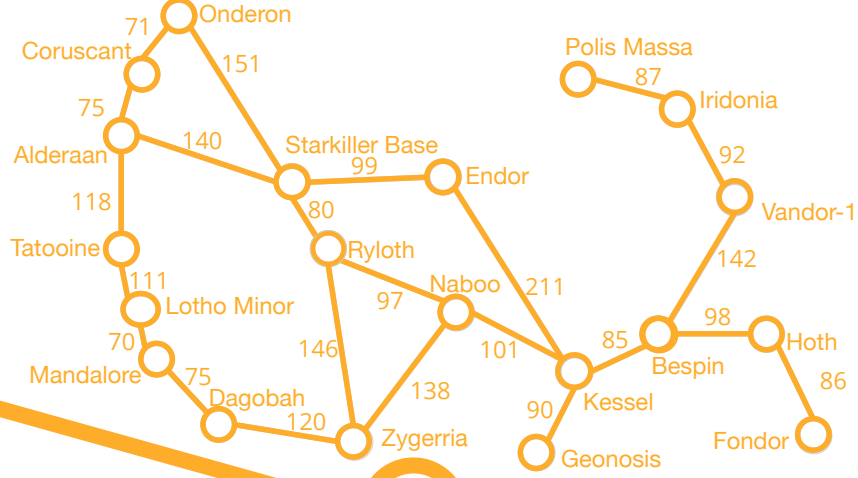

No, we apply the goal test to the **top of the queue**.

Frontier queue:

Naboo	**417 = 317 + 100**
Tatooine	**447 = 118 + 329**
Coruscant	**449 = 75 + 374**
Onderon	**526 = 146 + 380**
Starkiller Base	**532 = 300 + 253**
Alderaan	**646 = 280 + 366**
Starkiller Base	**591 = 338 + 253**
Kessel	**450 = 450 + 0**

Do we apply the goal test to the **children** like in BFS?

Planet	SLD	Planet	SLD
Alderaan	366	Lotho Minor	244
Bespin	80	Mandalore	241
Coruscant	374	Naboo	100
Dagobah	242	Onderon	380
Endor	176	Polis Massa	234
Fondor	161	Ryloth	193
Geonosis	77	Starkiller Base	253
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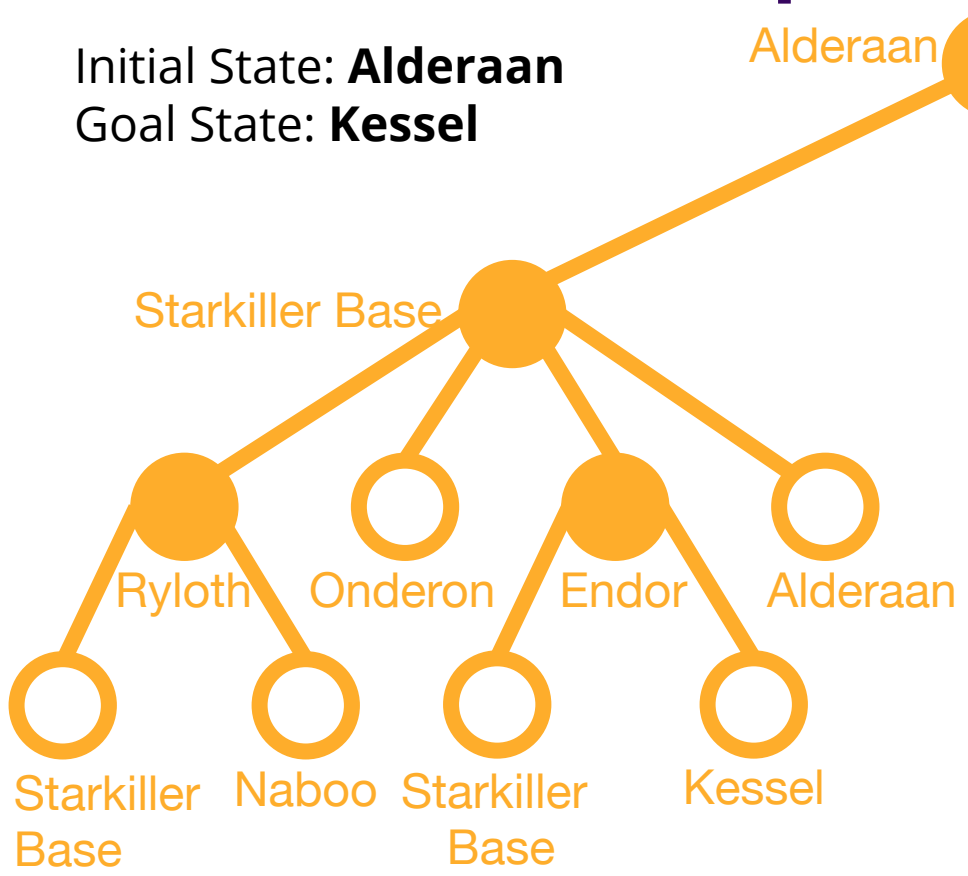
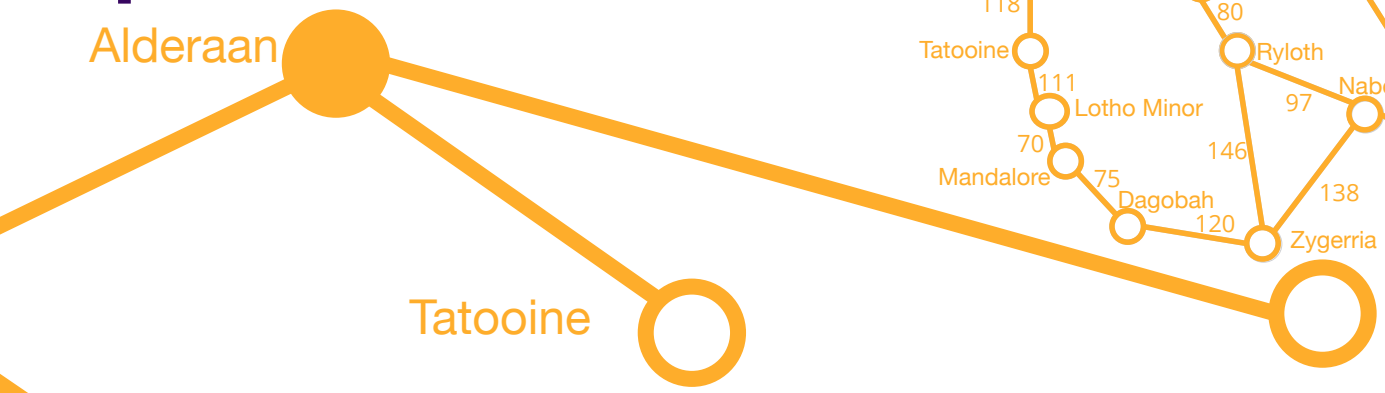
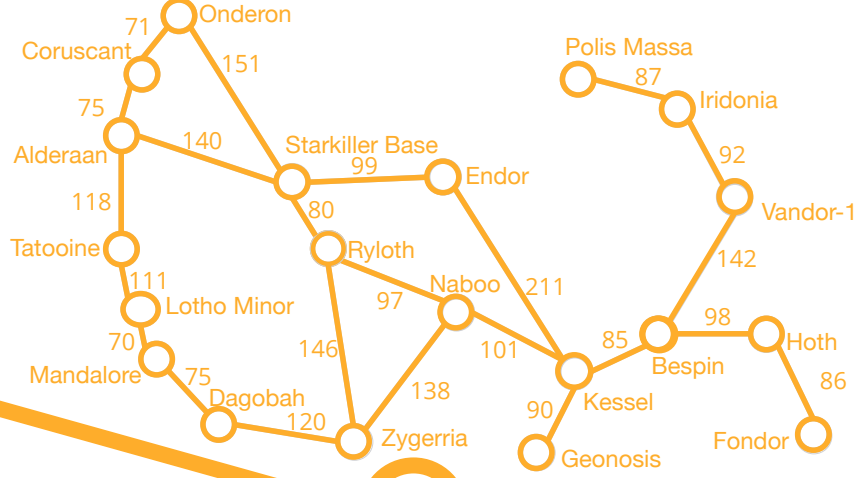
A* search example

Initial State: **Alderaan**
Goal State: **Kessel**







Kessel is not yet at **top of the queue**, so there may be a **shorter path**...

Frontier qu

Naboo	**417 = 317 + 100**
Tatooine	**447 = 118 + 329**
Coruscant	**449 = 75 + 374**
Kessel	**450 = 450 + 0**
Onderon	**526 = 146 + 380**
Starkiller Base	**532 = 300 + 253**
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Planet	SLD	Planet	SLD
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A* search example

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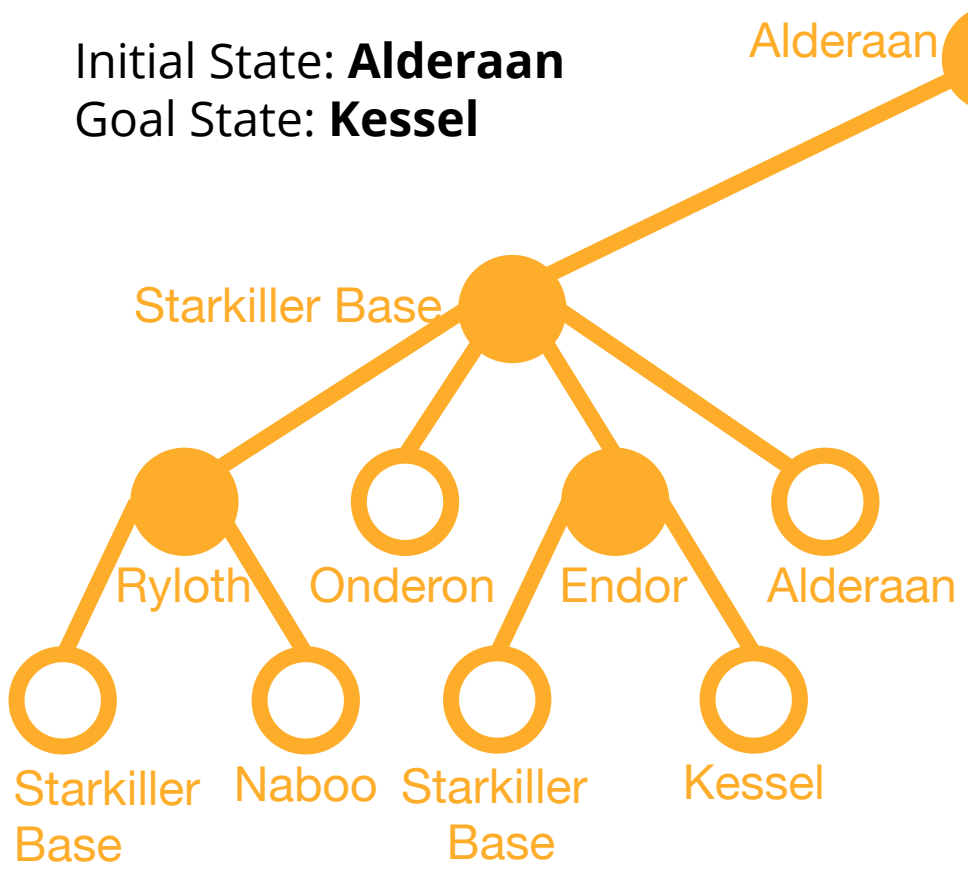

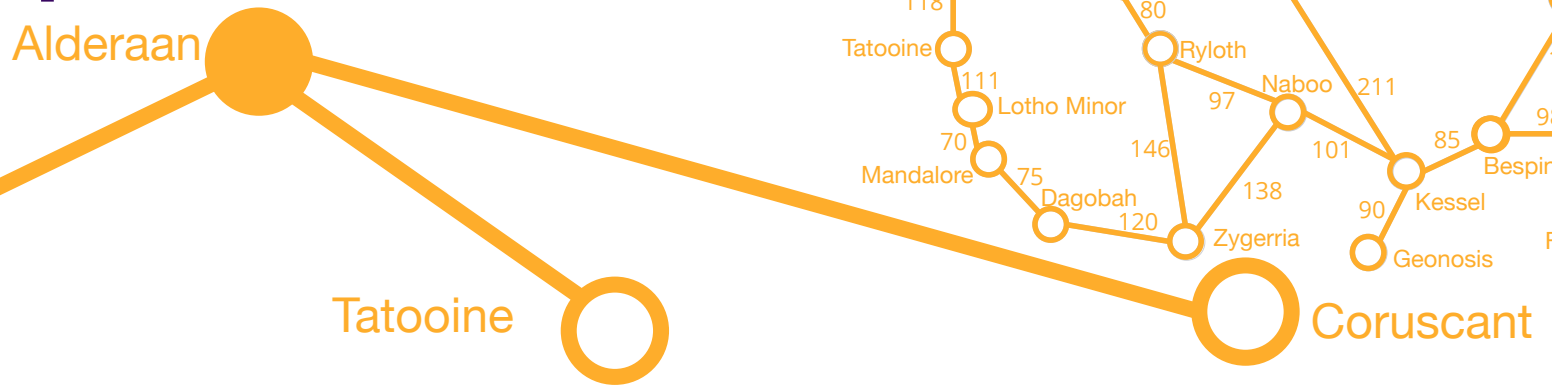

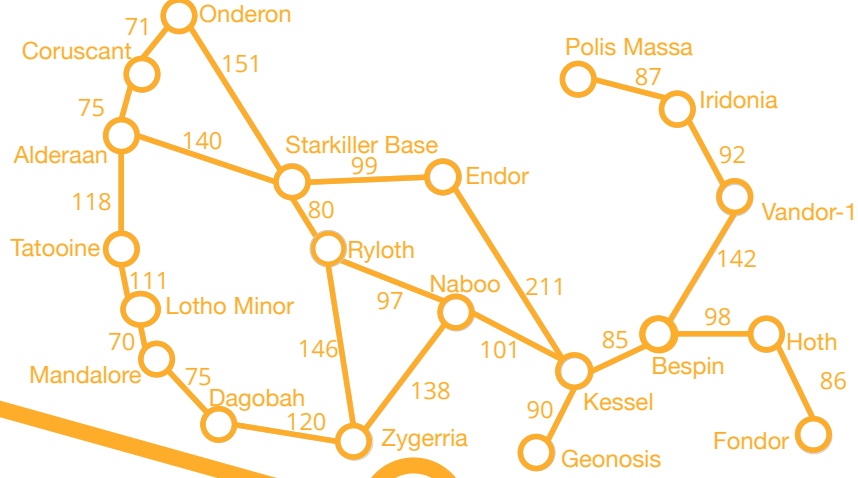

Frontier queue:

Naboo	**417 = 317 + 100**
Tatooine	**447 = 118 + 329**
Coruscant	**449 = 75 + 374**
Kessel	**450 = 450 + 0**
Onderon	**526 = 146 + 380**
Starkiller Base	**532 = 300 + 253**
Starkiller Base	**591 = 338 + 253**
Alderaan	**646 = 280 + 366**

Planet	SLD	Planet	SLD
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A* search example

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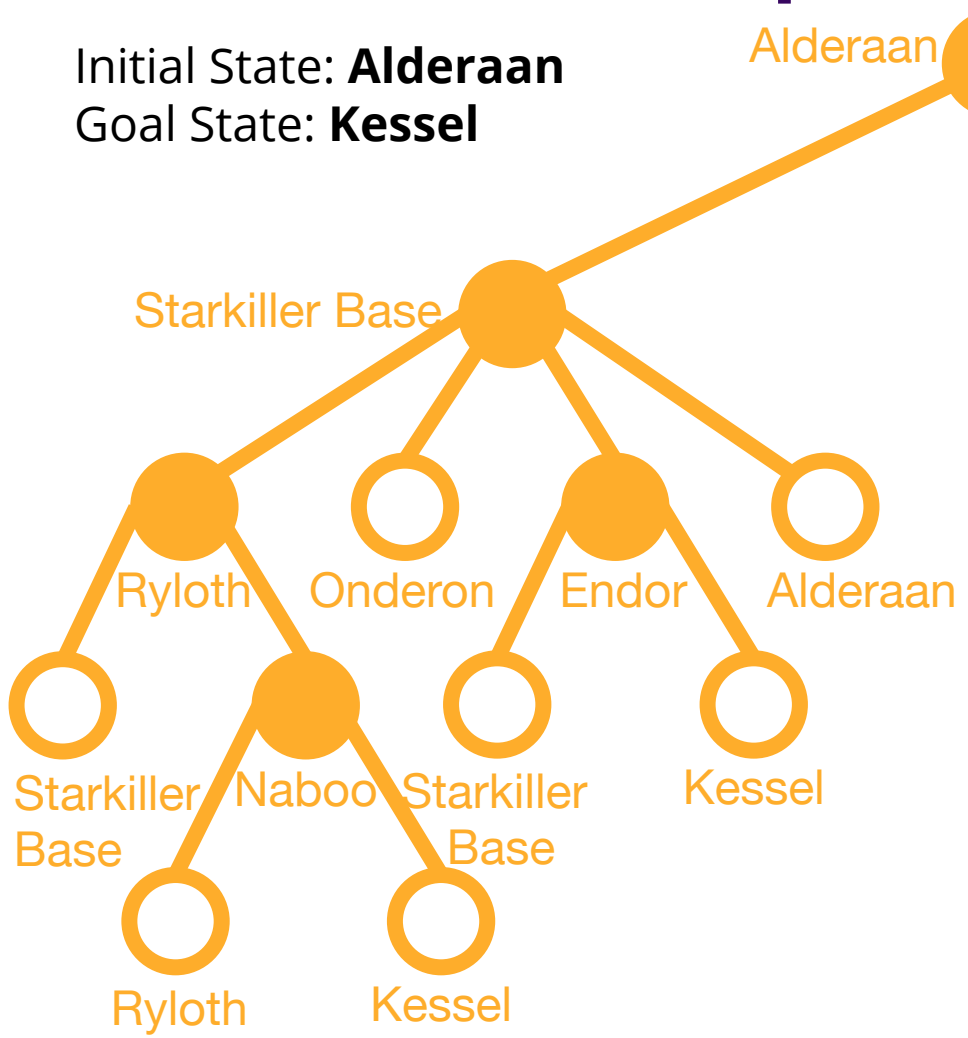

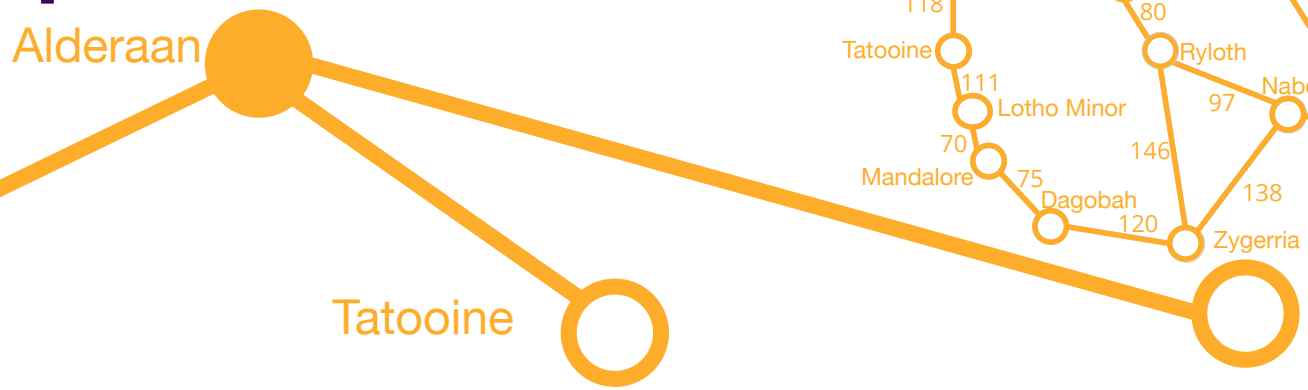

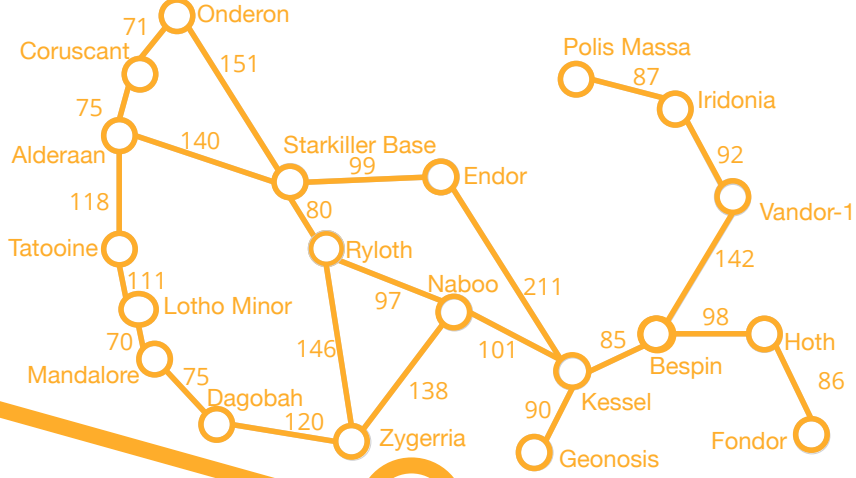

Frontier queue:

Tatooine	**447 = 118 + 329**
Coruscant	**449 = 75 + 374**
Kessel	**450 = 450 + 0**
Onderon	**526 = 146 + 380**
Starkiller Base	**532 = 300 + 253**
Starkiller Base	**591 = 338 + 253**
Alderaan	**646 = 280 + 366**
Ryloth	**607 = 414 + 193**
Kessel	**418 = 418 + 0**

Planet	SLD	Planet	SLD
Alderaan	366	Lotho Minor	244
Bespin	80	Mandalore	241
Coruscant	374	Naboo	100
Dagobah	242	Onderon	380
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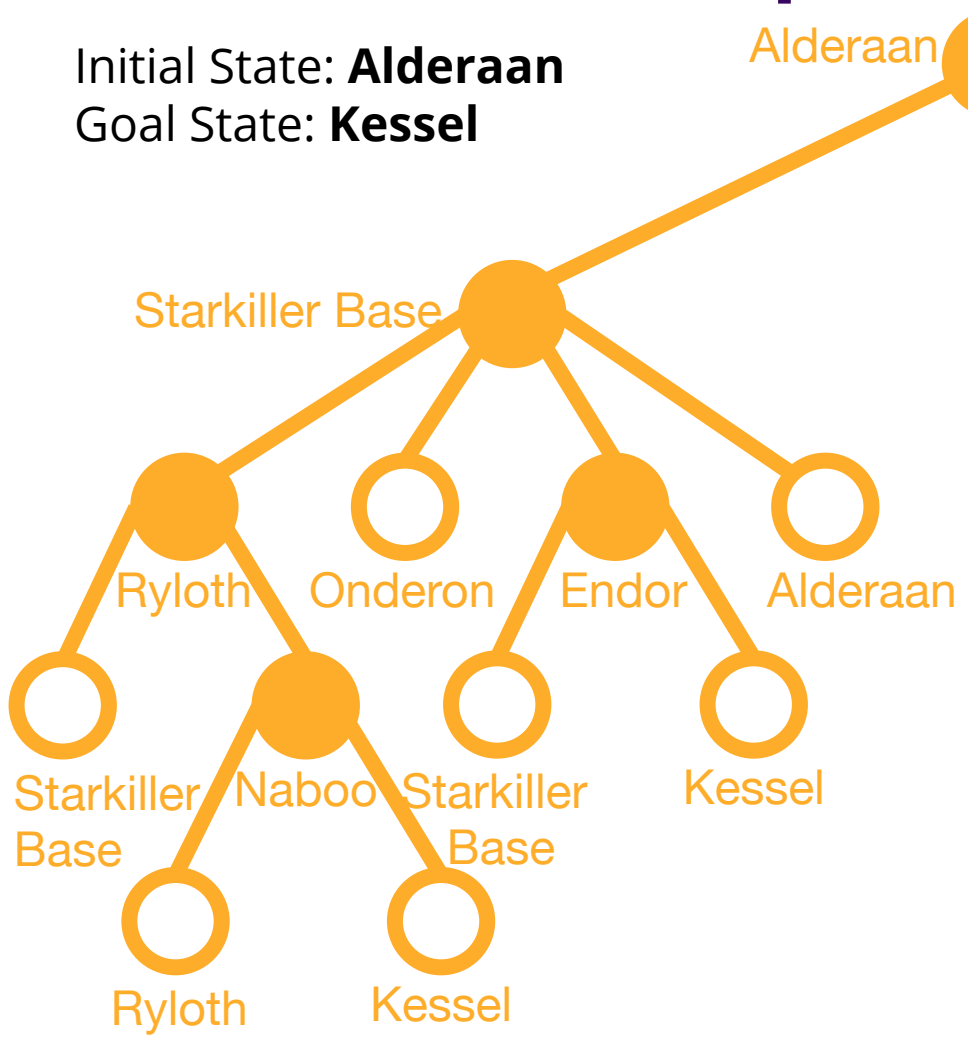
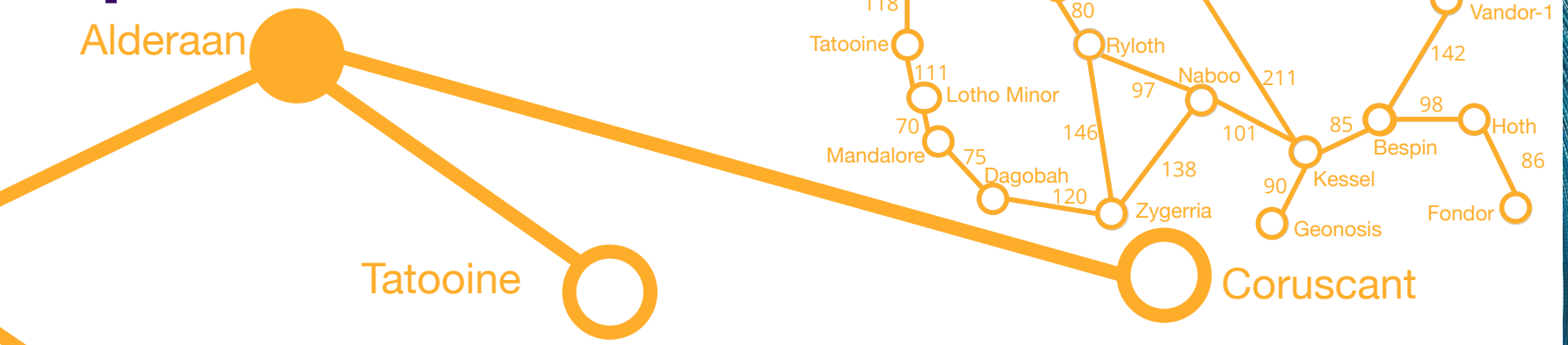
Initial State: **Alderaan**
Goal State: **Kessel**





Passes the goal test, so **this** is the shortest path to Kessel!

Frontier qu[illegible]:

Kessel	**418 = 418 + 0**
Tatooine	**447 = 118 + 329**
Coruscant	**449 = 75 + 374**
Kessel	**450 = 450 + 0**
Onderon	**526 = 146 + 380**
Starkiller Base	**532 = 300 + 253**
Starkiller Base	**591 = 338 + 253**
Ryloth	**607 = 414 + 193**
Alderaan	**646 = 280 + 366**

Planet	SLD	Planet	SLD
Alderaan	366	Lotho Minor	244
Bespin	80	Mandalore	241
Coruscant	374	Naboo	100
Dagobah	242	Onderon	380
Endor	176	Polis Massa	234
Fondor	161	Ryloth	193
Geonosis	77	Starkiller Base	253
Hoth	151	Tatooine	329
Iridonia	226	Vandor-1	199
Kessel	0	Zygerria	160

Idea: Admissibility

Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the frontier

Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

Heuristic functions

For the 8-puzzle

- **Avg. solution cost is about 22 steps**
  - **(branching factor ≤ 3)**
- **(branching factor ≤ 3)**
- **A good heuristic function can reduce the search process**

Start State

Goal State

Example Admissible heuristics

For the 8-puzzle:

$h_{oop}(n)$ = number of out of place tiles

$h_{md}(n)$ = total Manhattan distance (i.e., # of moves from desired location of each tile)

$h_{oop}(S) = 8$

$h_{md}(S) = 3+1+2+2+2+3+3+2 = 18$

Start State

Goal State

Relaxed problems

A problem with fewer restrictions on the actions than the original is called a *relaxed problem*

The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem

If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*, then $h_{oop}(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to *any adjacent square,* then $h_{md}(n)$ gives the shortest solution

Defining Heuristics: *h(n)*

Cost of an exact solution to a *relaxed* problem (fewer restrictions on operator)

Constraints on *Full* Problem:

A tile can move from square A to square B *if* A is adjacent to B *and* B is blank.

- Constraints on *relaxed* problems:
  - A tile can move from square A to square B *if* A is adjacent to B. (h_{md})
  - A tile can move from square A to square B *if* B is blank.
  - A tile can move from square A to square B. (h_{oop})

Dominance: A metric on better heuristics

If $h_2(n) \geq h_1(n)$ for all n (both admissible)

- then h_2 *dominates* h_1

So h_2 is optimistic, but more accurate than h_1

- $\boldsymbol{h_2}$ is therefore better for search
- Notice: $\boldsymbol{h_{md}}$ dominates $\boldsymbol{h_{oop}}$

Typical search costs (average number of nodes expanded):

d=12 Iterative Deepening Search = 3,644,035 nodes

$A^*(h_{oop})$ = 227 nodes, $A^*(h_{md})$ = 73 nodes

d=24 IDS = too many nodes

$A^*(h_{oop})$ = 39,135 nodes, $A^*(h_{md})$ = 1,641 nodes

The best and worst admissible heuristics

h*(n) - the (unachievable) Oracle heuristic

- h*(n) = the true distance from the n to goal

$h_{\text{we're here already}}(n) = h_{\text{teleportation}}(n) = 0$

Admissible: both yes!!!

h*(n) ***dominates all other heuristics***

$h_{\text{teleportation}}(n)$ ***is dominated by all heuristics***

A* search is Optimal

AIMA 3.5

A* search

Best-known form of best-first search.

Key Idea: avoid expanding paths that are already expensive, but expand most promising first.

Simple idea: *f(n)=g(n) + h(n)*

- ***g(n)*** the actual cost (so far) to *reach* the node
- ***h(n)*** estimated cost to *get from the node to the goal*
- ***f(n)*** estimated *total cost* of path through *n* to goal

Implementation: Frontier queue as priority queue by increasing *f(n)* *(as expected…)*

Key: Admissibility

Inadmissible (pessimistic) heuristics break optimality by pushing good plans too far back on the frontier, which means they may never get expanded.

Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs. That means that the true best plan will always be expanded.

Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

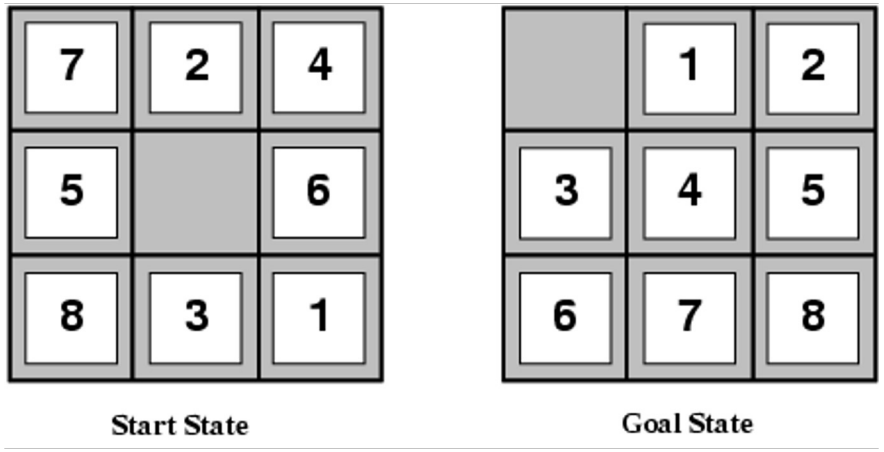
Admissible Heuristics

A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

Is Manhattan Distance admissible?



Coming up with admissible heuristics is most of what's involved in using A* in practice.

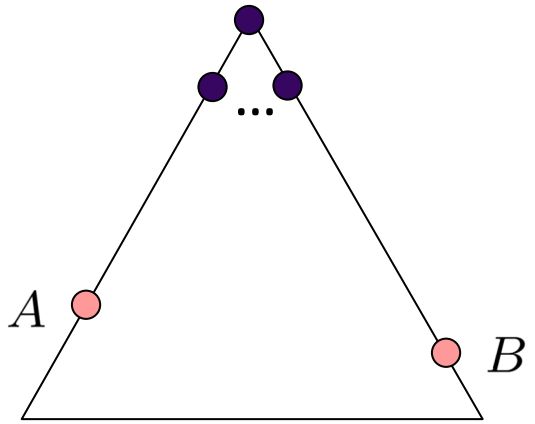
Optimality of A* Tree Search

Assume:

A is an optimal goal node

B is a suboptimal goal node

h is admissible



Claim:

A will exit the frontier before B

Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

Optimality of A* Tree Search

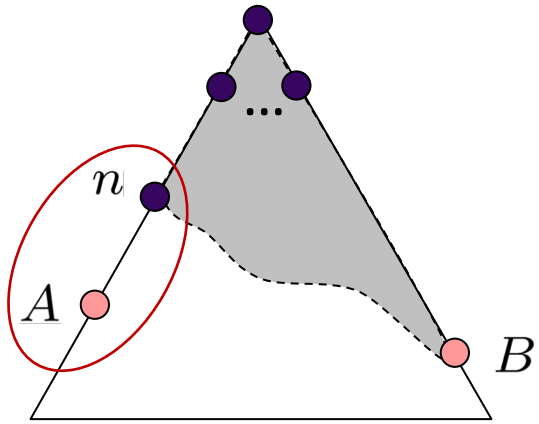
Proof:

Imagine B is on the frontier

Some ancestor *n* of A is on the frontier, too (maybe A!)

Claim: *n* will be expanded before B

- f(n) is less or equal to f(A)



$$f(n) = g(n) + h(n) \quad \text{Definition of f-cost}$$

$$f(n) \leq g(A) \quad \text{Admissibility of h}$$

$$g(A) = f(A) \quad \text{h = 0 at a goal}$$

Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

Optimality of A* Tree Search

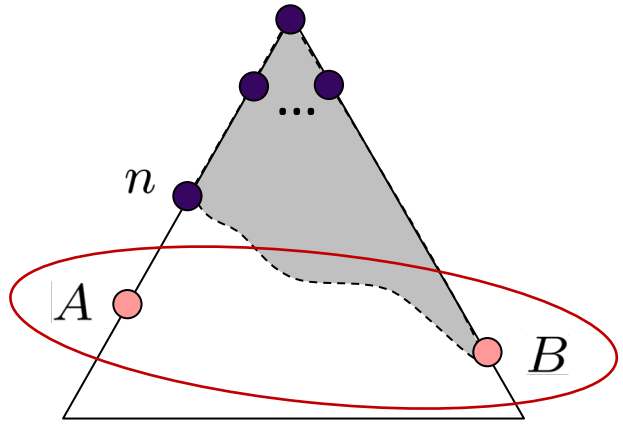
Proof:

Imagine B is on the frontier

Some ancestor *n* of A is on the frontier, too (maybe A!)

Claim: *n* will be expanded before B

- f(n) is less or equal to f(A)
- f(A) is less than f(B)



$g(A) < g(B)$ B is suboptimal

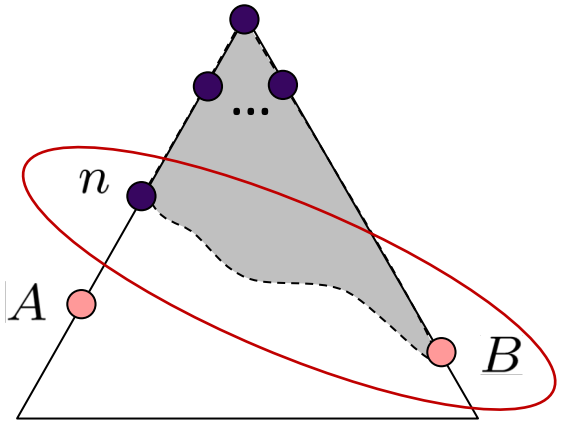
$f(A) < f(B)$ h = 0 at a goal

Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

Optimality of A* Tree Search

Proof:

- Imagine B is on the frontier
- Some ancestor *n* of A is on the frontier, too (maybe A!)
- Claim: *n* will be expanded before B
  - f(n) is less or equal to f(A)
  - f(A) is less than f(B)
  - *n* expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal



$$f(n) \leq f(A) < f(B)$$

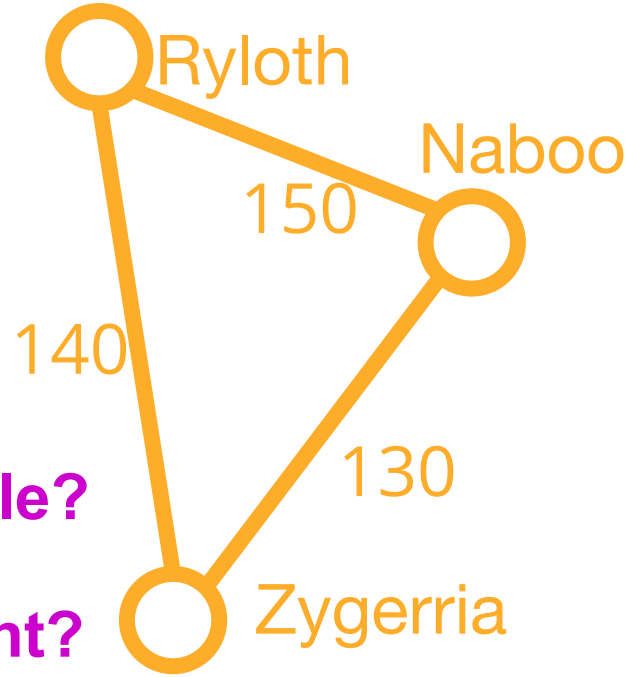
Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

Consistent Heuristics

This ensures that A* is optimal in **graph search** in addition to tree search.

Consistent heuristics are a subset of admis… …istics, which provide even stronger guarantees.

They obey the **triangle inequality**.



Planet	h1
Ryloth	150
Zygerria	5
Naboo	0

Is h1 admissible?

Is h1 consistent?

Planet	h2
Ryloth	150
Zygerria	120
Naboo	0

Is h2 admissible?

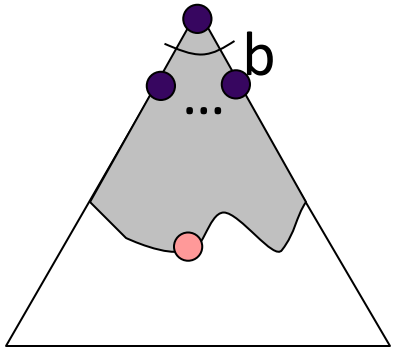
Is h2 consistent?

Properties of A*

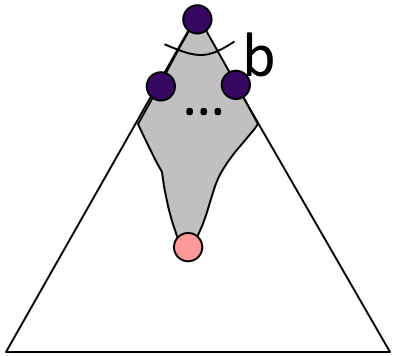
Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

Properties of A*

Uniform-Cost



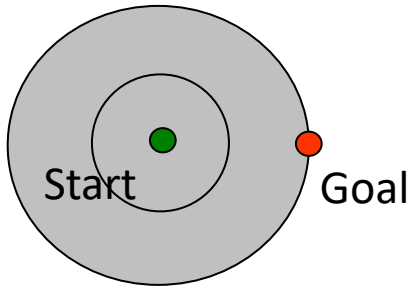
A*



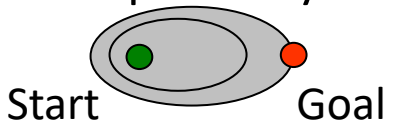
Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

UCS vs A* Contours

Uniform-cost expands equally in all “directions”



A* expands mainly toward the goal, but does hedge its bets to ensure optimality



Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

A* Applications

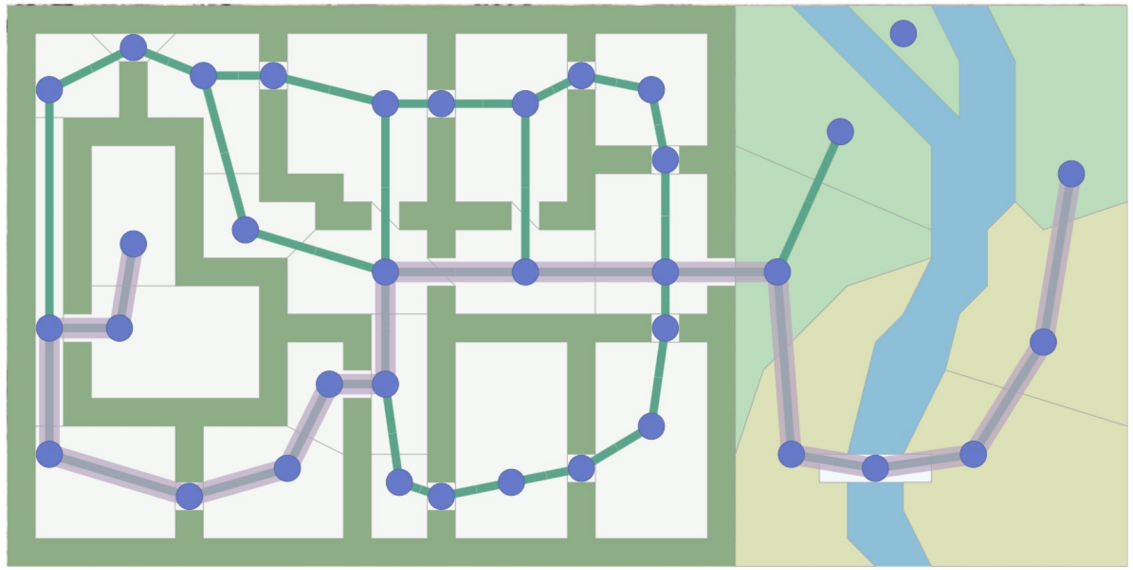
Pathing / routing problems (A* is in your GPS!)

Video games

Robot motion planning

Resource planning problems

…



Supplemental Reading

I recommend this A* tutorial by Amit Patel of Red Blob Games

This is not the solution to the HW assignment.

https://www.redblobgames.com/pathfinding/a-star/introduction.html

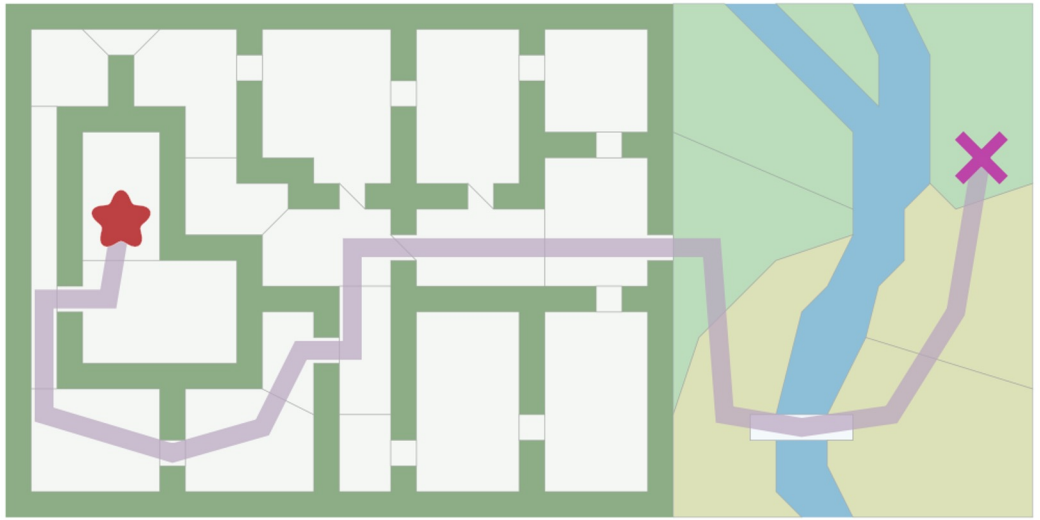
Introduction to the A* Algorithm

from Red Blob Games

Home Blog Links Twitter About Search

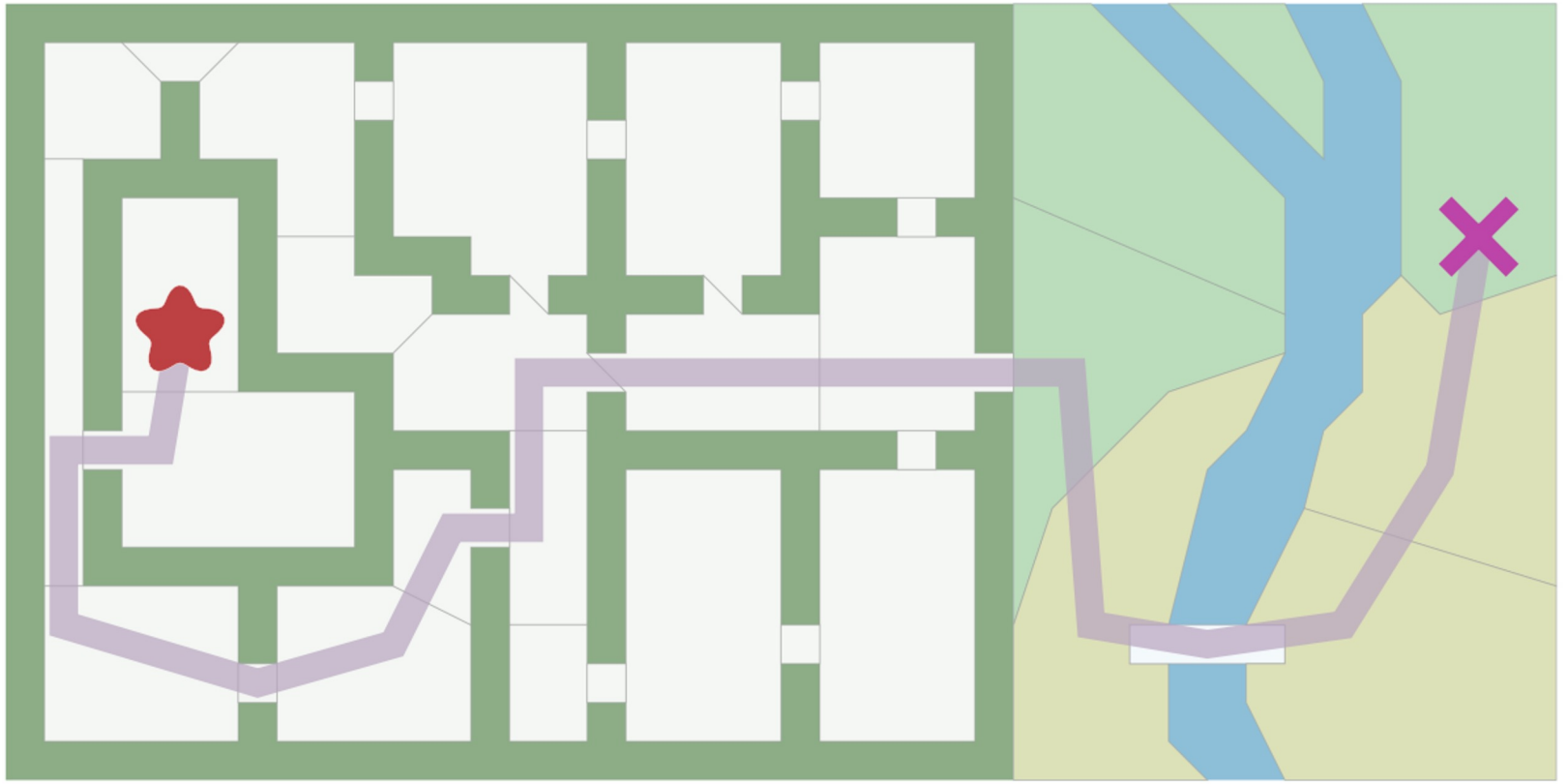
Created 26 May 2014, updated Aug 2014, Feb 2016, Jun 2016

In games we often want to find paths from one location to another. We're not only trying to find the shortest distance; we also want to take into account travel time. Move the blob (start point) and cross (end point) to see the shortest path.



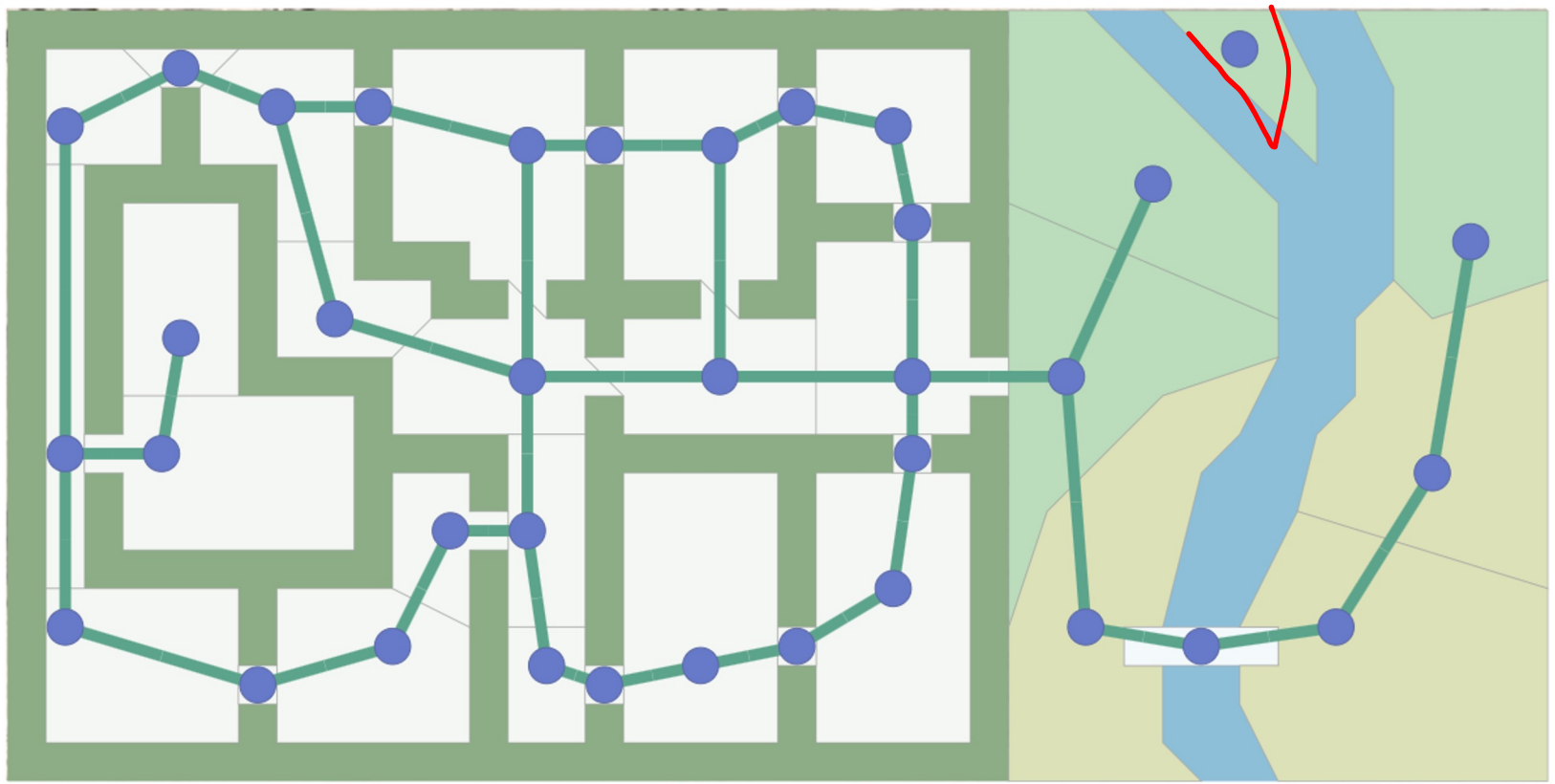
To find this path we can use a *graph search* algorithm, which works when the map is represented as a graph. **A*** is a popular choice for graph search. **Breadth First Search** is the simplest of the graph search algorithms, so let's start there, and we'll work our way up to A*.

Pathfinding in Games



https://www.redblobgames.com/pathfinding/a-star/introduction.html

Pathfinding in Games



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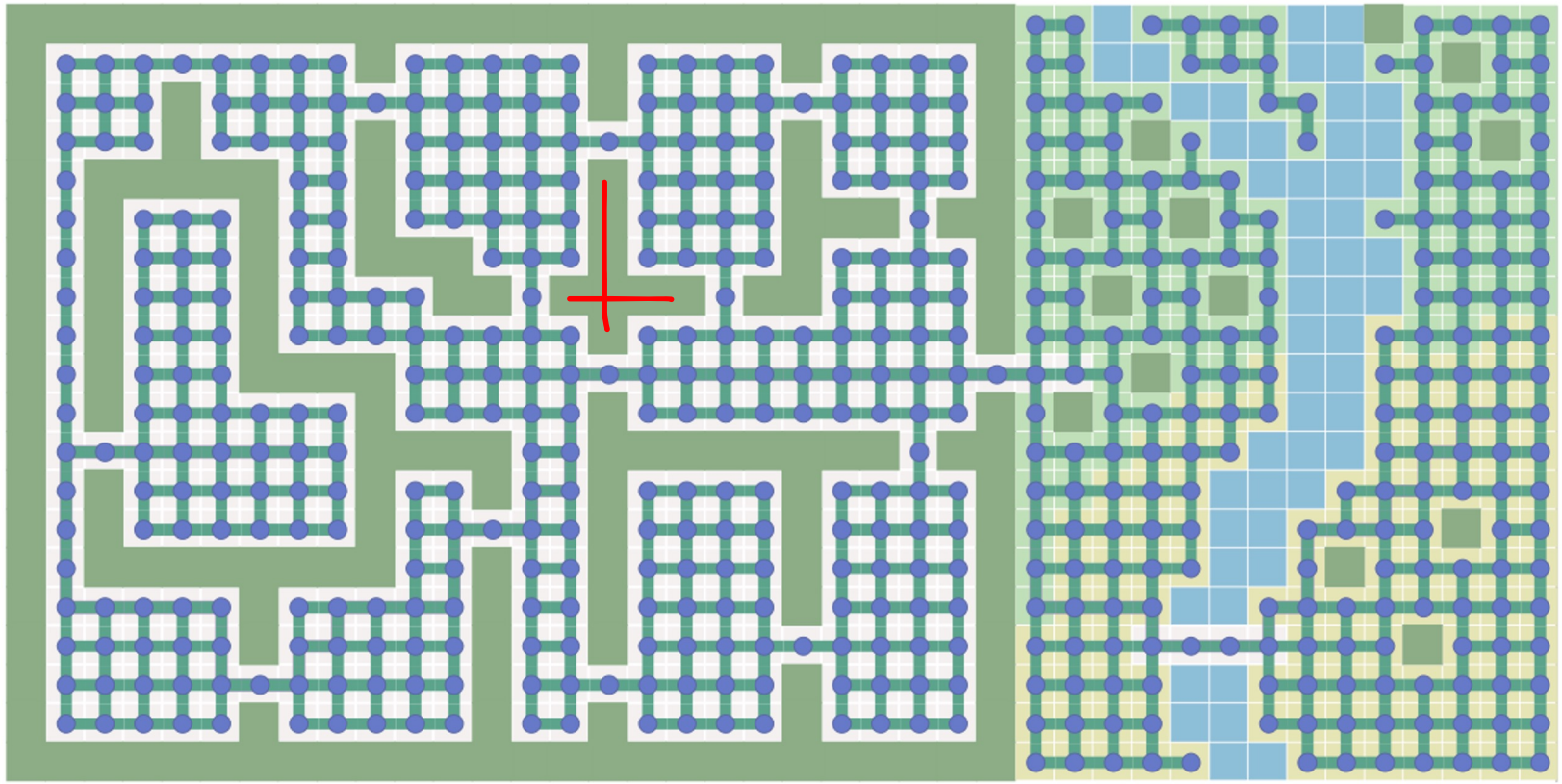
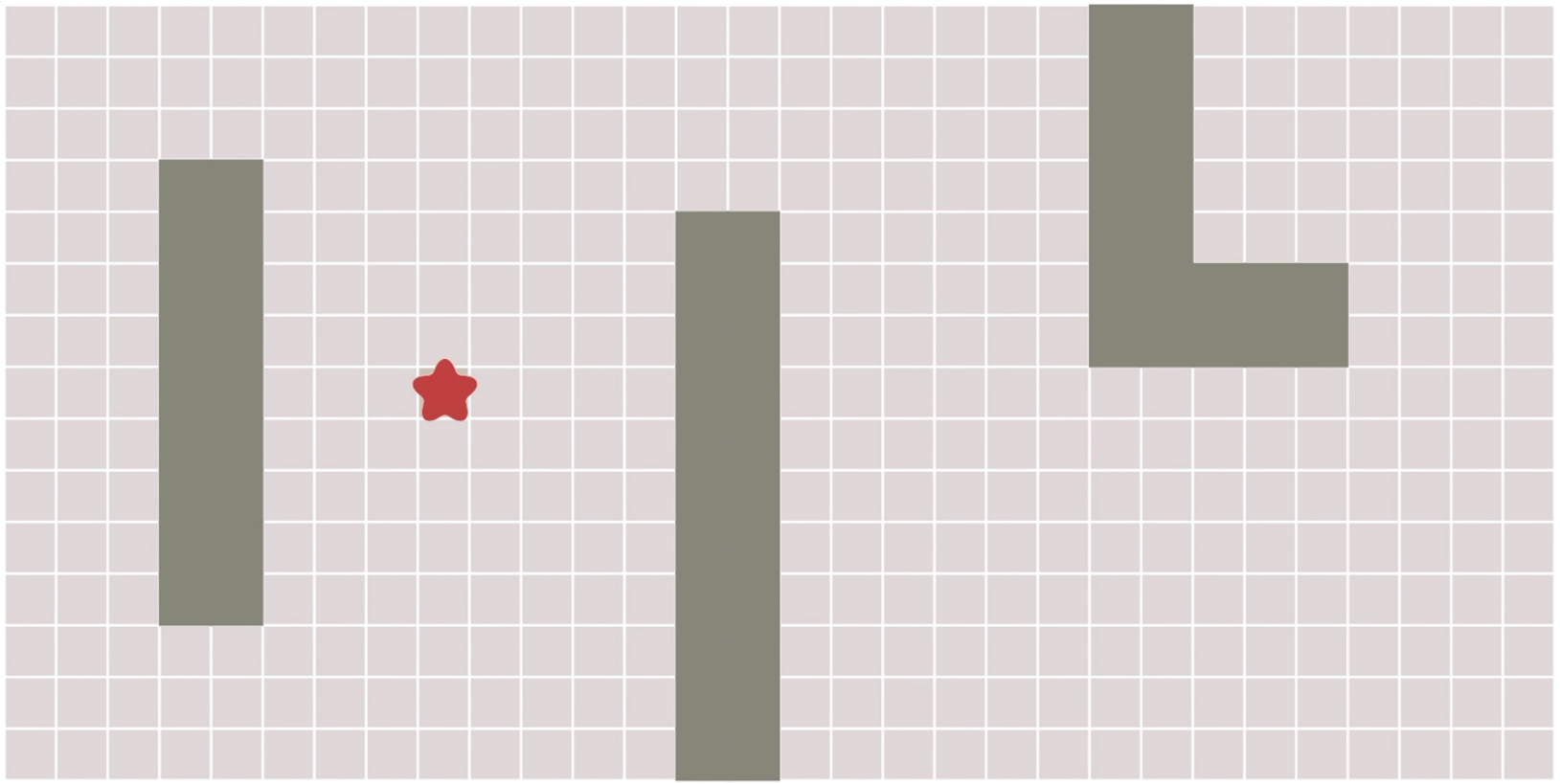
Pathfinding in Games



https://www.redblobgames.com/pathfinding/a-star/introduction.html

Breadth First Search



https://www.redblobgames.com/pathfinding/a-star/introduction.html

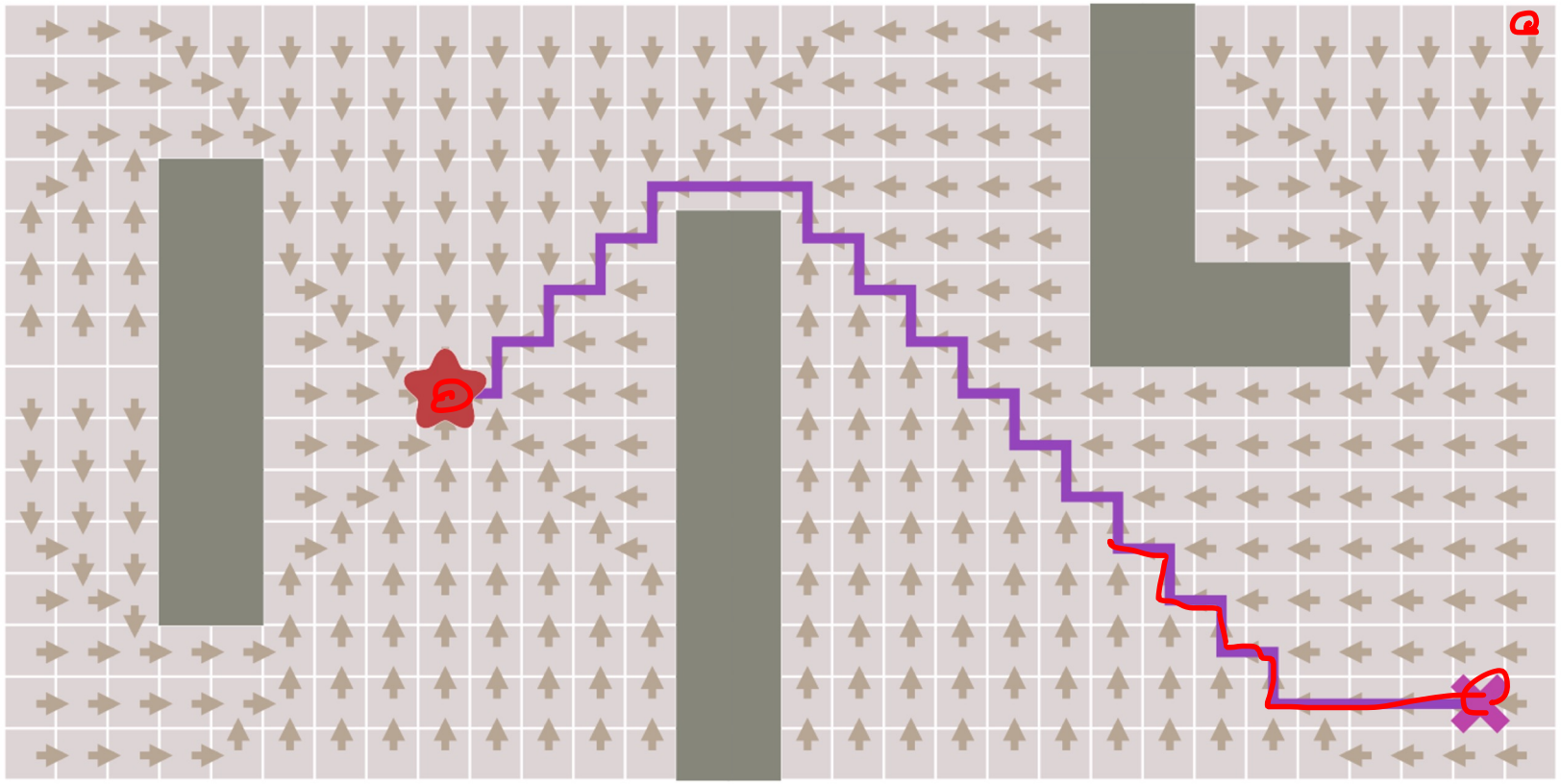
BFS in 10 lines of Python

```
frontier = Queue()
frontier.put(start)
visited = {}
visited[start] = True

while not frontier.empty():
    current = frontier.get()
    for next in graph.neighbors(current):
        if next not in visited:
            frontier.put(next)
            visited[next] = True
```

https://www.redblobgames.com/pathfinding/a-star/introduction.html

Finding the shortest path



https://www.redblobgames.com/pathfinding/a-star/introduction.html

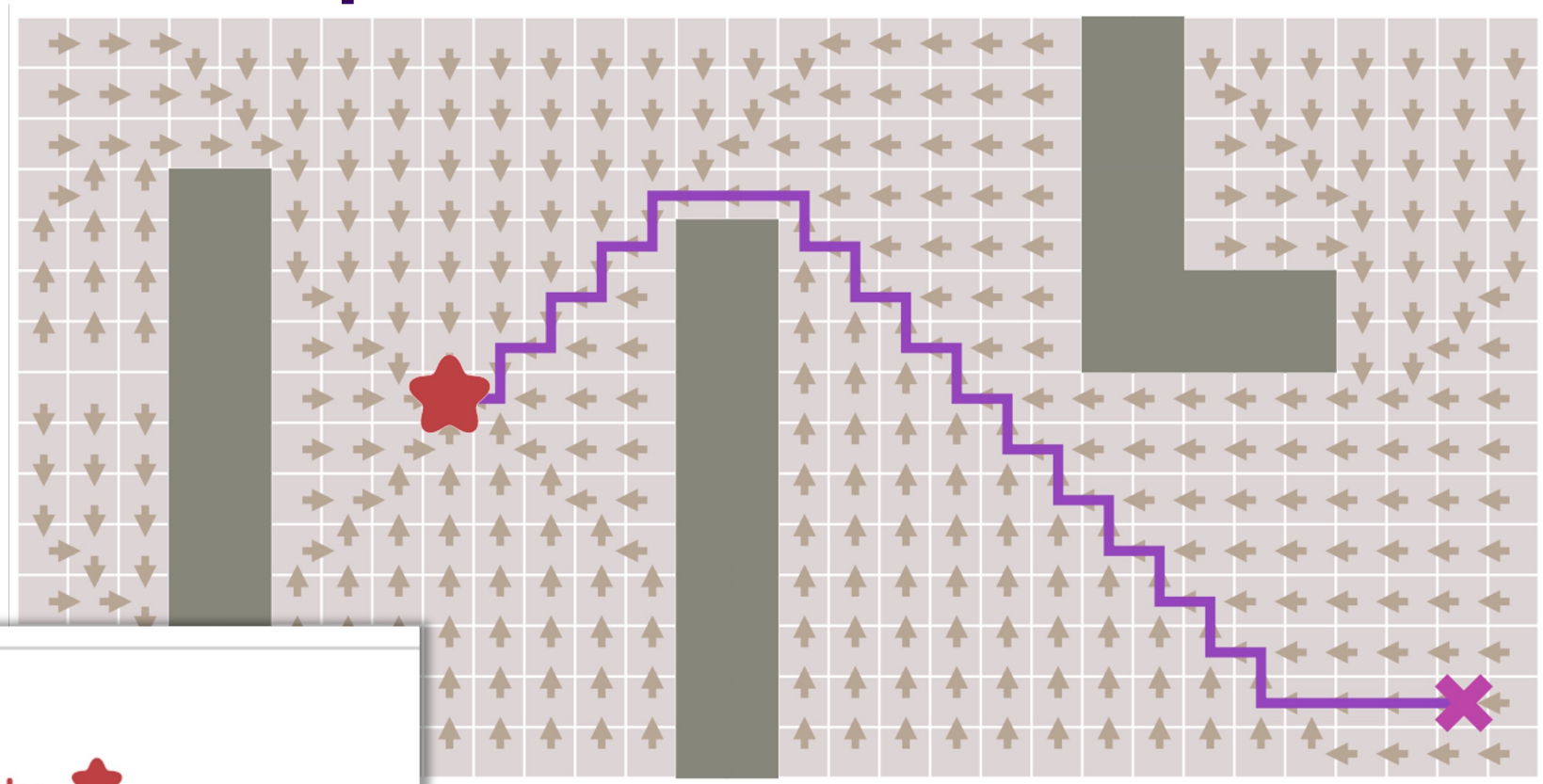
Finding the shortest path

```
frontier = Queue()
frontier.put(start )
came_from = {}
came_from[start] = None

while not frontier.empty():
    current = frontier.get()
    for next in graph.neighbors(current):
        if next not in came_from:
            frontier.put(next)
            came_from[next] = current
```

https://www.redblobgames.com/pathfinding/a-star/introduction.html

Finding the shortest path



```
current = goal
path = []
while current != start:
    path.append(current)
    current = came_from[current]
path.append(start) # optional
path.reverse() # optional
```