$$(A+UCV)^{-1} = A^{-1} - A^{-1}U\left(C^{-1}+VA^{-1}U\right)^{-1}VA^{-1}$$

$$\boxed{\begin{aligned} A &= \sigma_0^{-2} I_D \\ U &= \phi^T \\ C &= \sigma^{-2} I_N \\ V &= \phi \end{aligned}}$$

$$\Sigma = \left(\sigma_0^{-2} I_D + \sigma^{-2}\phi^T\phi\right)^{-1}$$

$$= \sigma_0^2 I_D - \sigma_0^2 I_D \cdot \phi^T \left(\sigma^2 I_N + \phi\, \sigma_0^2 I_D\, \phi^T\right)^{-1} \phi \cdot \sigma_0^2 I_D$$

$$\sigma^2(x^*) = \sigma^2 + \phi_{x^*}^T \Sigma \phi_{x^*}$$

$$= \sigma^2 + \underbrace{\sigma_0^2 \phi_{x^*}^T \phi_{x^*}}_{k(x^*,x^*)} - \underbrace{\sigma_0^2 \phi_{x^*}^T \phi^T}_{k(x^*,X)} \left(\sigma^2 I_N + \underbrace{\sigma_0^2 \phi\phi^T}_{k(X,X)}\right)^{-1} \underbrace{\phi\,\phi_{x^*}\,\sigma_0^2}_{k(X,x^*)}$$

$$= \sigma^2 + \underline{k(x^*,x^*)} - \underline{k(x^*,X)} \left(\underline{k(X,X)} + \sigma^2 I_N\right)^{-1} \underline{k(X,x^*)}$$

$$(K_{x,x} + \sigma^2 I) = L L^T$$

$$(K_{x,x} + \sigma^2 I)^{-1} y = (L L^T)^{-1} y$$

$$= \underbrace{(L^T)^{-1} \cdot \underbrace{L^{-1} y}}$$

$$K_{x^*,x} (K_{x,x} + \sigma^2 I)^{-1} K_{x,x^*} = K_{x^*,x} (L L^T)^{-1} K_{x,x^*}$$

$$= \underbrace{\boxed{K_{x^*,x} (L^T)^{-1}}}_{v^T} \cdot \underbrace{\underline{L^{-1} K_{x,x^*}}}_{v}$$

$$\vec{f} = \{ f_1, \dots \; f_N \} \qquad f_n = f(x_n)$$

$$p(\vec{f}) = N(\vec{f}; 0, \underbrace{K_{x,x}}_{N \times N})$$

$$p(y | \vec{f}) = \prod_n N(y_n; f_n, \sigma^2)$$

$$= \mathcal{N}(y;\ \vec{f},\ \sigma^2 I_n)$$

$$p(y \mid X, \theta) = \int p(y, \vec{f} \mid X, \theta)\, df$$

$$= \int p(y \mid \vec{f})\ p(\vec{f} \mid X, \theta) \cdot df$$

$$= \mathcal{N}(y\ ;\ 0,\ K_{xx} + \sigma^2 I_n)$$

$$f(z) \approx f(z_0) + \cancel{\left.\frac{df(z)}{dz}\right|_{z=z_0} (z - z_0)}$$

$$+ \frac{1}{2} \left.\frac{d^2 f(z)}{dz^2}\right|_{z=z_0} (z - z_0)^2$$

z_0 is the mode then $\left.\dfrac{df(z)}{dz}\right|_{z=z_0} = 0$

$=$

$$\ln f(z) \simeq \log f(z_0) - \frac{1}{2} A (z - z_0)^2$$

$$f(z) = \underline{f(z_0) \exp\left(-\frac{1}{2} A (z - z_0)^2\right)}$$

$$q(z) = N(z; \underline{z_0}, \underline{A^{-1}})$$

$$= \underbrace{\frac{1}{\sqrt{2\pi (A)^{-1}}} \exp\left(-\frac{1}{2} A (z - z_0)^2\right)}$$

$$= \frac{\sqrt{A}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} A (z - z_0)^2\right)$$

$$p(z) = \frac{f(z)}{Z}$$

$$\downarrow$$

$$\mathcal{N}(z;\, z_0,\, A^{-1})$$

$$Z = f(z)\Big|_{z=z_0} \cdot \sqrt{\frac{2\pi}{A}}$$

$$\int p(z)\, dz = 1$$

$$f(z) \simeq f(z_0)\exp\left(-\frac{1}{2}A(z-z_0)^2\right)$$

$$\int \frac{f(z)}{Z}\, dz = 1$$

$$Z = \int f(z)\, dz$$

$$= \int f(z_0) \cdot \exp\left(-\frac{1}{2} A (z - z_0)^2\right) dz$$

$$\boxed{Z = f(z_0) \cdot \sqrt{\frac{2\pi}{A}}}$$