

$$(A + UCU)^{-1} = A^{-1} - A^{-1}U(C^{-1} + VA^{-1}U)^{-1}VA^{-1}$$

$$\Sigma = (\sigma_0^{-2} \mathbb{I}_D + \sigma^{-2} \Phi^T \Phi)^{-1}$$

$$= \sigma_0^{-2} \mathbb{I}_D - \sigma_0^{-2} \mathbb{I}_D \cdot \Phi^T (\sigma^2 \mathbb{I}_N + \underbrace{\Phi \sigma_0^{-2} \mathbb{I}_D \Phi^T}_{\Phi \cdot \sigma_0^{-2} \mathbb{I}_D})^{-1}$$

$$\begin{aligned} A &= \sigma_0^{-2} \mathbb{I}_D \\ U &= \Phi^T \\ C &= \sigma^{-2} \mathbb{I}_N \\ V &= \Phi \end{aligned}$$

$$\sigma^2(x^*) = \sigma^2 + \Phi_{x^*}^T \Sigma \Phi_{x^*}$$

$$= \sigma^2 + \underbrace{\sigma_0^{-2} \Phi_{x^*}^T \Phi_{x^*}}_{h(x^*, x^*)} - \underbrace{\sigma_0^{-2} \Phi_{x^*}^T \Phi^T}_{h(x^*, X)} (\underbrace{\sigma^2 \mathbb{I}_N + \underbrace{\sigma_0^{-2} \Phi \Phi^T}_{\underbrace{\Phi \Phi_{x^*} \sigma_0^{-2}}_{h(X, x^*)}}})^{-1}$$

$$= \sigma^2 + \underline{h(x^*, x^*)} - \underline{h(x^*, X)} (\underline{h(X, X)} + \sigma^2 \mathbb{I}_N)^{-1}$$

$$\underline{h(X, x^*)}$$

$$(K_{x,x} + \sigma^2 I) = L L^T$$

$$(K_{x,x} + \sigma^2 I)^{-1} y = (L L^T)^{-1} y$$

$$= \underbrace{(L^T)^{-1} L^{-1} y}$$

$$K_{x^*, x} (K_{x,x} + \sigma^2 I)^{-1} K_{x,x^*} = K_{x^*, x} (L L^T)^{-1} K_{x,x^*}$$

$$= \underbrace{K_{x^*, x} (L^T)^{-1}}_{v^T} \cdot \underbrace{L^{-1} K_{x,x^*}}_v$$

$$\vec{f} = \{f_1, \dots, f_N\} \quad f_n = f(x_n)$$

$$p(\vec{f}) = N(\vec{f}; 0, K_{xx})$$

↪ N x N

$$p(y | \vec{f}) = \prod_n N(y_n; f_n, \sigma^2)$$

$$= N(y; \vec{f}, \sigma^2 \mathbf{I}_n)$$

$$p(y|x, \theta) = \int p(y, \vec{f} | x, \theta) d\vec{f}$$

$$= \int p(y | \vec{f}) p(\vec{f} | x, \theta) \cdot d\vec{f}$$

$$= N(y; 0, K_{nn} + \sigma^2 \mathbf{I}_n)$$

$$f(z) \approx f(z_0) + \frac{df(z)}{dz} \bigg|_{z=z_0} (z-z_0) + \frac{1}{2} \frac{d^2 f(z)}{dz^2} \bigg|_{z=z_0} (z-z_0)^2$$

$z_0$  is the mode then  $\frac{df(z)}{dz} \bigg|_{z=z_0} = 0$

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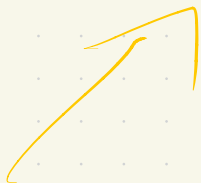
$$\ln f(z) \approx \ln f(z_0) - \frac{1}{2} A (z - z_0)^2$$

$$f(z) = \underline{f(z_0) \exp\left(-\frac{1}{2} A (z - z_0)^2\right)}$$

$$q(z) = N(z; \underline{z_0}, \underline{A^{-1}})$$

$$= \frac{1}{\sqrt{2\pi (A)^{-1}}} \exp\left(-\frac{1}{2} A (z - z_0)^2\right)$$

$$= \frac{\sqrt{A}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} A (z - z_0)^2\right)$$



$$p(z) = \frac{f(z)}{Z}$$



$$N(z, z_0, A^{-1})$$

$$Z = f(z) \Big|_{z=z_0} \cdot \sqrt{\frac{2\pi}{A}}$$

$$\int p(z) \, dz = 1$$

$$f(z) \approx f(z_0) \exp\left(-\frac{1}{2} A(z - z_0)^2\right)$$

$$\int \frac{f(z)}{z} dz = 1$$

$$z = \int f(z) \cdot dz$$

$$= \int f(z_0) \cdot \exp\left(-\frac{1}{2} \underline{A(z-z_0)^2}\right) dz$$

$$z = f(z_0) \cdot \sqrt{\frac{2\pi}{A}}$$