$$\underbrace{\underbrace{-\log p(\theta \mid y, X)}_{L}} = \frac{1}{2}(\theta - m_0)^T S_0^{-1} (\theta - m_0) + \sum_n \left[ y_n \log g_n + (1 - y_n) \log (1 - g_n) \right]$$

$$g_n = \sigma(\theta^T x_n)$$

$$\frac{dL}{d\theta} = S_0^{-1}(\theta - m_0) + \underbrace{\sum_n \underbrace{(g_n - y_n) \cdot x_n}_{l_n}}_{\theta} \rightarrow \theta_{MAP}$$

$$\underset{\sim}{(g_n} - y_n) x_n$$

$$\left[\frac{d^2 L}{d\theta \cdot d\theta^T}\right]_{i,j} = \frac{d^2 L}{d\theta_i \, d\theta_j}$$

$\hookrightarrow D \times D$

$$\frac{dl_n}{d\theta^T} = \frac{d\,(g_n x_n)}{d\theta^T}$$

$$\frac{dL}{d\theta \cdot d\theta^T} = S_0^{-1}$$

$$g_n = \frac{1}{1 + \exp(-\theta^T x_n)}$$

$$\frac{dg_n}{d\theta^T} = + \frac{\exp(-\theta^T x_n)\, x_n^T}{(1 + \exp(-\theta^T x_n))^2}$$

$$= g_n \cdot (1 - g_n) \cdot x_n^T$$

$$\frac{dl_n}{d\theta^T} = g_n \cdot (1 - g_n)\, x_n x_n^T$$

$$\frac{dL}{d\theta\, d\theta^T} = S_0^{-1} + \underbrace{\sum_n \underbrace{y_n(1-y_n)}_{\geq 0} \underbrace{x_n x_n^T}}$$

positive definite

$\underbrace{x_n x_n^T}$ rank 1 $D \times D$

positive semidefinite

positive definite

$-\log p(\theta \mid y, X)$ is convex

$\Rightarrow$ unique minimum

$$p(\underline{f}) = \mathcal{N}(\underline{f}; 0, K)$$

$$\underbrace{\log p(y \mid \underline{f}, x)}_{L} = \log \prod_n \underbrace{\sigma(f_n)^{y_n} [1 - \sigma(f_n)]^{1-y_n}}_{\ell_n}$$

$$\frac{d \log p(f)}{d \underline{f}} = -K^{-1} \cdot \underline{f}$$

$$\frac{d \ell_n}{d f_n} = \boxed{\frac{d \ell_n}{d f_n}} = (y_n - \sigma(\underline{f_n}))$$

$$\Rightarrow \frac{dL}{d\underline{f}} = y - \sigma(\underline{f})$$

$$\frac{d \log p(\underline{f} \mid y, x)}{d \underline{f}} = y - \sigma(f) - K^{-1} f$$

$$= 0$$

$$f = K (y - \sigma f)$$

$$m, n \quad \frac{d \log p(f \mid y, x)}{d f_n \, d f_m}$$

$$\frac{d \log p(\underline{f})}{d \underline{f} \, d \underline{y}^T} = -K^{-1}$$

$$\frac{d\ell_n}{\underbrace{df_n\, df_m}} = \begin{cases} 0 & \text{if } \underline{m} \neq \underline{n} \\ & \underline{m} = n \\ \underline{-\sigma(f_n)(1-\sigma(f_n))} & \end{cases}$$

$$H = \frac{d \log p(\underline{f} \mid x, y)}{d\underline{f}\, d\underline{f}^T}$$

$$= -\underbrace{K^{-1}} \cancel{+} \underline{\operatorname{diag}(\sigma(f_n) \cdot 1 - \sigma(f_n))}$$

$$S = \left(K^{-1} + \operatorname{diag} \sigma(f_n)(1-\sigma(f_n))\right)^T$$

$$\underbrace{q(\underline{f})} = \mathcal{N}(\underline{f}\,;\, m, S)$$

$$\boxed{p(y_t = 1 \mid x_t, x, y)} = \int p(y_t = 1 \mid f_t, x_t) \cdot p(f_t \mid x, y)\, df_t$$

$t$ = test

$$\Phi \leftarrow \underbrace{\sigma(f_t)} \quad \underbrace{\mathcal{N}(f_t;\, m_t, v_t)}$$

$$\underbrace{p(f_t \mid x, y)} = \int p(f_t \mid \underline{f}) \cdot p(\underline{f} \mid x, y)\, d\underline{f}$$

$$\simeq \int p(f_t \mid \underline{f})\; q(\underline{f})\; d\underline{f}$$

$$\mathcal{N}(f_t;\, \underbrace{K_{tx} K_{xx}^{-1}}_{A} \underline{f},\, \underbrace{K_{tt} - K_{tx} K_{xx}^{-1} K_{xt}}_{\Sigma})$$

$$\mathcal{N}(\underline{f};\, m, S)$$

$$\underbrace{p(f_t \mid x, y)} \simeq \mathcal{N}(f_t;\, \underbrace{Am},\, \underbrace{\Sigma + ASA^T})$$

$$p(\underline{f} \mid x, y) = \frac{p(\underline{f})\, p(y \mid x, \underline{f})}{p(y \mid x)}$$

$$p(y \mid x) = \int \underbrace{p(f) \cdot p(y \mid x, f)}\, dy$$

$$\log (p(f) \cdot p(y \mid x, f)) \simeq \log p(f_{map}) \cdot p(y \mid x, f_{map})$$

exponentiate both sides $+ \frac{1}{2} (f - f_{map})^T \cdot H (f - f_{map})$

$$p(y \mid x) \simeq p\, f_{map} \cdot p(y \mid x, f_{map})$$

$$\int \exp\underbrace{\left(+ \frac{1}{2} (f - f_{map})^T \cdot H (f - f_{map})\right)} dy$$

$\Downarrow$

$A^{-1} = H$

$$= \int \frac{1}{(2\pi)^{D/2} \cdot |A|^{1/2}} \exp\left(-\frac{1}{2}(f - f_{map})^T \cdot A^{-1} \cdot (f - f_{map})\right) \times (2\pi)^{D/2} |A|^{1/2}$$

$= 1$

$$p(y|x) = p(f_{map}) \cdot p(y|x, f_{map})$$

$$\times \underbrace{(2\pi)^{D/2}}_{} \cdot |A|^{1/2}$$

$$\frac{D}{2} \log(2\pi) + \frac{1}{2} \log(|A|)$$