

Session #1.8: Plan

- Vector operations
- Multiplication
- Matrix inverse

Vector operations

column vector

$$2 \cdot \begin{bmatrix} 2 \\ 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 10 \\ 12 \end{bmatrix}$$

Vector operations

$$\begin{matrix} u + v \\ \left[\begin{array}{c} 2 \\ 4 \\ 5 \\ 6 \end{array} \right] + \left[\begin{array}{c} 1 \\ 0 \\ 0 \\ 2 \end{array} \right] = \left[\begin{array}{c} 3 \\ 4 \\ 5 \\ 7 \end{array} \right] \end{matrix}$$

Multiplication

- Vector-vector multiplication
- Matrix-vector multiplication
- Matrix-matrix multiplication

Vector-vector multiplication (dot product)

$$\begin{array}{c}
 \text{u} \quad \text{v} \\
 \rightarrow \begin{bmatrix} 2 \\ 4 \\ 5 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix} \leftarrow \\
 \begin{array}{l}
 2 \cdot 1 \\
 + 4 \cdot 0 \\
 + 5 \cdot 0 \\
 + 6 \cdot 2
 \end{array} \left. \right\} 2+12=14
 \end{array}$$

$$\sum_{i=1}^n u_i v_i$$

Vector-vector multiplication (dot product)

v^T [2 4 5 6]

u ↴
row vector

column

$$v^T u = \sum_{i=1}^n u_i v_i$$

Matrix-vector multiplication

Diagram illustrating Matrix-vector multiplication:

Matrix U (dimensions 3x4) is multiplied by vector v (dimensions 4x1) to produce vector Uv (dimensions 3x1).

Matrix U is defined by columns:

u_0	$\begin{bmatrix} 2 & 4 & 5 & 6 \end{bmatrix}$
u_1	$\begin{bmatrix} 1 & 2 & 1 & 2 \end{bmatrix}$
u_2	$\begin{bmatrix} 3 & 1 & 2 & 1 \end{bmatrix}$

Vector v is defined by components:

$$v = \begin{bmatrix} 1 \\ 0.5 \\ 2 \\ 1 \end{bmatrix}$$

The result Uv is calculated as:

$$Uv = \begin{bmatrix} u_0^T v \\ u_1^T v \\ u_2^T v \end{bmatrix}$$

Matrix-vector multiplication

The diagram illustrates the computation of a matrix-vector product Uv . On the left, a matrix U with n columns is shown as a vertical stack of k rows. The top row is labeled u_0 , and the bottom row is labeled u_{k-1} . Ellipses between the rows indicate intermediate rows. To the right of U is a vector v enclosed in a bracket, with its dimension n indicated above it. An equals sign follows v . To the right of the equals sign is the result of the multiplication, represented as a vertical vector enclosed in a bracket. The entries of this vector are labeled $u_0^\top v$ at the top and $u_{k-1}^\top v$ at the bottom, with ellipses indicating intermediate entries.

Rows of matrix * vector

Matrix-matrix multiplication

$$U = \begin{bmatrix} 2 & 4 & 5 & 6 \\ 1 & 2 & 1 & 2 \\ 3 & 1 & 2 & 1 \end{bmatrix}$$

$$V = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0.5 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$Uv_0 = \begin{bmatrix} v_0 \\ \vdots \\ v_0 \end{bmatrix}$$

$$UV = \begin{bmatrix} | & | & | \\ u_0, v_0, v_0 & | & | \\ | & | & | \end{bmatrix}$$

$$UV =$$

Identity matrix

$$U \cdot I = U$$

$$I \cdot U = U$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

I

$$1 \cdot x = x$$
$$x \cdot 1 = x$$

Matrix inverse

$$A^{-1} A = I$$