Aniket R Vishwakarma
HPC 1 : Final Project

High Performance Computing 1: Final Project

Due Monday, Dec. 19, 2022 at 11:59 pm

Submission Instructions

Code may be written in `C`, `C++`, or `Fortran`. Clarity of code and plots will be a component of the credit you receive. Please `tar` or `zip` the components listed below into a SINGLE bundled file and submit it electronically through UBLearns. In the bundled submission please include:
(a) A PDF file containing your written responses, handwritten equations, and graphs.
(b) `README` file the states the contents of the `tar` or `zip` file and the steps to compile and run different experiments.
(c) The source code, text files generated, and makefiles (or shell scripts), if used.

Performance of the Poisson Problem

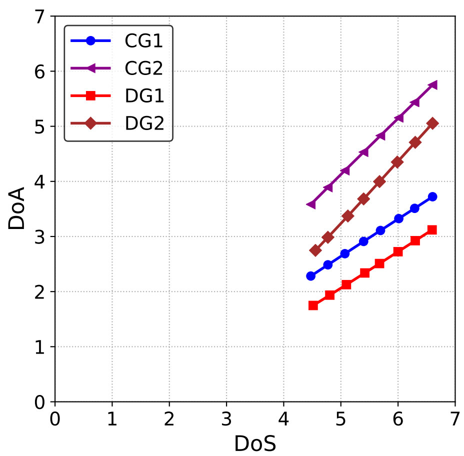
We will look at performance of a solver for the Poisson problem and include the notion of solution accuracy into our performance analysis. Recall that the Poisson equation can be written in Cartesian coordinates in three dimensions as

$$-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial z^2} = f.$$

In two dimensions, we would just drop the z-derivative. We will follow the analysis procedure laid out in Chang et al. 2018.

Part A: Mesh Convergence

In order to determine if our implementation is correct, we will use the method of manufactured solutions. This chooses an exact solution u^*, which determines an f so that the Poisson equation is satisfied. We can then evaluate the error in our computed solution u using the 2-norm of the difference, $e = \|u - u^*\|_2$. For example, this is plotted below where DoS stands for Digits of Size, defined as $\log_{10} N$ where N is the problem size, and DoA stands for Digits of Accuracy, defined as $-\log_{10} e$ where e is the error defined above.



1. Run SNES ex5 in serial using MMS 3 for a range of grid sizes, such as 64×64, 128×128, and 256×256. Solve the problem exactly using the LU preconditioner. The problem size and error will print out at the end of each run. Plot the negative logarithm of the L_2 error, $-\log e$, as a function of the logarithm of the problem size, $\log N$. Find the slope α of the line. This slope is called the *convergence rate* of the discretization. Remember that for a 64×64 grid with LU we would use

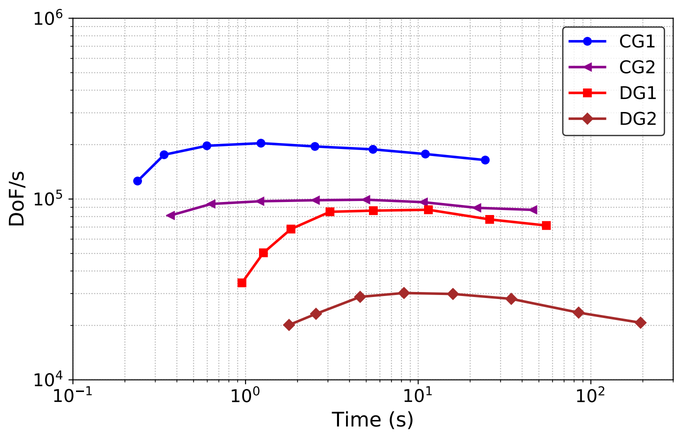
```
./ex5 -da_grid_x 64 -da_grid_y 64 -pc_type lu -mms 3
```

2. Repeat the above analysis for SNES ex13 using linear finite elements. You will need to use `-error_view` in order to get ex13 to print out the error. Both ex5 and ex13 analyses can be given on the same graph if desired. Remember that for a 64×64 grid with LU we would use

```
./ex13 -dm_plex_simplex 0 -potential_petscspace_degree 1 -dm_plex_box_faces 64,64 -pc_type lu
```

Part B: Static Scaling

Next, we look at a way to combine strong and weak scaling on a single plot. Instead of varying the number of processes, as we did for plots of strong and weak scaling in Homework 2, we will vary the problem size, and keep the number of processes fixed. We then plot the computation rate, defined as degrees of freedom (dof) solved per time or N/T, as a function of the runtime. For example,



1. Run SNES ex5 for a range of sizes, such as 256×256, 512×512, 1024×1024 and 2048×2048, on 64 processes and plot the computation rate, N/T, as a function of the runtime T, using logarithmic scales for both. The time T can be determined from the `SNESSolve` event using the `-log_view` output. Run this for three different solvers:
   a sparse direct factorization using LU (MUMPS or SuperLU_dist),

   ```
   ./ex5 -pc_type lu -pc_factor_mat_solver_type mumps
   ```

   Block-Jacobi with ILU(0) which is the default,

   ```
   ./ex5
   ```

   and Algebraic Multigrid,

   ```
   ./ex5 -pc_type gamg
   ```

   Make sure to have problem sizes small enough that the computation rate falls off, so you may need to decrease the mesh size to 64×64 if necessary.

2. Repeat the analysis in B.1 for SNES ex13. This means running with something like

   ```
   ./ex13 -dm_plex_simplex 0 -potential_petscspace_degree 1 -pc_type lu -pc_factor_mat_solver_type mumps
     -dm_plex_box_faces 256,256 -dm_distribute
   ```

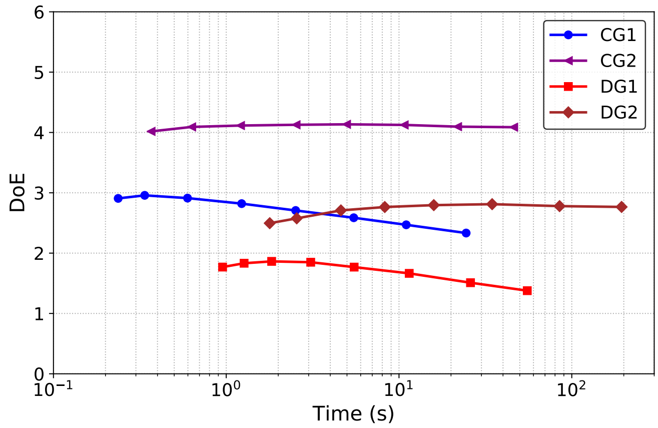
3. What is the minimum runtime for any size problem, for both examples?

4. Which (if any) solvers demonstrate weak scaling, indicated by a flat line to the right on the graph?

5. Which solver performs the best?

Part C: Efficacy

Last, we look at a way to combine accuracy with performance information. Again using a fixed number of processes, we will examine the *efficacy*, $1/(e \cdot T)$ where e is the error, as a function of the runtime T. For example,



where DoE means Digits of Efficacy, or $-\log_{10}(e \cdot T)$. We would like to simultaneously minimize both error and time, so higher efficacy is more desirable. This gives us a way to compare algorithms with different computational characteristics, meaning that a flop contributes differently to the accuracy of the result.

1. Run SNES ex5 for a range of sizes, such as 256×256, 512×512, 1024×1024 and 2048×2048, on 64 processes and plot the logarithm of the efficacy, $-\log(e \cdot T)$, as a function of the runtime T (on a log scale) for the same solvers as the prior problem. Make sure to have problem sizes small enough that the efficacy starts to fall off.

2. Repeat the above analysis for SNES ex13. This means running with something like

   ```
   ./ex13 -dm_plex_simplex 0 -potential_petscspace_degree 1 -pc_type lu -pc_factor_mat_solver_type mumps
     -dm_plex_box_faces 256,256 -dm_distribute -error_view
   ```

3. Does the efficacy metric change your choice of the best solver from Part B?

Aniket R Vishwakarma

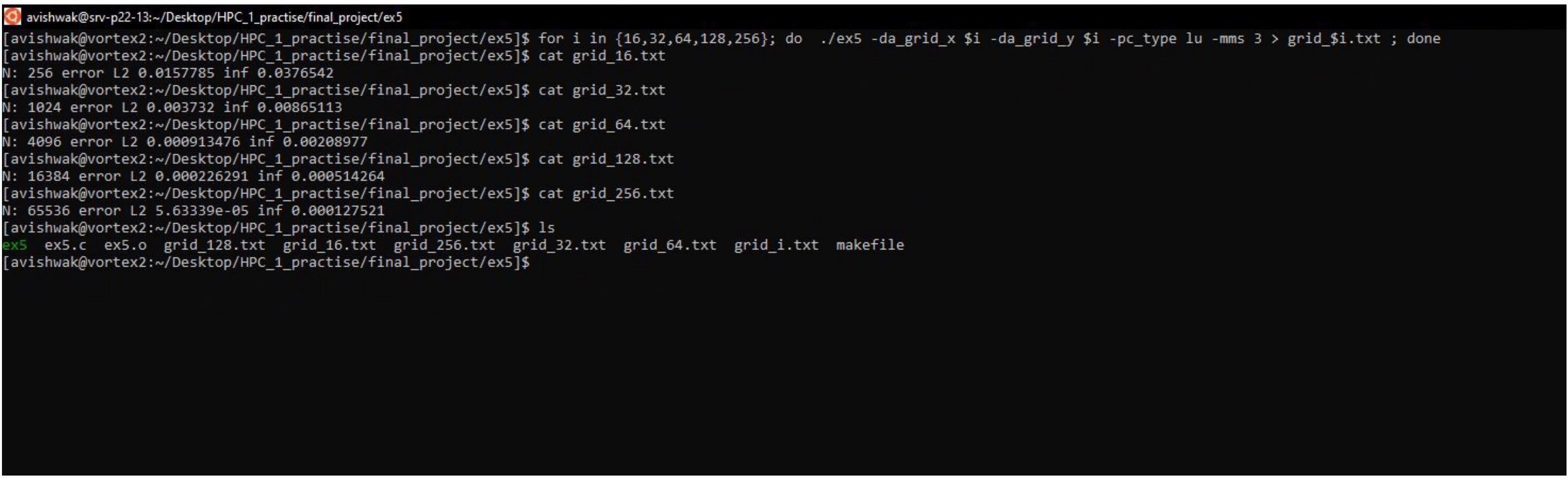
HPC Project

PART A

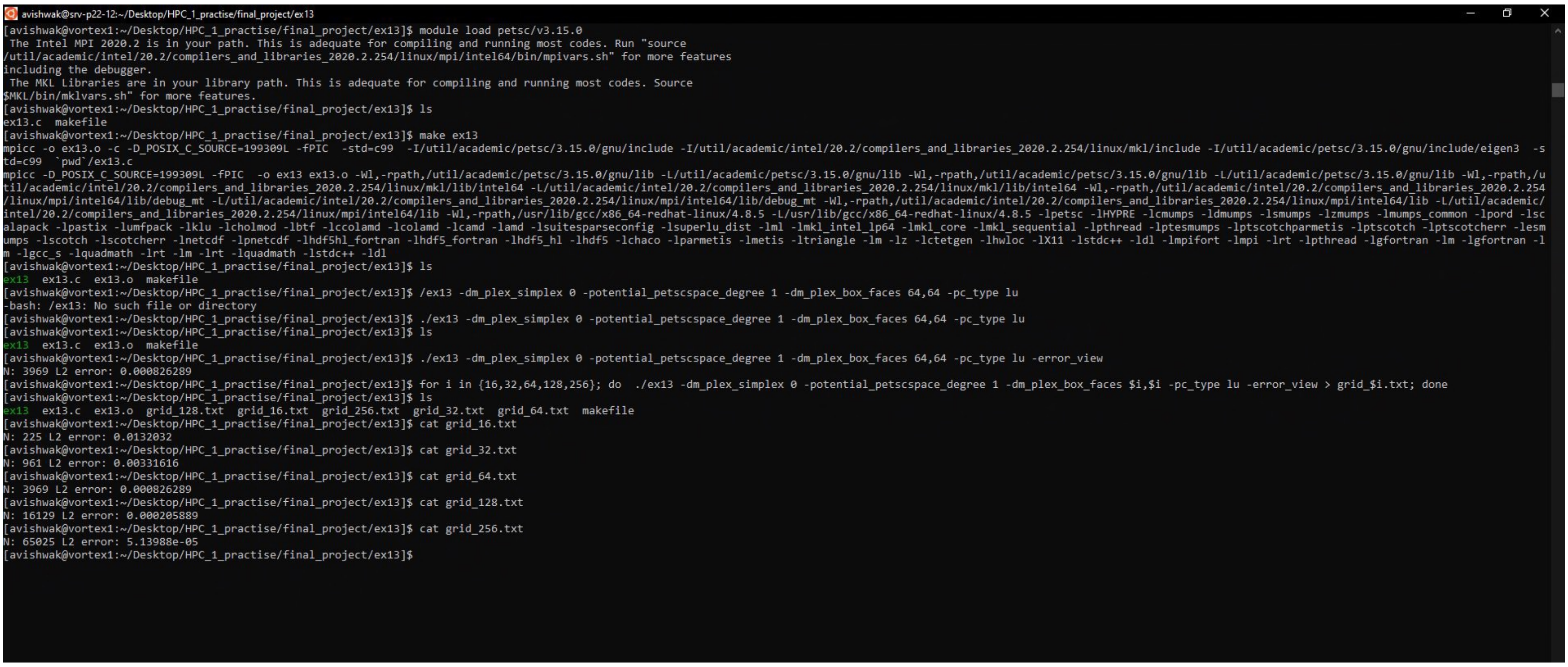
Step 1 : First Load petsc module on ccr head node using `module load petsc/v3.15.0
Step 2 : Then make file using make ex5 or make ex13
Step 3 : For ex5 varying grid size and run using :
`for i in {16,32,64,128,256}; do ./ex5 -da_grid_x $i -da_grid_y $i -pc_type lu -mms 3 > grid_$i.txt`

For ex13 use:
`for i in {16,32,64,128,256}; do ./ex13 -dm_plex_simplex 0 -potential_petscspace_degree 1 -dm_plex_box_faces $i,$i -pc_type lu -error_view > grid_$i.txt; done`

This would generate 5 text files containing results for their respective grid size in thier respective folders.



```
avishwak@srv-p22-13:~/Desktop/HPC_1_practise/final_project/ex5
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/ex5]$ for i in {16,32,64,128,256}; do  ./ex5 -da_grid_x $i -da_grid_y $i -pc_type lu -mms 3 > grid_$i.txt ; done
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/ex5]$ cat grid_16.txt
N: 256 error L2 0.0157785 inf 0.0376542
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/ex5]$ cat grid_32.txt
N: 1024 error L2 0.003732 inf 0.00865113
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/ex5]$ cat grid_64.txt
N: 4096 error L2 0.000913476 inf 0.00208977
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/ex5]$ cat grid_128.txt
N: 16384 error L2 0.000226291 inf 0.000514264
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/ex5]$ cat grid_256.txt
N: 65536 error L2 5.63339e-05 inf 0.000127521
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/ex5]$ ls
ex5  ex5.c  ex5.o  grid_128.txt  grid_16.txt  grid_256.txt  grid_32.txt  grid_64.txt  grid_i.txt  makefile
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/ex5]$
```



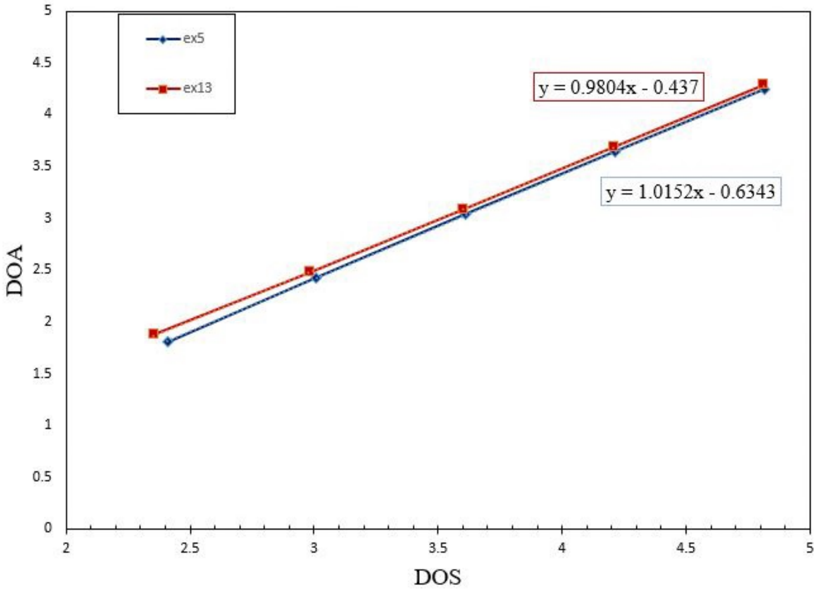
```
avishwak@srv-p22-12:~/Desktop/HPC_1_practise/final_project/ex13
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ module load petsc/v3.15.0
 The Intel MPI 2020.2 is in your path. This is adequate for compiling and running most codes. Run "source
/util/academic/intel/20.2/compilers_and_libraries_2020.2.254/linux/mpi/intel64/bin/mpivars.sh" for more features
including the debugger.
 The MKL Libraries are in your library path. This is adequate for compiling and running most codes. Source
$MKL/bin/mklvars.sh" for more features.
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ ls
ex13.c  makefile
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ make ex13
mpicc -o ex13.o -c -D_POSIX_C_SOURCE=199309L -fPIC  -std=c99  -I/util/academic/petsc/3.15.0/gnu/include -I/util/academic/intel/20.2/compilers_and_libraries_2020.2.254/linux/mkl/include -I/util/academic/petsc/3.15.0/gnu/include/eigen3  -s
td=c99  `pwd`/ex13.c
mpicc -D_POSIX_C_SOURCE=199309L -fPIC  -o ex13 ex13.o -Wl,-rpath,/util/academic/petsc/3.15.0/gnu/lib -L/util/academic/petsc/3.15.0/gnu/lib -Wl,-rpath,/util/academic/petsc/3.15.0/gnu/lib -L/util/academic/petsc/3.15.0/gnu/lib -Wl,-rpath,/u
til/academic/intel/20.2/compilers_and_libraries_2020.2.254/linux/mkl/lib/intel64 -L/util/academic/intel/20.2/compilers_and_libraries_2020.2.254/linux/mkl/lib/intel64 -Wl,-rpath,/util/academic/intel/20.2/compilers_and_libraries_2020.2.254
/linux/mpi/intel64/lib/debug_mt -L/util/academic/intel/20.2/compilers_and_libraries_2020.2.254/linux/mpi/intel64/lib/debug_mt -Wl,-rpath,/util/academic/intel/20.2/compilers_and_libraries_2020.2.254/linux/mpi/intel64/lib -L/util/academic/
intel/20.2/compilers_and_libraries_2020.2.254/linux/mpi/intel64/lib -Wl,-rpath,/usr/lib/gcc/x86_64-redhat-linux/4.8.5 -L/usr/lib/gcc/x86_64-redhat-linux/4.8.5 -lpetsc -lHYPRE -lcmumps -ldmumps -lsmumps -lzmumps -lmumps_common -lpord -lsc
alapack -lpastix -lumfpack -lklu -lcholmod -lbtf -lccolamd -lcolamd -lcamd -lamd -lsuitesparseconfig -lsuperlu_dist -lml -lmkl_intel_lp64 -lmkl_core -lmkl_sequential -lpthread -lptesmumps -lptscotchparmetis -lptscotch -lptscotcherr -lesm
umps -lscotch -lscotcherr -lnetcdf -lpnetcdf -lhdf5hl_fortran -lhdf5_fortran -lhdf5_hl -lhdf5 -lchaco -lparmetis -lmetis -ltriangle -lm -lz -lctetgen -lhwloc -lX11 -lstdc++ -ldl -lmpifort -lmpi -lrt -lpthread -lgfortran -lm -lgfortran -l
m -lgcc_s -lquadmath -lrt -lm -lrt -lquadmath -lstdc++ -ldl
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ ls
ex13  ex13.c  ex13.o  makefile
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ /ex13 -dm_plex_simplex 0 -potential_petscspace_degree 1 -dm_plex_box_faces 64,64 -pc_type lu
-bash: /ex13: No such file or directory
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ ./ex13 -dm_plex_simplex 0 -potential_petscspace_degree 1 -dm_plex_box_faces 64,64 -pc_type lu
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ ls
ex13  ex13.c  ex13.o  makefile
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ ./ex13 -dm_plex_simplex 0 -potential_petscspace_degree 1 -dm_plex_box_faces 64,64 -pc_type lu -error_view
N: 3969 L2 error: 0.000826289
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ for i in {16,32,64,128,256}; do  ./ex13 -dm_plex_simplex 0 -potential_petscspace_degree 1 -dm_plex_box_faces $i,$i -pc_type lu -error_view > grid_$i.txt; done
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ ls
ex13  ex13.c  ex13.o  grid_128.txt  grid_16.txt  grid_256.txt  grid_32.txt  grid_64.txt  makefile
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ cat grid_16.txt
N: 225 L2 error: 0.0132032
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ cat grid_32.txt
N: 961 L2 error: 0.00331616
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ cat grid_64.txt
N: 3969 L2 error: 0.000826289
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ cat grid_128.txt
N: 16129 L2 error: 0.000205889
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$ cat grid_256.txt
N: 65025 L2 error: 5.13988e-05
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/ex13]$
```

Q1 : ex5

grid size	Problem size	Digit_of_Size(DOS)	L2_error norm	Digit_of_accuracy(DOA)
16x16	256	2.408239965	0.0157785	1.801934286
32x32	1024	3.010299957	0.003732	2.428058365
64x64	4096	3.612359948	0.000913476	3.039302858
128x128	16384	4.214419939	0.000226291	3.645332718
256x256	65536	4.816479931	5.63E-05	4.249230181

Q2 : ex13

grid size	Problem size	Digit_of_Size(DOS)	L2_error norm	Digit_of_accuracy(DOA)
16x16	225	2.352182518	0.0132032	1.879320798
32x32	961	2.982723388	0.00331616	2.479364524
64x64	3969	3.598681099	0.000826289	3.082868029
128x128	16129	4.207607442	0.000205889	3.686366856
256x256	65025	4.813080361	5.14E-05	4.28904702



Above are the results obtained from calculation.
As seen in the graph on the right hand side, we can see the results forming 2 lines along with their respective equations in its `slope-intercept` form.
In this plot ordinate represents the ` -log(e) i.e. DoA` whereas abscissa represents `log N`, where N is the problem size(i.e. grid size).
Here, slope of the line represents *convergence rate.*
Hence **slope (i.e. convergence rate) of ex13 is 0.9804 and for ex5 is 1.0152**

PART B

For this part we solve the similar example ex5 and ex13 using 3 different solvers namely,

1. spare direct factorization using LU (MUMPS or SuperLU_dist) :
   ./ex5 -pc_type -pc_factor_mat_solver_type mumps
2. Block-Jacobi with ILU(0) :
   ./ex5
3. Algebraic Multigrid :
   ./ex5 -pc gamg

For range of grid sizes such as 256x256,512x512, 1024x1024, also we will need runtime infomation which can be obtained using flag -log_view

Refer read me file in respective directory for further instructions

Obtained results for ex5 and ex15 are as follows,

```
avishwak@srv-p22-12:~/Desktop/HPC_1_practise/final_project/partB/ex5/3_Algebraic_multigrid
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/1_MUMPS]$ clear
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/1_MUMPS]$ ls
Poisson_ex5_-10549001.err  Poisson_ex5_-10549001.out  ex5  ex5.c  ex5.o  grid_1024.txt  grid_2048.txt  grid_256.txt  grid_512.txt  makefile  qPoisson.sh
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/1_MUMPS]$ cat Poisson_ex5_-10549001.out
DONE !!!!
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/1_MUMPS]$ awk 'NR==11{print $0}'  grid_256.txt
Time (sec):           1.216e+00     1.000   1.216e+00
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/1_MUMPS]$ awk 'NR==11{print $0}'  grid_512.txt
Time (sec):           3.648e+00     1.000   3.648e+00
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/1_MUMPS]$ awk 'NR==11{print $0}'  grid_1024.txt
Time (sec):           1.558e+01     1.000   1.558e+01
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/1_MUMPS]$ awk 'NR==11{print $0}'  grid_2048.txt
Time (sec):           6.882e+01     1.000   6.882e+01
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/1_MUMPS]$ cd ../2_Block_jacobi/
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/2_Block_jacobi]$ ls
Poisson_ex5_-10549085.err  Poisson_ex5_-10549085.out  ex5  ex5.c  ex5.o  grid_1024.txt  grid_2048.txt  grid_256.txt  grid_512.txt  makefile  qPoisson.sh
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/2_Block_jacobi]$ pwd
/user/avishwak/Desktop/HPC_1_practise/final_project/partB/ex5/2_Block_jacobi
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/2_Block_jacobi]$ awk 'NR==11{print $0}'  grid_256.txt
Time (sec):           6.282e-01     1.001   6.279e-01
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/2_Block_jacobi]$ awk 'NR==11{print $0}'  grid_512.txt
Time (sec):           5.141e+00     1.000   5.141e+00
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/2_Block_jacobi]$ awk 'NR==11{print $0}'  grid_1024.txt
Time (sec):           2.232e+01     1.000   2.232e+01
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/2_Block_jacobi]$ awk 'NR==11{print $0}'  grid_2048.txt
Time (sec):           8.472e+01     1.000   8.472e+01
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/2_Block_jacobi]$ cd ../3_Algebraic_multigrid/
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/3_Algebraic_multigrid]$ ls
Poisson_ex5_-10549083.err  Poisson_ex5_-10549083.out  ex5  ex5.c  ex5.o  grid_1024.txt  grid_2048.txt  grid_256.txt  grid_512.txt  makefile  qPoisson.sh
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/3_Algebraic_multigrid]$ pwd
/user/avishwak/Desktop/HPC_1_practise/final_project/partB/ex5/3_Algebraic_multigrid
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/3_Algebraic_multigrid]$ awk 'NR==11{print $0}'  grid_256.txt
Time (sec):           3.868e-01     1.000   3.867e-01
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/3_Algebraic_multigrid]$ awk 'NR==11{print $0}'  grid_512.txt
Time (sec):           4.842e-01     1.000   4.842e-01
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/3_Algebraic_multigrid]$ awk 'NR==11{print $0}'  grid_1024.txt
Time (sec):           1.245e+00     1.000   1.245e+00
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/3_Algebraic_multigrid]$ awk 'NR==11{print $0}'  grid_2048.txt
Time (sec):           4.255e+00     1.000   4.255e+00
[avishwak@vortex1:~/Desktop/HPC_1_practise/final_project/partB/ex5/3_Algebraic_multigrid]$
```

Fig : Part B - ex5 results

MUMPS or SupeLU_dist			
grid size	Problem size(N)	Runtime (T)	N/T
128x128	16384	0.5053	32424.30239
256x256	65536	1.216	53894.73684
512x512	262144	3.648	71859.64912
1024x1024	1048576	15.58	67302.69576
2048x2048	4194304	68.82	60946.00407

Block-Jacobi with ILU(0)			
grid size	Problem size(N)	Runtime (T)	N/T
128x128	16384	0.2482	66011.28122
256x256	65536	0.6282	104323.4639
512x512	262144	5.141	50990.85781
1024x1024	1048576	22.32	46979.21147
2048x2048	4194304	84.72	49507.83758

Algebraic Multigrid			
grid size	Problem size(N)	Runtime (T)	N/T
128x128	16384	0.3759	43586.06012
256x256	65536	0.3868	169431.2306
512x512	262144	0.4842	541396.1173
1024x1024	1048576	1.245	842229.7189
2048x2048	4194304	4.255	985735.3702

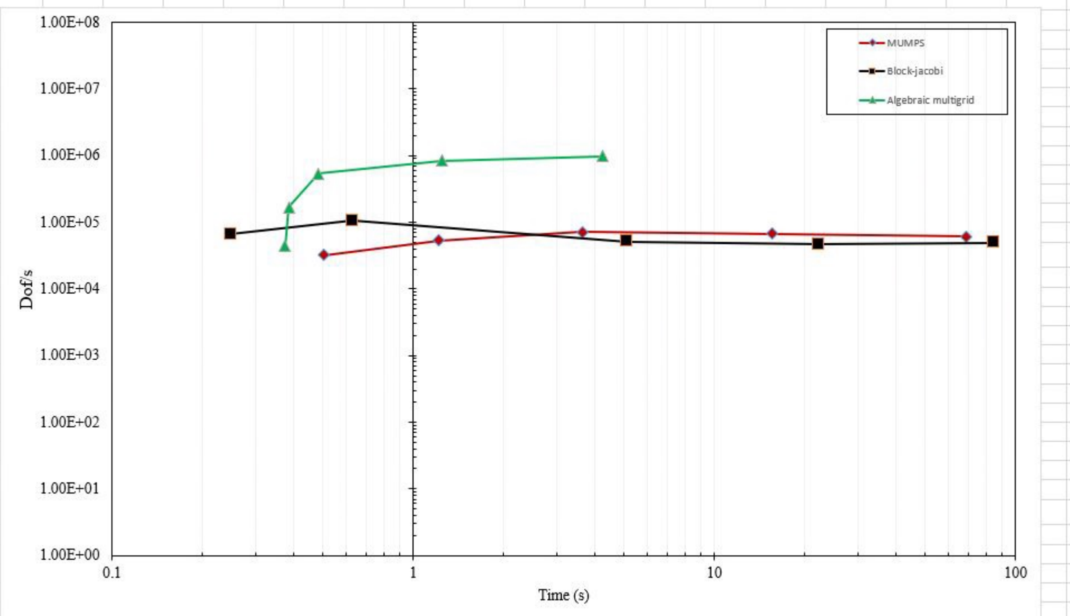


Fig: Part B ex5 graph

```
avishwak@srv-p22-13:~/Desktop/HPC_1_practise/final_project/partB/ex13/3_Algebraic_multigrid
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/1_MUMPS]$ pwd
/user/avishwak/Desktop/HPC_1_practise/final_project/partB/ex13/1_MUMPS
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/1_MUMPS]$ ls
Poisson_ex5_-10550337.err  Poisson_ex5_-10550337.out  ex13  ex13.c  ex13.o  grid_1024.txt  grid_128.txt  grid_2048.txt  grid_256.txt  grid_512.txt  makefile  qPoisson.sh
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/1_MUMPS]$ awk 'NR==11{print $0}' grid_128.txt
Time (sec):           1.200e+00     1.000   1.200e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/1_MUMPS]$ awk 'NR==11{print $0}' grid_256.txt
Time (sec):           4.227e+00     1.000   4.226e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/1_MUMPS]$ awk 'NR==11{print $0}' grid_512.txt
Time (sec):           1.751e+01     1.000   1.751e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/1_MUMPS]$ awk 'NR==11{print $0}' grid_1024.txt
Time (sec):           7.620e+01     1.000   7.620e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/1_MUMPS]$ awk 'NR==11{print $0}' grid_2048.txt
Time (sec):           3.861e+02     1.000   3.861e+02
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/1_MUMPS]$ cd ../2_Block_jacobi/
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/2_Block_jacobi]$ ls
Poisson_ex5_-10550333.err  Poisson_ex5_-10550333.out  ex13  ex13.c  ex13.o  grid_1024.txt  grid_128.txt  grid_2048.txt  grid_256.txt  grid_512.txt  makefile  qPoisson.sh
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/2_Block_jacobi]$ awk 'NR==11{print $0}' grid_128.txt
Time (sec):           1.077e+00     1.000   1.077e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/2_Block_jacobi]$ awk 'NR==11{print $0}' grid_256.txt
Time (sec):           3.579e+00     1.000   3.579e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/2_Block_jacobi]$ awk 'NR==11{print $0}' grid_512.txt
Time (sec):           1.414e+01     1.000   1.414e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/2_Block_jacobi]$ awk 'NR==11{print $0}' grid_1024.txt
Time (sec):           6.421e+01     1.000   6.421e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/2_Block_jacobi]$ awk 'NR==11{print $0}' grid_2048.txt
Time (sec):           3.794e+02     1.000   3.794e+02
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/2_Block_jacobi]$ cd ../3_Algebraic_multigrid/
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/3_Algebraic_multigrid]$ ls
Poisson_ex5_-10550330.err  Poisson_ex5_-10550330.out  ex13  ex13.c  ex13.o  grid_1024.txt  grid_128.txt  grid_2048.txt  grid_256.txt  grid_512.txt  makefile  qPoisson.sh
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/3_Algebraic_multigrid]$ awk 'NR==11{print $0}' grid_128.txt
Time (sec):           1.046e+00     1.000   1.045e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/3_Algebraic_multigrid]$ awk 'NR==11{print $0}' grid_256.txt
Time (sec):           3.465e+00     1.000   3.464e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/3_Algebraic_multigrid]$ awk 'NR==11{print $0}' grid_512.txt
Time (sec):           1.354e+01     1.000   1.354e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/3_Algebraic_multigrid]$ awk 'NR==11{print $0}' grid_1024.txt
Time (sec):           5.763e+01     1.000   5.763e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/3_Algebraic_multigrid]$ awk 'NR==11{print $0}' grid_2048.txt
Time (sec):           2.970e+02     1.000   2.970e+02
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partB/ex13/3_Algebraic_multigrid]$
```

Fig : Part B - ex13 results

MUMPS or SupeLU_dist

grid size	Problem size(N)	Runtime (T)	N/T
128x128	16384	1.2	13653.33333
256x256	65536	4.227	15504.14005
512x512	262144	17.51	14971.10223
1024x1024	1048576	76.2	13760.8399
2048x2048	4194304	386.1	10863.25822

Block-Jacobi with ILU(0)

grid size	Problem size(N)	Runtime (T)	N/T
128x128	16384	1.077	15212.62767
256x256	65536	3.579	18311.26013
512x512	262144	14.14	18539.17963
1024x1024	1048576	64.21	16330.41582
2048x2048	4194304	379.4	11055.09752

Algebraic Multigrid

grid size	Problem size(N)	Runtime (T)	N/T
128x128	16384	1.046	15663.47992
256x256	65536	3.465	18913.70851
512x512	262144	13.54	19360.70901
1024x1024	1048576	57.63	18194.9679
2048x2048	4194304	297	14122.23569

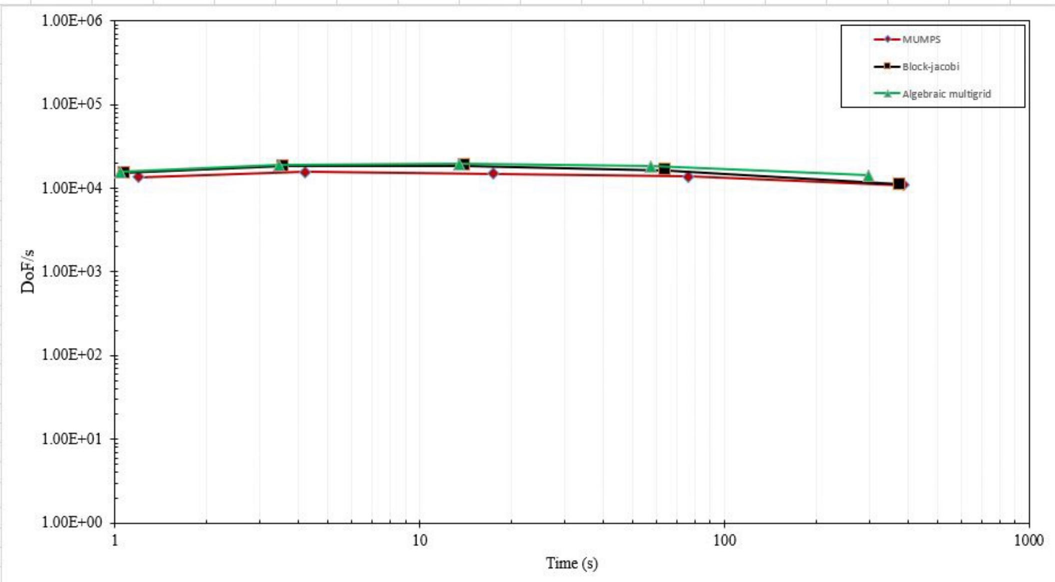


Fig : Part B ex13 graph

Q3.

For ex5: grid size 128x128 is with solver `Block-Jacobi` has the minimum runtime of 0.2482

For ex13: grid size 128x128 is with solver `Algebraic Multigrid` has the minimum runtime of 1.046

Q4.

Observation for ex5:

1. All the solvers in ex5, more or less indicate a flat line to the right side of the graph which means that its in a `optimal scaling range`.
2. Observe an increse in graph to the left side of the graph, which represents strong-scaling.
3. If we calculate and are going beyond the plotted data, possibly it would result in decrease Dof/s due to incorporation of memory-effects.

Observation for ex13:

1. All the solvers indicates almost flat-line trend indicating they are within the optimal range.

Q5.

for ex5: `Algebraic multigrid` is performs the best because it of it higher Dof/s value (i.e. higher degree of freedom solved per time, N/T)

For ex13: Although there is a slight performance difference, again `Algebraic multigrid` performs best,

PART C

Similar to part B, same we have to submit job to slurm in similar process, but here we result in terms of runtime as well as in error hence we use flag -error_view as well as -log_view ; refer read me file for more details

```
avishwak@srv-p22-13:~/Desktop/HPC_1_practise/final_project/partC/ex5/3_Algebraic_multigrid
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/1_MUMPS]$ ls
Poisson_ex5_-10551381.err  Poisson_ex5_-10551381.out  ex5  ex5.c  ex5.o  grid_1024.txt  grid_2048.txt  grid_256.txt  grid_512.txt  makefile  qPoisson.sh
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/1_MUMPS]$ pwd
/user/avishwak/Desktop/HPC_1_practise/final_project/partC/ex5/1_MUMPS
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/1_MUMPS]$ awk 'NR==1{print$0} NR==12{print $0}' grid_256.txt
N: 65536 error L2 5.63339e-05 inf 0.000127521
Time (sec):           9.532e-01     1.000   9.530e-01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/1_MUMPS]$ awk 'NR==1{print$0} NR==12{print $0}' grid_512.txt
N: 262144 error L2 1.40548e-05 inf 3.1753e-05
Time (sec):           3.764e+00     1.000   3.764e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/1_MUMPS]$ awk 'NR==1{print$0} NR==12{print $0}' grid_1024.txt
N: 1048576 error L2 3.5102e-06 inf 7.92267e-06
Time (sec):           1.622e+01     1.000   1.622e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/1_MUMPS]$ awk 'NR==1{print$0} NR==12{print $0}' grid_2048.txt
N: 4194304 error L2 8.77101e-07 inf 1.9787e-06
Time (sec):           7.122e+01     1.000   7.122e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/1_MUMPS]$ cd ../2_Block_jacobi/
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/2_Block_jacobi]$ ls
Poisson_ex5_-10551380.err  Poisson_ex5_-10551380.out  ex5  ex5.c  ex5.o  grid_1024.txt  grid_2048.txt  grid_256.txt  grid_512.txt  makefile  qPoisson.sh
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/2_Block_jacobi]$ awk 'NR==1{print$0} NR==12{print $0}' grid_256.txt
N: 65536 error L2 5.63338e-05 inf 0.000127521
Time (sec):           7.757e-01     1.000   7.755e-01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/2_Block_jacobi]$ awk 'NR==1{print$0} NR==12{print $0}' grid_512.txt
N: 262144 error L2 1.40548e-05 inf 3.17529e-05
Time (sec):           7.539e+00     1.000   7.538e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/2_Block_jacobi]$ awk 'NR==1{print$0} NR==12{print $0}' grid_1024.txt
N: 1048576 error L2 3.51015e-06 inf 7.92258e-06
Time (sec):           4.463e+01     1.000   4.463e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/2_Block_jacobi]$ awk 'NR==1{print$0} NR==12{print $0}' grid_2048.txt
N: 4194304 error L2 0.493371 inf 1.37351
Time (sec):           8.901e+01     1.000   8.901e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/2_Block_jacobi]$ cd ../3_Algebraic_multigrid/
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/3_Algebraic_multigrid]$ ls
Poisson_ex5_-10551294.err  Poisson_ex5_-10551294.out  Poisson_ex5_-10551379.err  Poisson_ex5_-10551379.out  ex5  ex5.c  ex5.o  grid_1024.txt  grid_2048.txt  grid_256.txt  grid_512.txt  makefile  qPoisson.sh
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/3_Algebraic_multigrid]$ awk 'NR==1{print$0} NR==12{print $0}' grid_256.txt
N: 65536 error L2 5.63339e-05 inf 0.000127521
Time (sec):           2.792e-01     1.001   2.790e-01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/3_Algebraic_multigrid]$ awk 'NR==1{print$0} NR==12{print $0}' grid_512.txt
N: 262144 error L2 1.40548e-05 inf 3.17529e-05
Time (sec):           4.756e-01     1.000   4.755e-01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/3_Algebraic_multigrid]$ awk 'NR==1{print$0} NR==12{print $0}' grid_1024.txt
N: 1048576 error L2 3.51044e-06 inf 7.92314e-06
Time (sec):           1.257e+00     1.000   1.257e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/3_Algebraic_multigrid]$ awk 'NR==1{print$0} NR==12{print $0}' grid_2048.txt
N: 4194304 error L2 8.77121e-07 inf 1.97873e-06
Time (sec):           5.436e+00     1.000   5.436e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex5/3_Algebraic_multigrid]$
```

Fig : Part C - ex5 results

ex5

MUMPS				
grid_size	problem size	L2 norm error	runtime	Digit of efficacy(DoE)
128x128	16384	0.000226291	0.4533	3.988946999
256x256	65536	5.63E-05	0.9532	4.270046148
512x512	262144	1.41E-05	3.764	4.276525715
1024x1024	1048576	3.51E-06	16.22	4.244617288
2048x2048	4194304	8.77E-07	71.22	4.204348425

Block jacobi					
grid_size	problem size	L2 norm error	runtime	Digit of efficacy(DoE)	
128x128	16384	0.000226291	0.2333	4.27741798	4.37E-05
256x256	65536	5.63E-05	0.7757	4.359537161	1.06E-04
512x512	262144	1.41E-05	7.539	3.974861587	1.57E-04
1024x1024	1048576	3.51E-06	44.63	3.805047437	4.39E+01
2048x2048	4194304	4.93E-01	89.01	-1.642612419	

Algebraic multigrid				
grid_size	problem size	L2 norm error	runtime	Digit of efficacy(DoE)
128x128	16384	0.000226291	0.2932	4.178168752
256x256	65536	5.63E-05	0.2792	4.803314768
512x512	262144	1.41E-05	0.4756	5.174933484
1024x1024	1048576	3.51E-06	1.257	5.355303168
2048x2048	4194304	8.77E-07	5.436	5.321661043

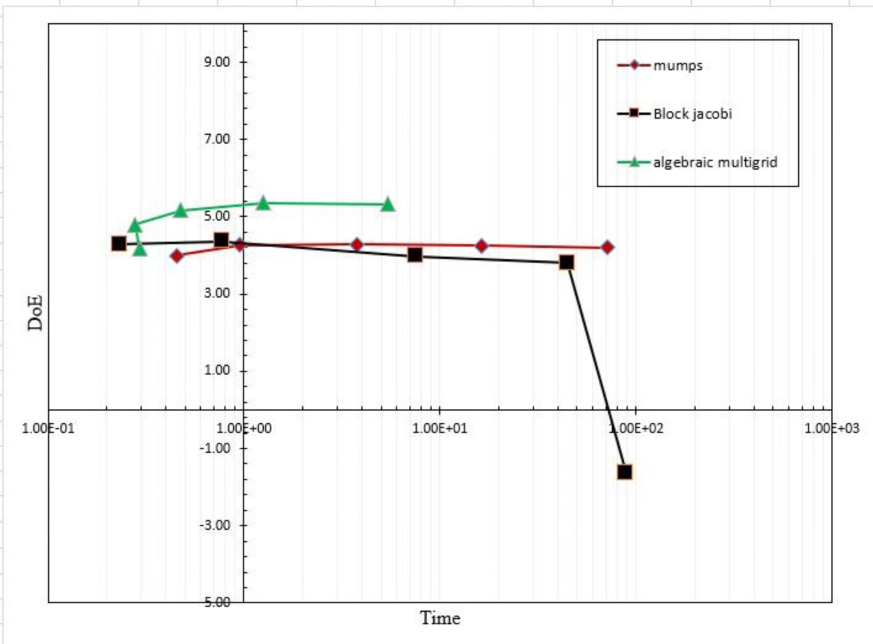


Fig : Part C - ex5 graph

```
avishwak@srv-p22-13:~/Desktop/HPC_1_practise/final_project/partC/ex13/3_Algebraic_multigrid
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/1_MUMPS]$ ls
Poisson_ex5_-10551300.err  Poisson_ex5_-10551300.out  ex13  ex13.c  ex13.o  grid_1024.txt  grid_128.txt  grid_2048.txt  grid_256.txt  grid_512.txt  makefile  qPoisson.sh
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/1_MUMPS]$ awk 'NR==1{print$0} NR==12{print $0}' grid_128.txt
N: 16129 L2 error: 0.000205889
Time (sec):           1.152e+00     1.000   1.152e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/1_MUMPS]$ awk 'NR==1{print$0} NR==12{print $0}' grid_256.txt
N: 65025 L2 error: 5.13988e-05
Time (sec):           4.165e+00     1.000   4.165e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/1_MUMPS]$ awk 'NR==1{print$0} NR==12{print $0}' grid_512.txt
N: 261121 L2 error: 1.28286e-05
Time (sec):           1.692e+01     1.000   1.692e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/1_MUMPS]$ awk 'NR==1{print$0} NR==12{print $0}' grid_1024.txt
N: 1046529 L2 error: 3.20535e-06
Time (sec):           7.301e+01     1.000   7.301e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/1_MUMPS]$ awk 'NR==1{print$0} NR==12{print $0}' grid_2048.txt
N: 4190209 L2 error: 8.00724e-07
Time (sec):           3.661e+02     1.000   3.661e+02
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/1_MUMPS]$ cd ../2_Block_jacobi/
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/2_Block_jacobi]$ ls
Poisson_ex5_-10551301.err  Poisson_ex5_-10551301.out  ex13  ex13.c  ex13.o  grid_1024.txt  grid_128.txt  grid_2048.txt  grid_256.txt  grid_512.txt  makefile  qPoisson.sh
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/2_Block_jacobi]$ awk 'NR==1{print$0} NR==12{print $0}' grid_128.txt
N: 16129 L2 error: 0.000205888
Time (sec):           1.028e+00     1.000   1.028e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/2_Block_jacobi]$ awk 'NR==1{print$0} NR==12{print $0}' grid_256.txt
N: 65025 L2 error: 5.13989e-05
Time (sec):           3.541e+00     1.000   3.541e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/2_Block_jacobi]$ awk 'NR==1{print$0} NR==12{print $0}' grid_512.txt
N: 261121 L2 error: 1.28366e-05
Time (sec):           1.418e+01     1.000   1.418e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/2_Block_jacobi]$ awk 'NR==1{print$0} N[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/2_Block_jacobi]$ awk 'NR==1{print$0}


[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/2_Block_jacobi]$ awk 'NR==1{print$0} NR==12{print $0}' grid_1024.txt
N: 1046529 L2 error: 3.2097e-06
Time (sec):           6.461e+01     1.000   6.461e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/2_Block_jacobi]$ awk 'NR==1{print$0} NR==12{print $0}' grid_2048.txt
N: 4190209 L2 error: 8.22512e-07
Time (sec):           3.974e+02     1.000   3.974e+02
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/2_Block_jacobi]$ cd ../3_Algebraic_multigrid/
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/3_Algebraic_multigrid]$ ls
Poisson_ex5_-10551305.err  Poisson_ex5_-10551305.out  ex13  ex13.c  ex13.o  grid_1024.txt  grid_128.txt  grid_2048.txt  grid_256.txt  grid_512.txt  makefile  qPoisson.sh
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/3_Algebraic_multigrid]$ awk 'NR==1{print$0} NR==12{print $0}' grid_128.txt
N: 16129 L2 error: 0.000205889
Time (sec):           1.049e+00     1.000   1.049e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/3_Algebraic_multigrid]$ awk 'NR==1{print$0} NR==12{print $0}' grid_256.txt
N: 65025 L2 error: 5.13987e-05
Time (sec):           3.580e+00     1.000   3.580e+00
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/3_Algebraic_multigrid]$ awk 'NR==1{print$0} NR==12{print $0}' grid_512.txt
N: 261121 L2 error: 1.28286e-05
Time (sec):           1.412e+01     1.000   1.412e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/3_Algebraic_multigrid]$ awk 'NR==1{print$0} NR==12{print $0}' grid_1024.txt
N: 1046529 L2 error: 3.20528e-06
Time (sec):           6.476e+01     1.000   6.476e+01
[avishwak@vortex2:~/Desktop/HPC_1_practise/final_project/partC/ex13/3_Algebraic_multigrid]$ awk 'NR==1{print$0} NR==12{print $0}' grid_2048.txt
N: 4190209 L2 error: 8.00732e-07
Time (sec):           3.163e+02     1.000   3.163e+02
```

Fig : Part C - ex13 results

ex13

MUMPS				
grid_size	problem size	L2 norm error	runtime	Digit of efficacy(DoE)
128x128	16129	2.06E-04	1.152	3.624914377
256x256	65025	5.14E-05	4.165	3.669432015
512x512	161121	1.28E-05	16.92	3.663420377
1024x1024	1046529	3.21E-06	73.01	3.630742193
2048x2048	4190209	8.01E-07	366.1	3.532917425

Block jacobi				
grid_size	problem size	L2 norm error	runtime	Digit of efficacy(DoE)
128x128	16129	2.06E-04	1.028	3.67437585
256x256	65025	5.14E-05	3.541	3.739920249
512x512	161121	1.28E-05	14.18	3.739873761
1024x1024	1046529	3.21E-06	6.46E+01	3.683235817
2048x2048	4190209	8.23E-07	397.4	3.485629894

Algebraic multigrid				
grid_size	problem size	L2 norm error	runtime	Digit of efficacy(DoE)
128x128	16129	2.06E-04	1.049	3.665591368
256x256	65025	5.14E-05	3.58	3.735164839
512x512	161121	1.28E-05	14.12	3.741986039
1024x1024	1046529	3.21E-06	64.76	3.682827186
2048x2048	4190209	8.01E-07	316.3	3.596413623

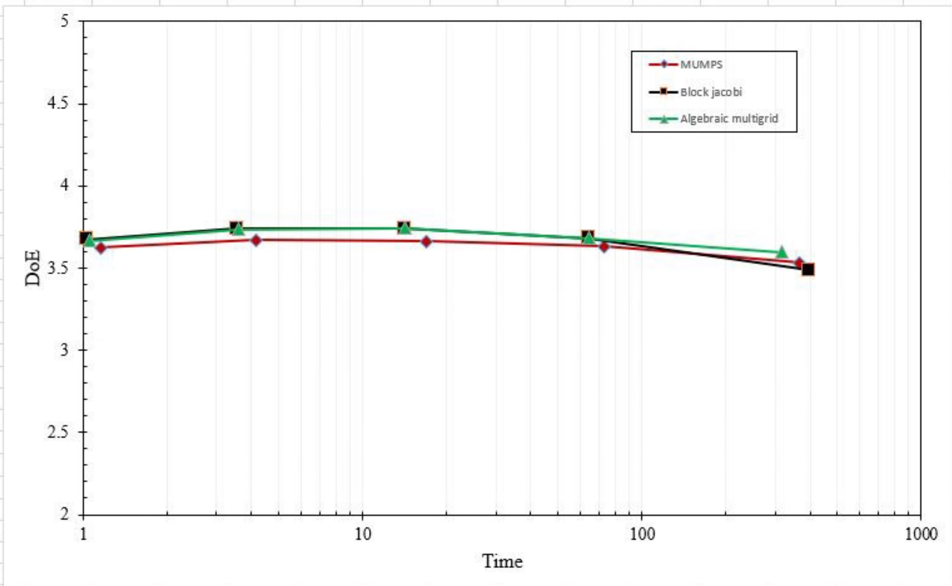


Fig : Part C - ex13 graph

Q3. On plotting efficacy metric w.r.t time again we can see that solver `Algebraic multigrid` performs best with a higher value or Digit of Efficacy(DoE), indicating lower L2 norm error, with lower runtime relative to other solvers for same grid size, thus being a proper choice. Hence, efficacy does not changes the choice of solver from part B