COM 2545 Order of Growth

Raziel Siegman ()

September 18, 2020

Question 1.

1) Allow $n_0 = 100$ and c=33.
For n>n_0, $n^2 \geq 17n + 1$.
Therefore, $0 \leq 32n^2 + 17n + 1 \leq 32n^2 + n^2 = 33n^2 = cx^2$.
Therefore, $32n^2 + 27n + 1$ is $O(n^2)$.

2) If $32n^2 + 17n + 1$ were $O(n)$, then there would be values c and n_0 where $0 \leq 32n^2 + 17n + 1 \leq cn$ for $n \geq n_0$.
This implies that $32n + 17 + \frac{1}{n} \leq c$, which is not true when $n \geq c$.

If $32n^2 + 17n + 1$ were $O(n \log n)$, then there would be values c and n_0 where $0 \leq 32n^2 + 17n + 1 \leq c(n \log n)$ for $n \geq n_0$. This implies that $\frac{32n^2+17n+1}{nlogn} \leq c$ and $\frac{32n+17+\frac{1}{n}}{logn} \leq c$. However, this is not true when n > c.

Question 2.

1) Allow $n_0 = 1$ and c=1.
$3n^2 + 17n + 1 \geq n^2 = cn^2$ for $n \geq n_0$ and is therefore $(\Omega(n^2))$.

Allow $n_0 = 1$ and c=1.
$3n^2 + 17n + 1 \geq n = cn$ for $n \geq n_0$ and is therefore $(\Omega(n))$.

2) If $32n^2+17n+1$ were $\Omega(n^3)$, then there would be values c and n_0 where $32n^2+17n+1 \geq cn^3 \geq 0$ for $n \geq n_0$.
This implies that $\frac{32}{n} + \frac{17}{n^2} + \frac{1}{n^3} \geq c$.
However, $\lim_{n\to+\infty}(\frac{32}{n} + \frac{17}{n^2} + \frac{1}{n^3}) = 0$, which implies that at as n approaches infinity, $c \leq 0$, which is impossible, as c must be a positive constant.

Question 3.

1) Allow $n_0 = 100$, $c_1 = 1$ and $c_2 = 33$.
For n>n_0, $n^2 \geq 17n + 1$.
Therefore, $0 \leq 32n^2 + 17n + 1 \leq 32n^2 + n^2 = 33n^2 = c_2x^2$.
Additionally, $3n^2 + 17n + 1 \geq n^2 = c_1n^2$.
Therefore, $0 \leq c_1n^2 \leq 3n^2 + 17n + 1 \leq c_2n^2$.
Therefore, $3n^2 + 17n + 1$ is $\Theta(n^2)$

2) If $32n^2 + 17n + 1$ were $\Theta(n)$, then there would be values c_1, c_2 and n_0 where $0 \leq c_1n \leq 32n^2 + 17n + 1 \leq c_2n$ for $n \geq n_0$.
This implies that $32n^2 + 17n + 1 \leq c_2n$, and $32n + 17 + \frac{1}{n} \leq c_2$, which is not true when $n \geq c_2$.

If $32n^2 + 17n + 1$ were $\Theta(n^3)$, then there would be values c_1, c_2 and n_0 where $0 \leq c_1n^3 \leq 32n^2 + 17n + 1 \leq c_2n^3$ for $n \geq n_0$.
This implies that $c_1n^3 \leq 32n^2 + 17n + 1$, and therefore $c_1 \leq \frac{32}{n} + \frac{17}{n^2} + \frac{1}{n^3}$.
However, $\lim_{n\to+\infty}(\frac{32}{n} + \frac{17}{n^2} + \frac{1}{n^3}) = 0$, which implies that at as n approaches infinity, $c \leq 0$, which is impossible, as c must be a positive constant.