COM 2545 Drill Comparing Orders of Growth 1

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Question 1.

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1) n^2
$\lim_{n\to+\infty}\frac{(2n)^2}{n^2}=4$
$\lim_{n\to+\infty}\frac{(n+1)^2}{n^2}=1$
Therefore, when the input size is doubled, it is slower by a factor of 4, and when the input size is increased by 1, it is slower by a factor of 1.

2) n^3
$\lim_{n\to+\infty}\frac{(2n)^3}{n^3}=8$
$\lim_{n\to+\infty}\frac{(n+1)^3}{n^3}=1$
Therefore, when the input size is doubled, it is slower by a factor of 8, and when the input size is increased by 1, it is slower by a factor of 1.

3) $100n^2$
$\lim_{n\to+\infty}\frac{(100)(2n)^2}{100n^2}=4$
$\lim_{n\to+\infty}\frac{(100)(n+1)^2}{100n^2}=1$
Therefore, when the input size is doubled, it is slower by a factor of 4, and when the input size is increased by 1, it is slower by a factor of 1.

4) $n\log n$
$\lim_{n\to+\infty}\frac{(2n)\log(2n)}{n\log n}=\lim_{n\to+\infty}\frac{(2n)(2)\frac{1}{2n}+2\log(2n)}{n(\frac{1}{n})+\log n}=\lim_{n\to+\infty}\frac{2+2\log(2n)}{1+\log n}=\lim_{n\to+\infty}\frac{2(2)(\frac{1}{2n})}{\frac{1}{n}}=2$
$\lim_{n\to+\infty}\frac{(n+1)\log(n+1)}{n\log n}=\lim_{n\to+\infty}\frac{(n+1)(\frac{1}{n+1})+\log(n+1)}{n(\frac{1}{n})\log n}=\lim_{n\to+\infty}\frac{log(n+1)}{logn}=\lim_{n\to+\infty}\frac{\frac{1}{n+1}}{\frac{1}{n}}=1$
Therefore, when the input size is doubled, it is slower by a factor of 2, and when the input size is increased by 1, it is slower by a factor of 1.

5) 2^n
$\lim_{n\to+\infty}\frac{2^{2n}}{2^n}=\infty$
$\lim_{n\to+\infty}\frac{2^{n+1}}{2^n}=2$
Therefore, when the input size is doubled, it is slower by a factor of (up to) infinity, and when the input size is increased by 1, it is slower by a factor of 2.

Question 2.

The computer is able to perform $10^{10}(60)(60)=3.6E13$ operations per hour.

1) n^2
$\sqrt{3.6E13}=6,000,000$. Therefore,$n=6,000,000$.

2) n^3
$\sqrt[3]{3.6E13}=33,019.3$. Therefore, $n=33,019$.

3) $100n^2$
$100n^2 = 3.6E13 \rightarrow n^2 = 3.6E11 \rightarrow \sqrt{3.6E11} = 600,000$. Therefore, $n = 600,000$.

4) $n \log n$
$n \log n = 3.6E13 \rightarrow$ Therefore, $n = 2,889,069,821,506$ (trial and error).

5) 2^n
$2^n = 3.6E13 \rightarrow \log_2 3.6E13 = 45.03$. Therefore, $n = 45$.

6)2^{2^n}
$2^{2^n} = 3.6E13 \rightarrow \log_2 3.6E13 = 2^n \rightarrow 45.03 = 2^n \rightarrow \log_2 45.03 = 5.49$. Therefore, $n = 5$.