

Arrays in Data structures

data types → int, float, char, double, string

int a = 5
data type name value

Memory - Long tape of bytes (8 bits)

For int - 4 bytes = $8 \times 4 \rightarrow 32$ bits

→ At one instance, we are able to store one value only.

→ Some data will stored in array

int a[60]; char b[10]; float c[5]

data example - [5, 10, 0, -2, 11]

['j', 'e', 'i', 'o', 'u']

[5, 'j', 'k', 2, 3] X (Not valid)

→ For arrays in run time we can't exceed the size.

int a[5] = {6, 2, 4, 3, 0}

Address formulae $a[0] = 6$

$a[1] = 2$

$a[2] = 4$

$a[3] = 3$

$a[4] = 0$

Drawback of Array:

waste of memory occupation.

char b[10] = {'s', 'a', 't', 'i', 's', 'h'};

s	a	t	i	s	h	\0	\0	\0	\0
---	---	---	---	---	---	----	----	----	----

For displaying:

```
int b[10];
```

```
printf("Enter the elements - ");
```

```
for (i=0; i<9; i++)
```

```
scanf("%d", &b);
```

→ Array is a fixed size sequential collection of data which are in same type.

→ Access a Particular element = arrayname[index]

→ data items are stored in continuous locations

→ Random access taking constant time.

In arrays we can't insert more than size of its own.

Eg:- int a[3];

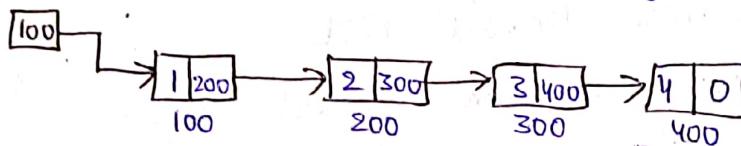
Above array there is no possibility to insert more than 3 elements.

Otherwise we assigned a[100]; and used for 5 elements only then remaining memory is wasted

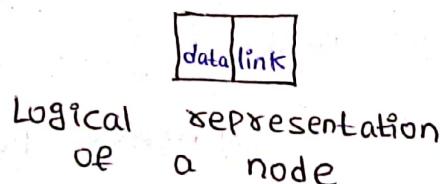
To reduce memory wastage linked list will be introduced.

Linked List

- linked list is a collection of different data types.
- data is not stored in consecutive locations. it may be stored anywhere in the memory.
- To find the address of a particular value we have to traverse to that node by using head pointer.
- int data type occupies 8 bytes.



- we have to store pointer also. For suppose we want to store extra data then memory manager can allocate space where 8 bytes are present.
- No need to specify the size.



```
struct node  
{  
    int data;  
    struct node *next; // creating pointer  
}
```

- here data type is struct node
- The node storing first node address is called as head/base, Using head we can point the whole nodes.

- Linked list is a linear data structure.
- Time complexity is order of n $O(n)$
- Extra space will be required to store address of next node.
- Binary search is not possible.
- It is present in dynamic size, memory allocation happens in run time.

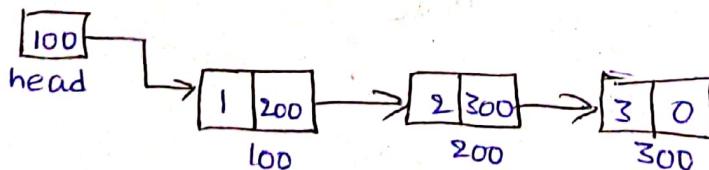
Linked list is of 4 types.

- single linked list
- double linked list
- circular linked list
- Doubly circular linked list.

Single Linked List

Single linked list is a type of linked list in which having link in one side with the next node.

Node = Data part + Address part.



→ head represents first node's Address.

struct node
{

int data;

struct node *next;

};

} → For SLL creation.

creation of SLL:

```
#include <stdio.h>
#include <stdlib.h>

struct node{
    int data;
    struct node *next;
};

void main()
{
    struct node *head, *temp, *n1;
    head=0;
    int choice;
    printf("Enter 1 to create SLL else enter 0-");
    scanf("%d", &choice);
    while(choice==1)
    {
        n1=(struct node *) malloc(sizeof(struct node));
        printf("Enter data-");
        scanf("%d", &n1->data);
        n1->next=0;
        // For creating first node
        if(head==0)
        {
            head=temp=n1;
        }
        else
        {
            temp->next=n1;
            temp=n1;
        }
        printf("Enter 1 to create another else 0-");
        scanf("%d", &choice);
    }
}
```



// in this area we do all operations -->

// for displaying

temp = head;

while (temp != 0)

{

printf ("%d ", temp->data);

temp = temp->next;

}

} // main closed.

In single linked there are many functions
They are:

Insertion - At beginning

At ending

At a specified Place (index)

Deletion - At beginning

At ending

At a specified Place (index)

Concatenation - Adding two linked lists

Searching - To search specified element.
finding its index position.

Insertion

→ Insertion is used to insert data in beginning, ending (or) at specified position.

→ we are now writing code for these
all -->

Insertion at beginning:

```
#include <stdio.h>
#include <stdlib.h>
```

```
struct node{
```

```
    int data;
```

```
    struct node *next;
```

```
};
```

```
void main()
```

```
{
```

```
    struct node *head, *temp, *n1, *begin;
    head=0;
```

```
    int choice;
```

```
    printf("Enter 1 to create else enter 0-");
    scanf("%d", &choice);
```

```
    while (choice == 1)
```

```
{
```

```
    n1 = (struct node *) malloc (sizeof (struct node));
```

```
    printf("Enter data-");
```

```
// scanf ("%d", &choice);
```

```
    scanf ("%d", &n1->data);
```

```
    n1->next=0;
```

```
    if (head == 0)
```

```
        temp=head=n1;
```

```
    else {
```

```
        temp->next=n1;
```

```
        temp=n1;
```

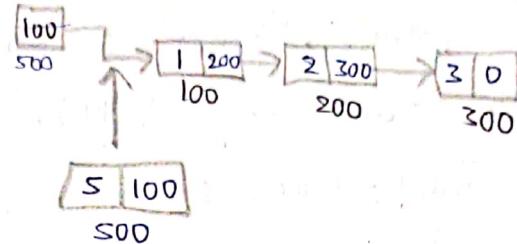
```
}
```

```
    printf("Enter 1 to create another else 0-");
```

```
    scanf ("%d", &choice);
```

```
}
```

```
// Asking user to insert at "beginning" or not
```



```
int opt;  
printf("Enter 1 to insert at begin, else 0-");  
scanf("%d", &opt);
```

```
while (opt == 1)  
{
```

```
begin = (struct node *) malloc(sizeof(struct node));  
printf("Enter data-");  
scanf("%d", &begin->data);
```

// Logic

```
begin->next = head;
```

```
head = begin;
```

```
printf("Enter 1 to insert another else 0-");  
scanf("%d", &opt);
```

// For displaying

```
temp = head;
```

```
while (temp != 0)  
{
```

```
printf("%d ", &temp->data);
```

```
temp = temp->next;
```

```
}
```

Inserting at ending

// creation is same for all
struct node *end;

```
int opt;  
printf("Enter 1 to insert at end else 0-");  
scanf("%d", &opt);  
while (opt == 1)  
{
```

```
end = (struct node *) malloc(sizeof(struct node));  
printf("Enter data-");  
scanf("%d", &end->data);  
end->next = 0;
```

// For traversing
temp = head;
while (temp != 0)
{

 temp1 = temp;
 temp = temp->next;
}

// Logic

 temp1->next = end;

 printf ("Enter 1 or 0 - ");

 scanf ("%d", &opt);

}

 if (opt == 1) {
 Inserting At a Specified Position

// creation is same

struct node *new;

int a; key;

 printf ("Enter 1 to insert at specified else 0 - ");
 scanf ("%d", &a);

 while (a == 1) {

 printf ("Enter position to insert - ");

 scanf ("%d", &key);

 new = (struct node *) malloc (sizeof (struct node));

 printf ("Enter data - ");

 scanf ("%d", &new->data);

// For traversing

 int i=0;

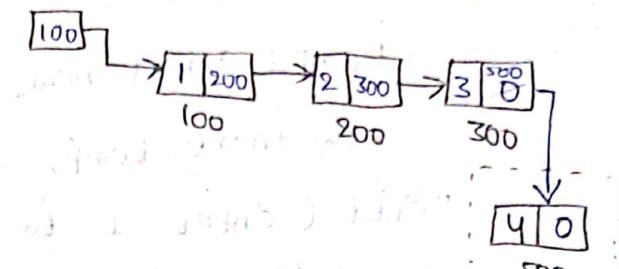
 temp = head;

 while (i < key) {

 temp1 = temp;

 temp = temp->next;

 i++;



// Logic

```
temp1 → next = new;
```

```
new → next = temp;
```

```
printf ("Enter 1 to insert another else 0-");  
scanf ("%d", &a);
```

Deletion

→ Deletion is generally used to delete unnecessary data in the LL.

→ This can be done in three ways:

Deletion at beginning, ending, At specified place

→ Creation is same as before.

deletion at beginning

// Asking user to delete or not

```
int k;
```

```
printf ("Enter 1 to delete at begin else 0-");
```

```
scanf ("%d", &k)
```

```
while (k == 1) {
```

// Logic

```
temp = head;
```

```
head = temp → next;
```

```
free(temp);
```

// For one more to delete

```
printf ("Enter 1 to delete else 0-");
```

```
scanf ("%d", &k);
```

```
}
```

deletion at ending

// creation is same as before

// Asking user to delete at end or not.

```
int a;
```

```
printf("Enter 1 to delete at end else 0-");
scanf("%d", &a);
```

```
while (a == 1)
{
```

// For traversing

```
temp = head;
```

```
while (temp != 0) {
```

```
temp1 = temp;
```

```
temp = temp->next;
}
```

// Logic

```
temp1->next = 0;
```

```
free(temp);
```

```
printf("Enter 1 or 0-");
scanf("%d", &a);
```

```
}
```

deleting at specified position

// Ask user to key value

// For traversing

```
int i = 0;
```

```
while (i < key) {
```

```
temp1 = temp;
```

```
temp = temp->next; i++;
}
```

// Logic

temp1 → next = temp → next;

free(temp);

temp → next = 0;

Pointf ("Enter 1 or 0-");

scanf ("%d", &a);

}

concatenation

// input two single linked lists

// For concatenation
// For traverse
temp = head;

while (temp → next != 0) {

temp = temp → next;
}

temp → next = head1;

// For printing

temp = head;

while (temp != 0) {

Pointf ("%d", temp → data);

temp = temp → next;

}

searching For element

→ ALSO finding its index

// input single linked list creation

```
int key, index=0, flag=0;
```

```
printf("Enter data to search-");  
scanf("%d", &key);
```

// For traversing

```
temp=head;
```

```
while (temp!=0) {
```

```
    temp=temp->next;
```

```
    if (key == temp->data)  
        {
```

```
            flag=1;
```

```
            printf("In position %d is %d", index+1, key);  
            break;
```

```
}
```

```
    index++;
```

```
}
```

```
if (flag==1)
```

```
    printf("searched element is found");
```

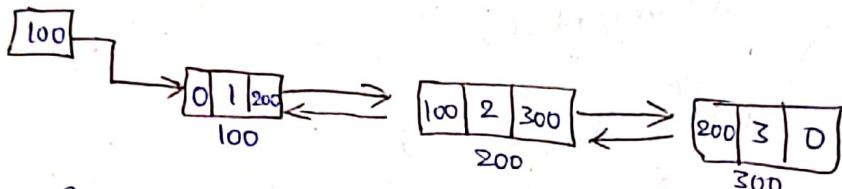
```
else
```

```
    printf("searched element is not found");
```

Double Linked List

→ Double linked list is a type of linked list in which is having data, 2 address parts.

One is for previous node address, another is for next node address to store.



→ Traversing is possible in both front and reverse.

→ Generally music apps like Wynk, Spotify etc., are used to play previous song or next song by using this double linked list.

→ Here also head represents first node's address.

→ Insertion, deletion, searching, concatenation, reverse

Creation of DLL

```
#include <stdio.h>
#include <stdlib.h>

struct node{
    int data;
    struct node *prev, *next;
}; // semi-colon is must

void main()
{
    struct node *head, *temp, *n1;
    head = 0; int choice;
    printf ("Enter 1 to create DLL else 0-");
    scanf ("%d", &choice);
```

```

while (choice == 1)
{
    n1 = (struct node *) malloc (sizeof (struct node));
    printf ("Enter data - ");
    scanf ("%d", &n1->data);
    n1->prev=0;
    n1->next=0;
    if
        while (head == 0)
    {
        if (temp == head)
            temp = head = n1;
        }
        else
        {
            // n1->next = temp;
            temp->next = n1;
            n1->prev = temp;
            temp = n1;
        }
    }
    printf ("Enter 1 to create another node else 0 - ");
    scanf ("%d", &choice);
}

// For printing
temp = head;
while (temp->next != 0) // while (temp != 0)
{
    printf ("%d", temp->data);
    temp = temp->next;
}
}
} // main closed.

```

Insertion at beginning

```
// create the DLL  
// Insertion at beginning  
int a;  
printf ("Enter 1 to insert at begin else 0-");  
scanf ("%d", &a);  
while (a==1){  
    begin=(struct node *) malloc (sizeof (struct node));  
    printf ("Enter data-");  
    scanf ("%d", &begin->data);  
    new->prev=0;  
    new->next=0;  
    // Logic  
    temp=head;  
    new->next=temp;           // new=begin  
    temp->prev=new;  
    head=new;  
    printf ("Enter 1 to insert another else 0-");  
    scanf ("%d", &a);  
}  
}
```

Insertion at ending

```
while (a==1){  
    end=(struct node *) malloc (sizeof (struct node));  
    printf ("Enter data-");  
    scanf ("%d", &end->data);  
    // For traversing to last node
```

```
temp = head;  
while (temp->next != 0) {  
    temp1 = temp;  
    temp = temp->next;  
}
```

// Logic

```
temp->next = new;
```

```
new->prev = temp;
```

```
new->next = 0;
```

```
printf ("Enter 1 or 0 -");
```

```
scanf ("%d", &a);
```

```
}
```

Insertion at Specified Position

```
// Input DLL creation  
// Allocate memory to new node first  
// Ask user to insert at which position.
```

// traversing

```
int i=0;
```

```
temp = head;
```

```
while (i < key) {
```

```
    temp1 = temp;
```

```
    temp = temp->next; i++;
```

```
}
```

// Logic

```
temp1->next = new;
```

```
new->prev = temp1;
```

```
new->next = temp;
```

```
temp->prev = new;
```



```
temp = head;
```

```
while (temp->next != 0) {
```

```
    temp1 = temp;
```

```
    temp = temp->next;
```

```
}
```

```
// Logic
```

```
temp->next = new;
```

```
new->prev = temp;
```

```
new->next = 0;
```

```
printf ("Enter 1 or 0 -");
```

```
scanf ("%d", &a);
```

```
}
```

Inse~~s~~tion at specified Position

```
// Input DLL creation
```

```
// Allocate memory to new node first
```

```
// Ask user to insert at which position.
```

```
// Traversing
```

```
int i=0;
```

```
temp = head;
```

```
while (i < key) {
```

```
    temp1 = temp;
```

```
    temp = temp->next; i++;
```

```
}
```

```
// Logic
```

```
temp1->next = new;
```

```
new->prev = temp1;
```

```
new->next = temp;
```

```
temp->prev = new;
```



Deletion at beginning

// input DLL creation.

// Asking user to delete at begin or not.
int k;

printf ("Enter 1 to delete at begin or 0-");
scanf ("%d", &k);

while (k == 1) {

temp = head;

head = temp->next;

free (temp);

Deletion at ending

// For traversing

temp = head;

while (temp->next != 0) {

temp1 = temp;

temp = temp->next;

}

// Logic

temp1->next = 0;

free (temp);

Deletion at specified position

// Ask position i.e., key to delete

// For traversing

int i=0;

temp = head;

while (temp->next != 0) {



```
temp1 = temp;  
temp = temp->next; i++;  
}
```

```
temp1->next = temp->next;  
temp->next->prev = temp1;  
temp->prev = 0;  
temp->next = 0;  
free(temp);
```

concatenation

```
// input two double linked lists head, head1  
head->next = head1
```

```
// concatenation
```

```
// For traversing
```

```
temp = head;  
while (temp->next != 0) {  
    temp = temp->next;  
}
```

```
temp->next = head1;  
Point the new double linked list.
```

searching for an element

// input double linked list

// for searching

int k, index=0, flag=0;

printf ("Enter element to search-");
scanf ("%d", &k);

// for traversing

temp = head;

while (temp->next != 0)
{

 temp = temp->next;

 if (k == temp->data) {

 flag = 1;

 printf ("In position %d is %d", index+1),
 break;

}

 index++;

}

if (flag == 1)

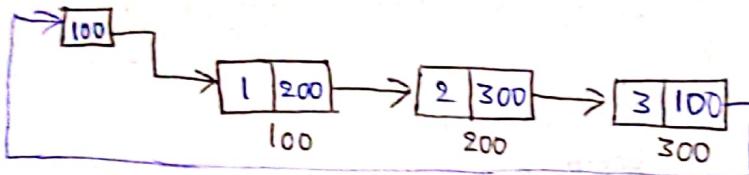
 printf ("searched element is found");

else

 printf ("searched element is not found");

Circular Linked List

- Circular linked list is a type of linked list which having data part, address part.
- It is similar to single linked list but it have link, last node to first node.
- It appears in circular shape, so it is called as circular linked list.



- Traversing is possible in only one way, but it have ability to reach to the first node.
- Used mostly in Playlist, they will repeat after completion of last song in the list, it means plays first song after completion of last song.
- here head and last also contains first node's address.

Creation of circular linked list

```
#include<stdio.h>
#include<stdlib.h>

struct node{
    int data;
    struct node *next;
};

Void main()
{
    struct node *head, *temp, *n1;
    head=0;
    int choice;
    printf ("Enter 1 to create CLL else 0-");
    scanf ("%d", &choice);
```

```

while (choice==1) {
    n1 = (struct node *) malloc(sizeof(struct node));
    printf("Enter data - ");
    scanf("%d", &n1->data);
    n1->next=0;
    if (head==0)
        temp=head=n1;
    else {
        temp->next=n1;
        temp=n1;
    }
    n1->next=head;
}

Pointf("Enter 1 to create another else 0-");
scanf("%d", &choice);
// For printing
temp=head;
while (temp->next!=head) {
    Pointf("%d", temp->data);
    temp=temp->next;
}
Pointf("%d", temp->data);
}
// main closed.

```

Insertion at beginning

```

// input : CLL creation
// Asking user to insert at beginning or not
int k;
printf("Enter 1 to insert at begin else 0-");
scanf("%d", &k);
while (k == 1) {
    new = (struct node *) malloc(sizeof(struct node));
    printf("Enter data");
    scanf("%d", &new->data);
}

```

<pre> // Logic temp = head; new->next = temp; temp->prev = new; head = new; </pre>	<pre> // Logic begin->next = head; new->next = head; // For traversing temp = head; while (temp->next != head) { temp = temp->next; } head = begin; temp->next = head; </pre>
--	--

```

printf("Enter 1 to insert another else 0-");
scanf("%d", &k);
}

```

Insertion at ending

```

// input : CLL creation
int choice;
printf("Enter 1 to insert at ending, else 0-");
scanf("%d", &choice);
while (choice == 1) {
    end = (struct node *) malloc(sizeof(struct node));
    printf("Enter data-");
    scanf("%d", &end->data);
}
// For traversing

```

```
temp = head;
```

```
while (temp->next != head) {
```

```
    temp1 = temp;
```

```
    temp = temp->next;
```

```
}
```

```
// Logic
```

```
temp->next = new;
```

```
new->prev = temp;
```

```
temp = temp->next;
```

```
new->next = 0;
```

```
printf("Enter 1 or 0 - ");
```

```
scanf("%d", &flag);
```

```
}
```

Insertion at Specified Place

```
// input CLL creation
```

```
k // Ask user to insert at key value or not
```

```
// Ask key value to insert.
```

```
while (k == 1) {
```

```
    printf("Enter key value to insert");
```

```
    scanf("%d", &key);
```

```
    new = (struct node *), malloc (sizeof(struct node));
```

```
    printf("Enter data - ");
```

```
    scanf("%d", &new->data);
```

```
// For traversing
```

```
int i = 0;
```

```
temp = head;
```

```
while (i < key) {
```

```
    temp1 = temp;
```

```
    temp = temp->next; i++;
```

```
}
```



// Logic

```
temp1->next = new;
new->prev = temp1;
new->next = temp;
temp->prev = new;
```

```
printf("Enter 1 to insert one more else 0-");
scanf("%d", &k);
```

}

// Logic

```
temp1->next = key;
key->next = temp;
```

deletion at beginning

// input CLL creation

```
int choice;
```

```
printf("Enter 1 to delete node at begin else 0-");
scanf("%d", &choice);
```

```
while (choice == 1) {
```

```
    temp = head;
    head = temp->next;
    free(temp);
}
```

```
printf("Enter 1 or 0-");
scanf("%d", &choice);
```

```
temp = head;
while (temp->next == head) {
    temp = temp->next;
}
d = head;
head = head->next;
temp->next = head;
free(d);
}
```

}

deletion at ending

// input CLL creation

// Ask user to delete node at end or not.

// for traversing

```
temp = head;
```

```
while (temp->next != head) {
    temp1 = temp;
    temp = temp->next;
}
```

temp->next = head;



```

    // Logic
    temp1->next = head;
    free(temp);
}

```

```

printf("Enter 1 or 0 to delete -");
scanf("%d", &choice);
}

```

deletion At specified position

// input CLL creation

// Ask user to delete at specified position

// Ask key value to delete

X // For traversing

```
int i=0;
```

```
while(i<key) {
```

```
temp1=temp;
```

```
temp = temp->next;
i++;
}
```

// Logic

```
temp1->next = temp->next;
```

```
temp->next->prev = temp1;
```

```
temp->prev = 0;
```

```
temp->next = 0;
```

```
free(temp);
```

// For traversing

```
int i=0;
```

```
temp=head;
```

```
while(i<k) {
```

```
temp1=temp;
```

```
temp = temp->next;
i++;
}
```

// Logic

```
temp1->next = temp->next;
```

```
free(temp);
```

```

printf("Enter 1 or 0 to delete -");
scanf("%d", &choice);
}

```

concatenation

```

// Input two circular linked lists say
    head, head1
    (i)      (ii)          temp->next = head1;
                           // 2nd list end
                           n2->next = head;

// concatenation
// For traversing
    temp = head;
    while (temp->next != head) {
        temp = temp->next;
    }
    temp->next = head1; // Logic
  
```

searching

```

// Input CLL creation
// For searching
int key, index=0, flag=0;
printf ("Enter element to search - ");
scanf ("%d", &key);
// For traversing
temp = head;
while (temp->next != head) {
    temp = temp->next;
    if (key == temp->data)
        flag = 1;
    printf ("In position of searched
            break; element is %d", index);
    index++;
}
  
```

```
if (flag == 1) {  
    printf ("searched element is found in");  
}  
else {  
    printf ("In searched element not found");  
}
```

STACK

Introduction to Stack:

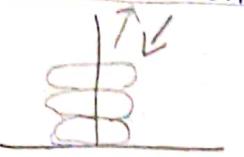
→ stack is a linear data structure, it is a container which follows insertion, deletion follow some rules.

Rule:

insertion
deletion

} possible from one end only

Eg:-

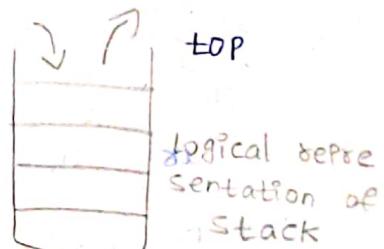


LIFO - Last in first out

FIFO - first in last out

insertion - Push(x)

deletion - POP() } possible at top of stack



→ Fundamental operations are Push, POP.

Operations:

Push(x) → insertion of data in stack

Pop() → has no arguments

Peek() / top() - returns top most element in stack

IsEmpty() - checks empty or not returns true (or) False.

IsFull() - check stack is full or not returns T (or) F

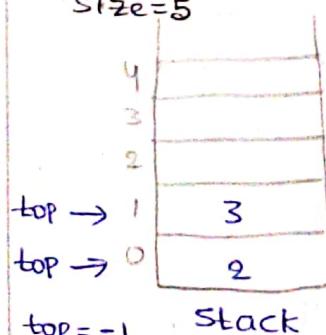
Implementation of stack we have two methods:

(i) static - Array

(ii) dynamic - linked list

At initial top = -1

→ when stack is empty, it returns "Underflow" condition



Push & POP

Push(2)

top++;

Push(3)

top++;

Pop()

top--;

Pop()

top--;

Push(4)

"Underflow"

"Overflow"

condition

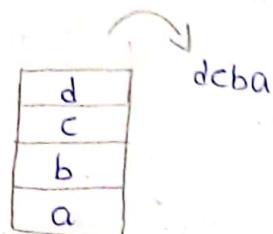
→ If stack is filled, its maximum size & it returns "overflow" condition. $\rightarrow \text{top} = n - 1$

`isFull()` } returns true or False
`isEmpty()`

Applications:

1) Reverse a string:

$$\text{abcd} = \text{dcba}$$



2) Undo: (`ctrl+z`)

This mechanism is performed using stack in our Text editor.

3) Recursion/Function call:

↓
Function calling itself is recursion.

4) TO check the balance of Paranthesis.

{ } { } { }

} { }

} { }

}

5) Infix to Postfix/Prefix.

6) Evaluation of Postfix expression.

Implementation of Stack using Array:-

→ Using static memory allocation

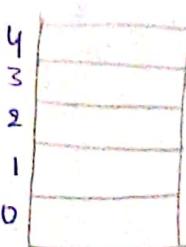
int a[5];

$5 \times 4 = 20$ bytes

0	1	2	3	4
100	104	108	112	116

int stack[5];

→ Suppose we have to insert more than 5 elements, then create new array with that size.



Code:

```
#include <stdio.h>           else use  
                                #define NS  
int stack[100], choice, n, top, x, i;  
  
Void Push()  
{  
    if (top == n-1)  
    {  
        printf("In STACK IS over flow");  
    }  
    else  
    {  
        printf("Enter element to Push-");  
        scanf("%d", &x);  
        top++;  
        stack[top] = x;  
    }  
}  
  
Void POP()  
{  
    if (top == -1)  
    {  
        printf("In STACK IS UNDER Flow");  
    }  
    else  
    {  
        printf("Init The Popped element is %d", stack[top]);  
        top--;  
    }  
}  
  
Void display()  
{  
    if (top >= 0)  
    {  
        printf("The elements in STACK is - ");  
        for (i = top; i >= 0; i--)  
        {  
            printf("%d", stack[i]);  
            printf(" In Press next choice");  
        }  
    }  
    else { printf("The stack is empty\n"); }  
}
```

```

Void Peek()
{
    if (top == -1)
    {
        printf("In the stack is Underflow");
    }
    else
    {
        printf("Peek Value: %d", stack[top]);
    }
}

Int main()
{
    top = -1;
    printf("Enter size of the stack [MAX=100]: ");
    scanf("%d", &n);
    printf("Init STACK OPERATIONS USING ARRAY");
    printf("Init -----");
    printf("Init 1.PUSH 2.POP 3.DISPLAY 4.Peek 5.EXIT");
    do
    {
        Int choice;
        printf("Enter the choice- ");
        scanf("%d", &choice);
        switch (choice)
        {
            case 1:
            {
                Push();
                break;
            }
            case 2:
            {
                Pop();
                break;
            }
        }
    }
}

```

```
case 3:  
    display();  
    break;  
}  
  
case 4:  
{  
    Peek();  
    break;  
}  
  
case 5:  
{  
    Pointe("In It EXIT POINT");  
    break;  
}  
  
default:  
{  
    Pointe("In It Please enter valid choice  
(1/2/3/4)");  
}  
} // switch  
} // do loop closed  
while (choice != 5);  
return 0;  
} // main closed....
```

Implementation of stack using linked list:



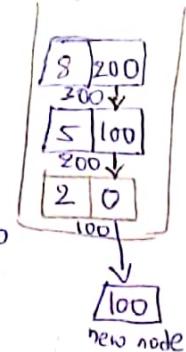
rule: LIFO principle

→ In Linked list we follow for insertion or deletion $O(n)$

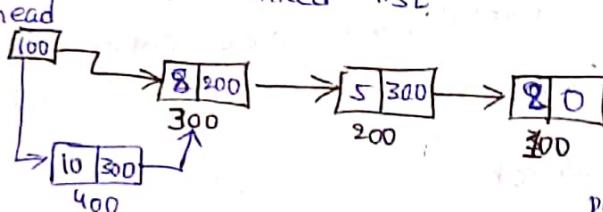
But in stack it is $O(1)$ only for any operation
i.e., Push()
Pop()

Push(2)
Push(5)
Push(8)

top = 100
top = 0



above as in form of linked list.



push(10)

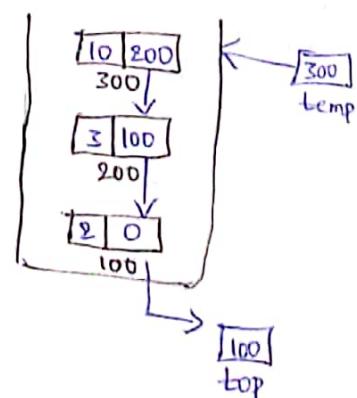
[10]
400

Code:

```
struct node {
    int data;
    struct node *next;
};

struct node *top = 0;

void Push(int x) {
    // creating node
    struct node *newnode;
    newnode = (struct node *) malloc(sizeof(struct node));
    newnode->data = x;
    newnode->next = top;
    top = newnode;
}
```



```
Void display()
{
    struct node *temp;
    temp=top;
    if (top==0)
        printf("STACK IS EMPTY");
    else
        while (temp!=0); // doubt for semicolon
    {
        printf("%d", temp->data);
        temp=temp->next;
    }
}
```

```
Void Peek()
{
    if (top==0)
        printf("The STACK IS EMPTY");
    else
        printf("TOP element is %d", top->data);
}
```

```
Void POP()
{
    struct node *temp;
    temp=top;
    if (top==0)
    {
        printf("STACK IS UNDERFLOW");
    }
    else
    {
        printf("The popped element is %d", top->data);
        top=top->next;
        free(temp);
    }
}
```



```

Void main()
{
    int c=1, n;
    while(c==1)
    {
        printf("1. Push 2. display 3. Pop 4. Peek");
        scanf("%d", &n);
        printf("Enter value");

        switch(n)
        {
            case 1:
            {
                push();
                break;
            }
            case 2:
            {
                display();
                break;
            }
            case 3:
            {
                pop();
                break;
            }
            case 4:
            {
                peek();
                break;
            }
        }
    }
    printf("press 1 to continue-");
    scanf("%d", &c);
}

```

3 //loop closed

3 //main closed...

INFIX, PREFIX AND POSTFIX EXPRESSIONS

Expression:

→ It contains operators, operands, constants, symbols, Paranth

Eg:- $5+1$, $P+Q$, $(a+b)^*c$

Binary :- INFIX:
operators $<\text{operator}> <\text{operator}> <\text{operator}>$ → infix notation

Precedence Order:

- ① $() \{ \} []$
- ② $\wedge \rightarrow R-L$
- ③ $* / \% \rightarrow L-R$
- ④ $+ - \rightarrow L-R$

Eg:- $5+1 * 6$

$$\begin{array}{ccc} (5+1) * 6 & \xrightarrow{\quad} & 6 * 6 = 36 \\ 5+1 * 6 & \xrightarrow{\quad} & 5+(1*6) \\ 5+(1*6) & \xrightarrow{\quad} & 5+6 = 11 \end{array}$$

Q. Evaluate $1+2*5+30/5$

We ASSOCIativity $\rightarrow L-R$

$$\begin{aligned} & 1+2*5+30/5 \\ & = 1+10+30/5 \\ & = 1+10+6 \\ & = 11+6 \Rightarrow 17 \end{aligned}$$

$$\begin{aligned} & 2^{\wedge} 2^{\wedge} 3 \\ & 2^{\wedge} 8 \quad R \rightarrow L \\ & = 256 \quad \checkmark \\ & 4^{\wedge} 3 = 64 \times \quad L \rightarrow R \end{aligned}$$

PREFIX: (Polish notation)

$<\text{operator}> <\text{operator}> <\text{operator}>$

Eg: $5+1 \Rightarrow +51$

(infix) (prefix)

$$\begin{array}{ccc} a * b + c & \Rightarrow & *ab+c \\ (\text{infix}) & & \boxed{+abc} \rightarrow \text{prefix} \end{array}$$

POSTFIX: (reverse Polish notation)

$<\text{operator}> <\text{operator}> <\text{operator}>$

$5+1 \Rightarrow 51+$

→ This may be complex expression

$$a * b + c \Rightarrow ab*c+ \boxed{ab*c+}$$



Infix to Postfix conversion:

- to Postfix
- 1) Print the operands (a, b, c), as they arrive
 - 2) If stack is empty (or) contains a left parenthesis on top, Push the incoming operator onto the stack.
 - 3) If incoming symbol is '(', Push it onto stack
 - 4) If incoming symbol is ')', Pop the stack & print the operators until '(' is found.
 - 5) If the symbol is an operator, then check the stack is empty or not.
 - (i) If the stack is empty then push the operator onto the top of the stack.
 - (ii) If the stack is not empty, then we have to check the priority of the operator.
 - a) If the priority of incoming operator is having higher priority of the top of the stack then push that operator onto the top of the stack.
 - b) If the priority of incoming operator is having lesser priority than top of the stack, then test the incoming operator with new top of the stack.
 - c) If the priority of incoming operator having equal precedence with top of stack, use associativity rule.
 - 6) After reading entire infix expression, if stack is not empty, then pop all the symbols in stack and add it to Postfix.

Associativity:

If it is $L \rightarrow R$ then Pop & print the top of stack and push the incoming operator

If it is $R \rightarrow L$ then Push the incoming operator

Ex: convert this Infix to Postfix expression.

Stack

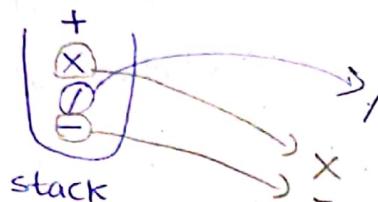


A+B/C

Postfix

ABC/+

→ A-B/C × D+E



Postfix

ABC/DX-E+

→ A+(B*C)

A+ BC*

ABC * + → Postfix

1. Convert Infix to Postfix k+l-m*n+(o*p)*w/u/v*t+q

Input Exp

Stack

Postfix Exp

k	-	k
+	+	k
l	-	kl
-	-	kl+
m	*	kl+m
*	*	kl+m
n	-*	kl+mn
+	+	kl+mn*-
(+()	kl+mn*-
O	+()	kl+mn*-o
^	+(^)	kl+mn*-o
P	+(^ -)	kl+mn*-op
)	+	kl+mn*op
*	+	kl+mn*op
w	+	kl+mn*opaw
/	+/	kl+mn*opaw*



\wedge	$+/\overrightarrow{L}$	$KL+mn*OP\Lambda W*$
/	$+/_L$	$KL+mn*OP\Lambda W*_L$
\vee	$+/_L$	$KL+mn*OP\Lambda W*_L/v$
\wedge	$+/\overrightarrow{L}$	$KL+mn*OP\Lambda W*_U$
/	$+/_L$	$KL+mn*OP\Lambda W*_U/_L$
\vee	$+/\overrightarrow{L}$	$KL+mn*OP\Lambda W*_U/v$
$*$	$+*_L$	$KL+mn*OP\Lambda W*_U/v/_L$
T	$+T/\overrightarrow{R}$	$KL+mn*OP\Lambda W*_U/v/T$
$+$	$+/_L$	$KL+mn*OP\Lambda W*_U/v/T*_L$
Q	$+$	$KL+mn*OP\Lambda W_X*_U/v/T*_L$

$$\text{Postfix} = KL+mn*OP\Lambda W*_U/v/T*_L + Q$$

If it have some $R \rightarrow L$ condition \boxed{AJAA}

\wedge	$+/\overrightarrow{A}$	- - - - -
J	$+/\overrightarrow{A}$	- - - - - J
\wedge	$+/\overrightarrow{A}$	- - - - - J
A	$+/\overrightarrow{A}$	- - - - - J + AA

2. Convert this infix expression to Postfix.
 $A - B + (m \wedge n) \times (o + p) - q / r \wedge s \times t + z$

	Stack	Postfix
A		A
-	-	A
B	-	AB
+	-	AB-
(+	AB-
M	+(C)	AB-M

\wedge	$+/\overrightarrow{A}$	AB
N	$+/\overrightarrow{A}$	AB
)	$+/\overrightarrow{x}$	AB
X	$+x$	AB
C	$+x(C$	AB
O	$+x(C$	AB
+	$+x(+$	AB
P	$+x(+$	AB
)	$+x$	AB
-	$-$	AB
Q	$-$	AB
/	$-/$	A
R	$-/$	A
\wedge	$-/\wedge$	A
S	$-/\wedge$	AB
X	$-x$	A
T	$-x$	A
+	$+$	A
Z	$+$	A

Finally, The postfix expression is $AB -$

\wedge	$+(\wedge$	$AB - M$
N	$+(\wedge$	$AB - MN$
)	$+)\wedge x$	$AB - M N \wedge$
X	$+x$	$AB - M N \wedge$
($+x($	$AB - M N \wedge$
O	$+x($	$AB - M N \wedge O$
$+$	$+x(+$	$AB - M N \wedge O$
P	$+x(+$	$AB - M N \wedge O P$
)	$+x$	$AB - M N \wedge O P +$
$-$	$-$	$AB - M N \wedge O P + *$
Q	$-$	$AB - M N \wedge O P + * + Q$
$/$	$- /$	$AB - M N \wedge O P + * + Q$
R	$- /$	$AB - M N \wedge O P + * + Q R$
\wedge	$- / \wedge$	$AB - M N \wedge O P + * + Q R$
S	$- / \wedge$	$AB - M N \wedge O P + * + Q R S$
X	$- x$	$AB - M N \wedge O P + * + Q R S \wedge$
T	$- x$	$AB - M N \wedge O P + * + Q R S \wedge / T$
$+$	$+$	$AB - M N \wedge O P + * + Q R S \wedge / T * -$
Z	$+$	$AB - M N \wedge O P + * + Q R S \wedge / T * - Z$

Finally, The Postfix

expression =

$AB - M N \wedge O P + * + Q R S \wedge / T * - Z +$

Infix \equiv Prefix \downarrow Using stack:

Eg:

Pehle
me operator hai
<operator> <operands> <operator>

A + B * C

A + (B * C)

A + * BC

[+ A * BC]

I. convert. infix expression to prefix exp.

$k + L - M * N + (O \wedge P) * w / u / v * T + Q$

write it in reverse order firstly to solve

$Q + T * V / U / W *) P \wedge O (+ N * M - L + K$

infix

stack

+ +

T +

* +

V +*

/ +*

[+ /] * +*/

/ +*/

w +*/

* +*/

) +*//*

P +*//*)

^ +*//*)

O +*//*)^

+*//*)^

prefix

Q

Q

QT

QT

QTV

QTV*

QTV*U

QTV*U)

QTVUW

QTVUW

RTVUW

RTVUW

QTVUWP

QTVUWP

QTVUWPO



($+ * / / *$	QTVUWPOA
+	$++$	QTVUWPOA * // *
N	$++$	QTVUWPOA * // *
*	$++ *$	QTVUWPOA * // *
M	$++ *$	QTVUWPOA * // *
-	$++ -$	QTVUWPOA * // * m
L	$++ -$	QTVUWPOA * // * m *
+	$++ - +$	QTVUWPOA * // * m * L
k	$\underline{++ - +}$	QTVUWPOA * // * m * L
we get, QTVUWPOA * // * m * L k + - + +		
$\boxed{\text{Prefix} = ++ - + k L * m * // * \wedge O P W U V T Q}$		

Algorithm:-

- 1) Reverse the infix expression and write it.
- 2) If operand is these simply put into prefix.
- 3) If stack is empty (or) contains a right parenthesis ')', push the incoming operator onto the stack.
- 4) If incoming symbol is ')', push it onto the stack.
- 5) If incoming symbol is '(', pop the stack until we get ')'. Print the operators.
- 6) If the stack is empty (or) not.

(i) If stack is empty then push that onto the stack.

(ii) If the stack is not empty, then check the priority of the operator.

a) If the priority of incoming operator having higher precedence than top of the stack, then Push that operator onto top of the stack.

b) If the priority of incoming operator having lesser precedence than top then POP & Print the top. Then test the incoming operator with new top of the stack.

c) If the priority of incoming operator is equals to top of stack, use Associativity rule.

d) After reading entire infix expression, if stack is not empty, then POP all the operators in the stack using LIFO method and add it to Prefix.

e) Then reverse the expression to get Prefix expression.

Associativity:

If its precedence is $L \rightarrow R$ then push the incoming operator.

If its Precedence is $R \rightarrow L$ then POP & point the top of the stack and Push the incoming operator.

Evaluation of Prefix Expression

infix - $a + b * c - d / e \wedge f$

$$\begin{aligned}\text{Prefix} &= - + a * b c / d \wedge e f \\ &\Rightarrow - + 2 * 3 4 / 16 \wedge 2 3\end{aligned}$$

$$a=2, b=3, c=4, d=16$$

$$e=2, f=3$$

$\langle \text{operator} \rangle \langle \text{operand} \rangle \langle \text{operand} \rangle \rightarrow \text{Binary Operator}$
Postfix → Read exp from Right to left

Postfix → Read exp from left to right.

$$- + 2 * 3 4 / 16 \wedge 2 3$$



$$- + 2 * 3 4 / 16 8$$

$$- + 2 * 3 4 / 16$$

$$16/8 = 2$$

$$- + 2 * 3 4 2$$

$$- + 2 12 2$$

$$- 14 2$$

$$\boxed{\text{Prefix} = 12}$$

Not getting $\langle \text{operator} \rangle \langle \text{operand} \rangle \langle \text{operator} \rangle \langle \text{operand} \rangle$ that expression may be not valid expression.

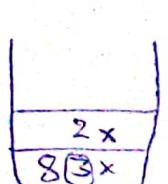
To get prefix expression in only one scan, then have to use stack for this.

Using Stack:

$$- + 2 * 3 4 / 16 \wedge 2 3$$

→ If the incoming is operand so, then push it onto stack.

→ If the incoming is operator. then pop the 2 top most elements from stack, do that operation and result will be pushed onto the top of the stack.



$$\begin{array}{c} \swarrow 1 \\ 2 \wedge 3 \\ = 8 \end{array}$$

$$\begin{array}{|c|} \hline \text{OP1=2} \\ \text{OP2=3} \\ \hline \end{array}$$

Algorithm:

- 1) scan the ~~prefix~~ expression from right to left.
- 2) for each char in Prefix exp
do
 - if operand is there, push it onto stack
 - else if operator is there, pop 2 elements
 - OP1 = top element
 - OP2 = next to top element
 - result = OP1 operator OP2
 - push result onto stack
- return stack[top]

Evaluation of Postfix Expression

Infix - $a+b*c-d/e+f$

$$a=2, b=3, c=4, d=16$$

Postfix - abc * + d e f / +

$$e=2, f=3$$

$$2 3 4 * + 16 2 3 / +$$

Postfix - Read the expression from L \rightarrow R
 <operator> <operator> <operator>

$$\rightarrow 2 3 4 * + 16 2 3 / +$$

$$2 12 + 16 2 3 / +$$

$$OP1 = 4$$

$$OP2 = 3$$

$$14 16 2 3 / +$$

$$3 \times 4 = 12$$

$$14 16 8 / +$$

$$OP1 = 16$$

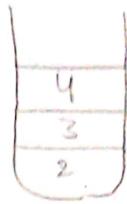
$$14 2 -$$

$$OP2 = 8$$

$$16/8 = 2$$

Postfix = 12

top value = OP 2
next top = OP 1 } in Postfix



$$\begin{aligned}OP_2 &= 4 \\OP_1 &= 3\end{aligned}$$

OP1 OP OP2

$$3 * 4 = 12$$

Algorithm:

- 1) scan the postfix expression from left to right.
- 2) for each char in postfix expression do
 - if operand is there, push it onto stack
 - else if operator is there, pop 2 elements from top.
 - $OP_1 = \text{next to top element}$
 - $OP_2 = \text{top element}$
 - $\text{result} = OP_1 \text{ operator } OP_2$
 - push result onto stack
 - return stack [top]

4.1

Introduction to Queue

- Queue is a linear data structure
- ADT - Abstract Data type.

Eg:- Buying a cinema ticket



which follows FIFO (First In First Out)

Rule:

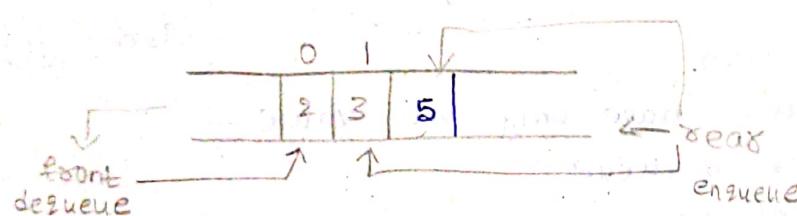
Insertion - Rear/Tail → enqueue()

deletion - Front/Head → dequeue()

Queue is a linear data structure in which Insertion (enqueue), deletion (dequeue) are possible at different ends.

enqueue at Rear

dequeue at front



for enqueue we have to rear++

we can't directly dequeue 5, it is possible when firstly dequeue 2,3 after that it is possible.
also called as LIFO (Last In Last Out)

Operations:-

enqueue(2)

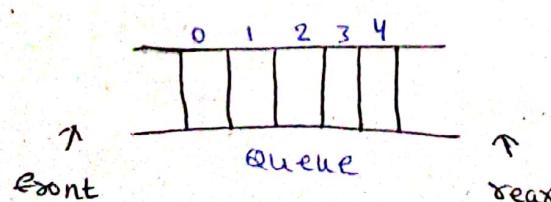
dequeue()

front() / peek()

isFull()

isEmpty()

size=5



$\text{front} = \text{rear} = -1$ (queue is empty)

// for inserting in 0th index

$\text{front}++, \text{rear}++;$

// For enqueue in 1st index

$\text{rear}++;$

enqueue(10)

// for enqueue in 2nd index

$\text{rear}++;$

enqueue(-1)

// For dequeue

$\text{front}++;$

$\text{rear} = \text{size} - 1$ Time complexity - O(n)

{ Pointe("overflow"); }

$\text{front} = \text{rear} = -1;$

Pointe("Underflow");

$\text{front} == \text{rear}$

// queue have only one value

after dequeue

$\text{front} = \text{rear} = -1$

(or)

$\text{front}++;$

Time complexity - O(1)

→ Queue have wastage of memory

Applications:-

→ Printer



→ customer care, they put on hold until representative is free.

→ Processor

Implementation of Queue Using Array

Queue



enqueue - rear

dequeue - front

→ By static memory Allocation

FIFO $O(1)$

```
int a[5];
```

```
int queue[5];
```

Code:

```
#include<stdio.h>
#include <stdlib.h>
#define N 5
int queue[N];
int front=-1;
int rear=-1;
```

```
void enqueue(int x)
{
```

```
if (rear == N-1)
```

```
{
```

```
printf ("Queue is full");
```

```
}
```

```
else if (front == -1 && rear == -1)
```

```
{
```

```
front=rear=0;
```

```
queue[rear] = x;
```

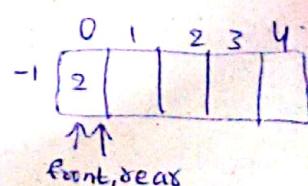
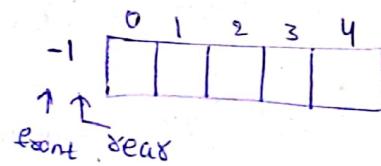
```
else {
```

```
//For inserting at 1st index
```

```
rear++;
```

```
queue[rear] = x;
```

```
}
```



```

Void dequeue
{
    if (front == -1 && rear == -1)
    {
        printf("Queue is empty");
    }
    else if (front == rear)
    {
        front = rear = -1;
    }
    else
    {
        printf("The dequeued element is %d", queue[front]);
        front++;
    }
}

Void display()
{
    int i;
    if (front == -1 && rear == -1)
    {
        printf("Queue is empty");
    }
    else
    {
        for (i = front; i <= rear  

             (or)  

             i < rear + 1; i++)
        {
            printf("%d", queue[i]);
        }
    }
}

Void peek()
{
    if (front == -1 && rear == -1)
    {
        printf("Queue is empty");
    }
    else
    {
        printf("%d", queue[front]);
    }
}

```

→ Although we have space after dequeue but we never fill that space. Using circular it possible.

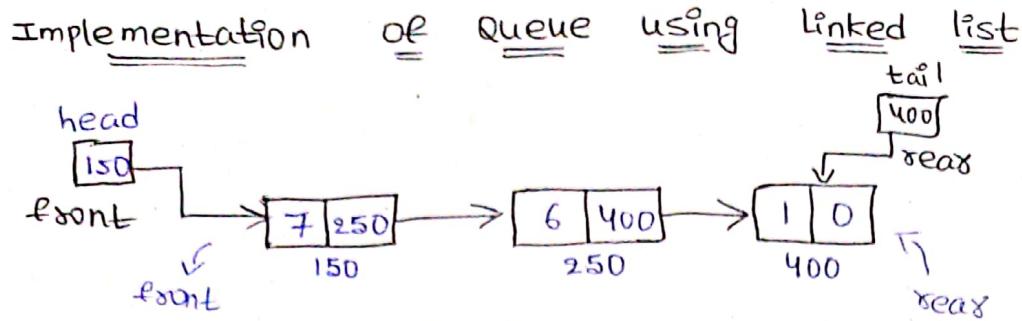


```

Void main()
{
    int c=1, n;
    while(c==1)
    {
        printf("1. Int 1.enqueue\n2. dequeu 3. Peek\n4. display\nIn Enter number:");
        scanf("%d", &n);
        switch(n)
        {
            case 1:
            {
                enqueue();
                break;
            }
            case 2:
            {
                dequeue();
                break;
            }
            case 3:
            {
                peek();
                break;
            }
            case 4:
            {
                display();
                break;
            }
            default:
            {
                printf("check the entered value");
            }
        }
    }
}

```

4.3



→ Dynamic memory Allocation.

FIFO rule, Time complexity $O(1)$



```

struct node
{
    int data;
    struct node *next;
};

struct node *front=0;
struct node *rear=0;

Void enqueue (int x)
{
    struct node *newnode;
    newnode = (struct node *)malloc (sizeof (struct node));
    newnode->data=x;
    newnode->next=0;

    if (front==0 & rear==0)
    {
        front=rear=newnode;
        rear->next=front;
    }
    else
    {
        rear->next=newnode;
        rear=newnode; rear->next=front;
    }
}

```

Diagram illustrating the enqueue operation:

- The queue initially has one node with data 5 and 0.
- A new node with data 100 is being enqueued.
- The 'front' pointer points to the current head node (5, 0).
- The 'rear' pointer points to the new node (100).
- The new node's 'next' pointer is set to the current 'front' node.
- The 'front' pointer is updated to point to the new node.

```
Void dequeue()
```

```
{
```

```
    struct node *temp;
```

```
    temp = front;
```

```
    if (front == 0 & & rear == 0) // only one node is there
```

```
        printf("Queue is empty");
```

```
    else {
```

```
        printf("%d deleted data", front->data);
```

```
        front = front->next;
```

```
        free(temp);
```

```
}
```

```
Void display()
```

```
{
```

```
    struct node *temp;
```

```
    if (front == 0 & & rear == 0)
```

```
        printf("Queue is empty");
```

```
    else {
```

```
        }
```

```
        temp = front;
```

```
        while (temp != 0) while (temp != front)
```

```
            printf("%d", temp->data);
```

```
            temp = temp->next;
```

```
}
```

```
Void peek()
```

```
{
```

```
    if (front == 0 & & rear == 0)
```

```
        printf("Queue is empty");
```

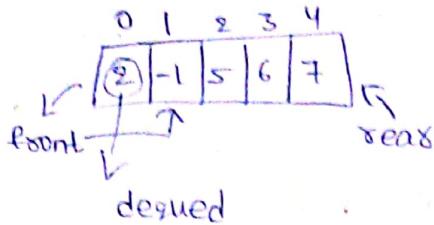
```
    else {
```

```
        }
```

```
        printf("%d", front->data); // = 0
```

```
}
```

```
Time complexity is O(n)
```

Circular Queue

front=0
rear=4 } Queue is full

Again calling enqueue(), it shows full but we have some space at 0th index in this case we didn't allow to insert any value.
→ This is drawback of Queue.

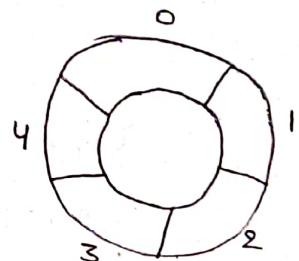
To overcome this drawback we have introduced circular Queue.

Implementation of Circular Queue Using Arrays

```
#define N 5
```

```
int Queue[N];
int front=-1;
int rear=-1;
```

```
void enqueue()
```



```
if (front == -1 & rear == -1)
```

```
{
```

```
front=rear=0;
```

```
queue[rear]=x;
```

```
else if ((rear+1)%N == front) // Not rear==N-1
```

```
{
```

```
printf("Queue is full");
```

some elements present

```
else
```

```
{
```

// Not rear++

```
rear=(rear+1)%N
```

```
queue[rear]=x;
```

A queue is full

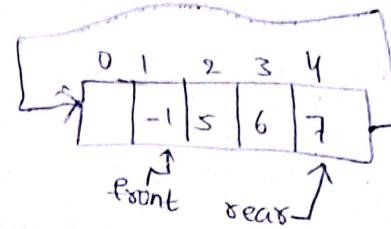
```

void dequeue()
{
    if (front == -1 && rear == -1)
        printf("Queue is empty");
    else if (front == rear)
    {
        front = rear = -1;
    }
    else
    {
        printf("%d the dequeued element", queue[front]);
        // Not front++;
        front = (front + 1) % N;
    }
}

void display()
{
    if (front == -1 && rear == -1)
        printf("Queue is empty");
    else
    {
        int i = front;
        printf("Queue is :");
        while (i != rear)
        {
            printf("%d", queue[i]);
            i = (i + 1) % N;
        }
        printf("%d", queue[rear]);
    }
}

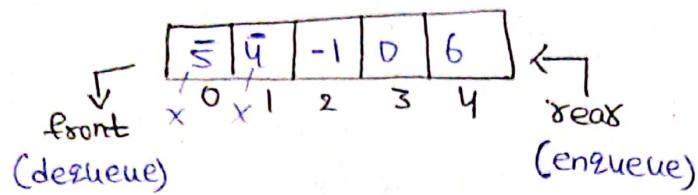
void peek()
{
    if (front == -1 && rear == -1)
        printf("Queue is empty");
    else
        printf("%d", queue[front]);
}

```



Implementation of Circular Queue Using LL:-

Queue

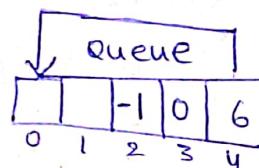


FIFO

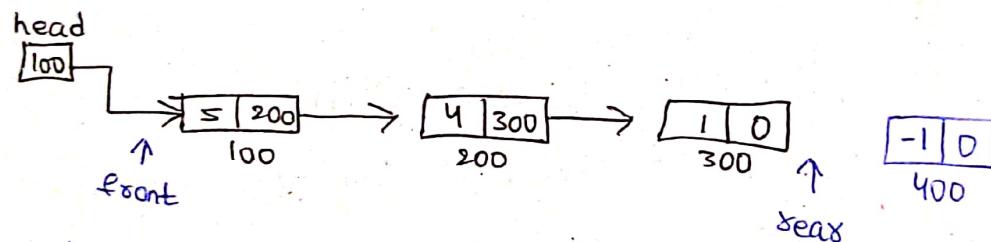
Although we have space in linear queue we can't enqueue elements, circular Queue it is possible

circular queue

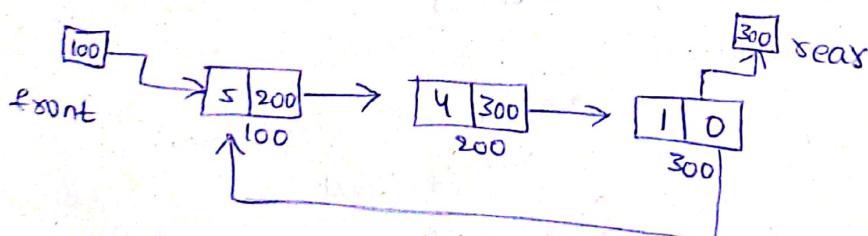
No wastage of space



Using linked list:-



Using head time complexity is $O(n) \times$
→ we are using *tail, *head it may be $O(1) \checkmark$



Code:

```
#include <stdio.h>
#include <stdlib.h>

struct node
{
    int data;
    struct node *next;
};

struct node *front=0;
struct node *rear=0;
struct node *new;
typedef struct node n;

void enqueue()
{
    new = (n *) malloc(sizeof(n));
    printf("Enter data");
    scanf("%d", &new->data);
    if (front == 0 && rear == 0)
        front=rear=new;
    else
    {
        rear->next=new;
        rear=new;
    }
    rear->next=front;
}

void dequeue()
{
    n * temp = front;
    if (front == 0 && rear == 0)
        printf("queue is empty");
}
```

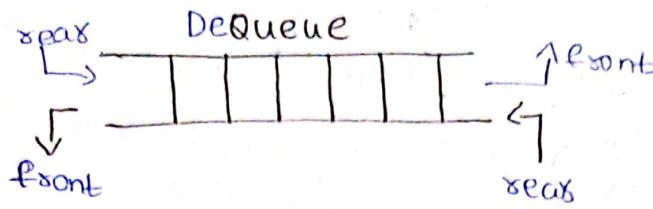
```
else if (front == rear)
{
    front = rear = 0;
    free(temp);
}

else
{
    printf("The deleted element = %d\n", front->data);
    front = front->next;
    rear->next = front;
    free(temp);
}
```

```
void peek()
{
    if (front == 0 & rear == 0)
        printf("Queue is empty\n");
    else
        printf("Peek value = %d\n", front->data);
}
```

```
void display()
{
    *temp = front;
    if (front == 0)
        printf("Queue is empty");
    else
        while (temp != rear)
    {
        printf("%d ", temp->data);
        temp = temp->next;
    }
    printf("%d\n", temp->data);
}
```

Dequeue (Double ended Queue)

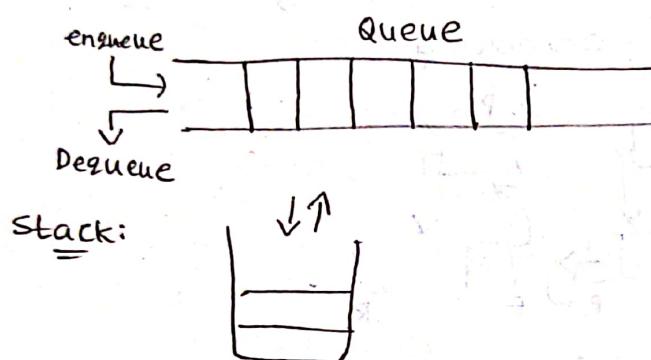


→ Insertion and deletion is possible at both ends.

Properties:

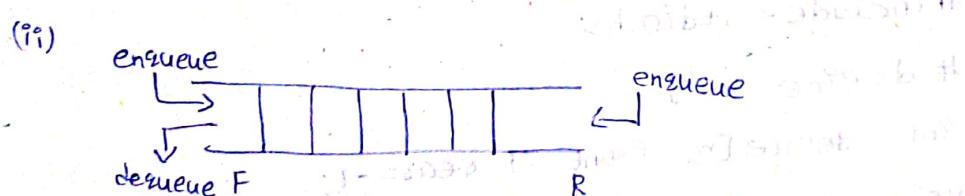
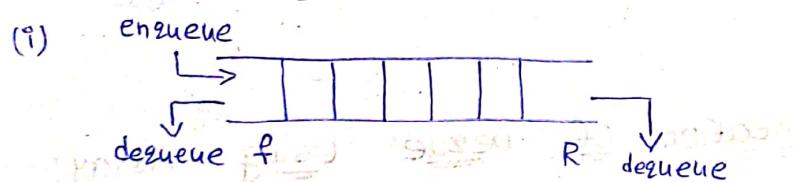
Stack - LIFO

Queue - FIFO



(i) Input-restricted

(ii) Output-restricted



Operations in Dequeue:

insert at front

delete at front

insert at rear

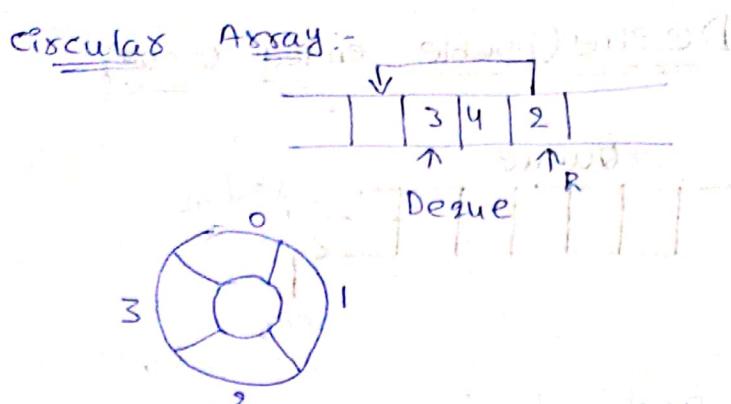
delete at rear

get front

get rear

is full() → by dequeue

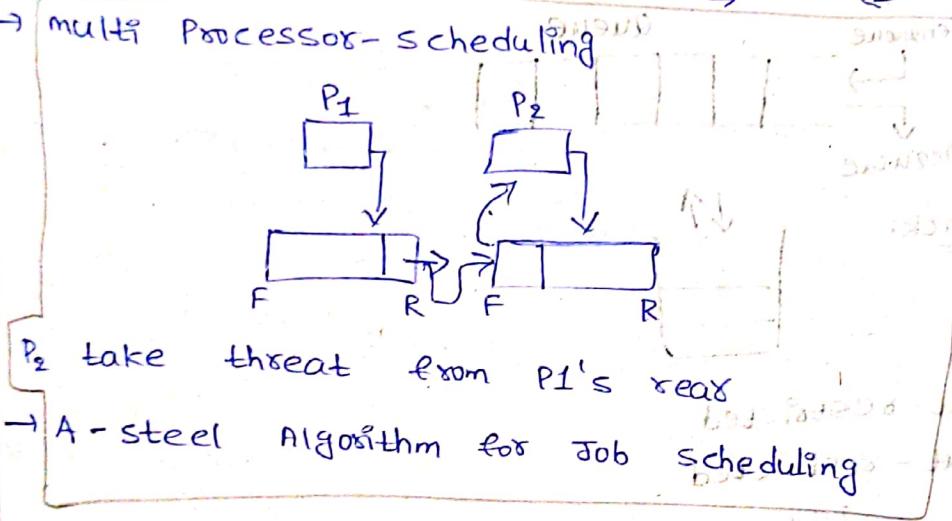
is empty() → by dequeue



Time complexity - $O(1)$

Applications:-

- used for Redo & Undo operations
- checking for Palindrome → radar
- multi Processor-scheduling



Implementation of Deque Using Array:-

```
#include<stdio.h>
#define n 5
int deque[n], front=-1, rear=-1;
void enqueuefront()
{
    int a;
    printf("enter data-");
    scanf("%d", &a);
    if((rear+1)%n == front)
        printf("overflow\n");
    else if(front == -1 && rear == -1)
        front=rear=0;
}
```

```

dequeue[front]=a;
}

else
{
    front = (front + (n-1))%n;
    dequeue[front]=a;
}

void enqueueear()
{
    int a;
    printf("Enter data-");
    scanf("%d", &a);
    if ((rear+1)%n == front)
        printf("overflow\n");
    else if (front == -1 & rear == -1)
    {
        front=rear=0;
        dequeue[rear]=a;
    }
    else
    {
        rear=(rear+1)%n;
        dequeue[rear]=a;
    }
}

void dequeuefront()
{
    if (front == -1 & rear == -1)
        printf("underFlow\n");
    else if (front == rear)
    {
        printf("Deleted element at front=%d\n", dequeue[front]);
        front=rear=-1;
    }
    else
    {
        printf("Deleted element at front=%d\n", dequeue[front]);
        front=(front+1)%n;
    }
}

```



```
void dequeueearl()
{
    if (front == -1 && rear == -1)
        printf ("Underflow \n");
    else if (front == rear)
    {
        printf ("Deleted element at rear=%d \n", deque[rear]);
        front = rear = -1;
    }
    else
    {
        printf ("Deleted element at rear=%d \n", deque[rear]);
        rear = (rear + (n - 1)) % n;
    }
}
```

```
void display()
{
    int i = front;
    if (front == -1 && rear == -1)
        printf ("Underflow \n");
    else
    {
        while (i != rear)
        {
            printf ("%d \t", deque[i]);
            i = (i + 1) % n;
        }
        printf ("%d \n", deque[i]);
    }
}
```

```
void getfront()
{
    if (front == -1 && rear == -1)
        printf ("Underflow \n");
    else
        printf ("Front=%d \n", deque[front]);
}
```

```
Void getrear() {  
    If (front == -1 && rear == -1)  
        printf("Underflow\n");  
    else  
        printf("Rear = %d \n", deque[rear]);  
}  
  
Void main()  
{  
    enqueuefront();  
    enqueuerear();  
    enqueuefront();  
    enqueuerear();  
    enqueuefront();  
    dequeuefront();  
    dequeuerear();  
    display();  
    getfront();  
    getrear();  
    dequeuefront();  
    dequeuerear();  
    dequeuefront();  
    dequeuerear();  
    display();  
}
```

Dequeue Using Linked List

```
#include <stdio.h>
#include <stdlib.h>

struct node
{
    struct node *prev;
    int data;
    struct node *next;
} *front=0, *rear=0, *new, *temp;

void create()
{
    int c=1;
    while(c==1)
    {
        new=(struct node *)malloc(sizeof(struct node));
        printf("Enter data-");
        scanf("%d", &new->data);
        new->prev=0;
        new->next=0;
        if(front==0)
            front=rear=new;
        else
        {
            rear->next=new;
            new->prev=rear;
            rear=new;
        }
        printf("Enter 1 to continue-");
        scanf("%d", &c);
    }
}

void enqueuefront()
{
    new=(struct node *)malloc(sizeof(struct node));
    printf("Enter data=");
    scanf("%d", &new->data);
```

```

new->prev=0;
new->next=front;
front->prev=new;
front=new;
}

void enqueueear()
{
    new = (struct node *)malloc(sizeof(struct node));
    printf("Enter data=");
    scanf("%d", &new->data);
    new->next=0;
    rear->next=new;
    new->prev=rear;
    rear=new;
}

void dequeefront()
{
    if (front==0)
        printf("Queue is empty");
    else
    {
        printf("dequeued element=%d\n", front->data);
        temp=front;
        front=front->next;
        front->prev=0;
        free(temp);
    }
}

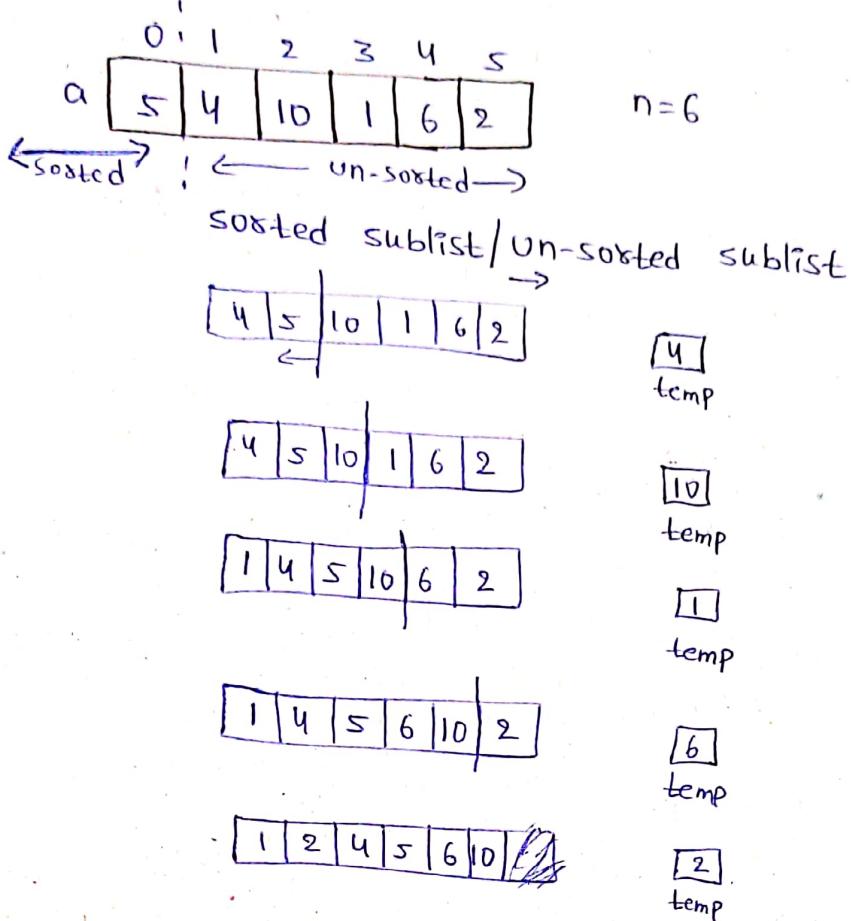
void dequeear()
{
    if (front==0)
        printf("Queue is empty\n");
    else
    {
        printf("dequeued element=%d\n", rear->data);
        temp=rear;
        rear=rear->prev;
        rear->next=0;
        free(temp);
    }
}

```

```
Void display()
{
    temp = front;
    while (temp != 0)
    {
        printf ("%d\n", temp->data);
        temp = temp->next;
    }
}
```

```
Void main()
{
    create();
    enqueuefront();
    enqueuerear();
    dequeuemax();
    dequeuemax();
    dequeuemax();
    dequeuemax();
    display();
}
```

Insertion sort:



why this is known as 'Insertion sort'?
 we are taken one value from Un-sorted sublist
 and we have inserted that value in sorted
 sublist, that's why this is insertion sort.

```
for (i=1; i<n; i++)
  {
```

```
    temp = a[i];
```

```
    j = i - 1;
```

```
// creating loop to reach 0th index
```

```
    while (j >= 0 && a[j] > temp)
      {
```

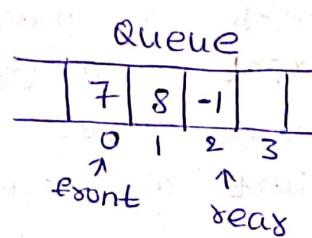
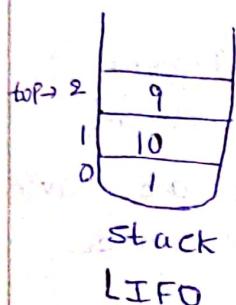
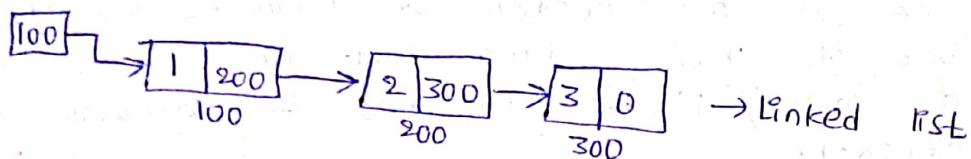
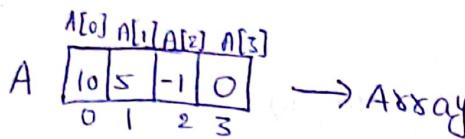
```
        a[j+1] = a[j];
```

```
        j--;
```

```
    a[j+1] = temp;
```

```
}
```

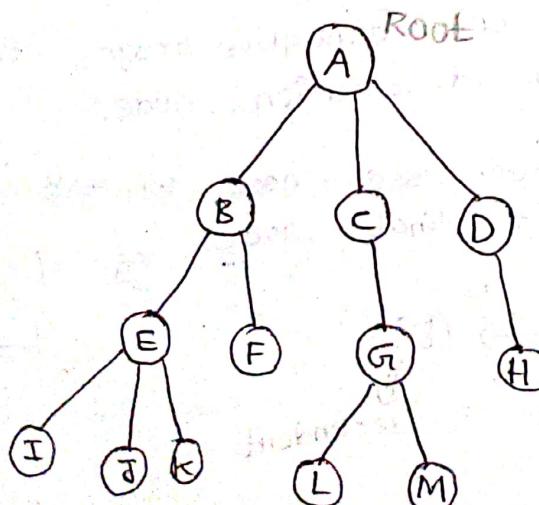
Data structures



FIFO

Non-Linear data structure

- Trees, Graphs are considered as Non-linear data structure.
 - It doesn't have sequence, it have multiple levels.
 - Data is in the form of hierarchy.
- eg:- college Management.



→ Data items which have hierarchy relations among them.

* Tree is a non-linear data structure which is going to represent this hierarchy.

→ Tree grows from top to bottom. We can go from B to A.

Root = A

nodes = B C D E F G (Having information)

⇒ Tree is a collection of elements (or) nodes (bcz it having data and link so, it called as node.) linked together to stimulate a hierarchy.

Terminology:-

The node which is having any parent is called as root node. (or) Top most element

Node - which stores the information and having data part and link to next node.

Parent - immediate predecessor ^{is any node} is called Parent. 'B' is the parent of E & F.

Child - immediate successor is called child node. B, C, D are children of A.

Leaf node - The node which doesn't have child I, J, K, L, M, F are leaf nodes. also called as external node.

Non-leaf node - which is having atleast one child A, B, C, D, E, G

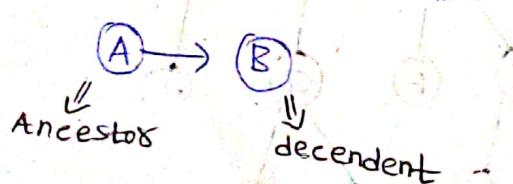
Internal nodes.

Path - sequence of connective edges from source node to destination node.

Ancestors - any predecessor node on the path from root to that node.

Eg: L = A, C, G

H = A, D



Internal node - A node with children

Descendent - Any successor node on the path from that node to the leaf node.

$$\text{Ex: } C = G, L, M$$

$$B = E, F, I, J, K$$

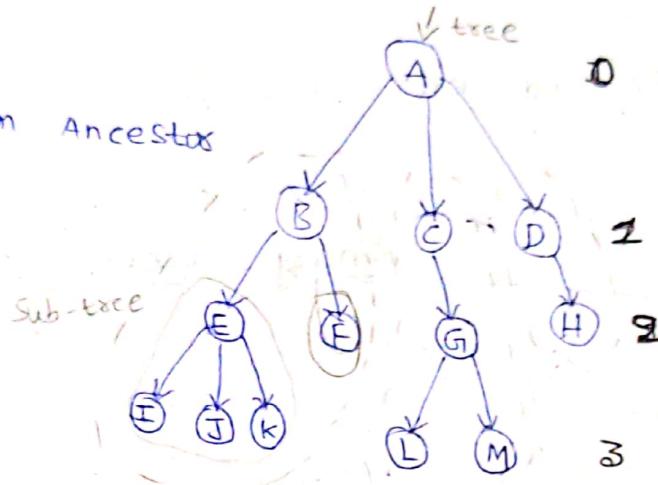
Q. what is the common ancestor of G and H?

$$G - C, A$$

$$H - D, A$$

'A' is common Ancestor

(0, 1, 2, 3 → Level)



Q. common descendent of C and D.

$$C - G, L, M$$

$$D - H$$

so, There is no common descendent for C and D.

Sub-tree - which is contains an node of this tree T and all its descendants is called as sub-tree.

Sibling - All children of same parent.

E, F are siblings

F, G are cousins not siblings.

Degree - The no. of children of that node

$$\text{degree of } A = 3, E = 3$$

$$\text{ " " } M = 0$$

For all leaf nodes it is '0'.

Tree degree = highest degree of any node.

Depth of node - Length of Path from root to that node.

$$F = 2 \quad D = 1 \quad \underline{\text{Root node}} \quad A = 0 \\ J = 3$$

height of node - no. of edges in the longest path from that node to leaf node
 $B = 2$, not 1

→ Height and depth of tree may be same.

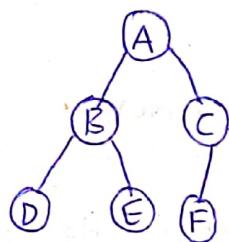
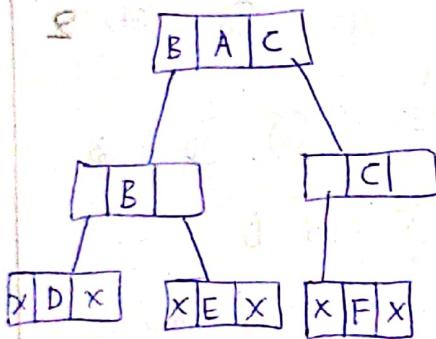
height of root node = height of tree

level of node = No. of edges from root to given node

Level of tree = level height of tree

To find edges: $(n-1)$ edges

Binary Tree



Memory Allocation:

struct node

{ int, float, char data;

struct node * left;

struct node * right;

disadv of array
static memory

Applications:

- It stores Hierarchical data, implements file system.
- Routing protocols.
- It organizes data for quick search, insertion and deletion i.e., Binary tree, heap tree etc.

depth of root node, height of leaf node is always "0".

→ Height and depth of node may or may not be same.

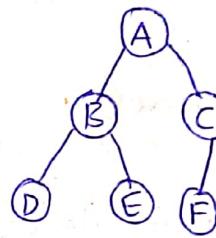
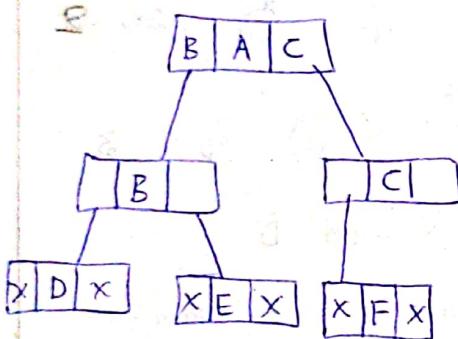
height of root node = height of tree

Level of node - No. of edges from root to given node

Level of tree = level height of tree

To find edges: $(n-1)$ edges

Binary Tree



Memory Allocation:

struct node

```
int, float char data;  
struct node *left;  
struct node *right;
```

Disadv of Array
static memory

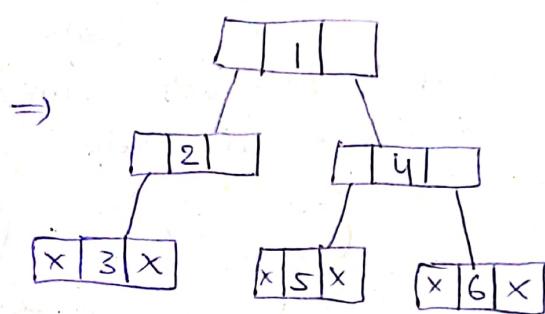
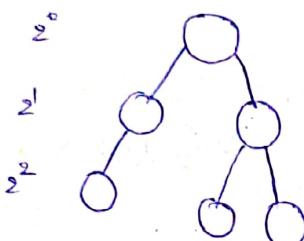
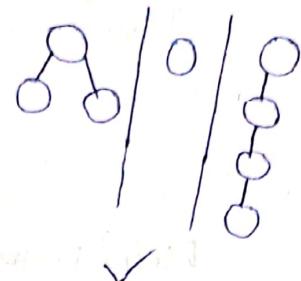
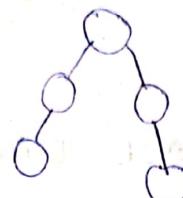
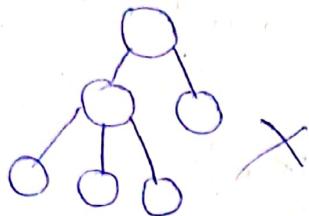
Applications:

- It stores hierarchical data, implements file system.
- Routing protocols.
- It organizes data for quick search, insertion and deletion i.e., Binary tree, heap tree etc.
- depth of root node, height of leaf node is always "0".

6.2

Binary tree & its types

→ It is a tree in which having each node (atmost) max of 2 children. i.e., having 0, 1, 2 children.



$$\text{Max possible nodes at a level} = 2^h$$

$$\begin{aligned} \text{Max no. of nodes of height } h &= 2^{h+1} - 1 \\ \text{Min no. of nodes of height } h &= 2^h \end{aligned}$$

Q. Find the max height of tree?

$$\text{Max height} =$$

$$\text{Min height} =$$

$$h = 2^{h+1} - 1$$

$$n+1 = 2^{h+1}$$

$$\log_2(n+1) = \log_2 2^{h+1}$$

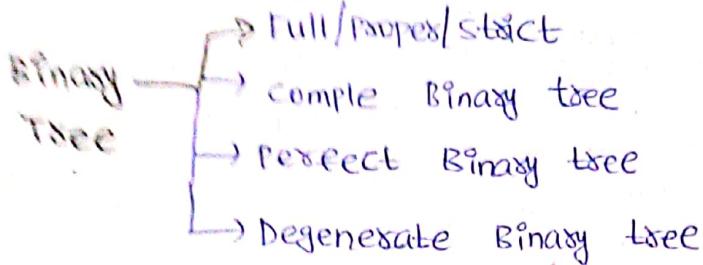
$$\log_2(n+1) = h+1$$

$$h = [\log_2(n+1) - 1] \rightarrow \text{min}$$

$$n = h+1$$

$$h = n-1 \rightarrow \text{max}$$

Types of Binary Trees:-

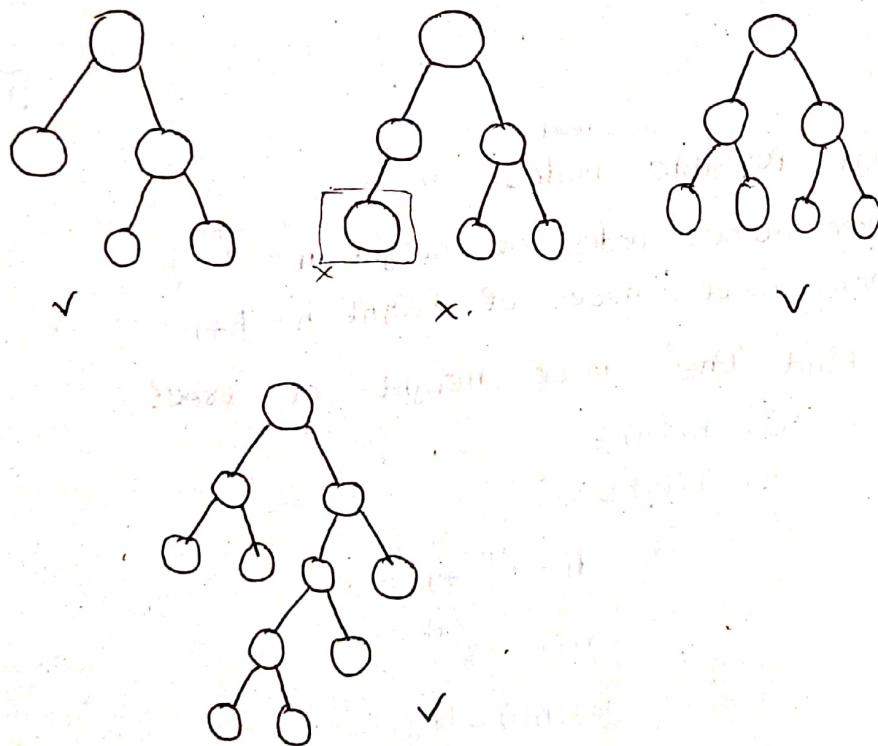


Full/Proper/Strict Binary Tree:-

→ It is binary tree where each node contains
0 or 2 children.

(or)

Each node will contain exactly 2 children
except leaf node.



$$\text{No. of leaf nodes} = \text{no. of internal nodes} + 1$$

$$6 = 5 + 1$$

$$6 = 6$$

$$\text{Max. no. of nodes} = \text{same as binary tree}$$

$$2^{h+1} - 1$$

$$\text{Min. no. of nodes} = \frac{h+1}{2}$$

$$\text{Min. height} = [\log_2(n+1)] - 1$$

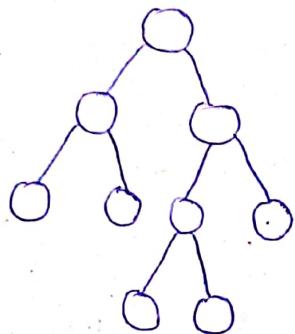
$$\text{Max height} = \left(\frac{n-1}{2}\right)$$

$$n = 2h+1$$

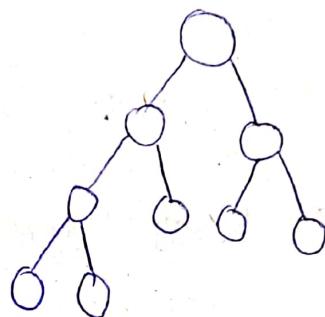
$$h = \frac{n-1}{2}$$

complete Binary Tree :-

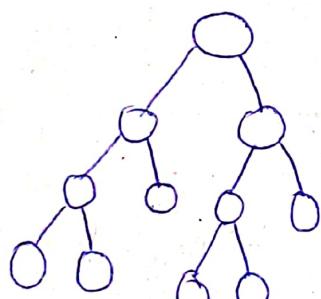
→ If a tree is a complete binary tree if all the levels are completely filled (except possibly the last level has nodes as left as possible)



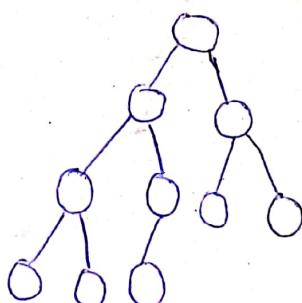
X



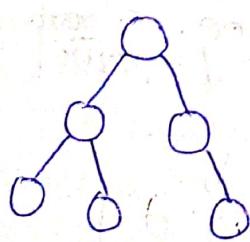
✓



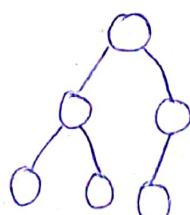
X



✓



X



✓

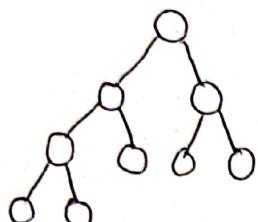
$$\text{Max nodes} = 2^{h+1} - 1$$

$$\text{min nodes} = 2^h$$

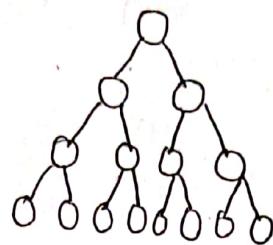
$$\text{Max height} = \lceil \log_2(n+1) \rceil - 1 \quad \text{min height} = \log n$$

Perfect Binary Tree:

- A tree can be a perfect binary tree if
- all the internal nodes having 2 children
 - all the leaf nodes are at same level.



X

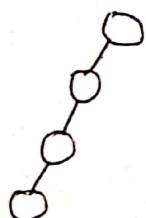


✓

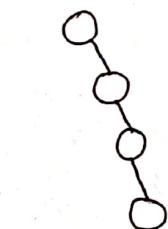
Every perfect binary tree can be full, complete binary tree. vice versa not possible.

Degenerate Binary Tree:

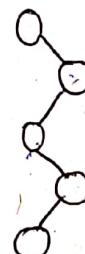
All the internal nodes have $\frac{1}{2}$ children & leaves are at same level.



Left-skewed BT



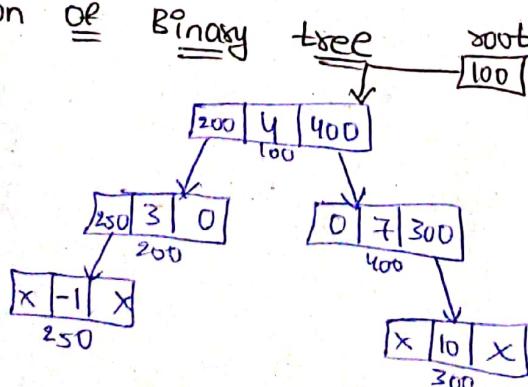
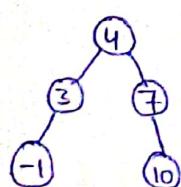
Right-skewed BT



✓ mixture of left, right skewed BT

5.3

Implementation of Binary tree



Contains 0, 1, 2 any can possible.

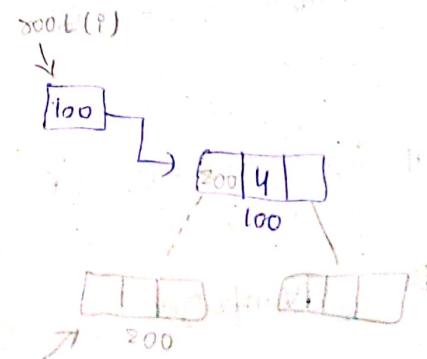
code:

```
#include <stdio.h>
#include <stdlib.h>

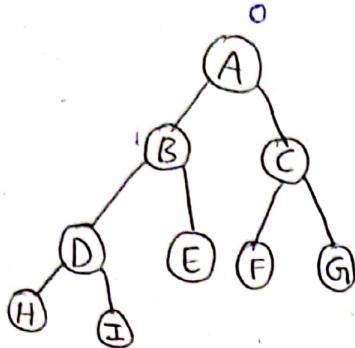
Struct node
{
    int data;
    struct node *left, *right;
};

x { // int create()
    {
        struct node *create()
        {
            int x;
            struct node *newnode;
            newnode=(struct node *) malloc(sizeof(struct node));
            printf("Enter data"); //for empty node
            scanf("%d", &x);
            if (x == -1)
            {
                return 0;
            }
            newnode->data = x;
            printf("Enter left child of %d", x);
            newnode->left = create();
            printf("Enter right child of %d", x);
            newnode->right = create();
            return newnode;
        }
    }
}

Void main()
{
    struct node *root;
    root = 0;
    root = create();
}
```



5.4 Array Representation of Binary Tree



A	B	C	D	E	F	G	H	I
0	1	2	3	4	5	6	7	8

case-I

A	B	C	D	E	F	G	H	I
1	2	3	4	5	6	7	8	9

case-II

case-(i)

If a node is at i^{th} index:-

→ left child would be at $(2 \times i) + 1$

→ right child would be at $(2 \times i) + 2$

→ Parent would be at $\left\lfloor \frac{i-1}{2} \right\rfloor$ → floor value

case-(ii)

If a node is at i^{th} index:

left child at $(2 \times i)$

right child at $= [(2 \times i) + 1]$

Parent at $\left[\frac{i}{2} \right]$

→ Binary Tree must be complete Binary tree
nodes must fill from left to right.

Having atmost one two children

Dis-advantage:

→ wastage of memory

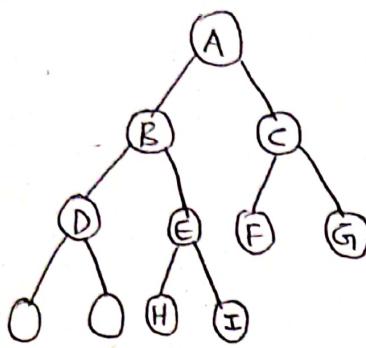
→ static memory allocation

Advantage:

easy to implement

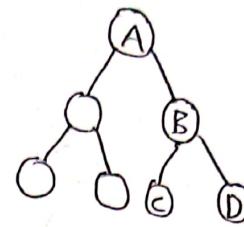
easy to traverse

Q. Represent the following Binary Tree in Array form.



A	B	C	D	E	F	G	-	-	H	I
0	1	2	3	4	5	6	7	8	9	10

A	B	C	D	E	F	G	-	-	H	I
1	2	3	4	5	6	7	8	9	10	11

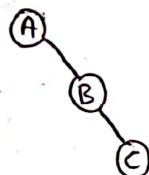


A	-	B	-	-	C	D
0	1	2	3	4	5	6

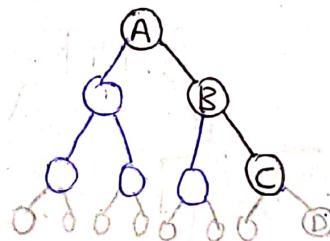
A	-	B	-	-	C	D
1	2	3	4	5	6	7

If H, I can placed at 7,8 (or) 8,9 positions then above formulas will not be applicable.

Q. Write array form:



Not complete B.T
Not possible



A	-	B	-	-	-	C
0	1	2	3	4	5	6

If the given tree is not complete B.T then, you have to make it as CBT using empty nodes.

A	-	B	-	-	-	C	-	-	-	-	D
0	1	2	3	4	5	6	7	8	9	10	11

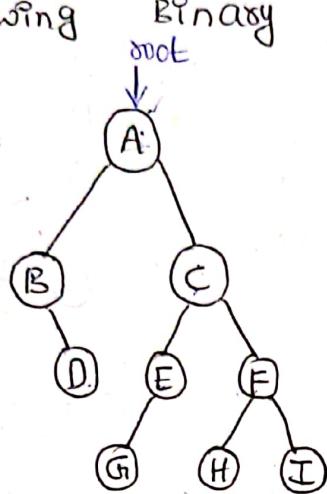
5.5 / Binary Tree Traversal

Inorder - Left Root Right

Pre-order - Root Left Right

Post-order - Left Right Root

Q. Write Inorder, Preorder and Post-order to the following Binary tree.



left, root, right

In-order = B D A G E C H F I

root, left, right

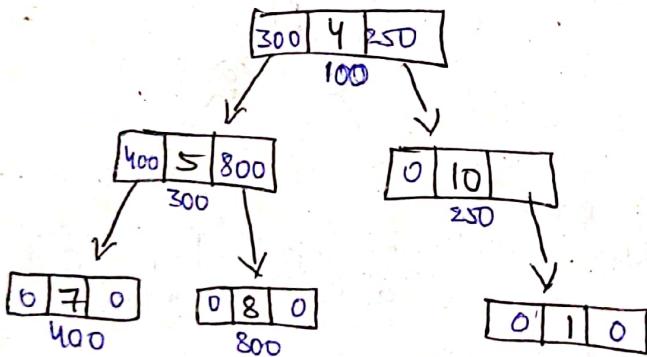
Pre-order = A B D C E G F H I

left, ~~root~~, right, root

Post-order = ~~E D A~~ D B G E H I F C A

Q. write Program for shown Tree.

5.6



In-order = left root right

7 5 8 4 10 1

Pre-order = root left right

4 5 7 8 10 1

Post-order = left right root

7 8 5 1 10 4

```

void main()
{
    struct node *root,
    printf ("Pre-order is-");
    Preorder (root);

}

void Preorder (struct node *root)
{
    if (root==0)
    {
        return;
    }
    printf ("%d", root->data);
    Preorder (root->left);
    Preorder (root->right);
}

void Postorder (struct node *root)
{
    if (root==0)
    {
        return;
    }
    Postorder (root->left);
    Postorder (root->right);
    printf ("%d", root->data);
}

void Inorder (struct node *root)
{
    if (root==0)
    {
        return;
    }
    Inorder (root->left);
    printf ("%d", root->data);
    Inorder (root->right);
}

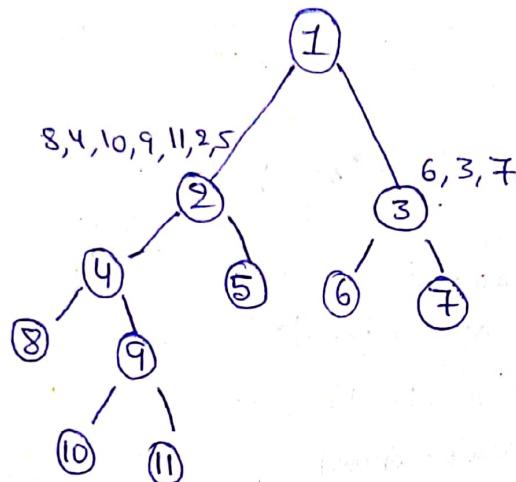
```

5.7

construct a Binary tree from Preorder & Inorder.

Preorder - 1, 2, 4, 8, 9, 10, 11, 5, 3, 6, 7 (R L right)

Inorder - $\xrightarrow{8, 4, 10, 9, 11, 2, 5, 1, 6, 3, 7}$ (L Root Right)

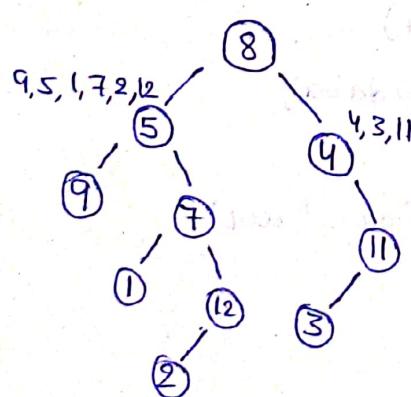


5.8

construct a Binary tree from Postorder & Inorder

Postorder - 9, 1, 2, 12, 7, 5, 3, 11, 4, 8 (L Right Root)

Inorder - 9, 5, 1, 7, 2, 12, 8, 4, 3, 11 (L Root Right)



5.10

Binary search Tree

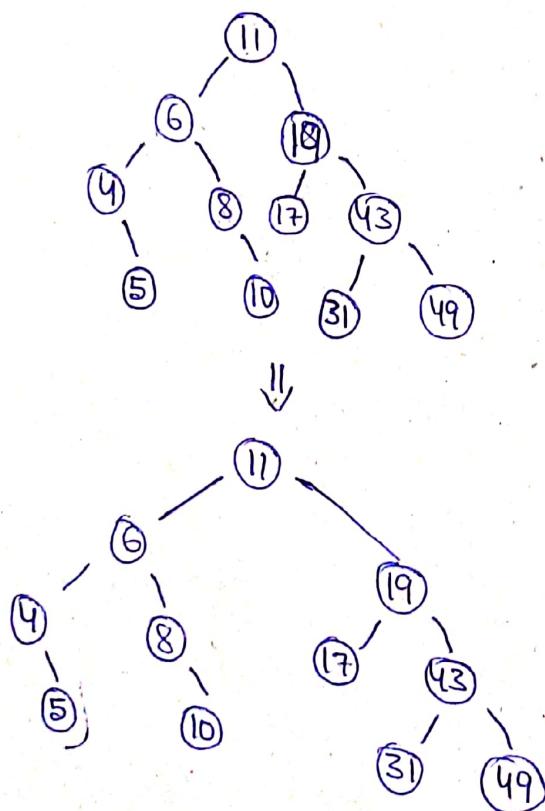
* It having atmost of two children

Eg:- having 0, 1 (or) 2 children.

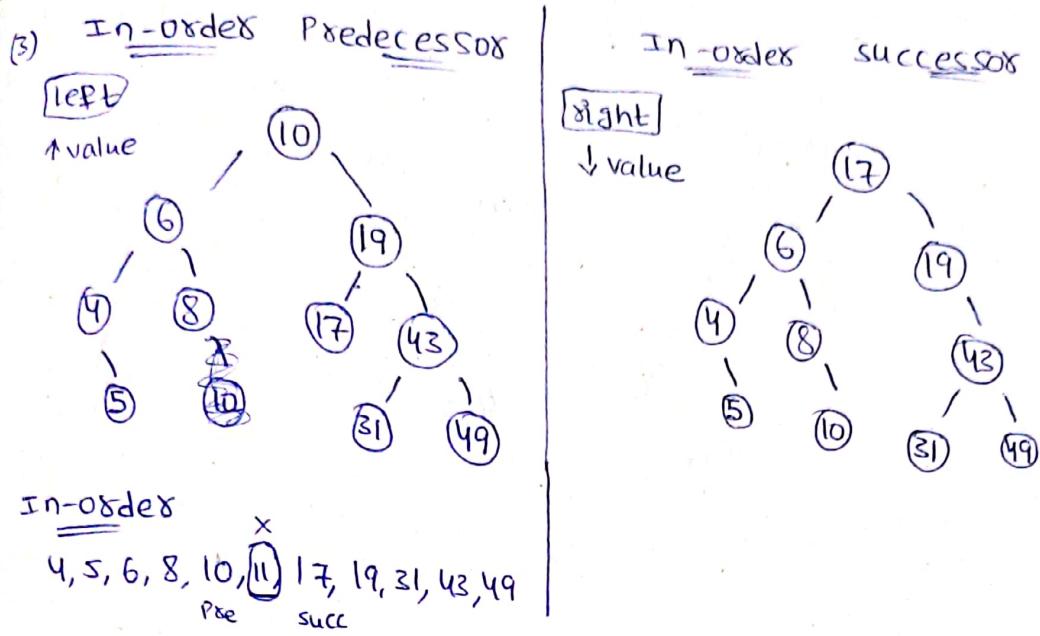
* Left sub-tree of any node having less values whereas right sub-tree of high values. to the root node

Draw Binary search tree by inserting the following numbers from left to right.

11, 6, 8, 19, 4, 10, 5, 17, 43, 49, 31

Deletion

- | | |
|--|--|
| ① 0 child
② 1 child
③ 2 children | → In the left we have to swap the highest number
→ In the right we have to swap the lowest number |
|--|--|
- (i) In-order Predecessor
 (ii) In-order Successor

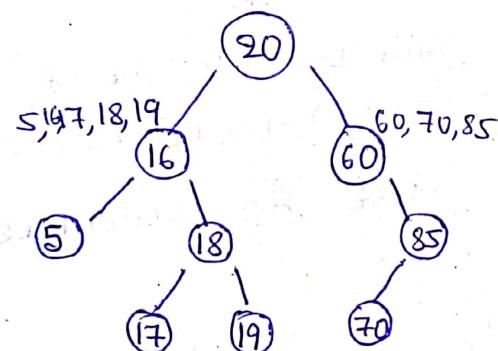


5.11

construct a Binary search tree from pre-order.

Pre-order = 20, 16, 5, 18, 17, 19, 60, 85, 70 (Root-left-right)

In-order = 5, 16, 17, 18, 19, 20, 60, 70, 85 (left-root-right)

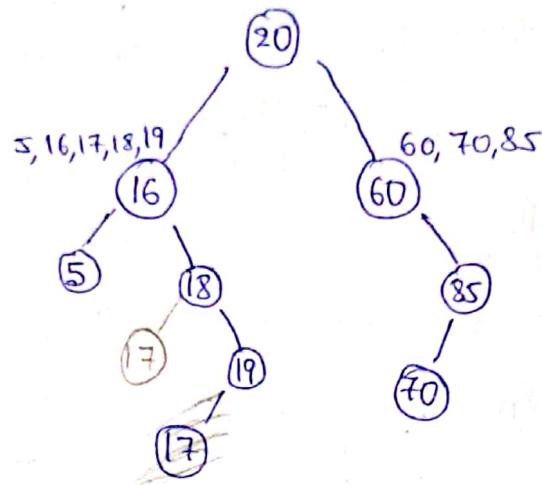


construct a Binary search tree from post-order.

Post-order = 5, 17, 19, 18, 16, 70, 85, 60, 20 (Left-right-root)

In-order = 5, 16, 17, 18, 19, 20, 60, 70, 85



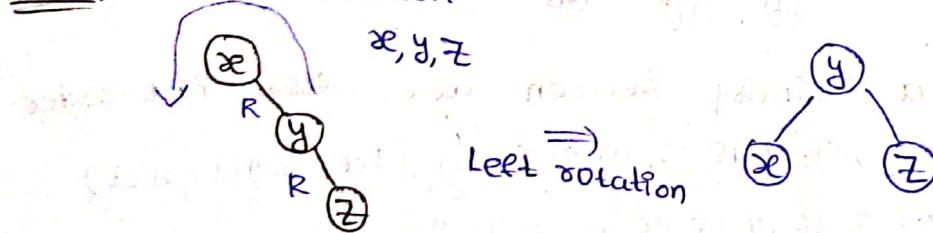


5.13

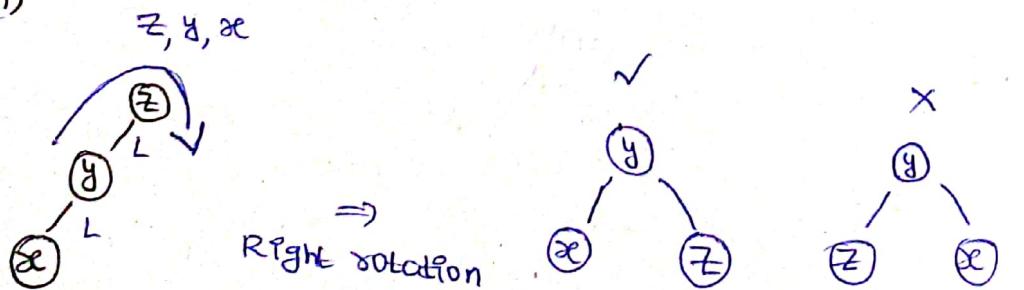
AVL Tree

- * AVL means Adelson Velsky and Landis tree - Jim Oliver
- self-balancing BST
- It is a BST, self balancing BST
- For each node in the tree:
 - height of left sub-tree - height of right sub-tree
= { -1, 0, 1 } → Balance factor
- Balance factors must be calculated for every level.

case-(i) Right rotation



case-(ii)

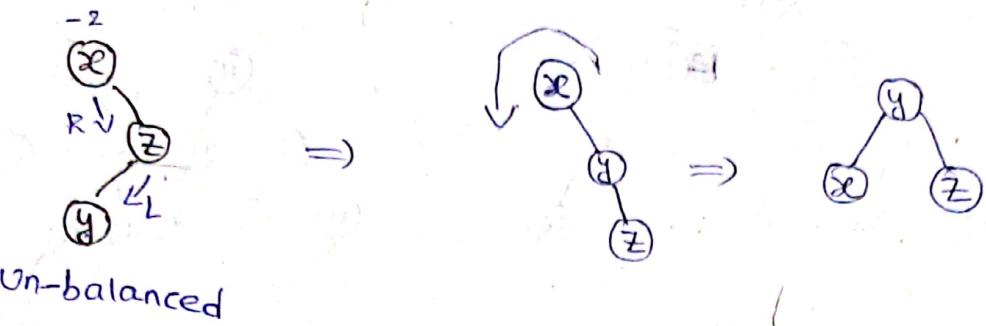


(Bcz it's an BST)

case-(iii)

x, z, y

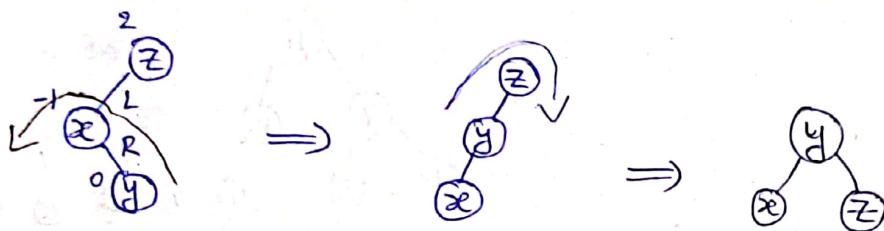
Right-left rotation.



case(iv)

Left-right rotation

z, x, y



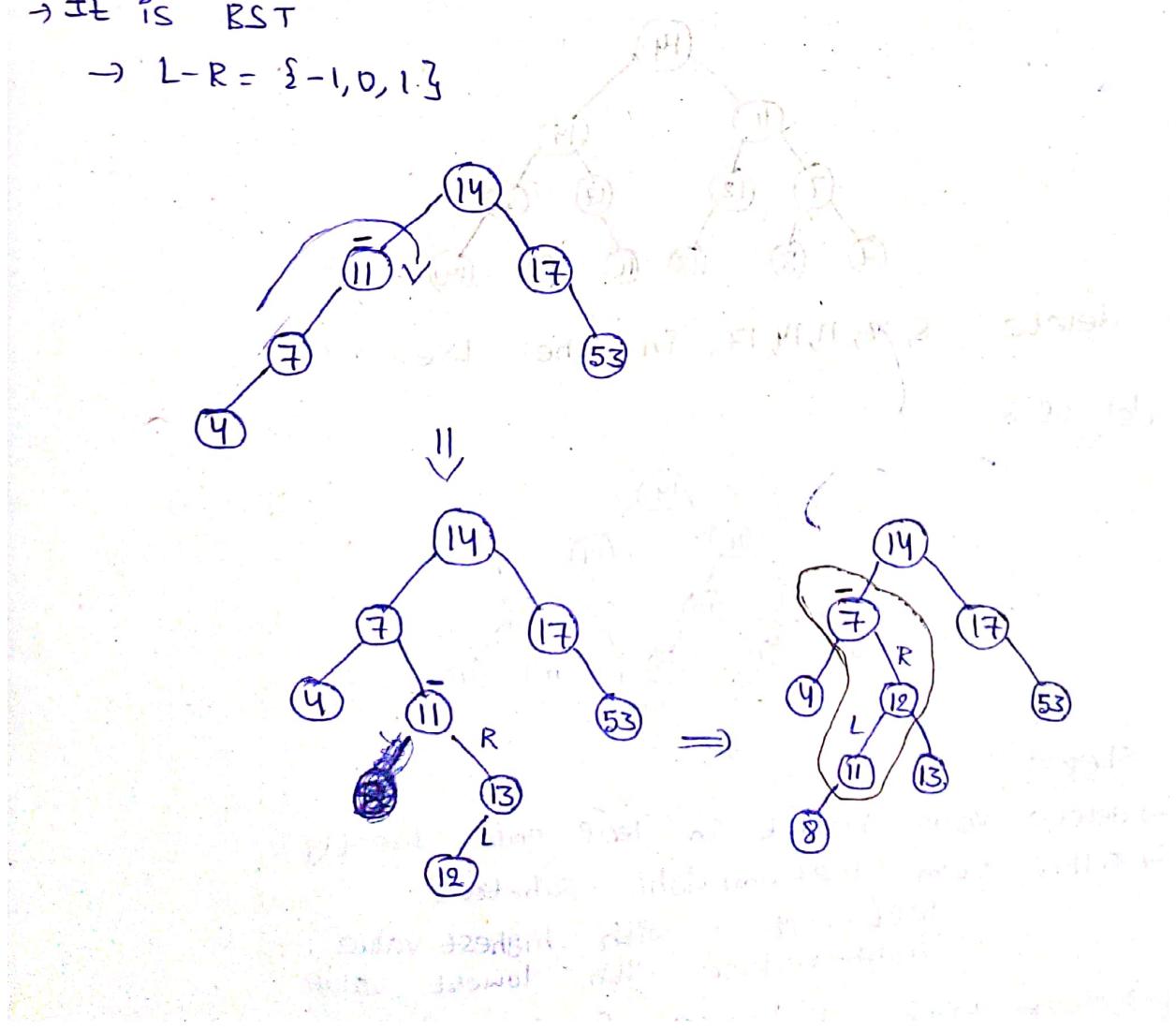
5.14

construct AVL tree by inserting the data:

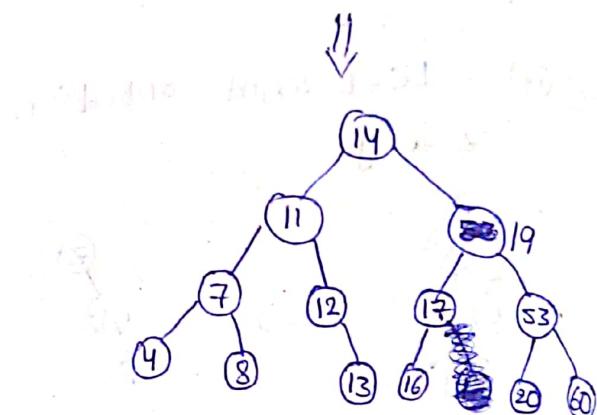
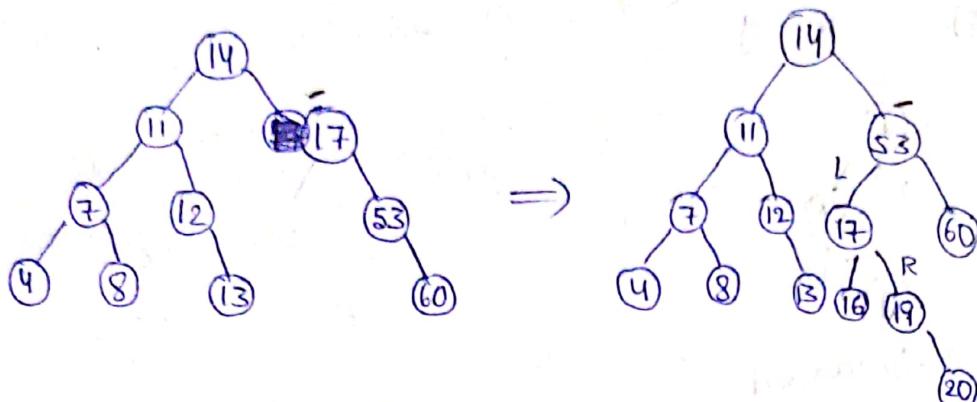
14, 17, 11, 7, 53, 4, 13, 12, 8, 60, 19, 16, 20

→ It is BST

→ L-R = {-1, 0, 1}

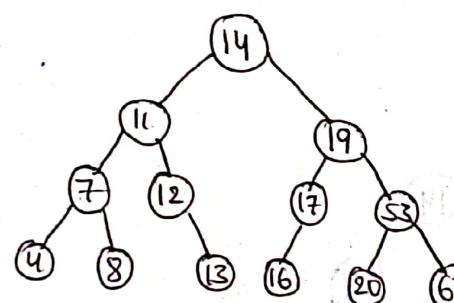


Scanned with OKEN Scanner



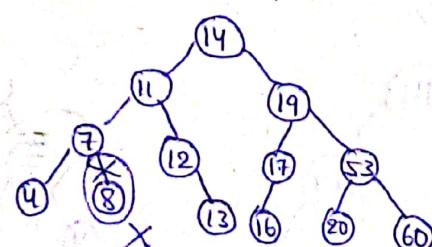
5.15

Deletion in AVL Tree



delete 8, 7, 11, 14, 17 in the tree.

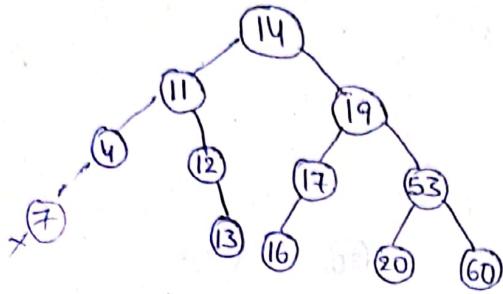
del of 8



steps:

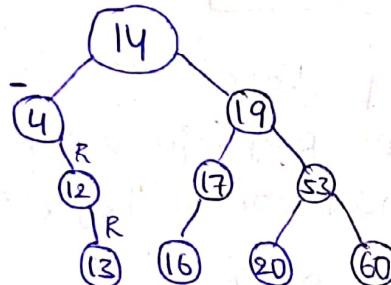
- delete value present in leaf node directly
- Either from left (or) right sub-tree
 - left-replace with highest value
 - right-replace with lowest value
- start with balance factor

del 7

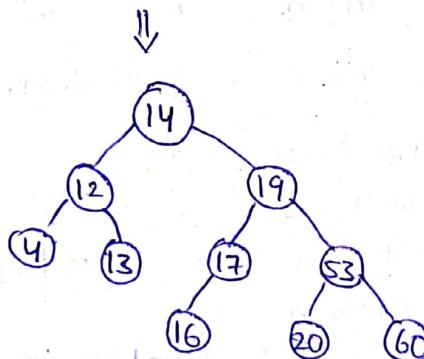


del 11

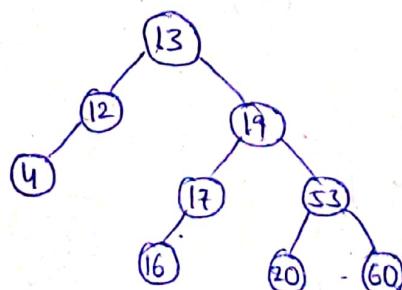
It have two children from left or right you will delete either sub-tree.
I'm taking left.



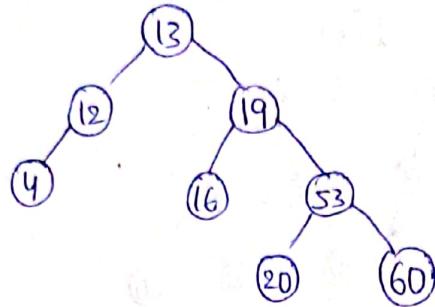
'4' is the critical node which is not balanced



del 14



del 17

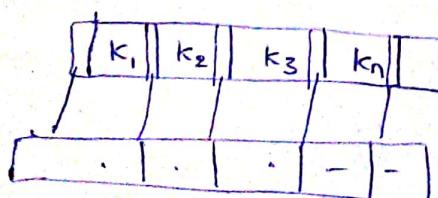


5.23

B-Trees

- * balanced m^{order} way tree
- * Generalization of BST in which a node can have more than one key & more than 2 children
- * maintains sorted data.
- * All leaf nodes must be at same level.
- * B-tree of order m has ~~max~~ m children
 - Every node has max m children.
 - min children - leaf $\rightarrow 0$
 - root $\rightarrow 2$
 - internal nodes $\rightarrow \lceil \frac{m}{2} \rceil \rightarrow$ ceiling value
 - Every node has max $(m-1)$ keys
 - min keys - root node $\rightarrow 1$
 - all other nodes $\rightarrow \lceil \frac{m}{2} \rceil - 1$

$m=5$



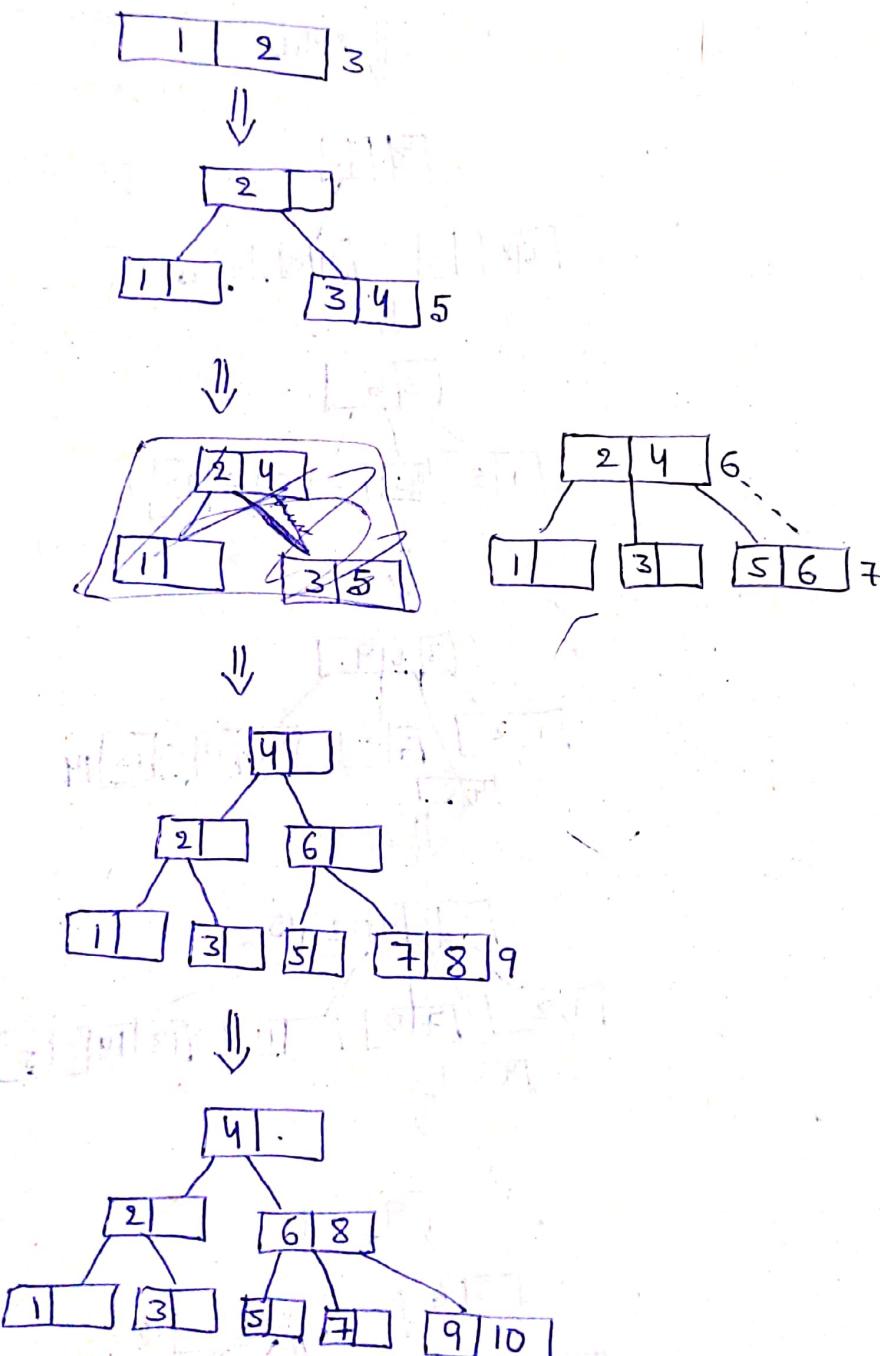
⇒ Insertion possible at only leaf node.

5.24

Create a B-tree of order-3 by inserting values from 1 to 10.

$$m = 3$$

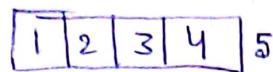
$$\text{keys} = 3-1 = 2$$



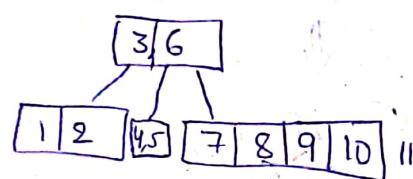
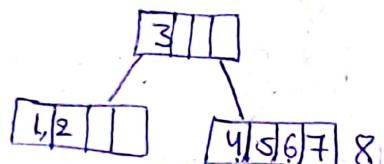
5.25

Create a B-tree of Order 5 by inserting values from 1 to 20.

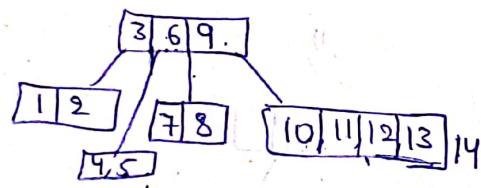
$$m = 5, \text{ keys} = 5 - 1 = 4$$



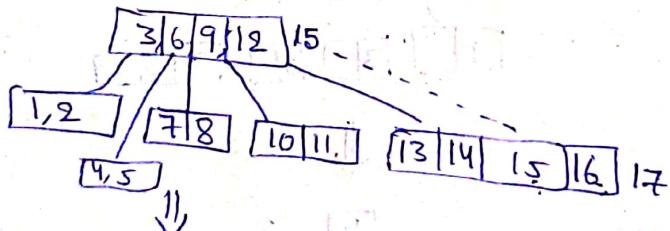
↓ split



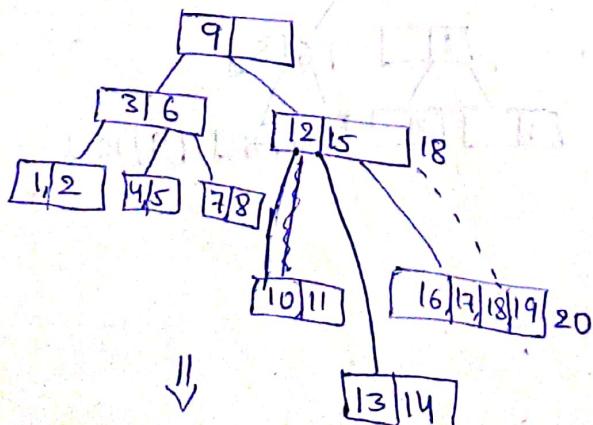
↓ split

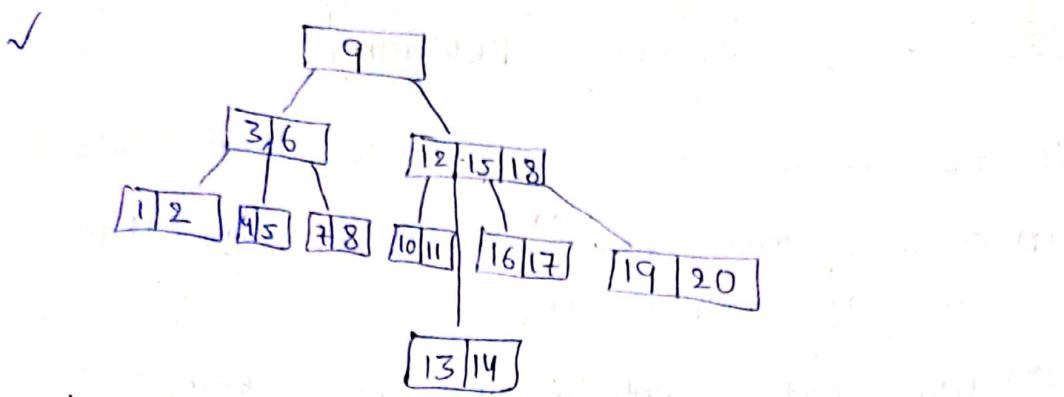


↓



↓

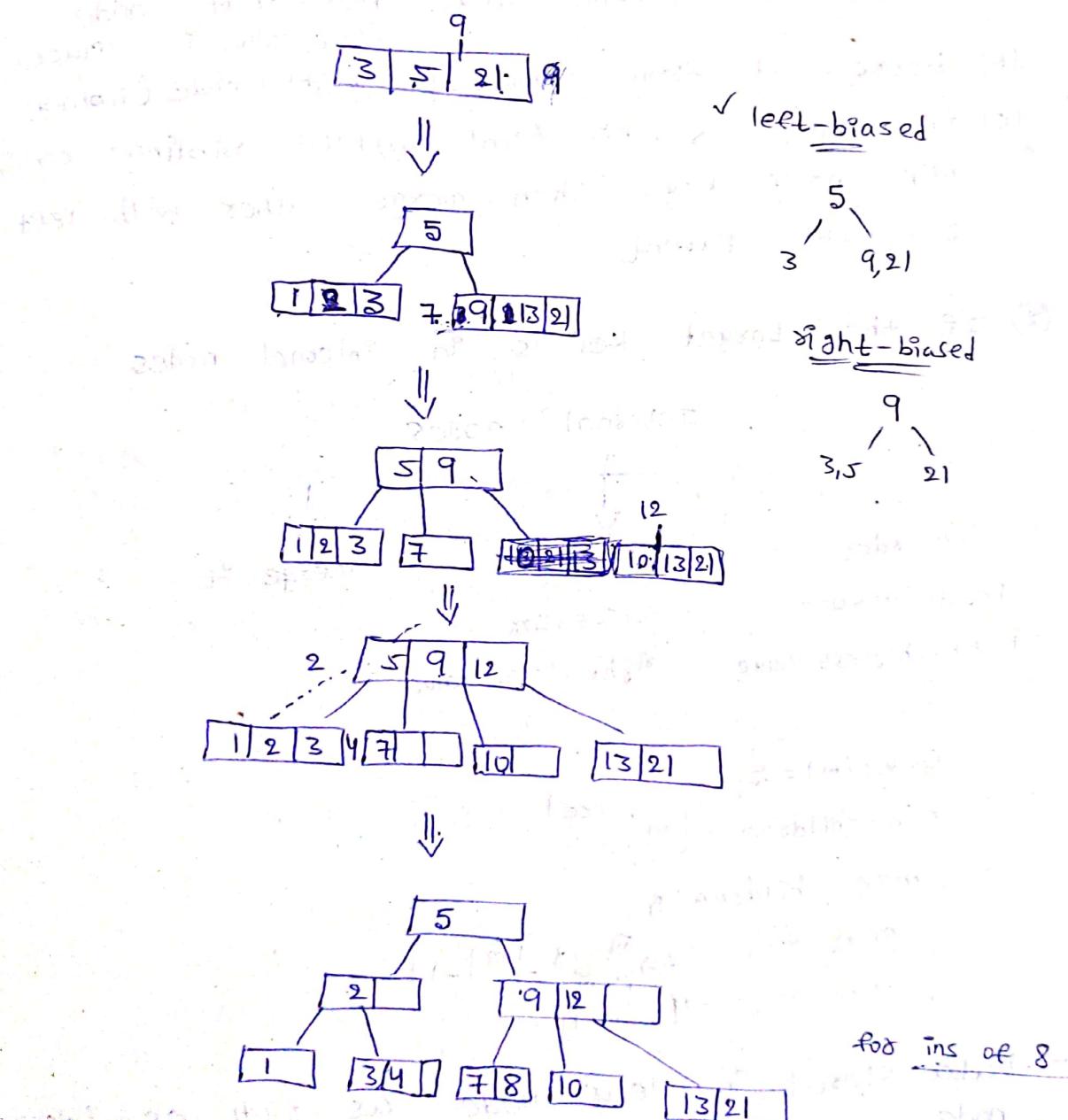




5.27

construct B-tree of order 4 by inserting
5, 3, 21, 9, 1, 13, 2, 7, 10, 12, 4, 8

$m=4$, keys = 3



even order \rightarrow 2 possible trees

- (i) left-biased
- (ii) right-biased

5.28

B-Tree Deletion

① If target key is in the leaf node

(i) contains (~~more than~~ ^{max} _{min} no.of) keys then directly delete it.

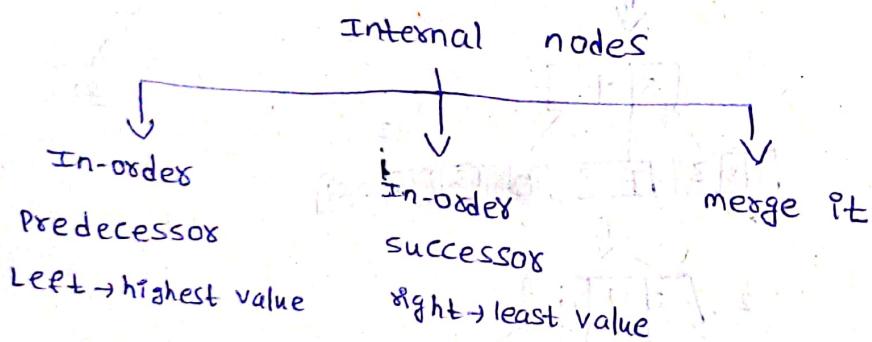
(ii) Leaf node contains min no.of keys

(a) borrow from immediate ^{highest value is replaced} left sibling node. Then take it from left and put it into the parent and parent value into that node.

(b) borrow it from immediate ^{lowest value is replaced} right node (sibling)

(c) if there is no right (or) left siblings containing min no.of keys then merge either with left (or) right parent.

② If the target key is in internal nodes



$\text{Order}(m) = 5$

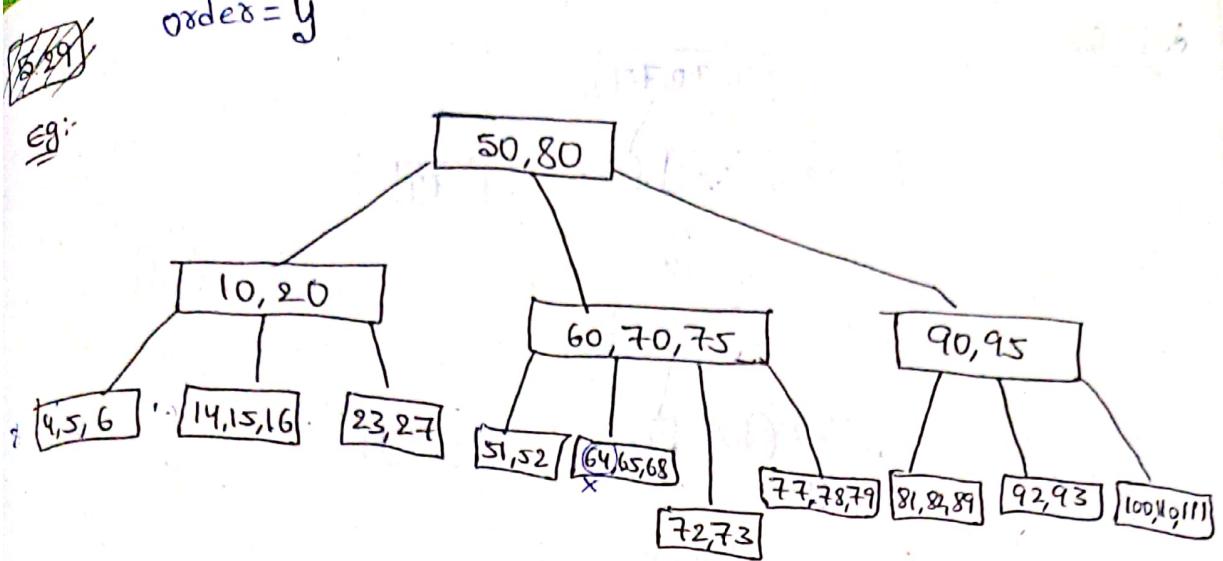
$$\text{min children} = \left\lceil \frac{m}{2} \right\rceil = 3$$

$$\text{max children} = 5$$

$$\text{min keys} = 2 \rightarrow \left\lceil \frac{m}{2} \right\rceil - \left(\left\lceil \frac{m}{2} \right\rceil - 1 \right)$$

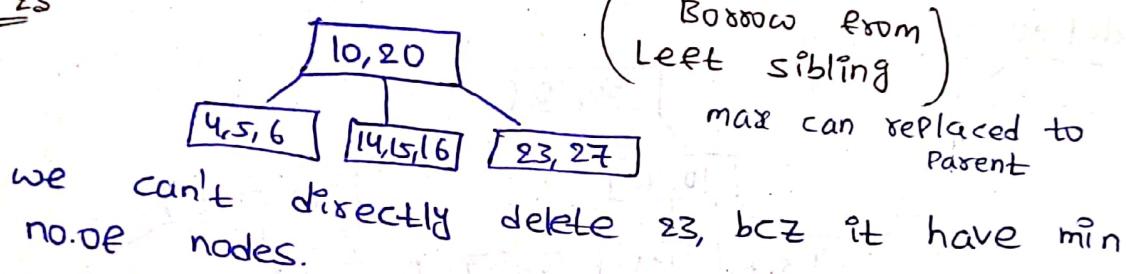
$$\text{max keys} = 4 \quad (m-1)$$

→ Data stored in leaf node as well as internal node.



del 64: 64 is a leaf node, having max keys, then directly delete 64

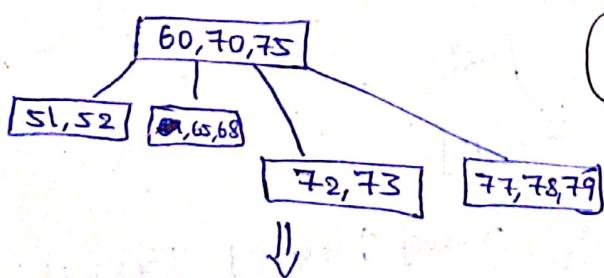
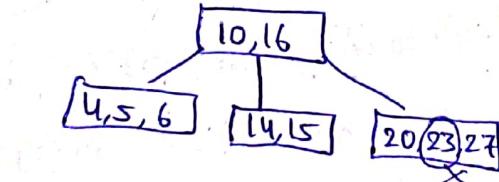
del 23



→ Borrow element from left sibling, cz it have more than min no.of keys

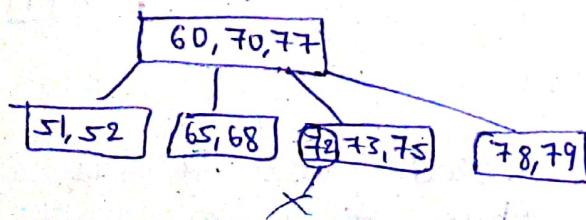
→ can't directly taken, max value is transferred to parent and common for both will come down.

del 72

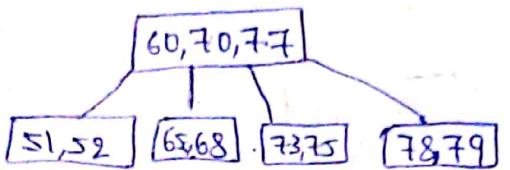


(Borrow from left sibling)

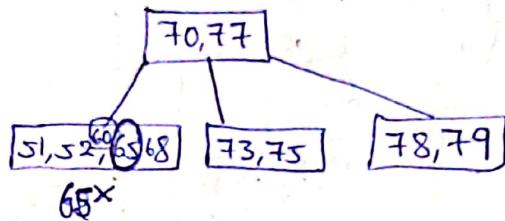
min can be replaced to parent



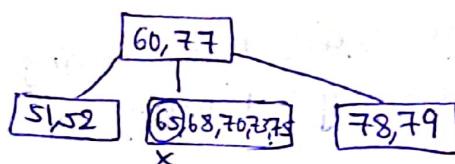
del 65



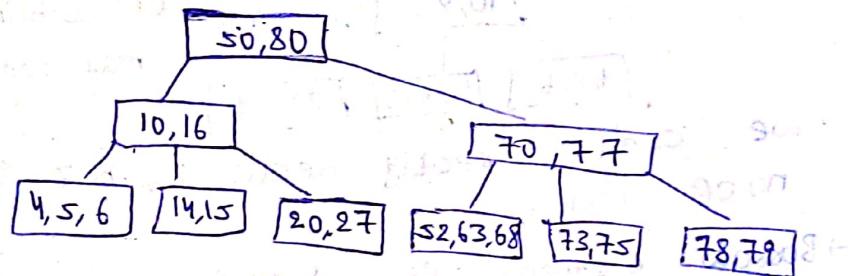
(merging)



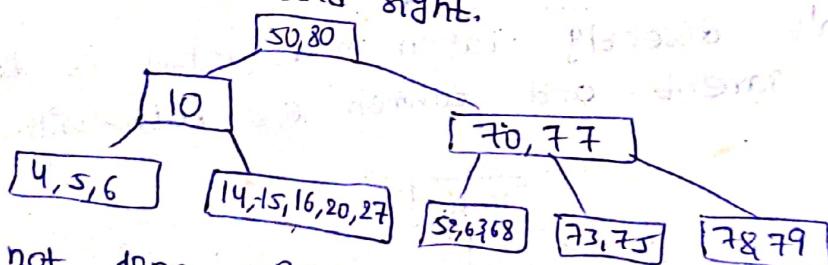
(o)



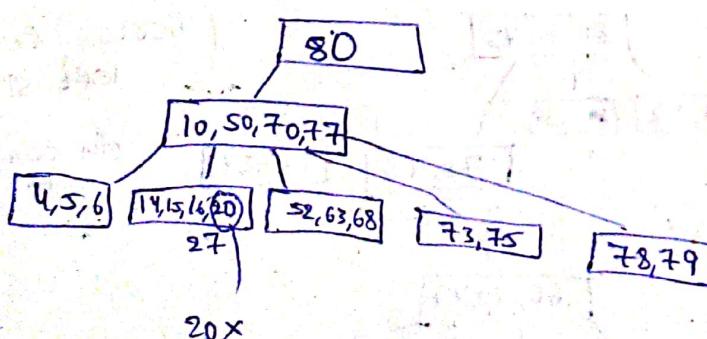
del 20



→ consider Parent node if there only min no. of nodes from left and right.

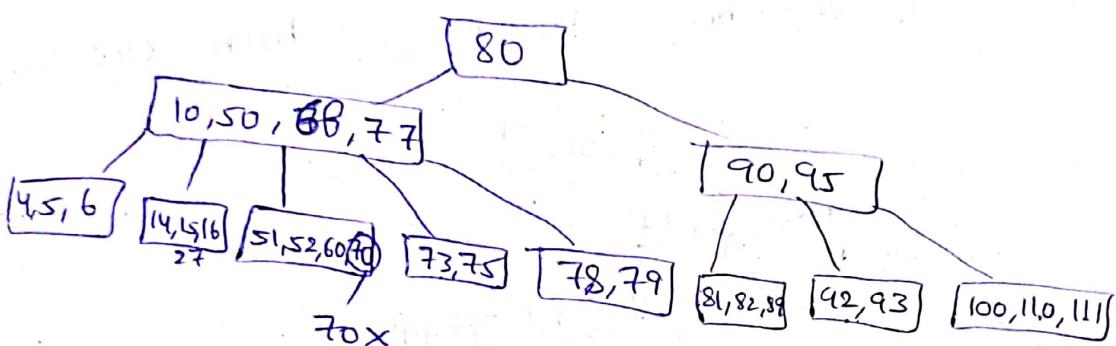
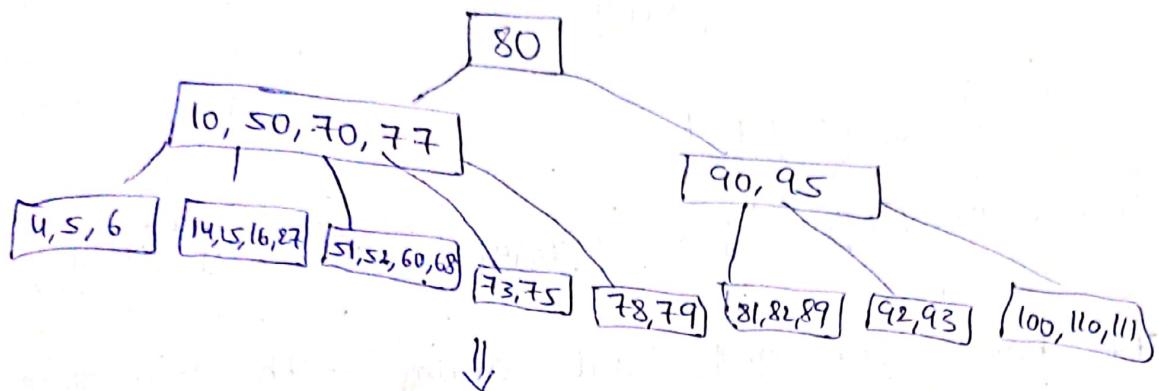


We're not done with this bcz every node must have min no. of nodes except root node.

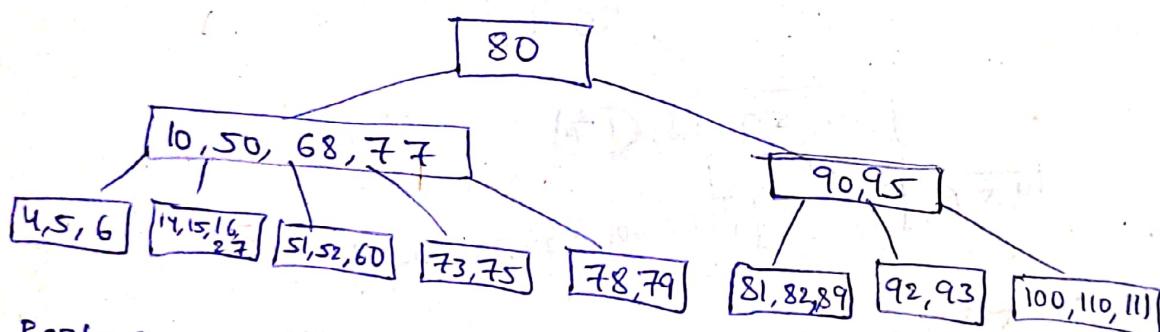


→ 20 can easily be deleted bcz it is in leaf node having more than min keys.

del 70

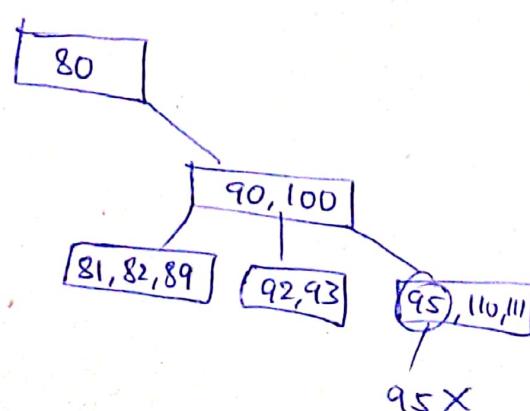


del 95



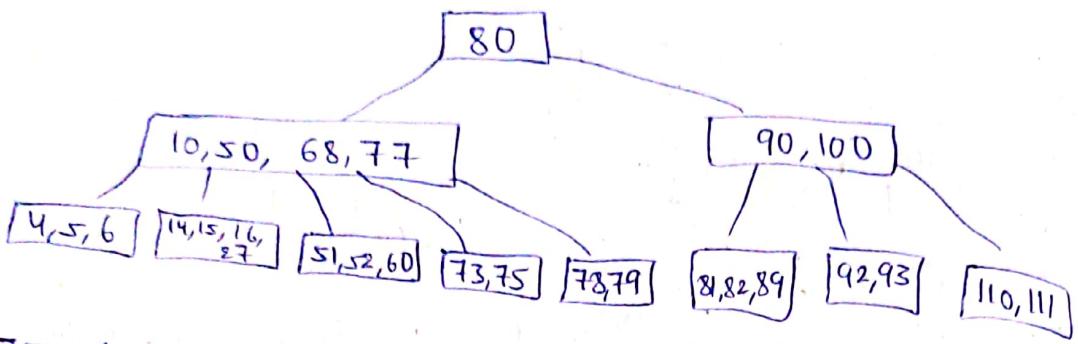
Replacing v^{th} in-order predecessor \rightarrow can't possible bcz
it having min no. of nodes

We are moving in-order successor

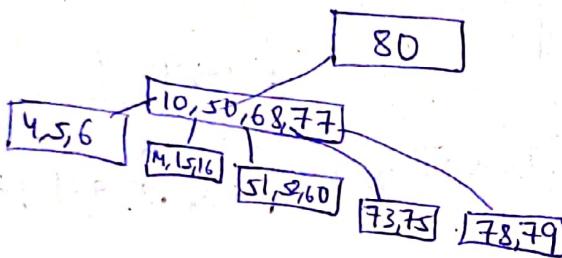


95X

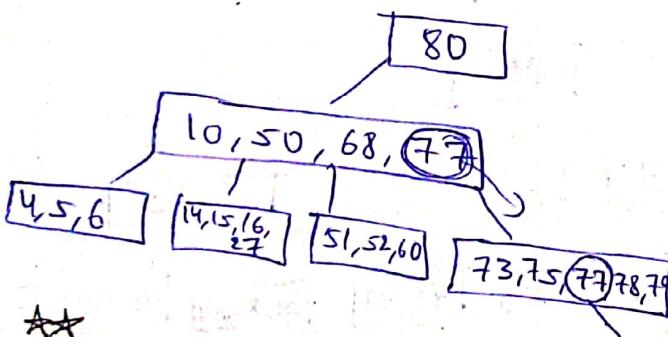
del 77



77 have left and right sibling with min no.of keys. so, we have to merge both our sibl.



↓



del 90 ★★

77x

5.29

B+ Tree

→ Data is stored only in leaf node.
 →

$$\text{Order } (m) = 4$$

$$\text{max children} = 4$$

min

$$\text{children} = \lceil \frac{m}{2} \rceil = 2$$

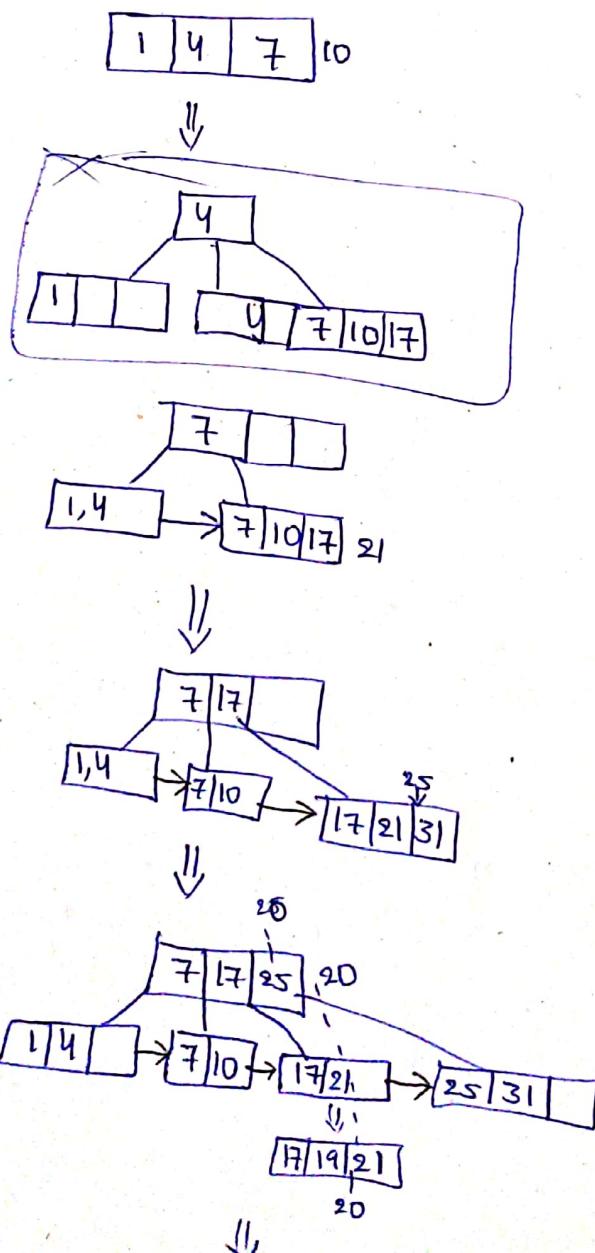
$$\text{max keys} = (m-1) = 3$$

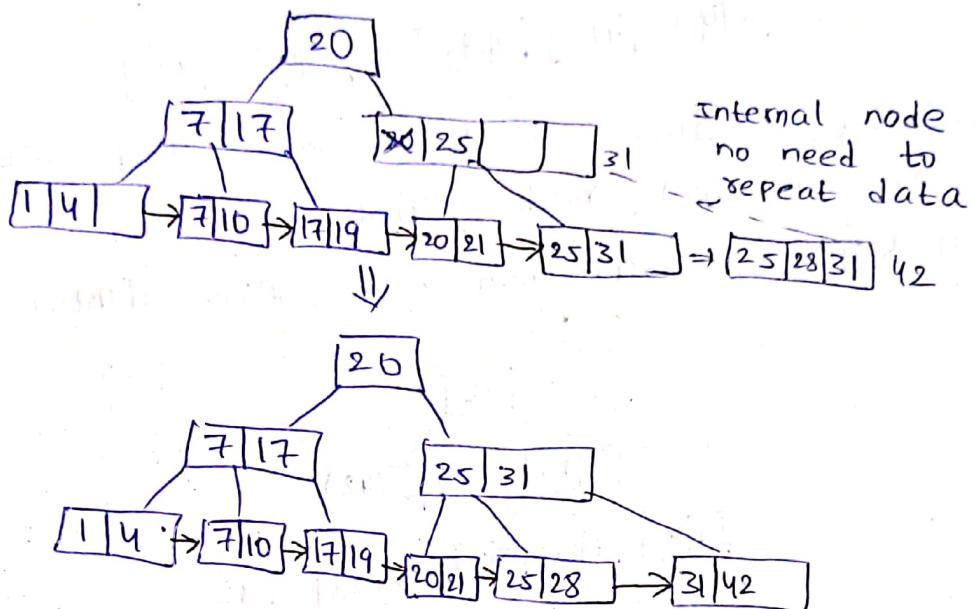
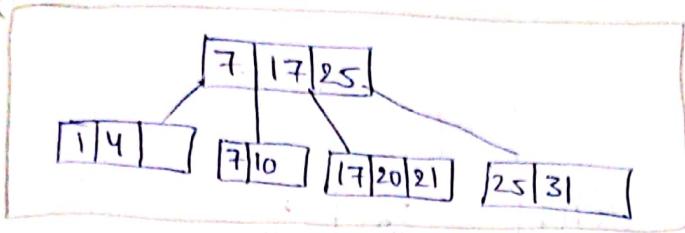
$$\text{min keys} = \lceil \frac{m}{2} \rceil - 1 = 1$$

} except root node

Creation of B+ tree :-

1, 4, 7, 10, 17, 21, 31, 25, 19, 20, 28, 42

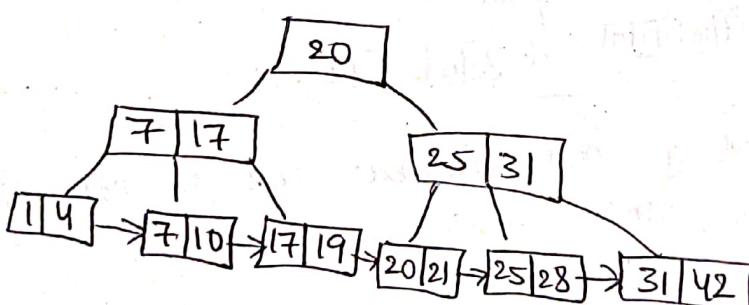




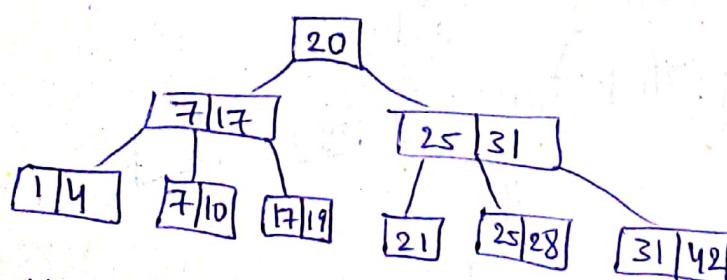
searching is easy with BT tree.

5.30

Deletion in BT tree

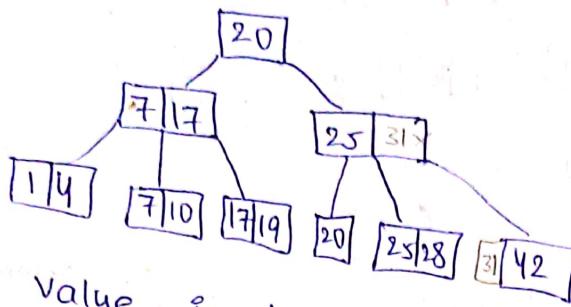


del 21



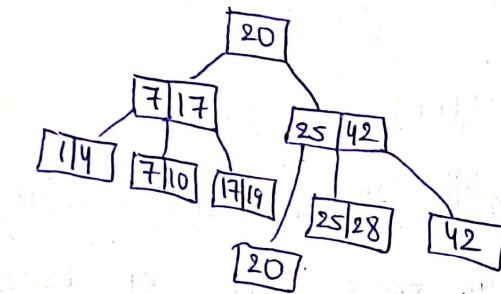
directly deleted Bcz it have more than
min no.of keys.

del 31

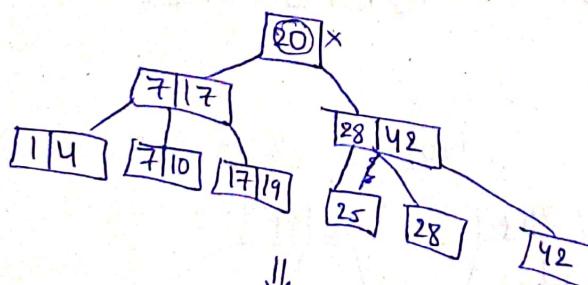


- (i) delete value in leaf node then after delete
in internal node also.
(ii) Replace with min value from right.

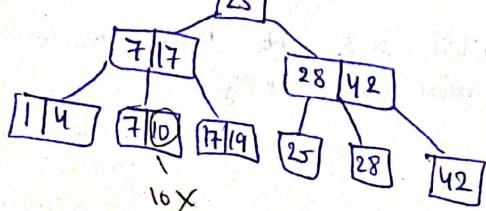
del 20



→ directly
deleting not possible bcz it have min.
borrow from right sibling

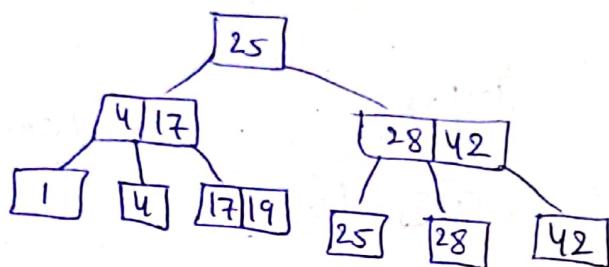
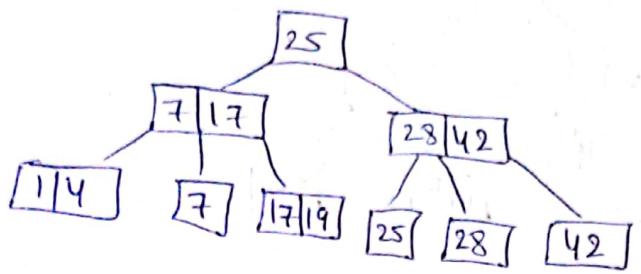


del =



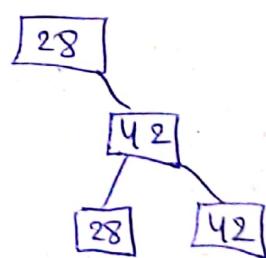
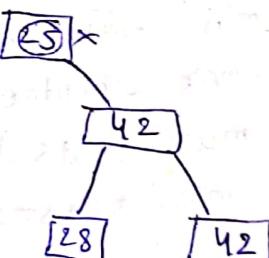
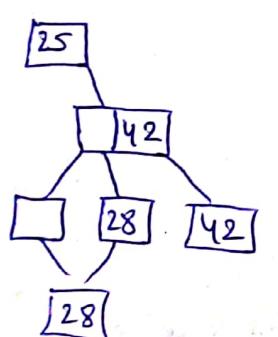
de 10 lox

del 7

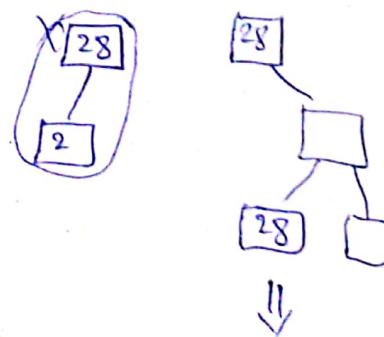
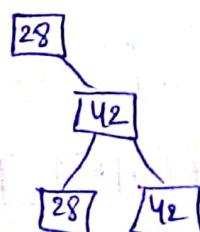


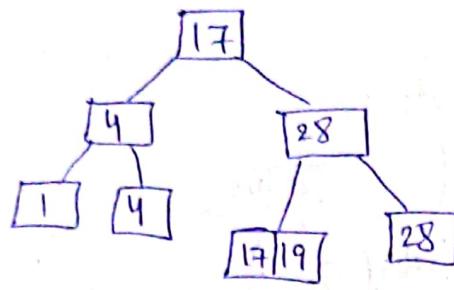
del 25

merging of nodes

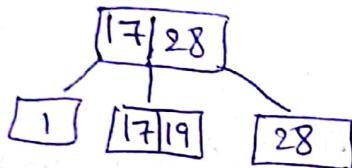


del 42





del 4



(shrinkage)
of the height

5.31

Insert below values in B+ tree of order = 5

$$\text{order}(m) = 5$$

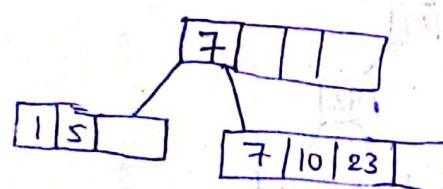
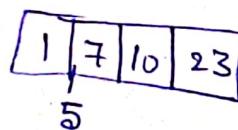
$$\text{min children } \lceil \frac{5}{2} \rceil = 3$$

$$\text{max children} = 5$$

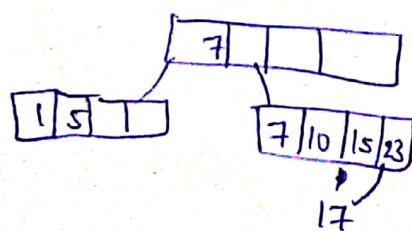
$$\text{max keys} = 4$$

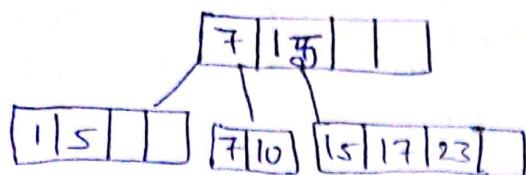
$$\text{min keys} = 2$$

7, 10, 1, 23, 5, 15, 17, 9, 11, 39, 35, 8, 40, 25

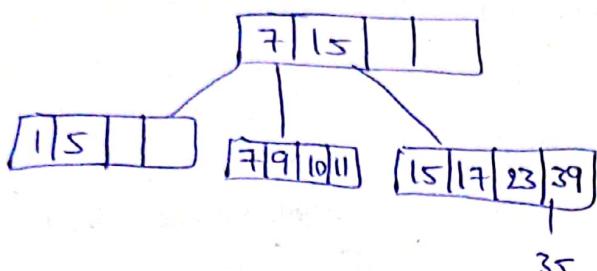


for 15

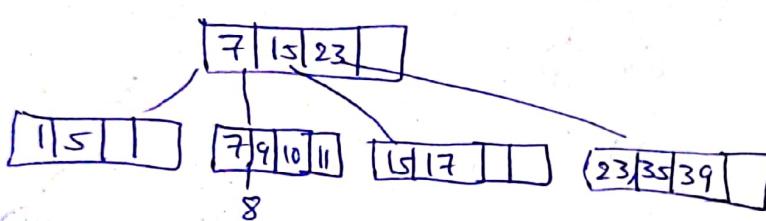




Ans 9, 11

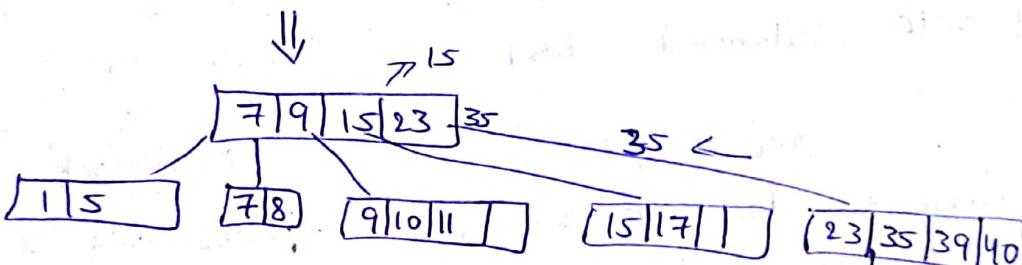


35

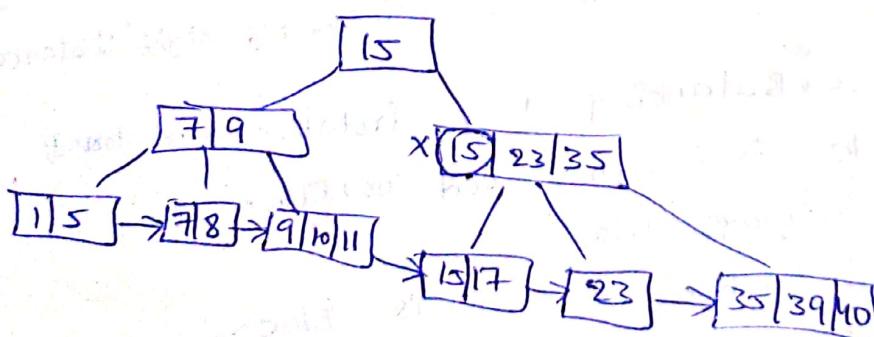


8

15



25



Note: Root node will not follow the min keys rule.

Red Black Tree

Introduction:

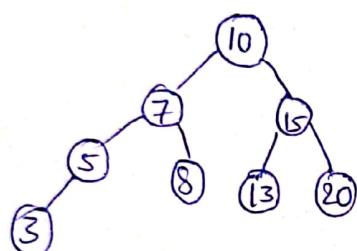
→ Tree is a data structure, it is a way of organizing data efficiently.

Average case - $O(\log_2 n)$

Best case - $O(1)$

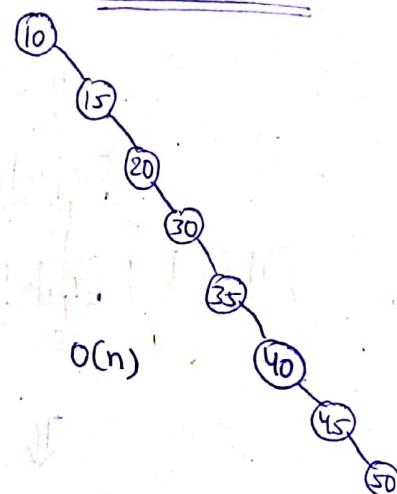
Worst case - $O(n)$

BST



$O(h)$

Sight-screwed BST



$O(n)$

→ self balanced BST

AVL

→ need to do more rotations to balance the tree

→ sub-set of R-B tree
→ strictly height balanced conditions:

→ It is a self balancing BST

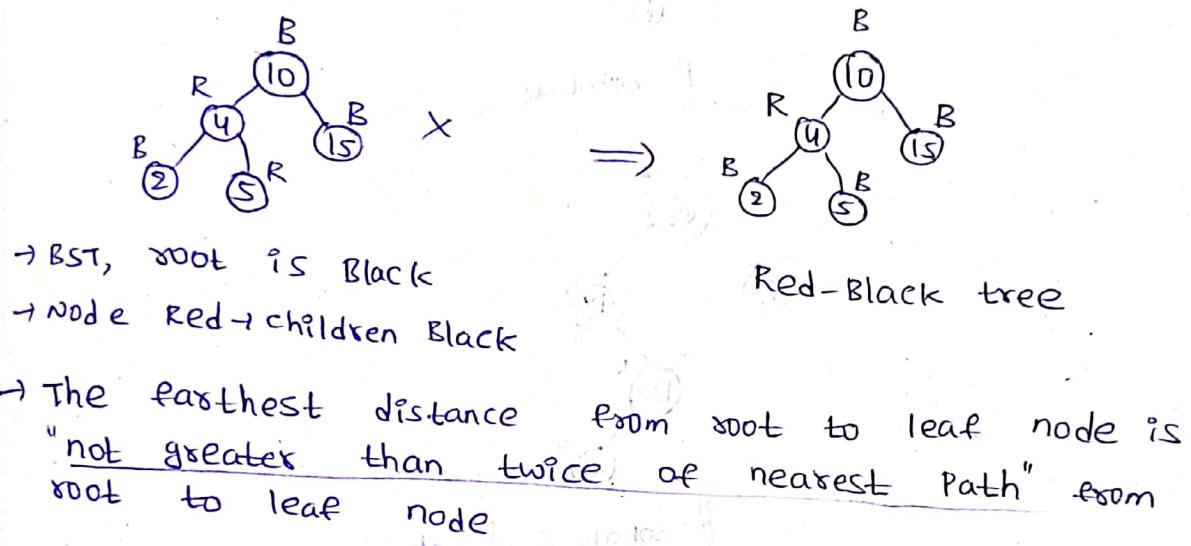
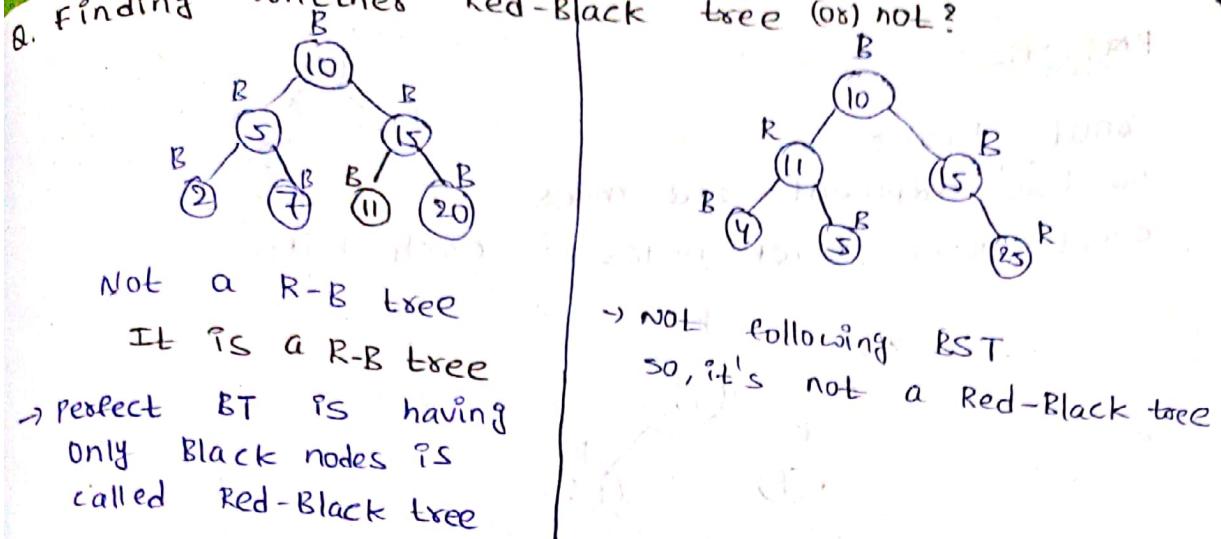
→ Every node is either red (or) Black.

→ Root is always black.

→ Every leaf which is NIL is Black.

→ If node is red then its children are Black.

→ Every path from a node to any of its descendant NIL node has same no. of Black nodes.



5.17

Insertion in Red-Black Tree:-

Rules:-

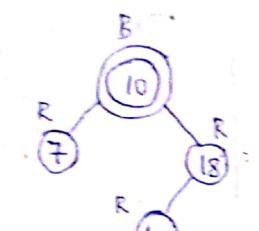
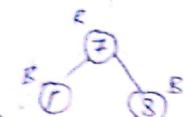
- 1) If tree is empty, create newnode as root node with color Black.
- 2) If tree is not empty, create newnode as leaf node with color Red.
- 3) If Parent of newnode is Black then exit.
- 4) If Parent of newnode is Red, then check the color of Parent's sibling of newnode.
 - (i) If color is Black (or) null then do suitable rotation & re-colour.
 - (ii) If color is Red then recolour & also check, if Parent's Parent of newnode is not root node then recolor it and Re-check.

Properties:

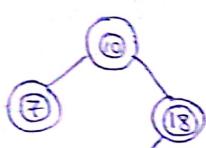
Root = Black

no two adjacent red nodes

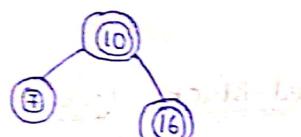
count no. of Black nodes in each Path



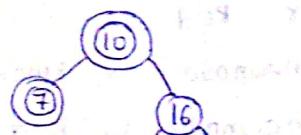
↓ de-colour



↓ rotates & de-colour



↓ de-colour, 16 also re-colour

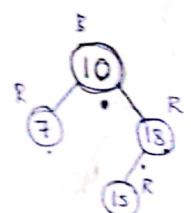


↓ Parent's sibling
Black (null)

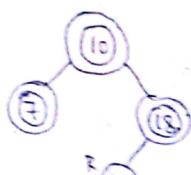
rotate & de-colour

Insert the elements in

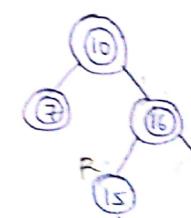
10, 18, 7, 15, 16, 30, 25, 4



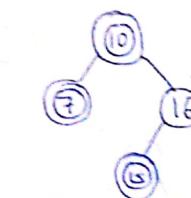
↓ R-R not check



↓



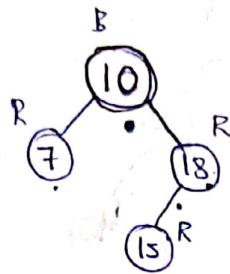
↓



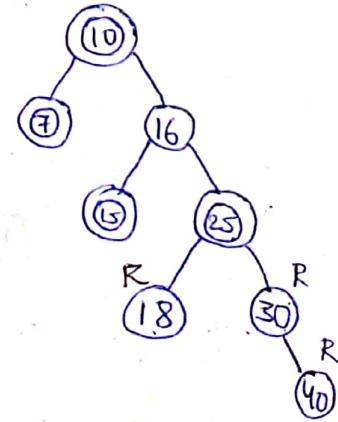
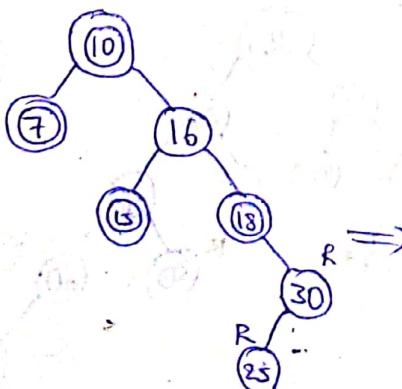
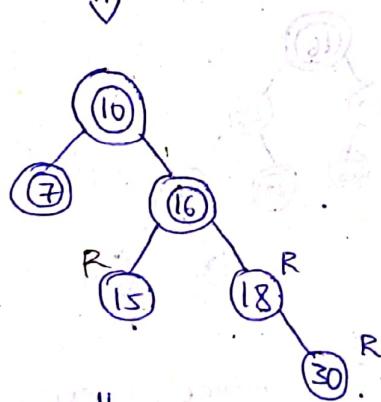
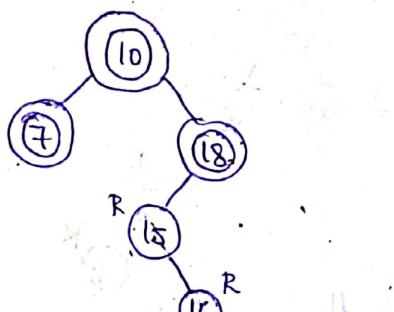
Scanned with OKEN Scanner

Insert the elements in Red-Black tree

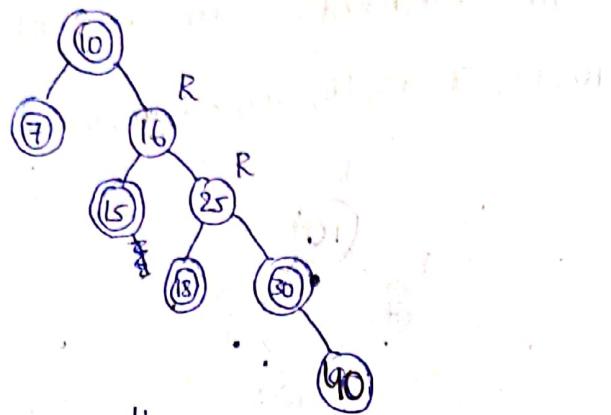
10, 18, 7, 15, 16, 30, 25, 40, 60, 2, 1, 70



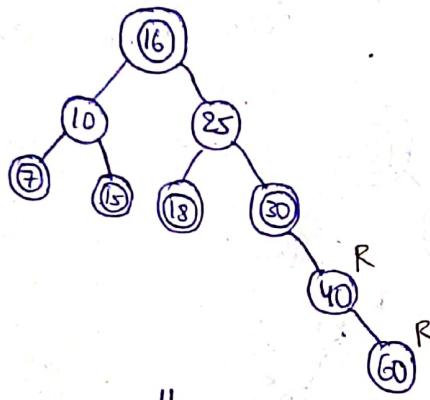
|| R-R not possible
check Parent's sibling



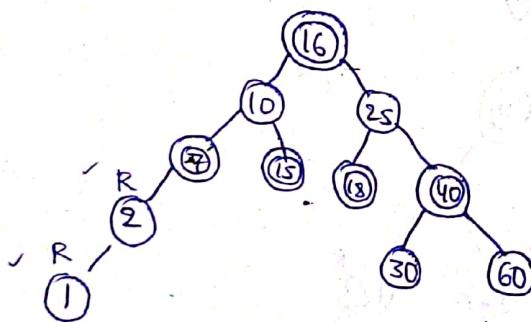
|| decolouring
re-check



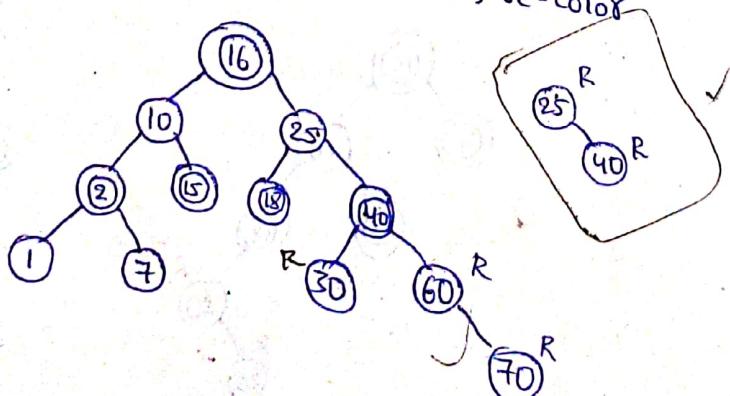
↓ rotate and recolor



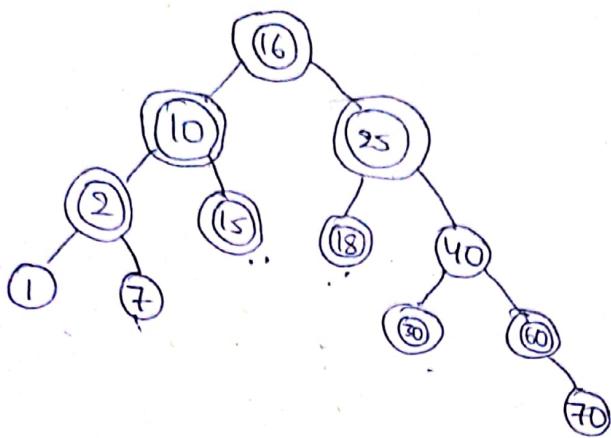
↓ rotate & recolor



↓ Parent's sibling is NULL
rotate & re-color



↓ Parent's sibling is R
recolor & check



Note:

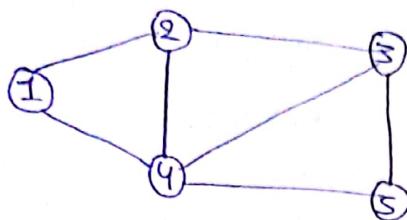
every Red-Black tree is a BST
But BST is need not to be Red-Black tree.

5.18

Deletion in Red-Black Tree

6. Graph Representation

6.1 Representation of a graph in computer:-



To represent graph in 2 methods

- (i) Adjacency matrix
- (ii) Adjacency list

(i) Adjacency matrix: (dense graph)

$i \downarrow$	1	2	3	4	5	$j \rightarrow$
1	0	1	0	1	0	
2	1	0	1	1	0	
3	0	1	0	1	1	
4	1	1	1	0	1	
5	0	0	1	1	0	

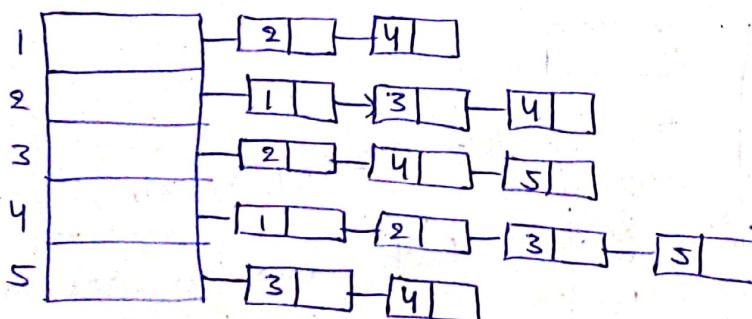
every node is connected with each other.

Time complexity $\Theta(n^2)$

5x5

$\{a[i][j] = 1 \text{ if } i \& j \text{ are adjacent}$
 $= 0 \text{ otherwise}\}$

(ii) Adjacency list: (sparse graph)



Time complexity $\Theta(n+e)$



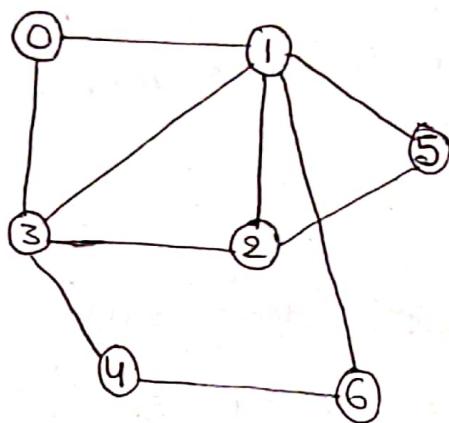
Scanned with OKEN Scanner

6.2

Graph Traversals

In Graph Traversal we have two methods:

- (i) BFS (Breadth first search)
- (ii) DFS (Depth first search)



(i) BFS:

Data structure used in BFS is Queue

Queue:-



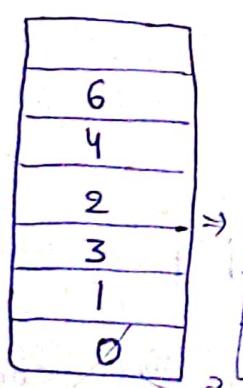
(FIFO)

Result = 0 1 3 2 5 6 4

(ii) DFS:

Data structure used in DFS is stack (LIFO)

Stack:-



Result = 0, 1, 3, 2, 4, 6, 5

Back track to get missed elements.

6.3

Types of edges in DFS

In Data structures, we have 4 types of edges in DFS (depth first search)

(i) Tree edge:-

Member of DFS traversal

(ii) Forward edge:-

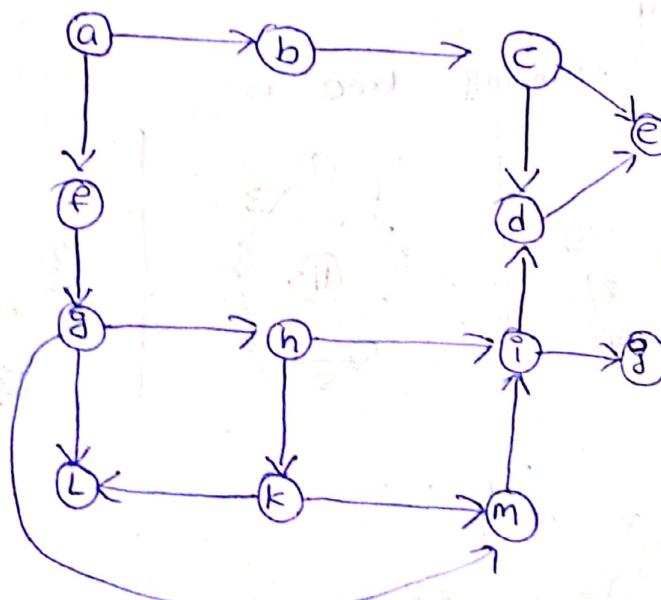
e: (x, y) where y appears after x and there is path from x to y.

(iii) Back edge:-

e: (x, y) where y appears before x and there is path from y to x.

(iv) Cross edge:-

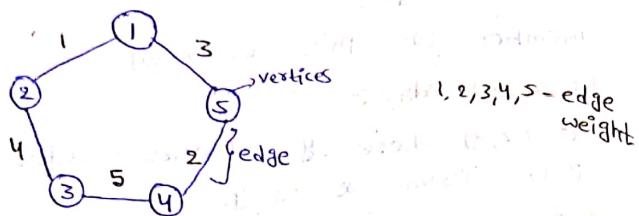
Any edge from x to y is a cross edge where there's no path from y to x.



6.4

MST (minimum spanning tree)

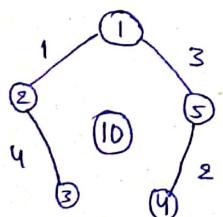
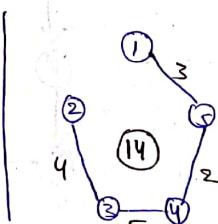
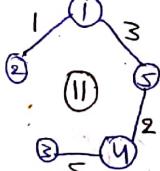
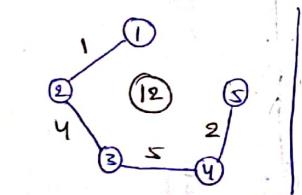
Spanning tree:

 $G(v, \epsilon)$ $G'(v', \epsilon')$

$$v' = v, \epsilon' \subset \epsilon, \epsilon' = |v|-1$$

$$\begin{aligned} \epsilon' &= |v|-1 \\ &= 5-1 \\ &= 4 \end{aligned}$$

resultant of spanning tree is:



This is the minimum spanning tree (MST)

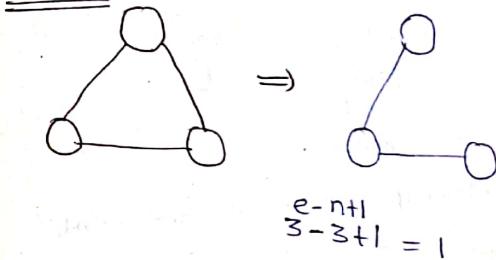
Note:

- Spanning tree should not contain any cycle.
- Spanning tree shouldn't be discontinuous.

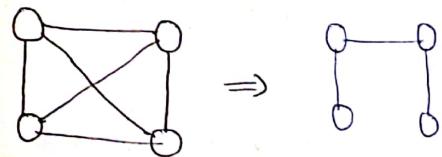
Properties of MST:

- Removing one edge from the will make it disconnected.
- Adding one edge to the
- If each edge have distinct there will be only one & unique
- A complete undirected graph can have many STs.
- Every connected & undirected graph has exactly one ST.
- Disconnected graph doesn't have any ST.
- From a complete graph by removing edges, we can construct a spanning tree.

- Q. From a complete graph by removing edges, we can construct a spanning tree.



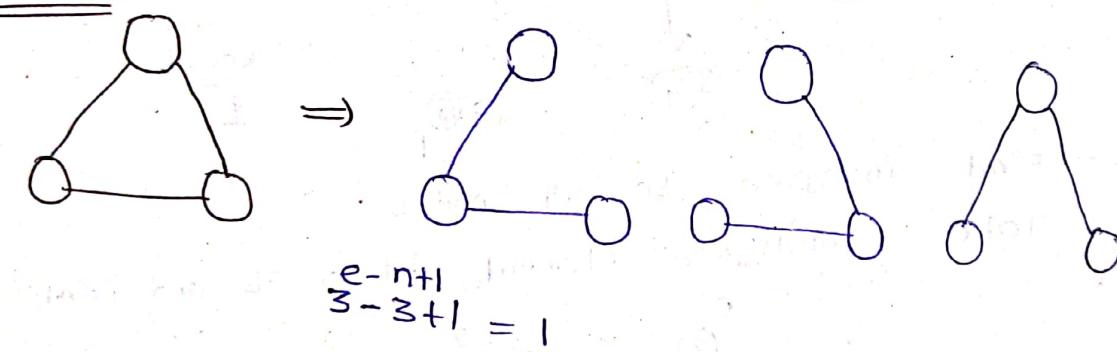
4 vertices



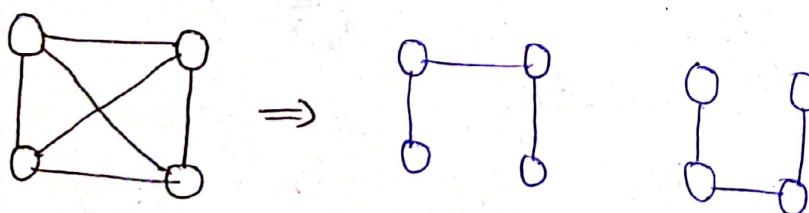
Properties of MST:

- Removing one edge from the spanning tree will make it disconnected.
- Adding one edge to the ST will create a loop.
- If each edge have distinct weight then there will be only one & unique MST
- A complete undirected graph can have n^{n-2} no. of ST
- Every connected & undirected graph has atleast one ST
- Disconnected graph doesn't have any ST
- From a complete graph by removing max edges ($e-n+1$) we can construct a ST

Q. From a complete graph by removing max($e-n+1$) edges, we can construct a spanning tree



4 vertices



$$e-n+1
6-4+1 = 3$$

max Possibilities n^{n-2}

$$= 4^{4-2}$$

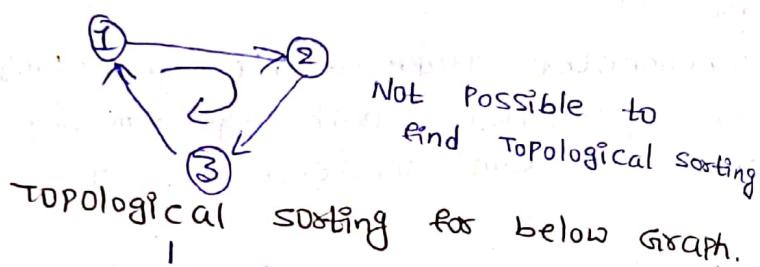
$$= 16$$

6.10

TOPOLOGICAL SORTING

- It is a linear ordering of its vertices such that for every directed edge uv for vertex u to v , u comes before vertex v in the ordering. ($u \rightarrow v$)
- Graph should be Directed Acyclic Graph
- Every DAG will have atleast one topological ordering.

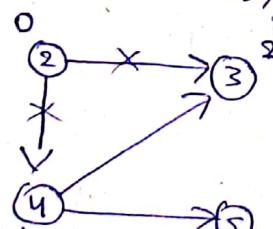
Q. Find



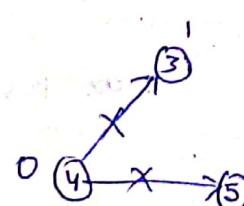
→ Find
Take

Indegree to all nodes:

0 in-degree element, delete it and print



Result
1, 2

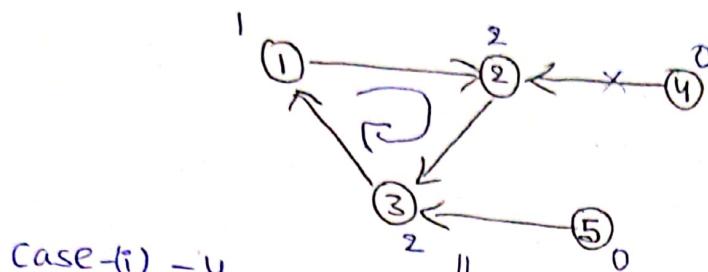


Result
1, 2, 4



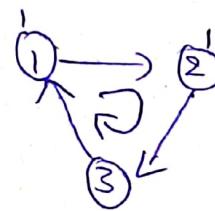
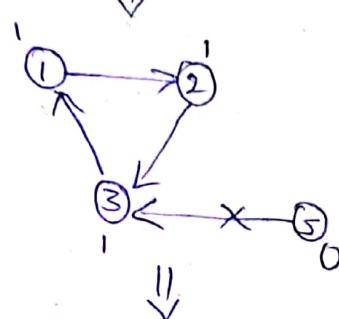
Result
1, 2, 4, 3, 5
1, 2, 4, 5, 3
2 TS is possible

Q. Topological sorting for below graph



Case-(i) - 4

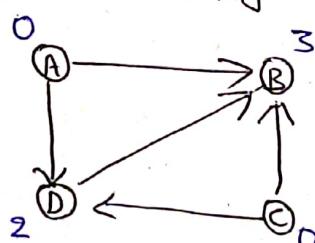
Case-(ii) - 5



In this we didn't get any vertex having in-degree '0'.

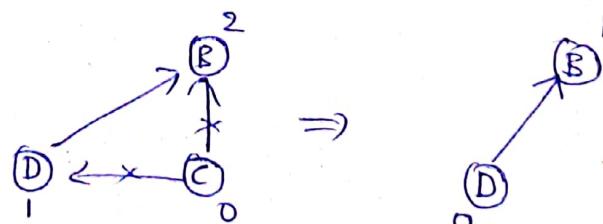
So, topological sorting is not possible for this graph bcz it is a cyclic graph.

Q. Find topological sorting for below graph.



Case-I

A, C, D, B



Case-II

C A D B

