Assignment 2

By Brandon Young and Ruicheng Wu

1 problem 1

2 problem 2

2.1 a.

We claim the provided tree correctly categorize the provided example since every example can be inducted from this decision tree. Like GPA above 3.6 is P and below 3.3 is N, then with publication P,otherwise check University Rank, Only rank 2 will be P,other ranks are N. And recommendation doesn't matter.

2.2 b.

Step I

$\mathbf{I}(\frac{6}{12}, \frac{6}{12})\mathbf{=1}$

GPA: $[3.9, 4.0]$ $3(PPP)$, $(3.2, 3.9)$ $5(PPPNN)$, $[3.0, 3.2]$ $4(NNNN)$

University: Rank 1— 5(PPPNN),Rank 2— 3(PPN) , Rank 3— 4(PNNN)

Publication: Yes 5(PPPNN) , No 7(PPPNNNN)

Recommendation: good 8(PPPPPNNN) , normal 4(PNNN)

Gain(GPA)=1-[$\frac{3}{12}$ B($\frac{3}{3}$)+$\frac{5}{12}$ B($\frac{3}{5}$)+$\frac{4}{12}$ B($\frac{0}{4}$)]=1-[0.0+0.404562747689+0.0] = 0.595437252311

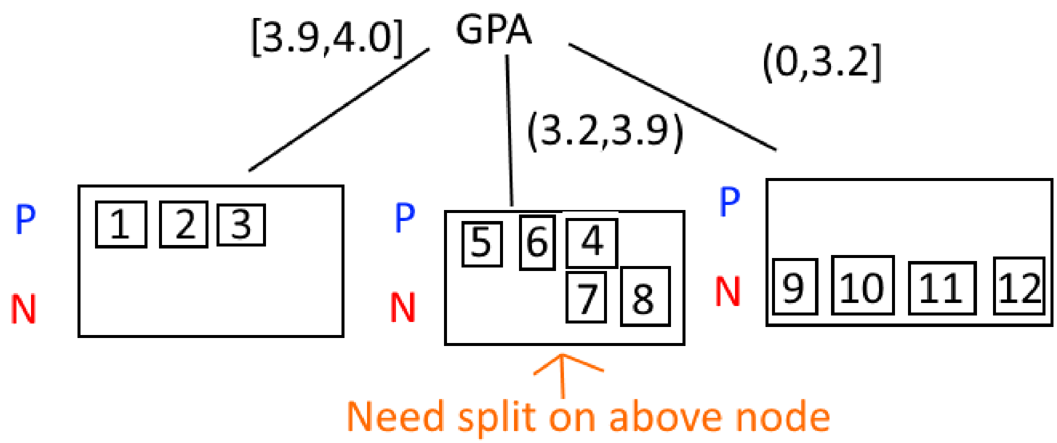
Gain(University)=1-[$\frac{5}{12}$ B($\frac{3}{5}$)+$\frac{3}{12}$ B($\frac{2}{3}$)+$\frac{4}{12}$ B($\frac{1}{4}$)]= 1-[0.404562747689+0.229573958514+0.270426041486] =0.095437252395

Gain(Publication)=1-[$\frac{5}{12}$ B($\frac{3}{5}$)+$\frac{7}{12}$ B($\frac{3}{7}$)]=1-[0.404562747689+0.574716412687]=

0.020720839624

Gain(Recommendation)=
1-[$\frac{8}{12}$ B($\frac{5}{8}$)+$\frac{4}{12}$ B($\frac{1}{4}$)]=1-[0.636289335283+0.270426041486]= 0.093284623231

So we pick GPA as the best Gain attribute in this level



Step II

I($\frac{2}{5}$, $\frac{3}{5}$)=0.970950594455

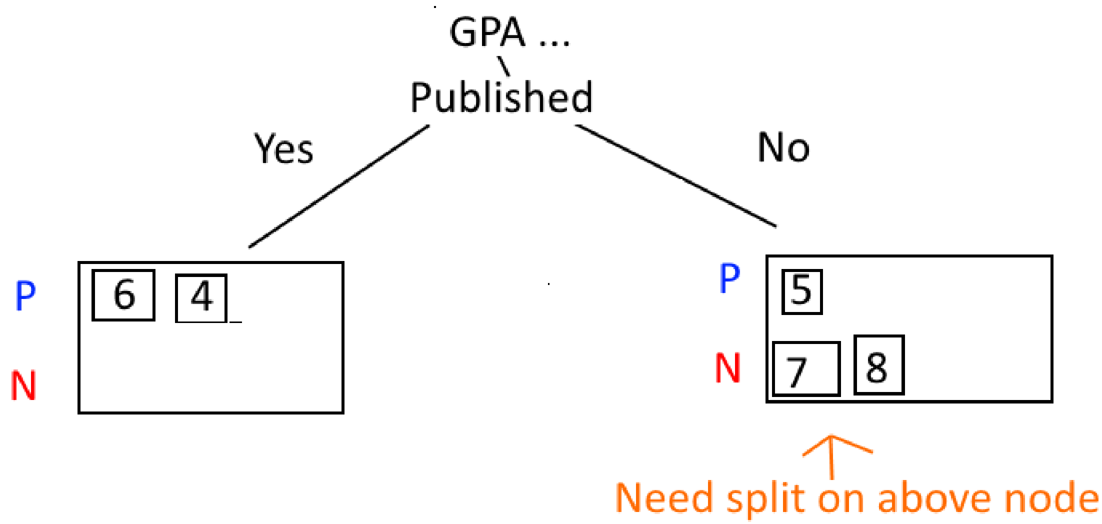
University: Rank 1— 2(PN),Rank 2— 1(P) , Rank 3— 2(PN)
Publication: Yes 2(PP) , No 3(PNN)
Recommendation: good 5(PPPNN)

Gain(University)=0.970950594455-[$\frac{2}{5}$ B($\frac{1}{2}$)+$\frac{1}{5}$ B($\frac{1}{1}$)+$\frac{2}{5}$ B($\frac{1}{2}$)]=
0.970950594455-[0.4+0.0+0.4] = 0.170950594455

Gain(Publication)=0.970950594455-[$\frac{3}{5}$ B($\frac{1}{3}$)+$\frac{2}{5}$ B($\frac{2}{2}$)]=0.970950594455-
[0.550977500433+0.0]= 0.419973094022

Gain(Recommendation)=0.970950594455-[$\frac{5}{5}$ B($\frac{3}{5}$)]=0.970950594455-[0.970950594455+0.0+0.0] = 0

So we pick Publication as the best Gain attribute in this level



Step III

I($\frac{1}{3}$, $\frac{2}{3}$)=0.918295834054

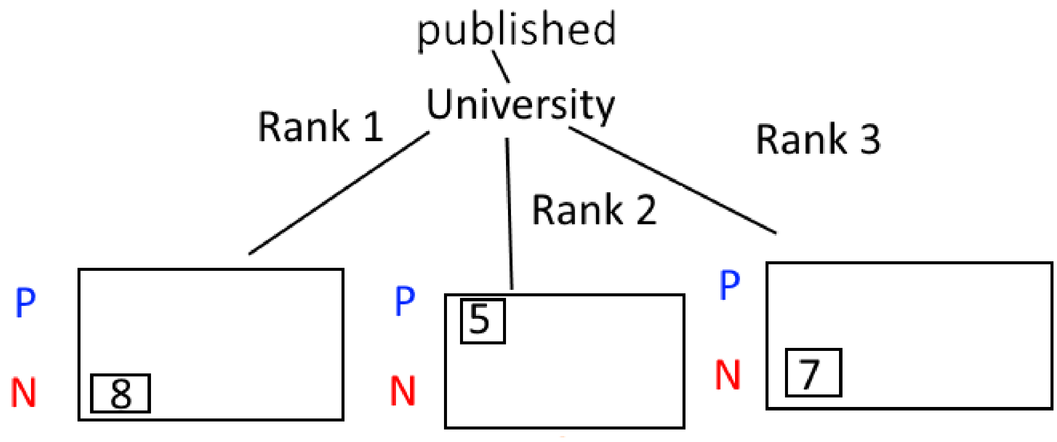
University: Rank 1 — 1(N),Rank 2 — 1(P) , Rank 3 — 1(N)

Recommendation: good 3(PNN)

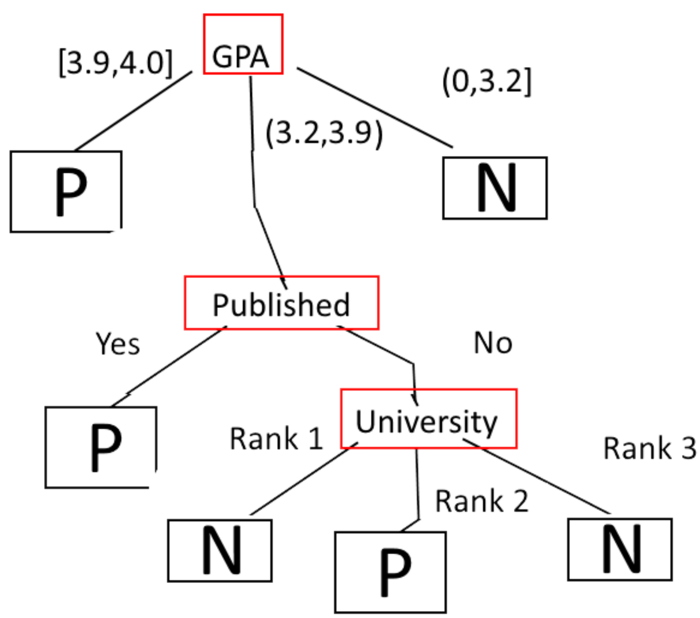
Gain(University)=0.918295834054-[$\frac{1}{3}$ B($\frac{0}{1}$)+$\frac{1}{3}$ B($\frac{1}{1}$)+$\frac{1}{3}$ B($\frac{0}{1}$)]=0.918295834054-[0.0+0.0+0.0]=0.918295834054

Gain(Recommendation)=0.918295834054-[$\frac{3}{3}$ B($\frac{1}{3}$)]=0.918295834054-[0.918295834054]= 0

So we pick University as the best Gain attribute in this level



And the final tree to be returned is



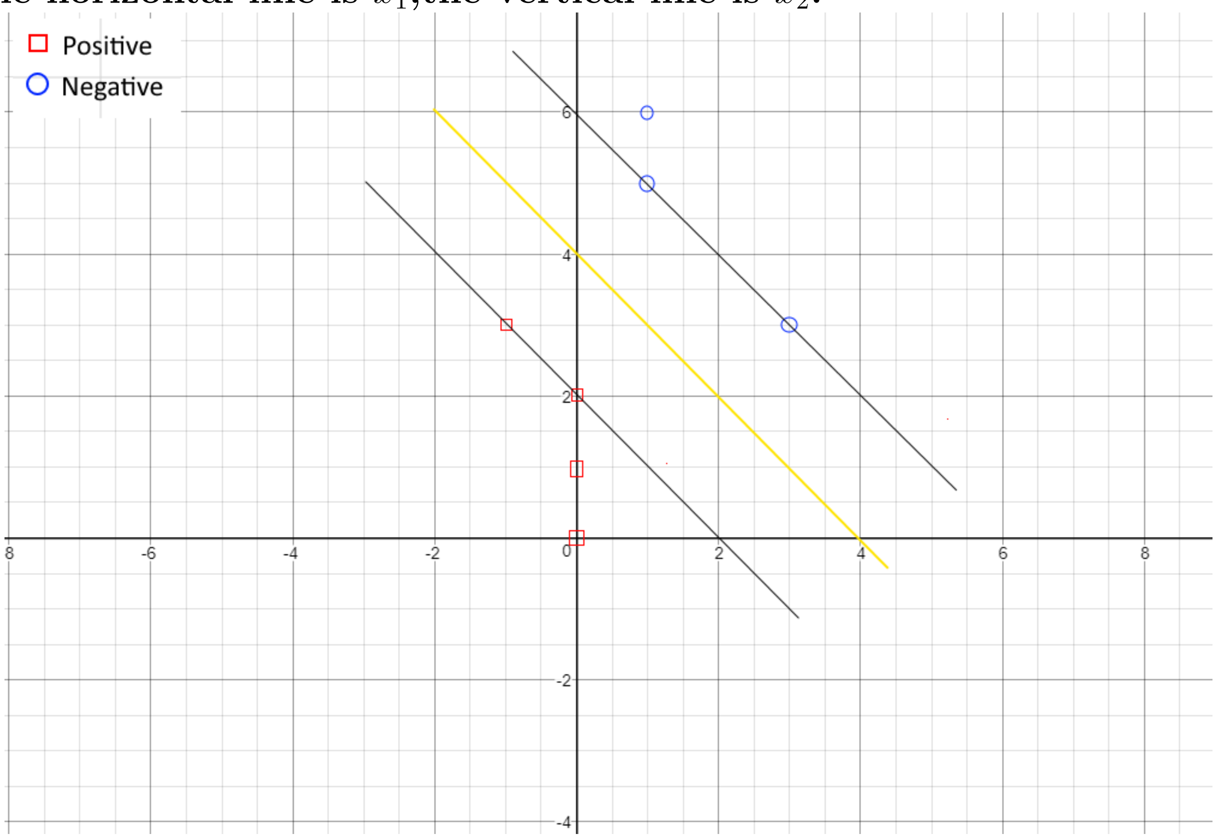
2.3 c.

The decision tree we got from b. is same from the provided one. This is not coincidence, it is believed both tree is generated by applying the decision tree algorithm.

3 problem 3

3.1 a.

The horizontal line is x_1,the vertical line is x_2.



3.2 b.

s_1 : $(1, 5)$, s_2 : $(-1, 3)$, w : $(1, 1)$, So decision boundary is: $y = (1, 1)x - 4$ Assume there is (a, a) for w^T, we will have

$$a + 5a + b = -1$$

$$-a + 3a + b = 1$$

Solve those equations for :

$$a = -0.5, b = 2$$

So we have w^T : $(-0.5, -0.5)$ and b : 2 for parameters. We can confirm the max Margin is $\frac{2}{||w^T||} = \frac{2}{(\frac{\sqrt{2}}{2})} = 2\sqrt{2}$, which verifies with the distance between s_1 : $(1, 5)$, s_2 : $(-1, 3)$, $\sqrt{(1-(-1))^2 + (5-3)^2} = 2\sqrt{2}$.

And also we verify with given data.

[-1 3]^T [0 2]^T [0 1]^T [0 0]^T, plug in: All positive , so those are 1;

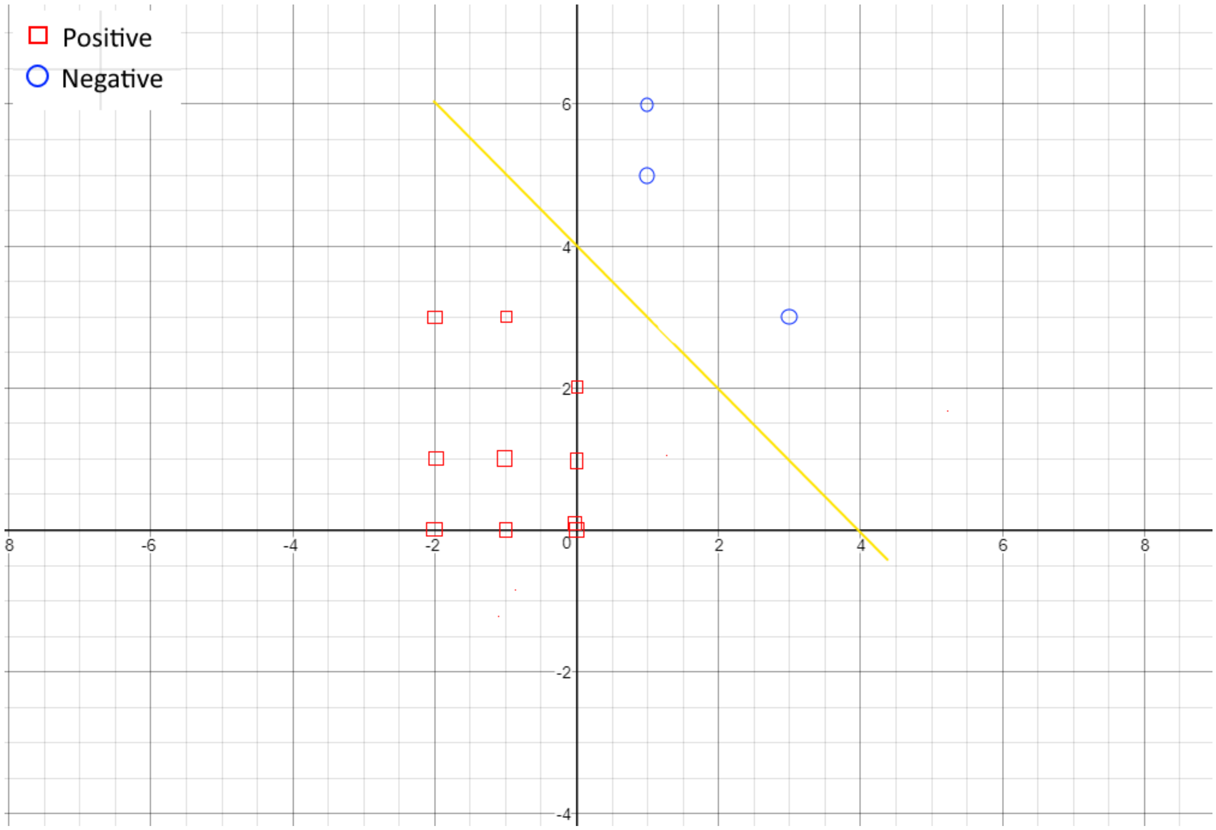
[1 5]^T [1 6]^T [3 3]^T , plug in: All negative , so those are -1;

3.3 c.

By inspection, the new added points are not support vectors(ie. the closest points to separating line)

We claim the w^T : $(-0.5, -0.5)$ and b : 2 are still same to parametrize with new data.

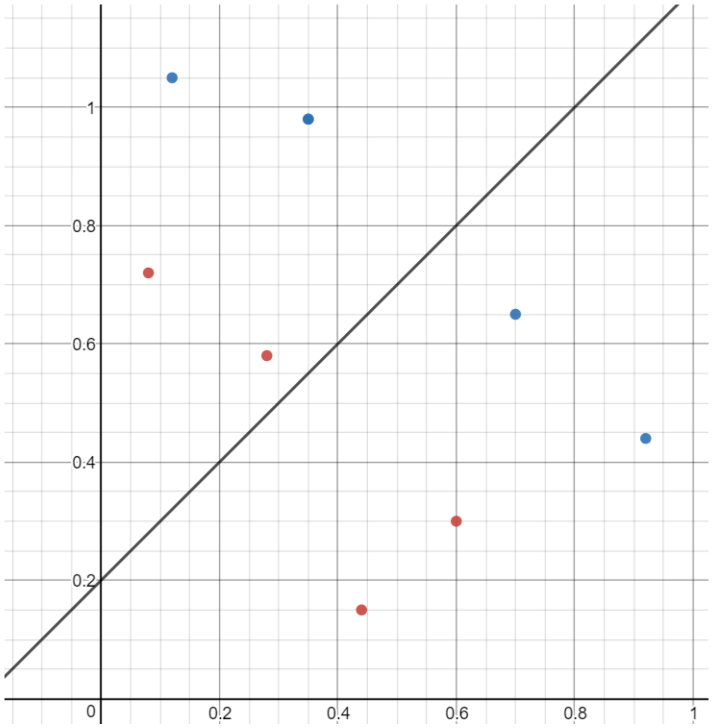
[-2 0]^T [-2 1]^T [-2 3]^T [-1 0]^T [-1 1]^T [0 0]^T, plug in: All positive , so those are 1;



4 problem 4

4.1 a.

The initial linear separator (line 1) is $i_2 = -\frac{i_1}{i_2} - \frac{0.2}{i_2}$ or $i_2 = i_1 + 0.2$:



Assume that the points are:

- A: $(0.08, 0.72)$
- B: $(0.28, 0.58)$
- C: $(0.44, 0.15)$
- D: $(0.6, 0.3)$
- E: $(0.12, 1.05)$
- F: $(0.35, 0.98)$
- G: $(0.7, 0.65)$
- H: $(0.92, 0.44)$

where A, B, C and D are in class 1 and E, F, G and H are in class -1.

Use $f(x) = w_1 i_1 + w_2 i_2 + 0.2$ to classify the samples. If $f(x) < 0$ then x is in class -1 and $h_w(x) = 0$. If $f(x) \geq 0$ then x is in class 1 and $h_w(x) = 1$. By the initial linear separator, 4 samples are misclassified (A, B, G and H):

- A: $(0.08) - (0.72) + 0.2 = -0.44 < 0, h_w(A) = 0$
- B: $(0.28) - (0.58) + 0.2 = -0.1 < 0, h_w(B) = 0$
- C: $(0.44) - (0.15) + 0.2 = 0.49 > 0, h_w(C) = 1$
- D: $(0.6) - (0.3) + 0.2 = 0.5 > 0, h_w(D) = 1$
- E: $(0.12) - (1.05) + 0.2 = -0.73 < 0, h_w(E) = 0$
- F: $(0.35) - (0.98) + 0.2 = -0.43 < 0, h_w(F) = 0$
- G: $(0.7) - (0.65) + 0.2 = 0.25 > 0, h_w(G) = 1$
- H: $(0.92) - (0.44) + 0.2 = 0.68 > 0, h_w(H) = 1$

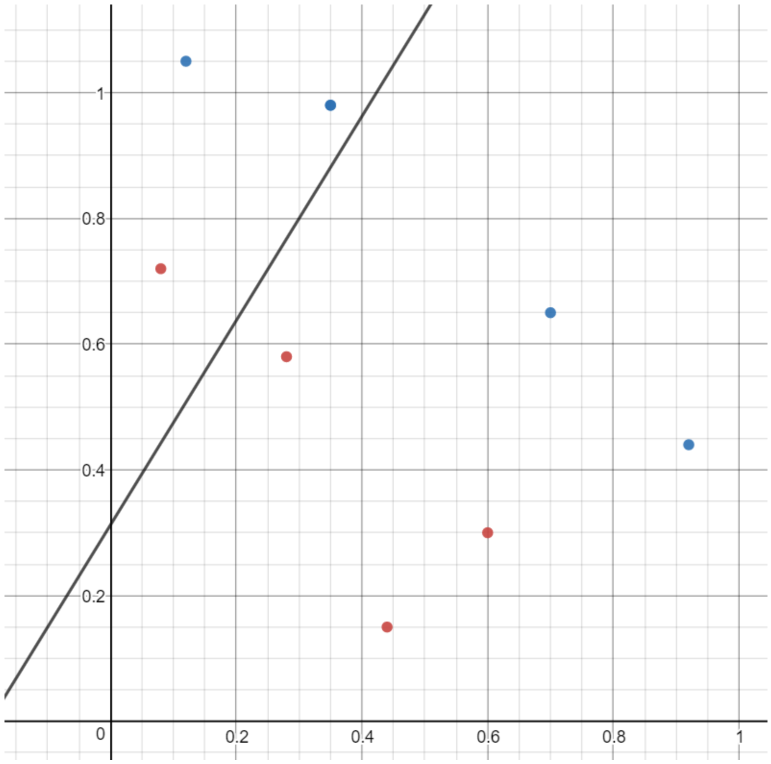
Let the learning rate, $\alpha = 0.5$ and use A to update the weights. Then $Err_A = y - h_w(A) = 1 - 0 = 1$.

$$w'_1 \leftarrow w_1 + \alpha * Err_A * i_1$$

$$w'_1 \leftarrow 1 + (0.5)(1)(0.08)$$
$$w'_1 = 1.04$$

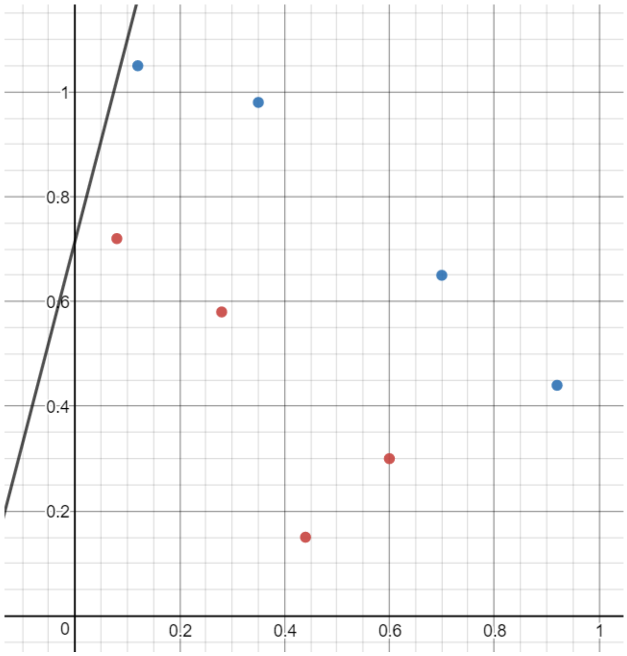
$$w'_2 \leftarrow (-1) + (0.5)(1)(0.72)$$
$$w'_2 = -0.64$$

Hence the new weights are $w_1 = 1.04$ and $w_2 = -0.64$. Line 2, then, is: $i_2 = 1.625(i_1) + 0.3125$, with 3 misclassified points:

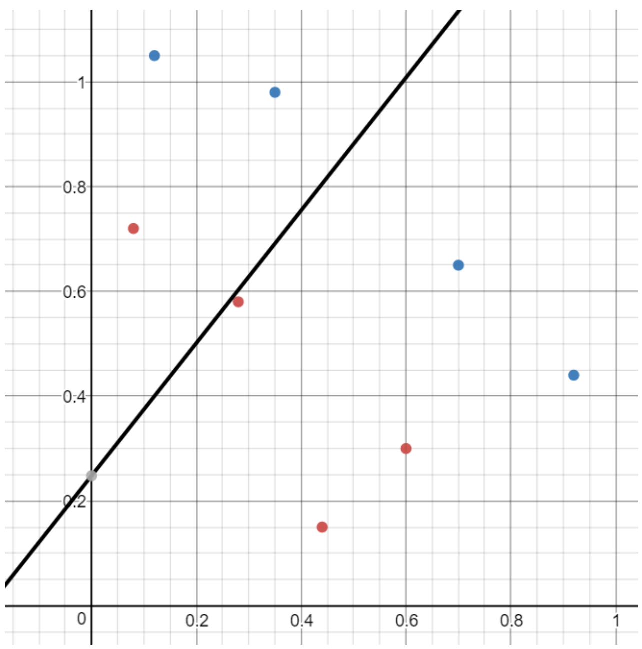


Following a similar process as above, the next few iterations generate the following plots:

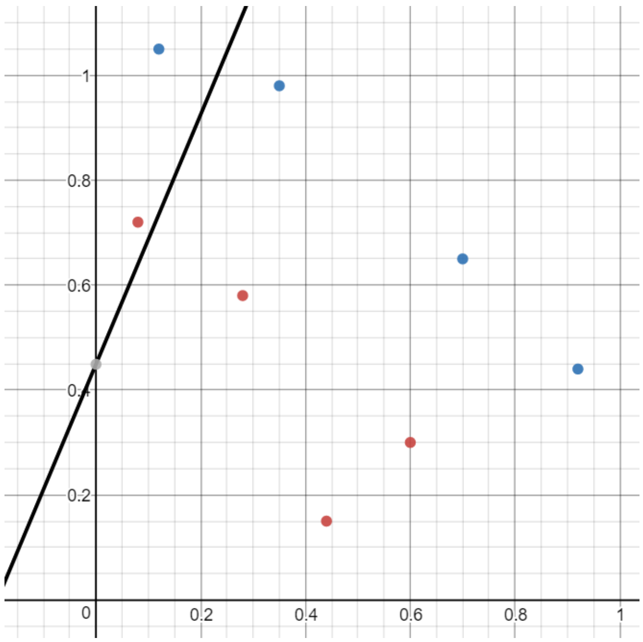
Update Line 2 with A to get $w_1 = 1.08$, $w_2 = -0.28$. Line 3: $i_2 = 3.857i_1 + 0.714$, with 4 misclassified points:



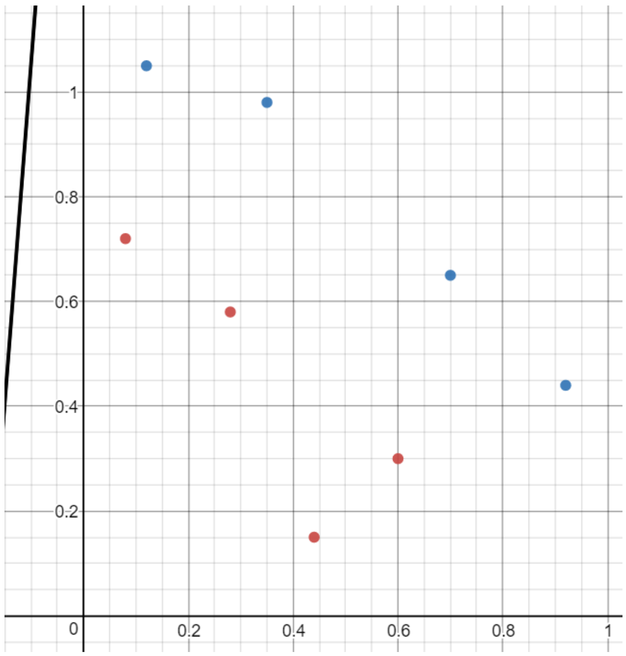
Update Line 3 with E to get $w_1 = 1.02$, $w_2 = -0.805$. Line 4, $i_2 = 1.267(i_1) + 0.248$, with 3 misclassified points:



Update Line 4 with A to get $w_1 = 1.06$, $w_2 = -0.445$. Line 5, $i_2 = 2.382(i_1) + 0.449$, with 4 misclassified points:



Update Line 5 with A to get $w_1 = 1.1$, $w_2 = -0.085$. Line 6, $i_2 = 12.941(i_1) + 2.35$, with 4 misclassified points:

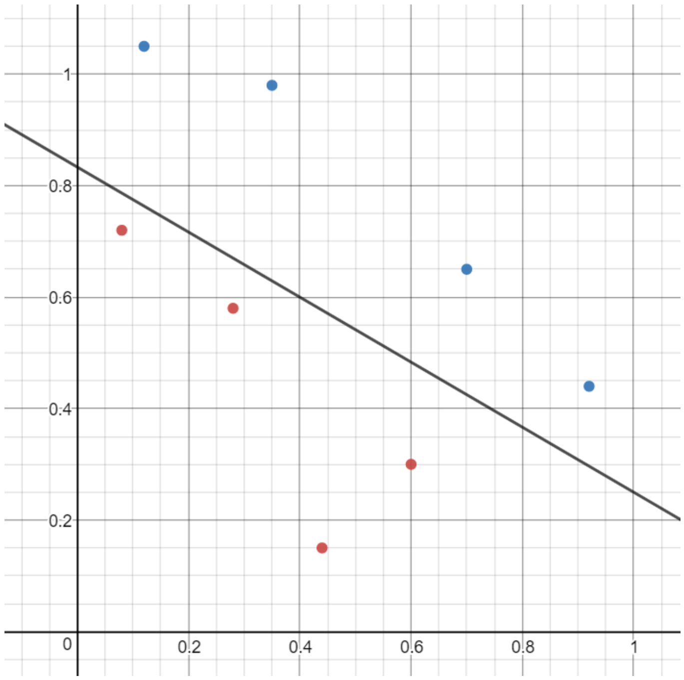


4.2 b.

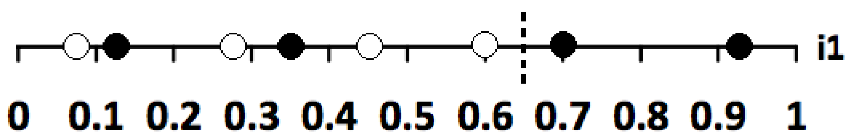
The linear separator that achieves perfect classification (found after 172 iterations) is:

$$i_2 = -0.58333 i_1 + 0.83333$$

where $w_1 = -0.14$ and $w_2 = -0.24$. The line in the graph:



4.3 c.



The diagram above shows only the i_1 value from each sample. The dotted line represents the location of the best possible split, which is at $i_1 = 0.65$, with only 2 misclassified points and a 25% error.

A separator in 1D is $w_1 i_1 + w_0 i_0$ or $w_1 i_1 + 0.2$ (assuming $w_0 = 0.2$) which can be used to solve for w_1:

$$w_1 = -\frac{0.2}{i_1} = -\frac{0.2}{0.65} = -0.3077$$

As a result, $w_1 = -0.3077$ is the weight that best classifies the samples.

5 problem 5

5.1 a.

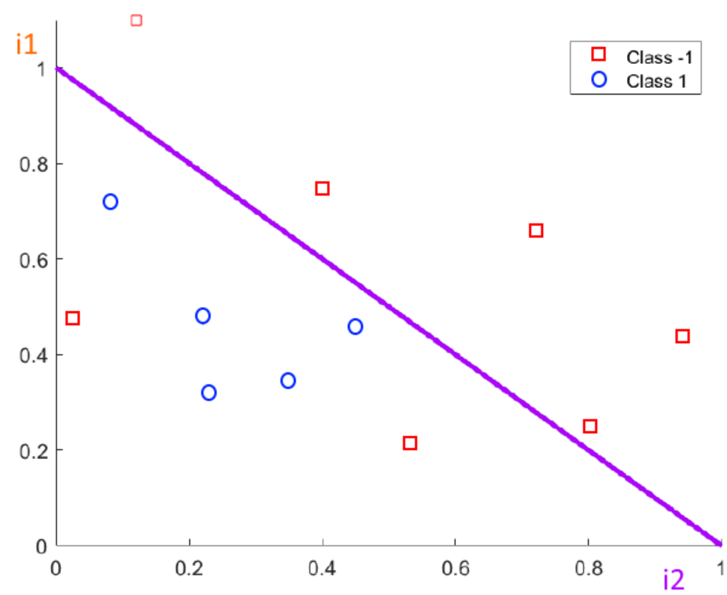
We define those points by inspection.

Class 1: (0.03,0.50),(0.11,1.2),(0.5,0.75),(0.54,0.23),(0.7,0.65),(0.8,0.25),(0.91,0.4

Class -1: (0.07,0.73),(0.23,0.49),(0.24,0.33),(0.35,0.35),(0.46,0.46);

The line can be observed from the graph, and we found the best single perceptron will have two unclassied points. So the minimum error will be $\frac{2}{12} = 0.167$;

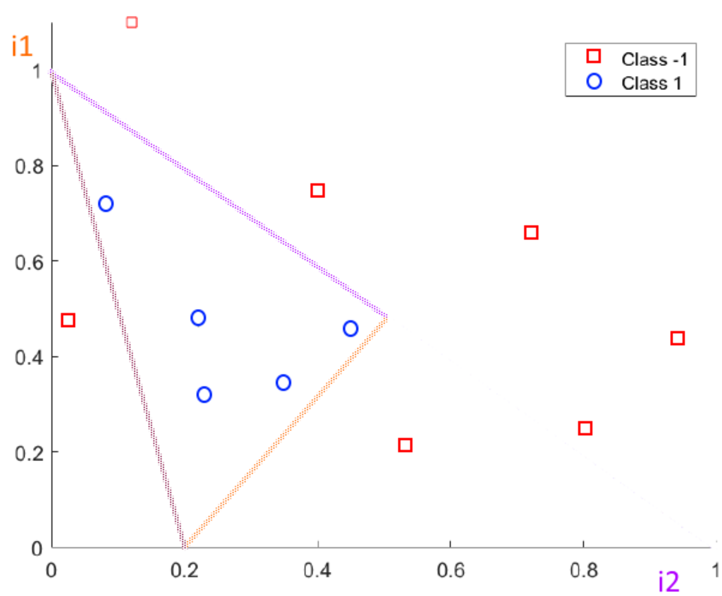
The formula for this dividing line is : $1 - 1 * x_1 - 1 * x_2 = 0$



5.2 b.

We claim that we need 3 separate lines to completely separate two classes. classes. So we need at least 3 perceptrons to compute classification functions. We will just use 3 perceptrons for simplicity.

Formula for line 1:
$1 - 1 * x_1 - 1 * x_2 = 0$
$W_{1,0} = 1, W_{1,1} = -1, W_{1,2} = -1$
Formula for line 2:
$0.2 - 1 * x_1 + 0.6 * x_2 = 0$
$W_{1,0} = 0.2, W_{1,1} = -1, W_{1,2} = 0.6$
Formula for line 3:
$-0.2 + 1 * x_1 + 0.2 * x_2 = 0$
$W_{1,0} = -0.2, W_{1,1} = 1, W_{1,2} = 0.2$



As we can see, a data point gives output as positive with all three perceptrons is class 1 and same applies to class -1. So in the next layer, we can simply apply "and" operation to the results of all thre eperceptrons. In that case, we only need one unit. And the output will be 1 if and only if all perceptrons in 1st layer output 1.

Hence we set

$$W_{4,0} = -2.5, W_{4,1} = 1, W_{4,2} = 1, W_{4,3} = 1$$