Prove that, for any $n+1$ integers $a_1$, $a_2$, ..., $a_n+1$, there exist two of the integers $a_i$ and $a_j$ with $i! = j$ such that $a_i a_j$ is divisible by $n$.

At first,if there exist at least two numbers $a_i$ and $a_j$ and they are equal to each other. Then their difference 0 are always divisible by n.

Now we proceed to another situation that no two numbers equal to each other.

When we want to find out if a number is divisible by n,the remainder can be any integer from 0 to $n-1$. Totally,that is $n$ numbers.

According to the pigeon hole theory.Think each remainder as a container, each number as a pigeon. Given $n+1$ numbers,there must be 2 numbers divide by $n$ that result in same remainder.

Let's assume those two numbers $a_i$ and $a_j$ are:

$a_i = an + r$

$a_j = bn + r$

When we take the absolute difference(Here I assume $a_i > a_j$): $a_i - a_j = (a-b)n$ Their difference has a factor $n$.That proves this question.