1.
Multiset Permutation:There are four options made to select 10 permutations: 10!/3!4!2!1! + 10!/3!4!1!2! + 10!/2!4!2!2! + 10!/3!3!2!2!

2.
a)By applying multinomial formula,$C((18),(5,13)) * 3^5 * (-2)^{13}$

the answer is 0,because the sum of coefficient is $8+9=17$,which is not 18.So by no means, there is not a term x^8y^9 in the expansion of the original function. So the coefficient is 0.
b)By applying multinomial formula,$C((9),(3,3,1,2)) * (-1)^3 * 2 * (-2)^2$

3.
Choice of diamond(0-7) 8
Choice of pearls(0-9) 10
Choice of doubloons(0-34) :35
Grand Total:$8 * 10 * 35 = 2800$

4
This is a distinguishable bins with undistinguishable balls,each horse is like bins,the money is like balls. So the answer is: $C(100+n-1,n-1)$

5
Think there are 13 balls,4 containers need 3 dividers.

a)$x_1 = 3, x_2 = 2$,there are 9 choices for x_3,and make x_4 fulfil the sum equal 13; by keep doing this until $x_1 = 3, x_2 = 10$,there is $(9+1) * 9/2 = 45$. $x_1 = 4, x_2 = 2$, this time starts from 8 choices,so this when $x_1 = 4$,the total is $(8+1) * 8/2 = 36$.By keep adding the total until $x_1 = 11, x_2 = 2$,then there is only 1 choices,and no more x_1 can choose.
The grand total should be $45+36+(7+1)*7/2+(6+1)*6/2+(5+1)*5/2+(4+1)*4/2+(3+1)*3/2+(2+1)*2/2+(1+1)*1/2 = 45+36+28+21+15+10+6+3+2+1 = 167$

b)First set up x_2, there are 5 options in total:
$x_2 = 5$,three containers need two dividers,according to formula : $C((8+3-1),(3-1))$;$x_2 = 4$, $C((9+3-1),(3-1))$;$x_2 = 3$, $C((10+3-1),(3-1))$;$x_2 = 2$, $C((11+3-1),(3-1))$;$x_2 = 1$, $C((12+3-1),(3-1))$;$x_2 = 0$, $C((13+3-1),(3-1))$;The grand total is :$C(10,2)+C(11,2)+C(12,2)+C(13,2)+C(14,2)+C(15,2)$

c)The answer is 0. Since the question is asking all $x_i's$ be multiple of 3. Assume

$$x_1 = 3 * j$$

$$x_2 = 3 * k$$

$$x_3 = 3 * l$$

$$x_4 = 3 * m$$

. Their sum is $3*(j+k+l+m)$, assume it be the LHS of $x_1+x_2+x_3+x_4 = 13$. 13/3 is not an integer , it contradicts the equation. So the number of non negative solution is 0.

6.Given there are 45 moves,the numbers of way to get from front lower left to back upper right is :
Choice 1: choose (1,0,0) moves,

$$C(45, 10)$$

Choice 2: choose (0,1,0) moves,

$$C(45, 15)$$

Choice 3: choose (0,0,1) moves,

$$C(45, 20)$$

By multiplication rule:

$$C(45, 10) * C(45, 15) * C(45, 20)$$