1.

Step1:the total is a permutation of 5 elements

Step2:a function f can map (Rainbow,Red Ballon,Clover,Horseshoe,Heart) to Rainbow,Red Ballon,Clover,Horseshoe,Heart(Order matters), it is a k to one relationship

Step3:we can tell

$$f(Rainbow, RedBallon, Clover, Horseshoe, Heart)$$

$$= f(Heart, Rainbow, RedBallon, Clover, Horseshoe)$$

$$= f(RedBallon, Clover, Horseshoe, Heart, Rainbow)$$

with total 5 cases map to 1 same result. So it is a 5 to 1 relationship.

Step4:hence, according to division principle, we take $5!/5 = 4! = 24$ different combinations

2.

a. with repetition, it is 5^5 sequences

b. with repetition,since first and last spot has assigned by A,the rest spot can have 5^3 sequences.Thus the total is $5^3 * 1 * 1$.

c. length 0,1 won't apply to this question.

Let's start with length 2: CD, 1 way to do this

length 3: choice one: CD is a whole part,now there are 2 options to put CD; choice two: no matter where to put CD, the rest spot can not be C, so there are 4 choices. By multiplication principle, Total is $2 * 4 = 8$

length 4: choice one: treat CD as a whole part, now there are 3 options; choice two: no more C,rest two spots can have a $4^2 = 16$ choices.By multiplication principle, Total is $3 * 16 = 48$

length 5: choice one: treat CD as a whole part, now there are 4 options; choice two: no more C,rest three spots can have a $4^3 = 64$ choices.By multiplication principle, Total is $4 * 64 = 256$

Grand total is $1 + 8 + 16 + 64 = 89$ according to addition principle.

d.

Choice 1: there are $(6, 2)$ combinations to put those two B's

Choice 2: other 4 spots can't have any B,so it is $4^4 = 256$

Total: by multiplication principle, the grand total is $6!/(2!4!) * 256 = 3840$

e.

Split the situations by length

First of all by pigeon hole principle, the length of the sequence can be more than 5 when each letter appears at most once.

length 0: if the empty sequence counts,otherwise ignore this case

length 1: $C(5, 1) = 5$, each letter appears once

length 2: $5^2 - 5 = 20$, subtraction principle, all permutation subtract case when sequence contains duplicate letters

length 3: choice 1 : choose 3 letters $C(5, 3) = 10$ choice 2 : permutation of those 3 letters $3! = 6$ total : $10 * 6 = 60$

length 4: choice 1 : choose 4 letters $C(5,4) = 5$ choice 2 : permutation of those 4 letters $4! = 24$ total : $5 * 24 = 120$

length 5: choice 1 : choose 5 letters $C(5,5) = 1$ choice 2 : permutation of those 5 letters $5! = 120$ total : $1 * 120 = 120$

3.

I will divide this question into two parts since it is asking numbers of distinct pairs of committees. For engineer committee: $C(13,5) = 1287$ For scientist committee:

$$C(7,2) + C(7,3) + C(7,4) + C(7,5) + C(7,6) + C(7,7) = 120$$

The result will be these two possibilities multiplying each other : $1287 * 120 = 154440$

4.

a. Assume there are two disjoint sets,A,B. A $= x_1, ..., x_m$ and B $= y_1, ..., y_n$. LHS is number of subsets of AB of size r. RHS counts same thing by selecting k elements from A ($C(m,k)$ ways), then selecting the remaining $r-k$ elements from B ($C(n, r-k)$ ways), k is integer ranging from 0 to r,that is all the cases which equals the LHS. Thus prove the equation.

b. Assume there are two disjoint sets,A,B with size n respectively. LHS counts a size 2 subset of AB. RHS first term counts a size 2 subset from each set A,B. By addition rule, $C(n,2) + C(n,2)$. Second term counts the situation when one element comes from A,one element comes from B. The grand total,add these up, counts the same thing as RHS. Thus prove the equation.