1.
Assume $a(n)$ is a function stands for "at least n couple are sitting next to each other (means either adjacent or facing each other,this function will be overcounted)".We observe that a(0) = $(2n)!$,sample space S,A_i stands for i_{th} couple sitting next to each other.For instance,$a(2) = \sum_{j<k} A_j \cap A_k$.By inclusion-exclusion principle,this is a derangement problem,$|S-(A_0 \cup A_1 ... \cup A_n)|$ is the answer to it. So $S = a(0) = 2n!$,

$$a(i) = C(n,i) * 2^i * ((2n-i)!)$$

,stands for "i couples in total,choose i from n couple,C(n,i);each couple will have 2 permutation,2^n;then treat those together couple as a whole entity,there are $2n-i$ entity to arrange,so $(2n-i)!$.

The actual function for this derangement is :$\sum_{i=0}^{n}(-1)^i * a(i)$,because $S - |A_1| - |A_2| ... - |A_n| + \sum_{|i_1,i_2|=2} |A_{i_1} \cap A_{i_2}| - ....$Here S is $2n!$,all other elements can be represented by $(-1)^i * a(i)$.

By summarize above, plug $a(i)$ in,$2n!$ also satisfies the summation ,$\sum_{i=0}^{n}(-2)^i * C(n,i) * ((2n-i)!)$

2.
Since it is a linear recurrence relationship,there must be a function $f(n)$ for b_n.$f(n)$= $q^n, q \neq 0$.Plug it in,

$$q^n = 2q^{n-1} + 2q^{n-2}$$

$$q^{n-2} * (q^2 - 2q - 2) = 0$$

,since $q \neq 0$,$(q^2 - 2q - 2) = 0$,$q_+ = 1 + \sqrt{3}, q_- = 1 - \sqrt{3}$.Obviously,we need some constants attached to each term.Suppose there are c_+, c_-.

$$f(n) = c_+ * (1+\sqrt{3})^n + c_- * (1-\sqrt{3})^n$$

Now plug $b_0 = 0, b_1 = 1$,$c_+ = \frac{\sqrt{3}}{6}, c_- = -\frac{\sqrt{3}}{6}$.So

$$f(n) = \frac{\sqrt{3}}{6} * (1+\sqrt{3})^n + (-\frac{\sqrt{3}}{6}) * (1-\sqrt{3})^n$$

3.
By observation,

A) we could place a domino on first row, so there are a_{n1} ways to tile the rest of the grid.

B) or we could place the domino horizontally on it, so we must place another domino horizontally below that and there are a_{n2} ways to tile the rest of the grid.

So,$a_0 = 1, a_1 = 1, a_2 = 2, a_3 = 3$,$a_n = a_{n-1} + a_{n-2}$

4.
By observation,

A) we could place a $1x1$ domino on first first row ,there is only one spot left on first row,then if we choose to place another $1x1$ domino,there is a_{n-1} ways to place rest tiles,regardless of we switch the order of both $1x1$ tile or not.But if we choose to place a 'L' domino on the first row,they make a $2x2$ square,in this case, there are a_{n-2} ways.However,the place where to put first $1x1$ matter in this case.Both cases gives out same ways,so it is $2 * a_{n-2}$ ways.The grand total of this situation is $an_1 + 2 * an - 2$

B) If we place a 'L' domino on first row and make it take over the whole first row.There are two ways to do that 'L' and upside-down 'L' on the board. No matter which case we can continue put one more $1x1$ tile to make a $2x2$ square with rest of an_2 ways or we put another 'L' to make a $2x3$ rectangle with composed with 2 $'L's$,which gives $an - 3$ ways.Since there are ways to start the first step, so the grand total of this situation is $2 * an_2 + 2 * an - 3$

By addition rule, the total is : $a_n = a_{n-1} + 4 * a_{n-2} + 2 * a_{n-3}$

We also find the base case: $a_0 = 1, a_1 = 1, a_2 = 4, a_3 = 10$,which satisfies the recurrence relation.