1. $E(n) = Z(n) + O(n) + T(n)$,E : all ternary sequence; Z sequence ends with 0 ; O sequence ends with 1;T sequence ends with 2;

$$E(n) = Z(n) + O(n) + T(n)$$

$$Z(n) = T(n-1)$$

$$O(n) = T(n-1)$$

$$T(n) = Z(n-1) + O(n-1) + T(n-1)$$

now substitute each term with RHS terms.

$$E(n) = Z(n-1) + O(n-1) + T(n-1) + 2 * T(n-1)$$

$$= E(n-1) + 2 * E(n-2)$$

.Thus proves the recurrence relation is true.

Given, $h_x - h_{x-1} - 2h_{x-2} = 0$.
Assume there is
$h(x) = h_0 + h_1 x + h_2 x^2 + h_3 x^3 ... + h_n x^n$
$-xh(x) = 0 - h_0 x - h_1 x^2 - h_2 x^3 - h_3 x^4 ... - h_{n-1} x^n$
$-2x^2 h(x) = 0 + 0 - 2h_0 x^2 - 2h_1 x^3 - 2h_2 x^4 - 2h_3 x^5 ... - 2h_{n-2} x^n$
Add them together:
$(1 - x - 2x^2)h(x) = h_0 + (h_1 - h_0)x$
$h(x) = \frac{1+2x}{(1-x-2x^2)} = \frac{c_1}{-2x+1} + \frac{c_2}{x+1}$
Solve for c_1, c_2:
$(c_1 - 2c_2)x + (c_1 + c_2) = 2x + 1$
$c_1 + c_2 = 1$
$c_1 - 2c_2 = 2$
$c_1 = \frac{4}{3}$,$c_2 = -\frac{1}{3}$
Now plug in c_1, c_2:
$h(x) = \frac{4}{3} * \frac{1}{-2x+1} - \frac{1}{3} * \frac{1}{x+1}$
convert into summation:

$$\frac{4}{3} * \sum_{k=0} 2^k x^k - \frac{1}{3} * \sum (-1)^k x^k$$

$$\sum_{k=0} [\frac{4}{3} * 2^k + (-\frac{1}{3}) * (-1)^k] x^k$$

Now we obtain the part for a_n,which is $[\frac{4}{3} * 2^k + (-\frac{1}{3}) * (-1)^k]$
2.
Guessing method:
Since the particular part is n, we guess solution to it is : $an + b$,plug in:
$an + b = 2(a(n-1) + b) + n$
$0 = (a+1)n - (2a - b)$
solve for a,b $a = -1, b = -2$

Now $-n-2$ is one solution to recurrence relation(Not the initial condition!),now we add a solution to the homogenous part, which is $c2^n$.By setting up the initial condition,we can solve for c:

$h_0 = -(0) - 2 + c2^{(0)} = 1$,$c = 3$

So $h_n = -n - 2 + 3 * 2^n$

Generating function:we have $h_n - 2h_{n-1} - n = 0$

$h(x) = h_0 + h_1 x + h_2 x^2 + h_3 x^3 ... + h_n x^n$

$-2xh(x) = 0 - 2h_0 x - 2h_1 x^2 - 2h_2 x^3 ... - 2h_{n-1} x^n$

$-\sum_{n>=0} n * x^n = 0 - 1x - 2x^2 - 3x^3 ... - nx^n$

By adding these equations together

$$(1-2x)h(x) - \sum_{n>=0} n * x^n = h_0 = 1$$

$$h(x) = \frac{1 + \frac{x}{(1-x)^2}}{(1-2x)} = \frac{x^2 - x + 1}{(1-x)^2 * (1-2x)}$$

$$= \frac{c_1}{(1-x)} + \frac{c_2}{(1-x)^2} + \frac{c_3}{(1-2x)}$$

$$= c_1 + c_2 + c_3 - (3c_1 + 2c_2 + 2c_3)x + (2c_1 + c_3)x^2$$

$$c_1 + c_2 + c_3 = 1$$

$$3c_1 + 2c_2 + 2c_3 = 1$$

$$2c_1 + c_3 = 1$$

set up matrix and solve them,$c_1 = -1, c_2 = -1, c_3 = 3$.

$$h(x) = \frac{-1}{(1-x)} + \frac{-1}{(1-x)^2} + \frac{3}{(1-2x)}$$

$$= \sum_n (-1) * x^n + \sum_n -1 * C(n+2-1, n)x^n + + \sum_n 3 * 2^n x^n$$

$$= \sum_n (-1 - n - 1 + 3 * 2^n)x^n$$

$$= \sum_n (-n - 2 + 3 * 2^n)x^n$$

So,$h_n = -n - 2 + 3 * 2^n$.

Since the g.f method produce the exact same answer as guessing method, they prove each other are correct answers amazingly !

3. Using Guessing method:

Since the particular part is $2n$, we guess solution to it is : $an + b$,plug in:

$an + b = 6(a(n-1) + b) - 9(a(n-2) + b) + 2n$

$(4a2)n + (4b12a) = 0$

so a = $\frac{1}{2}$,and b =$\frac{3}{2}$

Now $\frac{1}{2}n + \frac{3}{2}$ is one solution to recurrence relation(Not the initial condition!),now we add a solution to the homogenous part, which is $c_1 3^n + c_2 n 3^n$.By setting up the initial condition,we can solve for c_1, c_2:

$h_0 = c_1 3^0 + c_2 * 0 * 3^0 + \frac{1}{2} * 0 + \frac{3}{2}$

$h_1 = c_1 3^1 + c_2 * 1 * 3^1 + \frac{1}{2} * 1 + \frac{3}{2}$

solve for c_1, c_2:

$c_1 = -\frac{1}{2}$,$c_2 = -\frac{1}{6}$

$h(x) = -\frac{1}{2} * 3^n - \frac{1}{6} n * 3^n + \frac{1}{2} n + \frac{3}{2}$