1.total is $C(52,7) = 133784560$

a.

Choose the rank of four cards: 13 ways
Choose the suits of those four cards: 1 way
Choose the rank for rest three cards:12 ways
Choose the suits for these three cards : $C(4,3) = 4$ ways, thus the probability is

$$13 * 1 * 12 * 4/133784560 = 0.00000466421$$

b.

Choose the rank of three cards: 13 ways
Choose the suits of those three cards: 4 ways
Choose the two different ranks for rest cards:$C(12,2) = 66$ ways
Choose the suits for these each two cards : $C(4,2) * C(4,2) = 36$ ways, thus the probability is

$$13 * 4 * 66 * 36/133784560 = 0.00092351464$$

c.

Choose the three different rank of 6 cards: $C(13,3) = 286$ ways
Choose the suits of those 6 cards: $C(4,2) * C(4,2) * C(4,2) = 216$ ways
Choose the fourth rank for rest one card :10 ways
Choose the suits for last one card : 4 ways. Thus the probability is

$$286 * 216 * 10 * 4/133784560 = 0.01847029283$$

d.

Think the 7 consecutive ranks as a whole entity, there are $13 - 7 + 1 = 7$ spots to place the entity. Choose spot : 7 ways
Choose suits for this entity: every rank has 4 ways, so it is 4^7. Thus the probability is

$$4^7 * 7/133784560 = 0.00085725886$$

2.

Total is 6^6 cases

a.

There are 6! ways to have subset $(1,2,3,4,5,6)$,there are total 6^6 ways. Thus,the probability is $6!/6^6 = 0.01543209876$

b.

The total is $6^6 = 46656$,treat sum 15 as balls,6 dices are dividers. There are totally $15 - 1 = 14$ spaces to put $6 - 1 = 5$ dividers. There are $C(14,5) = 2002$ cases.However, we have to subtract total cases that is invalid

$(1,1,1,1,1,10), (1,1,1,1,2,9), (1,1,1,1,3,8)$

$(1,1,1,1,4,7), (1,1,1,2,2,8), (1,1,1,2,3,7), (1,1,2,2,2,7)$

All permutations are :$C(6,1) + C(6,2) * 3 * 2 + 3 * C(6,2) * C(4,1) + C(6,3) * 3 * 2 + C(6,3) * C(3,1) = 456$. So there are totally : $2002 - 456 = 1546$ ways to have sum of 15. Thus the probability is $1546/46656 = 0.0331361454$

c.

This time, first distribute 3 to green dice. Then there is only 12 as sum. By performing same method. $C(11, 5) = 462$ Cases that has at 4 at green dice. There are still invalid cases existing $(4, 1, 1, 1, 1, 7)$. There are 6 cases. By subtraction rule, $(462 - 6)/46656 = 0.00977366255$.

d.

Find situation with at least 1, choose dice to be 1 and rest to be any $6^5/6^6$.By subtraction, $1 - (6^5/6^6)$ is the event that number on each of dice is at least 2.

e.

Each number is 1/6. Choice 1: 2 dice to be $2C(6, 2)$. Choice 2:2 dice to be $4C(4, 2)$. Choice 3:2 dice to be $6C(2, 2)$. Grand total :$(1/6)^6 * C(6, 2) * C(4, 2) * C(2, 2) = 0.00192901234$

3.

Both sides contains r,n. $r \leq n$.

Both sides are counting the ways to choose team with r leaders from n people.

LHS:Think as choose a team with size of k from n people, $k \leq n$.Then from those k people, choose r leaders.r may equal k. RHS:In this case, first choose r leaders from n people : $C(n, r)$. Since $r \leq n$, the rest team member totally has 2^{n-r} ways to choose. By multiplication rule, the total is $C(n, r) * 2^{n-r}$. LHS choose k team members as well as choosing r leaders with all situation of $k \leq n$. RHS choose r leaders from n people,then counts all different situations for choosing rest team members.They are actually counting the same thing