3.

Firstly,we distinguish 5 edges of the outer C_5, 5 edges of the inner C_5, and 5 crossing edges.

A Hamiltonian cycle must end at its starting point, and so must use an even number (in this case ,2 or 4) of crossing edges.

If there are 2 crossing edges, then their ends in both the outer C_5 and the inner C_5 must be joined by paths of length 4. This implies that these ends must be adjacent in both the outer C_5 and the inner C_5, which is impossible.(In the graph,the inner vertex shown in-order is not adjacent connected)

If there are 4 crossing edges, then we must have three edges in both the outer C_5 and the inner C_5, including two through each of the points not on chosen crossing edges. There is a unique such configuration, but it consists of two 5-cycles rather than a Hamiltonian cycle.

By conclusion of above situations,there is no Hamiltonian cycle in a Petersen graph.