1.

As for every vertex ,the degree is $n-1$.According to ore's theorem, there is a Hamiltonian path in K_n. No matter a) or b), I will just consider a Hamiltonian path in K_n will stop by n vertices(think as n stops for a road trip).So the choices all permutation of these stops is $n!(K_n's$ property,every vertices connecting all remaining vertices). Besides,there are $n-1$ edges since there are n vertices for a Hamiltonian path.

Now we add another limitation,which requires a monochromatic path. We have that

$$E(X) = Pr(X = Red)1 + Pr(X = Blue)0 = Pr(X = Red) = \frac{1}{2^{n-1}}$$

This is like an linear expectation problem: n! * $\frac{1}{2^{n-1}}$ turns out to be the expectation of this question which is * $\frac{n!}{2^{n-1}}$, this proves for a) and b) since it is an expectation number.

4. Think this combinatorically:

We can consider this as a random coloring of K_n with each edge colored red or blue with probability 1/2. The probability from any groups of n vertices that has all edges on that clique(every two edges adjacent to each other) of same color is $2^{1-C(n,2)}$.The number of such group is $C(r,n)$.Hence the probability that there is at least one monochromatic clique is at most:

$$C(r,n) * 2^{1-C(n,2)}$$

,we know => $C(r,n) * 2^{1-C(n,2)} < 1$

And we are given the two estimation:

$$\frac{2r^n}{n! * 2^{\frac{n*(n-1)}{2}}}$$

in order to estimate this, i assume $r = 2^{\frac{n}{2}}$,plug in and discard some constants:

$$\frac{2^{1+\frac{n}{2}}}{n!} < 1$$

Hence, $r = 2^{\frac{n}{2}}$ satisfy this condition.

And since $r = O(L)$, $L = 2^n$