1. a) lack: reflexive property

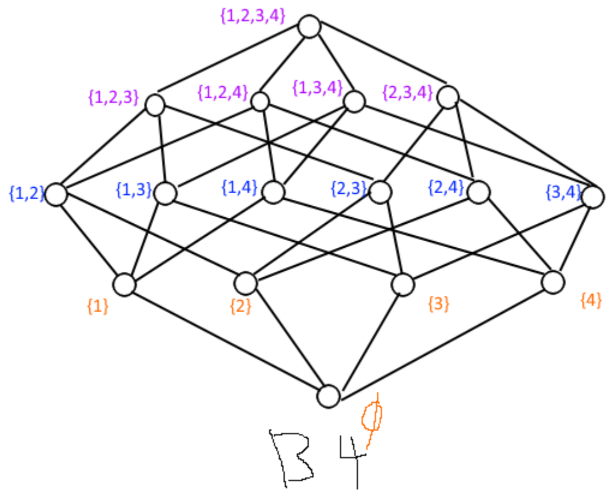
b) it is a partial order

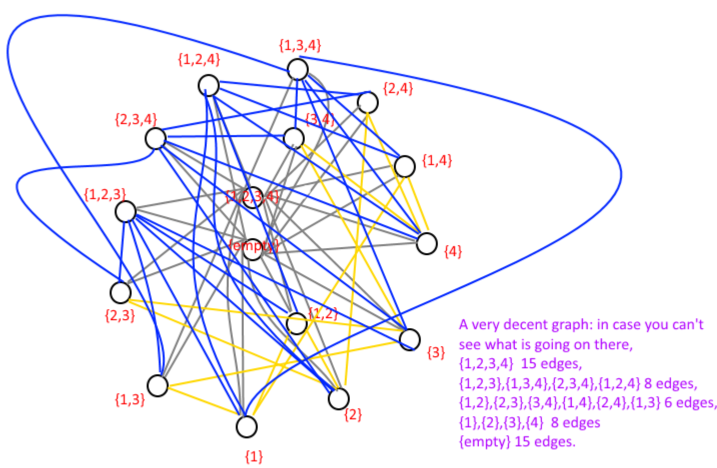
c) lack: antisymmetric property,because we have both $a \leq b$, $b \leq a$, which should implies $a = b$

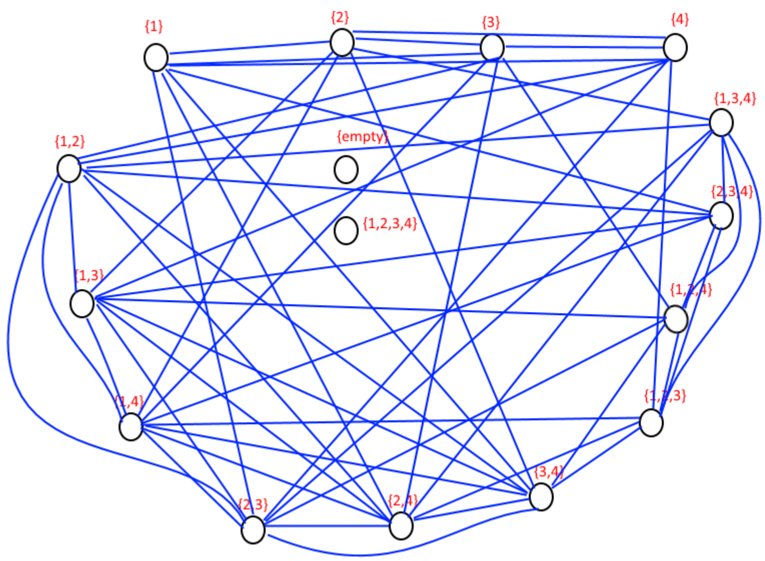
d) lack: transitive property,need $a \leq c$

2.(All graph shows in order of Hasse diagram, comparability graph and incomparability graph)

a)

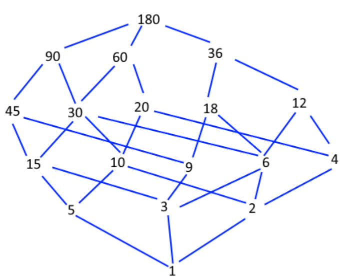


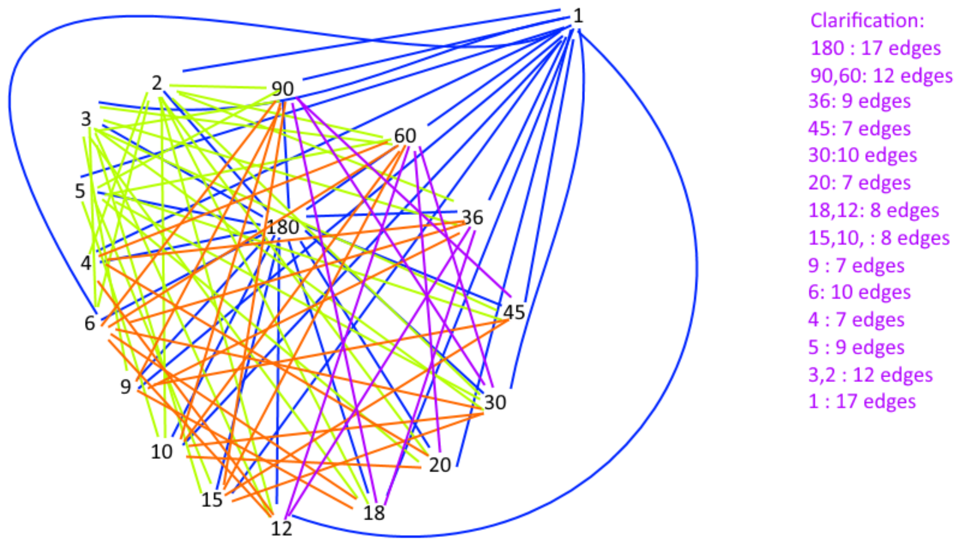


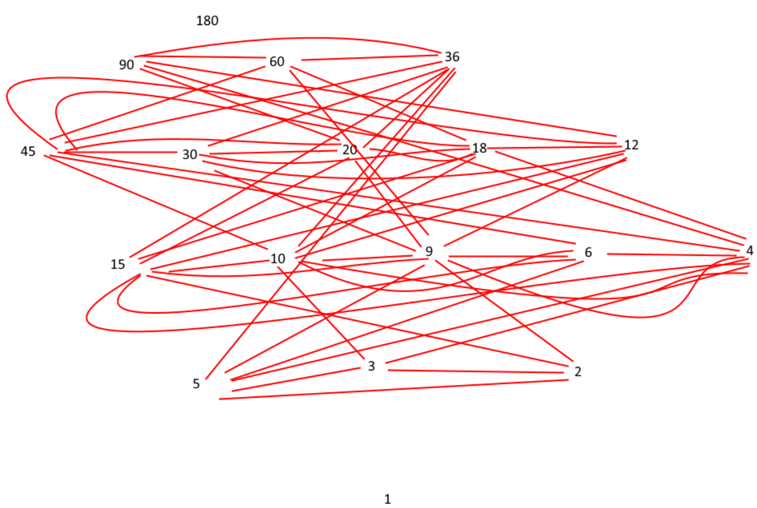


Clarification:
{1,2,3,4} 0 edges
{1,2,3}... capacity of 3 set : 7 edges
{1,2}...capacity of 2 set : 9 edges
{1}... capacity of 1 set: 7 edges
{empty} :0 edges

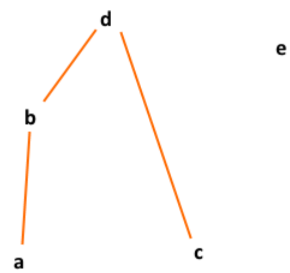
b)



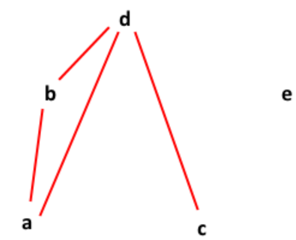

1
2
90
3
60
5
180
36
4
45
6
9
30
10
20
15
18
12
Clarification:
180 : 17 edges
90,60: 12 edges
36: 9 edges
45: 7 edges
30:10 edges
20: 7 edges
18,12: 8 edges
15,10, : 8 edges
9 : 7 edges
6: 10 edges
4 : 7 edges
5 : 9 edges
3,2 : 12 edges
1 : 17 edges


180
90
60
36
45
30
20
18
12
15
10
9
6
4
3
5
2
1

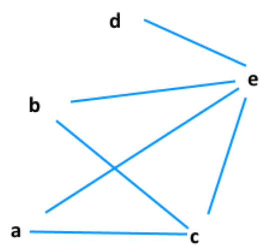
c)



Hasse Diagram



comparability graph



incomparability graph

3.

a)1,2,3,4,7,8,17

b)10,14,15,16,18

c)height is 5, if you follow (4,5,6,9,14)

d) According to Dilworths Theorem : It is possible to decompose a finite poset P into width(P), and no fewer, chains.

width(P) = the largest antichains

and we know "Fact 2.0.1. max(P) and min(P) are both antichains."

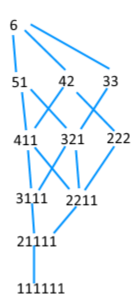
In our case, it is the —min— = 7.

So we claim it can be partitioned by 7 chains and no fewer.

e)

As said in d), 7.

4.



a)

b)

6 , from bottom to top by any path

c)

6, according to Anti-Dilworths Theorem: It is possible to decompose a finite poset P into height(P),and no fewer, antichains.And we know the height is the longest chain,in this case is 6.

d)

3,the antichains is in the middle of this lattice. (411,321,222) , (51,42,33)

e)

According to Dilworths Theorem : It is possible to decompose a finite poset P into width(P), and no fewer, chains.

width(P) = the largest antichains

and we know "Fact 2.0.1. max(P) and min(P) are both antichains."

In our case, it is the —min— = 6.

So we claim it can be partitioned by 6 chains and no fewer.

5.

This problem barely asks for largest antichain of a Boolean lattice, according to the Sperners Theorem,

the largest antichain of B_n is of size :

$$C(n, floor(n/2))$$

$$= C(5, floor(5/2)) = C(5, 2) = 10$$

So size of 10 collection is the largest collection ,$C(5, 2) = C(5, 3)$,so set contains 2 elements and 3 elements:

such collection could be :

$$(1, 2),(1, 3),(1, 4),(1, 5),(2, 3),(2, 4),(2, 5),(3, 4),(3, 5),(4, 5)$$

$$(1, 2, 3),(1, 2, 4),(1, 2, 5),(1, 3, 4),(1, 3, 5),(1, 4, 5),(2, 3, 4),(2, 3, 5),(2, 4, 5),(3, 4, 5)$$