1.

a.

Since both function are finite in a small upper bound,we just multiply them up. $f(x) * g(x) = 1 - 4x^2 + x^3 - 4x^5 + x^6 - 4x^8$.

Formula for x^k :

$k = 0, 3, 6$;coefficient is 1

$k = 2, 5, 8$;coefficient is -4

$k =$other values;coefficient is 0

b.

$f(x) = \sum_{j=0}^{n} C(n, j) * x^j$

$g(x) = \sum_{l=0} x^l$,according to Newton's binomial theorem.

$f(x) * g(x) = \sum_{k=0} \sum_{l=0}^{k} C(n, k - l) * x^k$.Thus the coefficient is $C(n, k - l)$

c.

$f(x) = \sum_{n=0} (2x)^n$

$g(x) = \sum_{m=0} \frac{(x)^m}{m!}$

$f(x) * g(x) = \sum_{n=0} \sum_{m=0} \frac{2^n}{m!} * x^{n+m}$,Thus the coefficient is $\frac{2^n}{m!}$

2.

a.

$7!/((4!)(3!)) = 35$

b.

according to formula,$C(-7, 4) = (-1)^4 * C(7 + 4 - 1, 4) = 210$

c.

$$\frac{\frac{1}{2} * -\frac{1}{2} * -\frac{3}{2} * -\frac{5}{2}}{4!} = \frac{1}{128}$$

d.

$$\frac{-\frac{1}{2} * -\frac{3}{2}}{2!} = \frac{3}{8}$$

3.Simply,this is a 6 types of object model,we can choose any number of each objects except for object 77.For object 77,we can only choose at most 1.This question is asking how many object values can be generated by following above rule.The generating function is:

$$(1 + x^{77}) * (1 + x + x^2 + x^3 + x^4 ... + x^n) * (1 + x^3 + x^6 + x^9 ... + x^{3n}) *$$

$$(1 + x^{17} + x^{34} + x^{51} ... + x^{17n}) * (1 + x^{19} + x^{38} + x^{57} ... + x^{19n}) * (1 + x^{42} + x^{84} ... + x^{42n})$$