Machine Problem 1 Kalman Filter Update Equations
Introduction to Computational Robotics
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From the system equation

$$x_t = \begin{pmatrix} x_{1,k} \\ x_{2,k} \end{pmatrix} = \begin{pmatrix} x_{1,k-1} \\ x_{2,k-1} \end{pmatrix} + \begin{pmatrix} u_{1,k-1} \\ u_{2,k-1} \end{pmatrix} + \omega_{k-1}$$

and the observer equation

$$z_t = \begin{pmatrix} z_{1,k} \\ z_{2,k} \end{pmatrix} = \begin{pmatrix} x_{1,k} \\ x_{2,k} \end{pmatrix} + v_k,$$

A, B and H are the identity matrix, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Q and R are also given to be:

$$Q = \begin{pmatrix} 10^{-4} & 2 * 10^{-5} \\ 2 * 10^{-5} & 10^{-4} \end{pmatrix}$$

$$R = \begin{pmatrix} 10^{-2} & 5 * 10^{-3} \\ 5 * 10^{-3} & 10^{-2} \end{pmatrix}$$

Plugging in A, B, H, Q and R, the time update equations are:

$$\hat{x}_k^- = \begin{pmatrix} \hat{x}_{1,k-1} \\ \hat{x}_{2,k-1} \end{pmatrix} + \begin{pmatrix} u_{1,k-1} \\ u_{2,k-1} \end{pmatrix}$$

$$P_k^- = P_{k-1} + \begin{pmatrix} 10^{-4} & 2 * 10^{-5} \\ 2 * 10^{-5} & 10^{-4} \end{pmatrix}$$

The measurement updates are:

$$K_k = \frac{P_k^-}{P_k^- + \begin{pmatrix} 10^{-2} & 5 * 10^{-3} \\ 5 * 10^{-3} & 10^{-2} \end{pmatrix}}$$

$$P_k = \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - K_k \right) P_k^-$$

$$\hat{x}_k = \begin{pmatrix} \hat{x}_{1,k} \\ \hat{x}_{2,k} \end{pmatrix} = \begin{pmatrix} \hat{x}_{1,k}^- \\ \hat{x}_{2,k}^- \end{pmatrix} + K_k \left(z_k - \begin{pmatrix} \hat{x}_{1,k}^- \\ \hat{x}_{2,k}^- \end{pmatrix} \right)$$