Examen scris la analiza matematica
-sesiune iarna 2021-

1. Studiati convergenta si absolut convergenta seriei

$$\sum_{n=1}^{\infty}(-1)^{n+1}\sin\frac{\pi}{\sqrt{n}}$$

2. Calculati integrala improprie

$$\int_{1}^{\infty}\frac{1}{x^3+x}\,\mathrm{d}x$$

3. Determinati punctele critice si punctele de extrem local (specificand tipul acestora) pentru functia

$$f:\mathbb{R}^2\to\mathbb{R},\quad f(x,y)=(x+xy+y^2)\sqrt{\mathrm{e}^x}$$

4. Fie $g:(0,+\infty)^2\to\mathbb{R}$ o functie de clasa C^1. Exprimati relatia

$$-u\frac{\partial g}{\partial u}(u,v)+\frac{v}{1+2u}\frac{\partial g}{\partial v}(u,v)=1,\quad \forall(u,v)\in(0,+\infty)^2$$

in variabilele $(x,y)\in(0,+\infty)^2$, efectuand transformarea $u=\frac{y}{x}$ si $v=x+2y$. Determinati apoi o functie g cu proprietatile de mai sus. Verificare.