Problema 9.3.5.

Simplificaţi următoarele funcţii booleene de patru variabile date prin valorile de 1, utilizând metoda lui Quine:

5. $f_5(1,1,1,1)=1$, $f_5(0,1,0,1)=1$, $f_5(0,1,1,1)=1$, $f_5(1,1,1,0)=1$, $f_5(1,1,0,0)=1$, $f_5(1,0,0,0)=1$, $f_5(1,0,0,1)=1$, $f_5(0,0,0,1)=1$;

Tabel de valori al functiei f5

x_1	x_2	x_3	x_4	f_5	
0	0	0	0	0	
0	0	0	1	1	m_1
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	1	m_5
0	1	1	0	0	
0	1	1	1	1	m_7
1	0	0	0	1	m_8
1	0	0	1	1	m_9
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	1	m_{12}
1	1	0	1	0	
1	1	1	0	1	m_{14}
1	1	1	1	1	m_{15}

Metoda analitică a lui Quine-Mc'Clusky

- se bazează pe completarea a două tabele ajutătoare
  - unul pentru factorizare, utilizat la calcularea mulţimii monoamelor maximale
  - unul pentru identificarea mulţimii monoamelor centrale
- se aplică formei canonice disjunctive a funcţiei
- poate fi utilizată pentru oricâte variabile

FCD (forma canonica disjunctiva)este:

$f(x_1,x_2,x_3,x_4) = m_1 \vee m_5 \vee m_7 \vee m_8 \vee m_9 \vee m_{12} \vee m_{14} \vee m_{15}$

Deci
Sf ={(1,1,1,1),(0,1,0,1),(0,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0),(1,0,0,1),(0,0,0,1)}

Pasul 1 : Se ordoneaza multimea suport a functiei cu 4 variabile descrescator sau crescator (am ales crescator) dupa numarul de valori de 1 continut de fiecare cvadruplu

Sf ={(0,0,0,1),(1,0,0,0),(0,1,0,1),(1,1,0,0),(1,0,0,1),(0,1,1,1),(1,1,1,0),(1,1,1,1)}

Pasul 2 : Construirea primei tabele si factorizarea

Grup	X_1	X_2	X_3	X_4	
I	0	0	0	1	m_1 ✓
	1	0	0	0	m_8 ✓
II	0	1	0	1	m_5 ✓
	1	1	0	0	m_{12} ✓
	1	0	0	1	m_9 ✓
III	0	1	1	1	m_7 ✓
	1	1	1	0	m_{14} ✓
IV	1	1	1	1	m_{15} ✓
V = I + II	0	-	0	1	$m_1 \vee m_5$
	-	0	0	1	$m_1 \vee m_9$
	1	-	0	0	$m_8 \vee m_{12}$
	1	0	0	-	$m_8 \vee m_9$
VI = II + III	0	1	-	1	$m_5 \vee m_7$
	1	1	-	0	$m_{12} \vee m_{14}$
VII = III + IV	-	1	1	1	$m_7 \vee m_{15}$
	1	1	1	-	$m_{14} \vee m_{15}$

Pasul 3 : Identificarea monoamelor maximale

Identificarea monoamelor maximale

- Mulțimea **monoamelor maximale** este formată din toate monoamele corespunzătoare liniilor nebifate din tabel.

$m_1 \vee m_5 = max_1 = \overline{x_1}\,\overline{x_3}x_4$

$m_1 \vee m_9 = max_2 = \overline{x_2}\,\overline{x_3}x_4$

$m_8 \vee m_{12} = max_3 = x_1\,\overline{x_3}\,\overline{x_4}$

$m_8 \vee m_9 = max_4 = x_1\,\overline{x_2}\,\overline{x_3}$

$m_5 \vee m_7 = max_5 = \overline{x_1}\,x_2x_4$

$m_{12} \vee m_{14} = max_6 = x_1x_2\,\overline{x_4}$

$m_7 \vee m_{15} = max_7 = x_2x_3x_4$

$m_{14} \vee m_{15} = max_8 = x_1x_2x_3$

$$M(f) = \{max_1, max_2, max_3, \ldots, max_8\}$$

Pasul 4 : Identificarea monoamelor centrale

monoame maximale / mintermi	max1	max2	max3	max4	max5	max6	max7	max8
m_1	*	*						
m_8			*	*				
m_5	*				*			
m_{12}			*			*		
m_9		*		*				
m_7					*		*	
m_{14}						*		*
m_{15}							*	*

$m_1 \vee m_5 = max_1 = \bar{x}_1 \bar{x}_3 x_4$

$m_1 \vee m_9 = max_2 = \bar{x}_2 \bar{x}_3 x_4$

$m_8 \vee m_{12} = max_3 = x_1 \bar{x}_3 \bar{x}_4$

$m_8 \vee m_9 = max_4 = x_1 \bar{x}_2 \bar{x}_3$

$m_5 \vee m_7 = max_5 = \bar{x}_1 x_2 x_4$

$m_{12} \vee m_{14} = max_6 = x_1 x_2 \bar{x}_4$

$m_7 \vee m_{15} = max_7 = x_2 x_3 x_4$

$m_{14} \vee m_{15} = max_8 = x_1 x_2 x_3$

Pastul 5: Identificarea formelor simplificate

Se observa ca nu exista nicio steluta unica pe linia sa, deci $C(f5) = \emptyset$

Este cazul al treilea al algoritmului de simplificare . Se vor cauta cele mai simple solutii , adica se vor alege monoame maximale astfel incat intersectia liniilor avand stelute comune cu coloanele monoamelor maximale selectate sa fie cat mai mica si numarul monoamelor maximale alese sa fie , de asemenea , cat mai mic , iar reuninea acestor linii sa fie egala cu multimea tuturor liniilor tabelului.

$m_1 \vee m_5 = max_1 = \bar{x}_1 \bar{x}_3 x_4$

$m_1 \vee m_9 = max_2 = \bar{x}_2 \bar{x}_3 x_4$

$m_8 \vee m_{12} = max_3 = x_1 \bar{x}_3 \bar{x}_4$

$m_8 \vee m_9 = max_4 = x_1 \bar{x}_2 \bar{x}_3$

$m_5 \vee m_7 = max_5 = \bar{x}_1 x_2 x_4$

$m_{12} \vee m_{14} = max_6 = x_1 x_2 \bar{x}_4$

$m_7 \vee m_{15} = max_7 = x_2 x_3 x_4$

$m_{14} \vee m_{15} = max_8 = x_1 x_2 x_3$

O forma simplificata este :

mintermi \ monoame maximale	max1	max2	max3	max4	max5	max6	max7	max8
m_1	*	*						
m_8			*	*				
m_5	*				*			
m_{12}			*			*		
m_9		*		*				
m_7					*		*	
m_{14}						*		*
m_{15}							*	*

$h_1(x_1, x_2, x_3, x_4) = max_1 \vee max_4 \vee max_6 \vee max_7$

$h_1(x_1, x_2, x_3, x_4) = \bar{x}_1 \bar{x}_3 x_4 \vee x_1 \bar{x}_2 \bar{x}_3 \vee x_1 x_2 \bar{x}_4 \vee x_2 x_3 x_4$

O alta forma simplificata este

mintermi \ monoame maximale	max1	max2	max3	max4	max5	max6	max7	max8
m_1	*	*						
m_8			*	*				
m_5	*				*			
m_{12}			*			*		
m_9		*		*				
m_7					*		*	
m_{14}						*		*
m_{15}							*	*

$m_1 \vee m_5 = max_1 = \bar{x}_1 \bar{x}_3 x_4$

$m_1 \vee m_9 = max_2 = \bar{x}_2 \bar{x}_3 x_4$

$m_8 \vee m_{12} = max_3 = x_1 \bar{x}_3 \bar{x}_4$

$m_8 \vee m_9 = max_4 = x_1 \bar{x}_2 \bar{x}_3$

$m_5 \vee m_7 = max_5 = \bar{x}_1 x_2 x_4$

$m_{12} \vee m_{14} = max_6 = x_1 x_2 \bar{x}_4$

$m_7 \vee m_{15} = max_7 = x_2 x_3 x_4$

$m_{14} \vee m_{15} = max_8 = x_1 x_2 x_3$

$$h_2(x_1, x_2, x_3, x_4) = max_2 \vee max_3 \vee max_5 \vee max_8$$

$$h_2(x_1, x_2, x_3, x_4) = \bar{x}_2 \bar{x}_3 x_4 \vee x_1 \bar{x}_3 \bar{x}_4 \vee \bar{x}_1 x_2 x_4 \vee x_1 x_2 x_3$$