Question **1**

Correct

Mark 0.50 out of 0.50

Use Fermat's method to determine the decomposition of the number $n = 7003$ into two factors.

Important note: All answer boxes should be filled in using the convention that those not applicable must be filled in with x.

Solution.

Initialization:

$t_0 = [\sqrt{n}]=$ 83

Iterations:

$t = t_0 + 1$: $t^2 - n=$ 53 perfect square (yes/no) no

$t = t_0 + 2$: $t^2 - n=$ 222 perfect square (yes/no) no

$t = t_0 + 3$: $t^2 - n=$ 393 perfect square (yes/no) no

$t = t_0 + 4$: $t^2 - n=$ 566 perfect square (yes/no) no

$t = t_0 + 5$: $t^2 - n=$ 741 perfect square (yes/no) no

$t = t_0 + 6$: $t^2 - n=$ 918 perfect square (yes/no) no

$t = t_0 + 7$: $t^2 - n=$ 1097 perfect square (yes/no) no

$t = t_0 + 8$: $t^2 - n=$ 1278 perfect square (yes/no) no

$t = t_0 + 9$: $t^2 - n=$ 1461 perfect square (yes/no) no

$t = t_0 + 10$: $t^2 - n=$ 1646 perfect square (yes/no) no

$t = t_0 + 11$: $t^2 - n=$ 1833 perfect square (yes/no) no

$t = t_0 + 12$: $t^2 - n=$ 2022 perfect square (yes/no) no

$t = t_0 + 13$: $t^2 - n=$ 2213 perfect square (yes/no) no

$t = t_0 + 14$: $t^2 - n=$ 2406 perfect square (yes/no) no

$t = t_0 + 15$: $t^2 - n=$ 2601 perfect square (yes/no) yes

$t = t_0 + 16$: $t^2 - n=$ x perfect square (yes/no) x

$t = t_0 + 17$: $t^2 - n=$ x perfect square (yes/no) x

$t = t_0 + 18$: $t^2 - n=$ x perfect square (yes/no) x

$t = t_0 + 19$: $t^2 - n=$ x perfect square (yes/no) x

$t = t_0 + 20$: $t^2 - n=$ x perfect square (yes/no) x

Values:

$s=$ 51 $t=$ 98

Conclusion:

The obtained two factors of n are (in increasing order!) 47 and 149 .