Seminar Nr. 1, Random Variables and Applications

```
clear all;
format longg;
```

2. Messages arrive at an electronic message center at random times, with an average of 9 messages per hour. What is the probability of
a) receiving *exactly* 5 messages during the next hour (event A)?
b) receiving *at least* 5 messages during the next hour (event B)?

```
% a)
prob_a = poisspdf(5, 9);
fprintf("P(A) = %g", prob_a);
```

```
P(A) = 0.0607269
```

```
% b)
prob_not_b = poisscdf(4, 9);
prob_b = 1 - prob_not_b;
fprintf("P(B) = %g", prob_b);
```

```
P(B) = 0.945036
```

3. After a computer virus entered the system, a computer manager checks the condition of all important files. He knows that each file has probability 0.2 to be damaged by the virus, independently of other files. Find the probability that
a) at least 5 of the first 20 files checked, are damaged (event A);
b) the manager has to check at least 6 files in order to find 3 that are undamaged (event B).

```
% a)
% success = damaged file, p=0.2
% binomial distribution
% n = 20, p = 0.2
% P(A) = P(X >= 5) = 1 - P(X<5) = 1 - P(x <= 4)
%       = 1 - binocdf(4, 20, 0.2) where k=4
prob_not_a = binocdf(4, 20, 0.2);
prob_a = 1 - prob_not_a;
fprintf("P(A) = %g", prob_a);
```

```
P(A) = 0.370352
```

```
% b)
% success = undamaged file, so p=0.8
% B: check at least 6 files in order to find 3 undamaged
% the 3rd success in at least 6 trials
% the 3rd success after at least 3 failures
% X - nr. of files found to be damaged (failures) before the 3rd undamaged one is
% found (success). X has negative binomial distribution n=3, p=0.8
% P(B) = P(X >= 3) = 1 - P(X < 3) = 1 - P(X <= 2) = 1 - F(2)
prob_not_b = nbincdf(2, 3, 0.8);
prob_b = 1 - prob_not_b;
fprintf("P(B) = %g", prob_b);
```

```
P(B) = 0.05792
```

4. Consider a satellite whose work is based on block A, independently backed up by a block B. The satellite performs its task until both blocks A and B fail. The lifetimes of A and B are Exponentially distributed with mean lifetime of 10 years. What is the probability that the satellite will work for more than 10 years (event E)?

```
% mean lifetime of 10 years (miu) => lambda = 1/10 years^(-1)
% P(E) = P((TA > 10) OR (TB > 10))  # union
%      = 1 - P((TA <= 10) AND (TB <= 10))
%      = 1 - P(TA <= 10) * P(TB <= 10)   because they are independent of each other
%      = 1 - F_TA(10) * F_TB(10)   cdfs!!
%      = 1 - expcdf(10,10)^2
prob_e = 1 - expcdf(10, 10)^2;
fprintf("P(E) = %g", prob_e);
```

```
P(E) = 0.600424
```

5. Compilation of a computer program consists of 3 blocks that are processed sequentially, one after the other. Each block takes Exponential time with the mean of 5 minutes, independently of other blocks. Compute the probability that the entire program is compiled in less than 12 minutes (event A). Use the Gamma-Poisson formula to compute this probability two ways.

```
% Solution 1
% T - total compilation time
% T is the sum of 3 independent exponential variables (for each block)
% lambda = 1/5 because we are given mean as time units (5 minutes)
% Gamma distribution with alpha = 3 and 1/lambda = 5
% P(A) = P(T < 12) = F_T(12) = gamcdf(12, 3, 5)
prob_a_method_1 = gamcdf(12, 3, 5);
fprintf("P(A_1) = %g", prob_a_method_1);
```

```
P(A_1) = 0.430291
```

```
% Solution 2 (with Gamma Poisson formula)
% P(T <= t) = P(X >= alpha) where X has Poisson distribution with parameter
% lambda t = 1/5 * 12 = 2.4
% Since T is continuous r.v. P(T < 12) = P(T <= 12)
% P(A) = P(X >= 3) = 1 - P(X < 3) = 1 - P(X <= 2) = 1 - F(2)
prob_a_method_2 = 1 - poisscdf(2, 2.4);

fprintf("P(A_2) = %g", prob_a_method_2);
```

```
P(A_2) = 0.430291
```

6. Under good weather conditions, 80% of flights arrive on time. During bad weather, only 50% of flights arrive on time. Tomorrow, the chance of good weather is 60%. What is the probability that your flight will arrive on time?

```
% Denote the events
% A: the flights arrives on time
% G: good weather
% B: bad weather
% P(B) + P(G) = 1 (the good and a bad weather forms a partition).
% P(A) = P(A | G) * P(G) + P(A | B) * P(B)

p_a_g = 0.8;
p_a_b = 0.5;
p_g = 0.6;
```

```
p_b = 1 - p_g;
p_a = p_a_g * p_g + p_a_b * p_b;
fprintf("P(flight arrives on time) = %g", p_a);
```

```
P(flight arrives on time) = 0.68
```