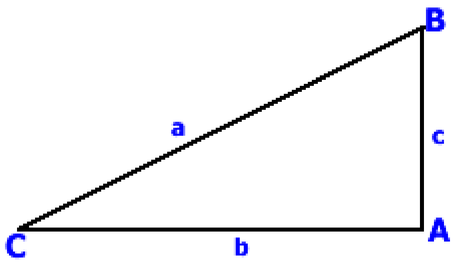
Elemente de trigonometrie - Pagina 1, Pagina 2

Definitii



Intr-un triunghi dreptunghic, considerand masura unui unghi ascutit numim:

sinusul=cateta opusa / ipotenuza
cosinusul=cateta alaturata / ipotenuza
tangenta=cateta opusa / cateta alaturata
cotangenta=cateta alaturata / cateta opusa

Sinusul, cosinusul, tangenta si cotangenta se numesc **functii trigonometrice** si se noteaza cu ***sin, cos, tg,*** si ***ctg.***

In triunghiul ABC de mai sus avem:

$$\sin B = \frac{b}{a} \quad tgB = \frac{b}{c} \quad \sin C = \frac{c}{a} \quad tgC = \frac{c}{b}$$

$$\cos B = \frac{c}{a} \quad ctgB = \frac{c}{b} \quad \cos C = \frac{b}{a} \quad ctgC = \frac{b}{c}$$

Simple formule trigonometrice

Fiind dat un triunghi ABC dreptunghic in A, sunt adevarate urmatoarele relatii:

$$\sin^2 B + \cos^2 B = 1$$ *formula fundamentala a trigonometriei*

$$tgB = \frac{\sin B}{\cos B} \qquad ctgB = \frac{\cos B}{\sin B} \qquad ctgB = \frac{1}{tgB}$$

$$\sin(90^0 - C) = \cos C \qquad \cos(90^0 - C) = \sin C$$

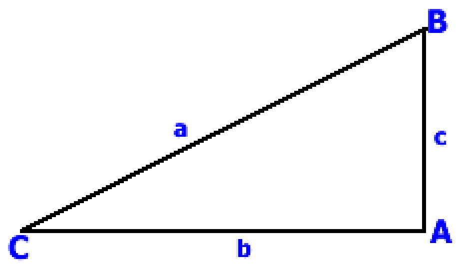
$$tg(90^0 - C) = ctgC$$

Tabele trigonometrice

Nu punem aici decât cele mai cunoscute valori ale functiilor trigonometrice (in tabelul de mai jos):

u	30^0	45^0	60^0
sin *u*	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos *u*	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tg *u*	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
ctg *u*	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$

Alte formule



Pentru triunghiul alăturat avem formulele:

$$1+tg^2 B = 1+\frac{b^2}{c^2} = \frac{c^2+b^2}{c^2} = \frac{a^2}{c^2} = \frac{1}{\cos^2 B}$$

$$1+tg^2 B = \frac{1}{\cos^2 B}$$

$$1+ctg^2 B = \frac{1}{\sin^2 B}$$

$$\sin B = \cos\left(\frac{\pi}{2}-B\right)$$

$$\cos B = \sin\left(\frac{\pi}{2}-B\right)$$

$$tgB = ctg\left(\frac{\pi}{2}-B\right)$$

$$ctgB = tg\left(\frac{\pi}{2}-B\right)$$

$$\sin(x+2k\pi) = \sin x$$

$$\cos(x+2k\pi) = \cos x \quad k \in Z$$

$$\sin(\pi - x) = \sin x$$

$$\cos(\pi - x) = -\cos x$$

$$\cos x = -\cos(\pi - x)$$

$$\sin(\pi + x) = -\sin x \qquad \sin(-x) = -\sin x$$

$$\cos(\pi + x) = -\cos x \qquad \cos(-x) = \cos x$$

$\sin(x+2\pi)=\sin x$
$\cos(x+2\pi)=\cos x$

Tangenta

$tg(x+\pi)=tgx$
$tg(\pi-x)=-tgx$
$tg(x+k\pi)=tgx, k\in Z$ $\qquad 1+tg^2x=\dfrac{1}{\cos^2 x}$ $\qquad tg(-x)=-tgx$

Cotangenta

$ctg(x+\pi)=ctgx$
$ctg(x-\pi)=ctgx$
$ctg(x+k\pi)=ctgx, (\forall)k\in Z$ $\qquad ctg(-x)=-ctgx$ $\qquad 1+ctg^2x=\dfrac{1}{\sin^2 x}$

$\cos\left(\dfrac{\pi}{2}-x\right)=\sin x$

$\sin\left(\dfrac{\pi}{2}-x\right)=\cos x$

$tg\left(\dfrac{\pi}{2}-x\right)=ctgx$

$ctg\left(\dfrac{\pi}{2}-x\right)=tgx$

$\cos(x-y)=cox\cos y+\sin x\sin y, (\forall)x,y\in R$
$\cos(x+y)=\cos x\cos y-\sin x\sin y$
$\sin(x+y)=\sin x\cos y+\cos x\sin y$
$\sin(x-y)=\sin x\cos y-\cos x\sin y$
$\cos 2x=\cos^2 x-\sin^2 x$ forma omogenă
$\cos 2x=2\cos^2 x-1$ numai în funcție de $\cos\alpha$
$\cos 2x=1-2\sin^2 x$ numai în funcție de $\sin\alpha$
$(\forall)x\in R$
$\sin 2x=2\sin x\cos x$

$1+\cos 2x=2\cos^2 x\rightarrow\cos^2 x=\dfrac{1+\cos 2x}{2}$

$1-\cos 2x=2\sin^2 x\rightarrow\sin^2 x=\dfrac{1-\cos 2x}{2}$

$tg(x+y)=\dfrac{tgx+tgy}{1-tgxtgy}$

$tg(x-y)=\dfrac{tgx-tgy}{1+tgxtgy}$

$tg2x=\dfrac{2tgx}{1-tg^2x}$

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$

$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

$$\cos a\cos b = \frac{\cos(a-b)+\cos(a+b)}{2}$$

$$\sin a\sin b = \frac{\cos(a-b)-\cos(a+b)}{2}$$

$$\sin a\cos b = \frac{\sin(a-b)+\sin(a+b)}{2}$$

Dacă $tg\frac{x}{2} = t$, avem:

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$tgx = \frac{2t}{1-t^2}$$

$$tg\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, (\forall)\ \alpha, \beta \in R$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}, (\forall)\ \alpha, \beta \in R$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, (\forall)\ \alpha, \beta \in R$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}, (\forall)\ \alpha, \beta \in R$$

$$\sin A + \sin B + \sin C = 4\cos\frac{C}{2}\cos\frac{A}{2}\cos\frac{B}{2}$$

$$\cos A + \cos B + \cos C - 1 = 4\sin\frac{C}{2}\sin\frac{A}{2}\sin\frac{B}{2}$$

, unde **A+B+C=π**

Cauchy Buniakovski

$$(ax+by)^2 \le (a^2+b^2)(x^2+y^2)$$

$$a^2x^2 + b^2y^2 + 2axby \le a^2x^2 + a^2y^2 + b^2x^2 + b^2y^2$$

$$a^2y^2 + b^2x^2 - 2axby \ge 0 \Leftrightarrow (ay-bx)^2 \ge 0$$

$$ay = bx \Leftrightarrow \frac{a}{x} = \frac{b}{y}$$