Ultimatum Game: Theoretical Analysis

Git link:

https://github.com/razvanbaboiucs/BMDC-game-theory-assginment/tree/master/3-ultimatum

Game Description

The Ultimatum Game is a classic economic experiment that demonstrates strategic decision-making and concepts of fairness:

1. Player 1 is given the opportunity to divide $10 with Player 2.
2. Player 1 makes a proposal to Player 2 about how much to share.
3. Player 2 decides whether to accept or reject.
4. Take it, or leave it. No back and forth bargaining.
5. If Player 2 accepts, each person receives the share agreed upon.
6. Otherwise, the money disappears and neither player gets anything.

Game Theory Analysis

Subgame Perfect Nash Equilibrium (SPNE)

The subgame perfect Nash equilibrium for the Ultimatum Game is:

- **Player 1**: Offers the minimum possible positive amount (e.g., $1)
- **Player 2**: Accepts any offer greater than $0

Reasoning:

1. **Backward Induction**: We start by analyzing Player 2's decision in the second stage.
   - For Player 2, any positive amount is better than nothing (positive utility > zero utility)
   - Therefore, a rational Player 2 should accept any offer greater than $0
2. **Player 1's Strategy**: Knowing that Player 2 will accept any positive offer, Player 1 maximizes their payoff by offering the minimum possible positive amount.
   - This allows Player 1 to keep almost all of the $10 while still ensuring Player 2 accepts

Nash Equilibria that are not Subgame Perfect

The Ultimatum Game also has multiple Nash equilibria that are not subgame perfect:

- **Player 1**: Offers amount $X (any amount)
- **Player 2**: Rejects any offer less than $Y where Y > X

Why this is a Nash equilibrium:

1. Given Player 2's strategy (reject if less than \$Y), Player 1 cannot improve by offering less than \$Y
2. Given Player 1's offer of \$X, Player 2 cannot improve by accepting if X < Y

Why it's not subgame perfect: This equilibrium relies on Player 2's threat to reject positive offers, but this threat is not credible when analyzed in the subgame. In the subgame where Player 2 must decide, rejecting a positive offer is irrational because: - Accepting gives Player 2 a positive payoff - Rejecting gives Player 2 zero payoff

Therefore, these Nash equilibria fail the subgame perfection criterion because they involve non-credible threats.