Analiza

1. $\sum_{n=1}^{\infty} (-1)^{n+1} \sin \frac{\pi}{\sqrt{n}}$, $x_n = \sin \frac{\pi}{\sqrt{n}}$

$$\left.\begin{array}{l} \lim_{n \to \infty} \sin \frac{\pi}{\sqrt{n}} = \sin 0 = 0 \\ x_n \text{ - descrescătoare} \end{array}\right\} \Rightarrow \sum_{n=1}^{\infty} (-1)^{n+1} \sin \frac{\pi}{\sqrt{n}}$$

$$\left.\begin{array}{l} x_n = \sin \frac{\pi}{\sqrt{n}} \\ y_n = \frac{\pi}{\sqrt{n}} \end{array}\right\} \Rightarrow \frac{x_n}{y_n} = \lim_{n \to 0} \frac{\sin \frac{\pi}{\sqrt{n}}}{\frac{\pi}{\sqrt{n}}} = 1 \in (0, \infty) \Rightarrow \text{au aceeași natură}$$

$$\underbrace{\sum_{n=1}^{\infty} \frac{1}{n}}_{\text{divergentă}} < \sum_{n=1}^{\infty} \frac{\pi}{n} \Rightarrow \sum_{n=1}^{\infty} \frac{\pi}{n} \text{ divergentă} \Rightarrow \sum_{n=1}^{\infty} (-1)^{n+1} \sin \frac{\pi}{\sqrt{n}} \text{ nu este absolut convergentă}$$

2. $$\int_1^{\infty} \frac{1}{x^3 + x} dx = \lim_{u \nearrow \infty} \int_1^u \frac{1}{x(x^2+1)} dx$$

$$\frac{1}{x(x^2+1)} = \frac{{}^{x^2+1)}A}{x} + \frac{{}^{x)}Bx + C}{x^2+1} = \frac{Ax^2 + A + Bx^2 + Cx}{x^2+1}$$

$A = 1$
$B = -1$
$C = 0$

$$\Rightarrow \lim_{u \nearrow \infty} \int_1^u \frac{1}{x(x^2+1)} dx = \lim_{u \nearrow \infty} \int_1^u \frac{1}{x} dx + \int_1^u \frac{-x}{x^2+1} dx =$$

$t = x^2+1 \,/()'$
$dt = 2x$

$$= \lim_{u \nearrow \infty} \ln x \Big|_1^u - \frac{1}{2} \int_1^u \frac{1}{t} dt = \lim_{u \nearrow \infty} \ln u - \frac{1}{2} \ln x^2+1 \Big|_1^u =$$

$$= \lim_{u\to\infty} \ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln 2 = \lim_{u\to\infty} \frac{1}{2}\ln 2 + \ln \underbrace{\frac{u}{(u^2+1)^{\frac{1}{2}}}}_{0}$$

$$= \frac{1}{2}\ln 2$$

3. $f:\mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = (x+xy+y^2)(e^x)^{\frac{1}{2}}$

$$\frac{\partial f}{\partial x} = (1+y)\sqrt{e^x} + (x+xy+y^2)\frac{1}{2}e^{x^{\frac{1}{2}-1}}\cdot e^x$$

$$= (1+y)\sqrt{e^x} + (x+xy+y^2)\frac{e^x}{2\sqrt{e^x}} = \sqrt{e^x}\left(1+y+\frac{(x+xy+y^2)\cancel{\sqrt{e^x}}}{2\cancel{\sqrt{e^x}}}\right)$$

$$\frac{\partial f}{\partial y} = (x+2y)\sqrt{e^x}$$

$$\left.\begin{aligned} &(1+y)\sqrt{e^x} + (x+xy+y^2)\frac{e^x}{2\sqrt{e^x}} = 0 \\ &(x+2y)\sqrt{e^x} = 0 \Rightarrow x+2y=0 \Rightarrow y = -\frac{x}{2} \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \left(\overset{3}{1}-\frac{x}{2}\right)\sqrt{e^x} + \left(\overset{4}{x}-\overset{2/2}{\frac{x^2}{2}}+\frac{x^2}{4}\right)\frac{e^x}{2\sqrt{e^x}} = 0$$

$$\frac{2-x}{2}\sqrt{e^x} + \frac{4x-x^2}{4}\frac{e^x}{2\sqrt{e^x}} = 0$$

$$\sqrt{e^x}\left(\overset{4}{\frac{2-x}{2}} + \frac{4x-x^2}{4}\frac{\cancel{\sqrt{e^x}}}{2\cancel{\sqrt{e^x}}}\right) = 0$$

$$\sqrt{e^x}\left(\frac{8\cancel{-4x}\cancel{+4x}-x^2}{8}\right) = 0$$

$$\sqrt{e^x}\left(\frac{-x^2+8}{8}\right) = 0 \Rightarrow -x^2+8=0 \Rightarrow x^2=8$$

$$x = \pm 2\sqrt{2} \Rightarrow$$

$\Rightarrow$ pt $x = 2\sqrt{2}$ $y = -\sqrt{2}$

$x = -2\sqrt{2}$ $y = \sqrt{2}$

$$H(f)(x,y) = \begin{pmatrix} \frac{\partial f}{\partial x^2} & \frac{\partial f}{\partial x \partial y} \\ \frac{\partial f}{\partial y \partial x} & \frac{\partial f}{\partial y^2} \end{pmatrix}$$

$$\frac{\partial f}{\partial y \partial x} = \left((x+2y)\sqrt{e^x}\right)'_x = \sqrt{e^x} + (x+2y)\,\frac{1}{2}\cdot e^{x^{\frac{1}{2}-1}} \cdot e^x = \sqrt{e^x} + (x+2y)\,\frac{e^x}{2\sqrt{e^x}} =$$

$$= \sqrt{e^x}\left(\frac{(1+x+2y)\cancel{\sqrt{e^x}}}{2\cancel{\sqrt{e^x}}}\right)$$

$$\frac{\partial f}{\partial x^2} = \left(\sqrt{e^x}\left(1+y+\frac{x+xy+y^2}{2}\right)\right)'_x = \frac{e^x}{2\sqrt{e^x}}\left(1+y+\frac{x+xy+y^2}{2}\right) +$$

$$+ \frac{\sqrt{e^x}}{2}\left(1+y\right) = \frac{\sqrt{e^x}}{2}\left(1+y+\frac{x+xy+y^2}{2}+1+y\right)$$