Analiza

1. $$\sum_{n=2}^{\infty} (-1)^n \frac{n}{n\sqrt{n}-1}$$

$$x_n = \frac{n}{n\sqrt{n}-1}, \quad x_{n+1} - x_n = \lim_{n\to\infty} \frac{n+1}{(n+1)\sqrt{n+1}-1} - \frac{n}{n\sqrt{n}-1} =$$

$$= \frac{\cancel{n}\left(1+\overset{\nearrow 0}{\frac{1}{n}}\right)}{\cancel{n}\left(1+\underset{\searrow 0}{\frac{1}{n}}\right)n\sqrt{\underset{\searrow 0}{\frac{1}{n}}+\underset{\searrow 0}{\frac{1}{n^2}}}-1} - \frac{\cancel{n}}{\cancel{n}\left(\sqrt{n}-\underset{\searrow 0}{\frac{1}{n}}\right)} = 0$$

Leibniz

x_n desc

$\lim x_n = 0$

$\Rightarrow (-1)^n x_n$ convergentă

$$\frac{n}{n\sqrt{n}-1} \overset{?}{\geq} \frac{1}{\sqrt{n}}$$

$$\frac{\cancel{n}}{\cancel{n}\left(\sqrt{n}-\frac{1}{n}\right)} \geq \frac{1}{\sqrt{n}}$$

$$\frac{1}{\sqrt{n}-\frac{1}{n}} \geq \frac{1}{\sqrt{n}}, \forall n \in [2, +\infty)$$

$\frac{1}{\sqrt{n}}$ divergentă ptc $\frac{1}{n^{\frac{1}{2}}}$ serie armonică $p=\frac{1}{2}<1$ $\Rightarrow$

$\Rightarrow \sum_{n=2}^{\infty} \frac{n}{n\sqrt{n}-1}$ divergentă $\Rightarrow \sum_{n=2}^{\infty} (-1)^n \frac{n}{n\sqrt{n}-1}$ nu este absolut convergentă

2. $\displaystyle\int_0^1 \frac{1}{e^{\sqrt{x}}-1}\,dx$

$$f(x)=\frac{1}{e^{\sqrt{x}}-1},\quad x\in(0,1],\quad f(x)>0$$

$$\lim_{\substack{x\to 0\\ x>0}}(x-0)^p\cdot f(x)=\lim_{\substack{x\to 0\\ x>0}} x^p\cdot\frac{1}{e^{\sqrt{x}}-1}\underset{p=1}{=}\lim_{\substack{x\to 0\\ x>0}}\frac{x}{e^{\sqrt{x}}-1}\underset{l'h}{\overset{\frac{0}{0}}{=}}$$

$$\lim_{\substack{x\to 0\\ x>0}}\frac{1}{e^{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}}=\lim_{\substack{x\to 0\\ x>0}}\frac{2\sqrt{x}}{e^{\sqrt{x}}}=0$$

$$\lim_{\substack{x\to 0\\ x>0}} x^p\cdot\frac{1}{e^{\sqrt{x}}-1}\underset{p=\frac{1}{2}}{=}\lim_{\substack{x\to 0\\ x>0}}\frac{\sqrt{x}}{e^{\sqrt{x}}-1}\underset{l'h}{\overset{\frac{0}{0}}{=}}\lim_{\substack{x\to 0\\ x>0}}\frac{\cancel{\frac{1}{2\sqrt{x}}}}{e^{\sqrt{x}}\cdot\cancel{\frac{1}{2\sqrt{x}}}}=1\Rightarrow\int_0^1\frac{1}{e^{\sqrt{x}}-1}\,dx$$

3. $$\frac{1}{(1+x^2)^2}=(1+x^2)^{-2}$$

$$(1+t)^{-k}=\sum_{m=0}^{\infty}\binom{m+k-1}{m}(-t^m)$$

$$\Rightarrow\frac{1}{(1+x^2)^2}=\sum_{m=0}^{\infty}\binom{m+2-1}{m}(-x^{2m})$$

$$=\sum_{m=0}^{\infty}\binom{m+1}{m}(-x^{2m})=\sum_{m=0}^{\infty}-\frac{m+1}{m}x^{2m}$$

4. $f(x,y,z)=x^a y^b z^c,\quad S=\{(x,y,z)\in(0,\infty)^3\mid x+y+z=1\}$

$F(x,y,z)=x+y+z-1=0$

$$L(x, y, z) = x^a y^b z^c + \lambda x + \lambda y + \lambda z - \lambda$$

$$\frac{\partial L}{\partial x}(x_1, x_2, x_3, \lambda) = 0$$

$$\frac{\partial L}{\partial y}(x_1, x_2, x_3, \lambda) = 0$$

$$\frac{\partial L}{\partial z}(x_1, x_2, x_3, \lambda) = 0$$

$$F(x_1, x_2, x_3) = 0$$

$$\Rightarrow \begin{cases} a x^{a-1} y^b z^c + \lambda = 0 \\ b y^{b-1} x^a z^c + \lambda = 0 \\ c z^{c-1} x^a y^b + \lambda = 0 \\ x + y + z - 1 = 0 \end{cases}$$