Analiza

1. a) $I = \int_0^\infty \frac{\operatorname{arctg} x}{1+x^2}\,dx = \int t\,dt = \frac{\operatorname{arctg}^2 x}{2}\Big|_0^\infty = \frac{\left(\frac{\pi}{2}\right)^2}{2} = \frac{\pi^2}{8}$

$t = \operatorname{arctg} x \quad dx \,/\, ()'$

$dt = \frac{1}{1+x^2}$

b) $I = \int_{-1}^{1} \frac{x+1}{\sqrt{1-x^2}}\,dx = \int_{-1}^{1} \frac{x}{\sqrt{1-x^2}}\,dx + \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\,dx$

$= -\sqrt{1-x^2}\Big|_{-1}^{1} + \arcsin\Big|_{-1}^{1} = \arcsin 1 - \arcsin -1 = \frac{\pi}{2} - \frac{3\pi}{2} =$

$= -\pi$

c) $J_n = \int_0^\infty x^n \cdot e^{-x}\,dx = \underbrace{x^n \cdot (-e^{-x})\Big|_0^\infty}_{0} + \underbrace{n\int_0^\infty e^{-x} \cdot x^{n-1}\,dx}_{J_{n-1}}$

$f' = e^{-x} \qquad f = -e^{-x}$

$g = x^n \qquad g' = n \cdot x^{n-1}$

$$\left.\begin{aligned} J_n &= n\,\cancel{J_{n-1}} \\ \cancel{J_{n-1}} &= n-1\,\cancel{J_{n-2}} \\ &\vdots \\ \cancel{J_2} &= 2 \cdot \cancel{J_1} \\ \cancel{J_1} &= 1 \cdot J_0 \end{aligned}\right\} \Rightarrow J_n = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 2 \cdot 1 \cdot J_0$$

$= n! \cdot (-1) = -n!$

d) $\mathcal{I} = \int_1^2 \frac{1}{\sqrt{x(2-x)}}\,dx = \int_1^2 \sqrt{\frac{1}{2x-x^2+1-1}}\,dx = \int_1^2 \sqrt{\frac{1}{-(x-1)^2+1}}$

$$= \int_1^2 \frac{1}{\sqrt{1-(x-1)^2}}\,dx = \arcsin t \Big|_0^1 = \frac{\pi}{2}$$

$x-1=t \quad /()'$

$1 \cdot dx = dt$

$x=2 \Rightarrow t=1$

$x=1 \Rightarrow t=0$

2. a) $\int_0^3 \frac{x^3+1}{\sqrt{9-x^2}}\,dx$

$f(x) = \frac{x^3+1}{\sqrt{9-x^2}}$

$f(x) : [0,3) \longrightarrow \mathbb{R}$

$$\lim_{x \nearrow 3} (3-x)^p \cdot f(x) = \lim_{x \nearrow 3} (3-x)^{p} \cdot \frac{x^3+1}{((\cancel{3-x})(3+x))^{\frac{1}{2}}} = \lim_{x \nearrow 3} (3-x)^{p-\frac{1}{2}} \cdot \frac{x^3+1}{\sqrt{3+x}}$$

caz I $\quad p=\frac{1}{2}, \; p<1 \quad \lambda = \frac{28}{\sqrt{6}} < +\infty \Rightarrow f(x)$ convergentă

caz II $\quad p=2, \; p>1 \quad \lambda = 0 \not> 0$

b) $\int_0^{\infty} \frac{\operatorname{arctg} x}{x} = \underbrace{\int_0^1 \frac{\operatorname{arctg} x}{x}}_{I_1} + \underbrace{\lim_{u \nearrow \infty} \int_1^u \frac{\operatorname{arctg} x}{x}}_{I_2}$

$f(x) = \frac{\operatorname{arctg} x}{x}$

$I_1: \lim_{x \searrow 0} (x-0)^p \cdot \frac{\operatorname{arctg} x}{x} = \lim_{x \searrow 0} (x-0)^p \cdot 1 = \lim_{x \searrow 0} x^{\frac{1}{2}} = 0 < \infty$

$\Downarrow$

$f(x)$ conv

$I_2: \lim_{x \nearrow +\infty} x^p \cdot f(x) = x^{\cancel{p}\,p-1} \cdot \frac{\operatorname{arctg} x}{\cancel{x}} = \lim_{x \nearrow +\infty} x^{p-1} \cdot \frac{\pi}{2}$ $\left.\right\}$ $\Rightarrow$

$p = 1$

$\Rightarrow 1 \cdot \frac{\pi}{2} = \frac{\pi}{2} \Rightarrow f(x)$ div

div + cov = div $\Rightarrow$ I div.