Verified Data Structures and Algorithms in Agda
Progress Report

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Part II

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Motivation

- Dependent typing enforces correctness of programs.
- Nested types maintain invariants of data structure.
- Together they present a challenge both for programming and verifying correctness
- The dissertation's purpose was:
-     Implementing the Finger Tree in agda
-     Proving the correctness of its methods.

Regular vs Nested Types

List is an example of a **regular** data type.

One can also declare **irregular (nested)** data types [0], where the recursive call to the type constructor takes as argument a different type (examples to follow)

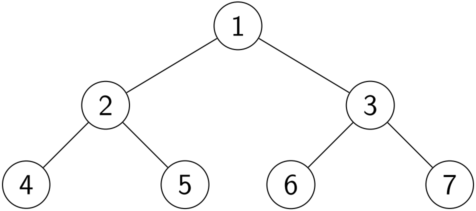
In older versions of Haskell and ML, although the declarations are valid, one couldn't write functions to operate on them. [0] (Hindley-Milner typing).

They turn out to be very useful in practice, as they can keep strong invariants on the data.

Example 1: Full binary tree

$$\textbf{data}\ \mathit{Nest}\ (A : \mathit{Set}) : \mathit{Set}\ \textbf{where}$$
$$\quad \mathit{Nil} : \mathit{Nest}\ A$$
$$\quad \mathit{Cons} : A \to \mathit{Nest}\ (A \times A) \to \mathit{Nest}\ A$$

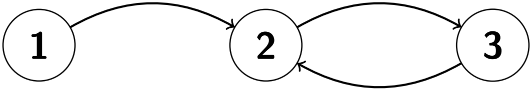
$$ex : \mathit{Nest}\ \mathbb{N}$$
$$ex = \mathit{Cons}\ 1\ (\mathit{Cons}\ (2, 3)\ (\mathit{Cons}\ ((4, 5), (6, 7))\ \mathit{Nil}))$$



Example 2: Cyclic list [5]

data *Clist* $(A : Set) : Set$ **where**
$\quad Var : A \rightarrow Clist\ A$
$\quad Nil : Clist\ A$
$\quad RCons : \mathbb{N} \rightarrow Clist\ (Maybe\ A) \rightarrow Clist\ A$

$ex : CList\ ()$
$clist2 = RCons\ 1\ (RCons2\ (RCons\ 3\ (Var\ (just\ nothing))))$



Example 3: Finger Trees

$$
\begin{array}{l}
\textbf{data}\ \mathit{FingerTree}\ \{a\}\ (A : \mathit{Set}\ a) : \mathit{Set}\ a\ \textbf{where} \\
\quad \mathit{Empty} : \mathit{FingerTree}\ A \\
\quad \mathit{Single} : A \rightarrow \mathit{FingerTree}\ A \\
\quad \mathit{Deep}\ \ : \mathit{Digit}\ A \rightarrow \mathit{FingerTree}\ (\mathit{Node}\ A) \rightarrow \mathit{Digit}\ A \rightarrow \\
\qquad\qquad \mathit{FingerTree}\ A
\end{array}
$$

Motivation

The original paper suggests implementing Views [6] in order to make implementations more straight-forward, while keeping the same complexity.

$$\begin{array}{l} \textbf{data}\ \mathit{ViewL}\ \{a\}\ (A : \mathit{Set}\ a) : \mathit{Set}\ a\ \textbf{where} \\ \quad \mathit{nilL} : \mathit{ViewL}\ A \\ \quad \mathit{consL} : A \to \mathit{FingerTree}\ A \to \mathit{ViewL}\ A \end{array}$$

And a function to transform between the views.

$$\begin{array}{l} \mathit{viewL} : \forall \{a\}\ \{A : \mathit{Set}\ a\} \to \mathit{FingerTree}\ A \to \mathit{ViewL}\ A \\ \mathit{viewL}\ \mathit{ft} = \{!!\} \end{array}$$

Motivation

However, implementing functions stops working because of termination checker.

$$
\begin{aligned}
&\mathit{reverse} : \forall \{a\}\ \{A : \mathit{Set}\ a\} \rightarrow \mathit{FingerTree}\ A \rightarrow \mathit{FingerTree}\ A \\
&\mathit{reverse}\ \mathit{ft}\ \mathit{with}\ \mathit{viewL}\ \mathit{ft} \\
&\mathit{reverse}\ \mathit{ft} \mid \mathit{nilL} = \mathit{ft} \\
&\mathit{reverse}\ \mathit{ft} \mid \mathit{consL}\ x\ \mathit{xs} = (\mathit{reverse}\ \mathit{xs}) \triangleright x
\end{aligned}
$$

Progress Made - Implementation

- Implemented the FingerTree in 3 iterations with operations like reduce, view, append etc.
- 1. The plain, non-dependent one (the declaration of which was above)
- 2. A modification that includes the concept of a measurement [3]
- 3. A dependent version of the second one.

Progress Made - Verification

- The dependent implementation contains internal verification of the axioms measurement should entail
- Wrote proofs of correctness for the cons operator.
- Specialized the data structure through measurements (e.g. Random Access Sequence).
- Implemented a 4th version of the FingerTree, following an Isabelle implementation [4], that has a very different approach

Progress Made - Overcoming Difficulties

- Using the principle of WF induction, I could write an implementation of the reverse function.
- I have presented an argument that Sized types cannot be used in this context.

Further work

- Produce more correctness proofs:
- Evaluate by seeing whether the dependent version hinders amortized performance (which is amortized O(1) for most operations [2])
- Explore the performance difference between the original and the isabelle implementation.
- Provide a more convincing argument that Sizes cannot be employed here.
- Attempt to generalize for other nested data structures [1].

References

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Thank you!