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Verified functional datastructures in Agda

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Proforma

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[1]This word count was computed by detex diss.tex | tr -cd '0-9A-Za-z \n' | wc -w

Declaration

I, [Name] of [College], being a candidate for Part II of the Computer Science Tripos [or the Diploma in Computer Science], hereby declare that this dissertation and the work described in it are my own work, unaided except as may be specified below, and that the dissertation does not contain material that has already been used to any substantial extent for a comparable purpose.

Signed [signature]

Date [date]

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Acknowledgements

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Chapter 1

Introduction

1.1 Functional Datastructures

There has always been an imballance between the use of functional programming versus imperative programming both in industry, as well as in terms of available resource. Functional programming has often been ruled out in the past because of it running slow, or simply because it was stigmatised as belonging into academia, regardless of its properties [?]. However, this paradigm is being introduced now at the forefront of business development. This is because of the persistency[0] of data-structures, useful for the ever-present multicore environment. The reliablity aspect, formal verification and keeping runtime errors to a minimum have also been relevant. The issue of the runtime speed has been addressed gracefully by Okasaki [?], which, introduced a reusable concept that can help designers build efficient data structures: the implicit recursive slowdown.

1.2 Dependent Typing

An important breakthrough in writing verifiably correct code is the introduction of dependent types.[?] In this setting the distinction between types and values becomes blurry, allowing us to define types that depend on values.

An immediate practical motivation is performing the sum of two vectors. The usual programming paradigm would be (in pseudocode):

```
def sum (l1, l2):
  if (l1.length != l2.length)
    raise ListsNotEqualException;
  ...
```

This can cause a runtime error and arguably disrupts the logical flow of the program. In a program that supports dependent types, we can construct lists that are both parametrized by a variable (as in the usual polymorphic programming), but also 'indexed'.

The usual definiton of lists would be (in Agda - but easily any functional language)

```
data List (A : Set) : Set where
  nil : List A
  _::_ : A → List A → List A
```

Compare this with the dependent definition:

```
data Vec (A : Set) : ℕ → Set where
  nil : Vec A 0
  _::_ : ∀ {n : ℕ} → A → Vec A n → Vec A (n + 1)
```

This allows us to write functions that require a 'proof' that the two arguments are of equal length.

```
sum : ∀ {n : ℕ} → Vec ℕ n → Vec ℕ n → Vec ℕ n
sum xs ys = ?
```

If this is not obvious from the context, the program will not type check. The developer is forced to only write correct programs.

Further details about agda syntax will be provided in section (Introduction to Agda)

1.3 Nested Types

Another way of maintaing invariants throughout the program is the trick or 'irregular' or 'nested' datatypes. They allow forcing strong structural invariants on the datastructure and have gained interest because of their practical implications. [**?**] Some difficulties come up

in recursive calls or inductive proofs, fact which will be covered and discussed in section (TODO implementation/section$_{nest_example}$).

For example, consider the next data structure, introduced in Bird and Meertens [**?**] as well, but slightly modified so I can build a pathological example on top of it later on:

> **data** Nest $\{a : \text{Level}\}$ $(A : \text{Set } a) : \text{Set } a$ **where**
>   nilN $:$ Nest A
>   consN $: (A \times A) \rightarrow \text{Nest } (A \times A) \rightarrow \text{Nest A}$

This can be thought of as the levels of a full binary tree (with the exeption of the binary tree with only one element, which I am ruling out for simplicity)

Another example, with a more interesting application is:

```
data BinTree (A : Set) : Set where
  empty : BinTree A
  single : A → BinTree A
  deep :  BinTree (Node A) → BinTree A
```

Where Node is simply:

```
data Node (A : Set) : Set where
  node : A → A → Node A
```

It can be seen from the declaration that the structure will be forced to be a sequence of deep constructors, followed by either a single (Node^n A) or an empty constructor. The number of elements stored in it has to be a power of two (2^n) making it equivalent to the leafs of a full binary tree.

This dissertation is mostly concerned with 2-3 trees, the basis for the FingerTrees, which will be studied in detail in the Implementation section. They are an example to show arising problems when proving properties of nested and dependent typed structures, what limits are imposed and how some of them can be overcome.

1.4 Introduction to Agda

Agda is a dependently typed programming language, developed in the spirit of Haskell, kept as simple as possible [**?**]. All the previous examples are written in Agda, and their syntax and the newly

introduced syntax will be described as we go on. Along other pro-
gramming languages like Coq [**?**] or Isabelle [**?**], Agda is used as an
interactive (or automatic) theorem prover. What makes it different
from the two previously mentioned system is the ability to write the
code and the proofs in the same environment. Its relative simplicity
also motivated its use in this project.

1.4.1 Types as Values

As said before, a dependently typed environment allows types to be
not only arguments to functions, as it is the case in generic data struc-
tures, but also returned values.

  In Agda, the base for all types is called Set, which can be simplis-
tically though of as the type of types.

  I will use, as a running example throughout this section, the con-
struction of natural numbers and lists. They should be sufficient for
introducing most of the concepts that will be needed throughout this
dissertation.

1.4.2 Declaring Data Structures

Data structures in agda follow the ADT (Algebraic Data Types)
paradigm. They group together constructors that can introduce the
given structure. Each constructor should be thought of as a function
which returns an instance of the data structure.

```
data ℕ : Set where
  zero : ℕ
  suc : ℕ → ℕ
```

  We declare the type of natural numbers, which is of type Set. It
has only two constructors; zero, which takes no argument, and suc
(successor) which, provided a natural number, can construct the
next natural number.

  As you probably noticed, Agda has full unicode support. This
makes writing proofs about mathematical objects nice and readable
since you can use the conventional symbols.

Another piece of elegant syntax is the presence of mixfix opera-
tors. Most unicode characters can be used in the name of the opera-
tor, and by _ you tell agda that's where you want to put an argument

```
(Example: _+_, if_then_else_, [_])
```

1.4.3 Agda's interactive help

Before writing the implementation of a function, as you stumble upon
the equals(=) sign, you can tell agda to place a hole (! !) instead of
an implementation. Here, you can perform a number of operations:

- See the types and values of variables in the scope

- Case-split
  For example, consider the addition of natural numbers.

  $$\_+\_ : \mathbb{N} \to \mathbb{N} \to \mathbb{N}$$
  $$n + m = \{!!\}$$

  Performing a case-split on the variable n shows me all the
  possible ways in which a natural number can be constructed.

  $$\_+\_ : \mathbb{N} \to \mathbb{N} \to \mathbb{N}$$
  $$zero + m = \{!!\}$$
  $$suc\ n + m = \{!!\}$$

  A consequence of this is that all functions in Agda must be total.
  If a possible constructor is not present in the definition, it will not
  type-check.

- Refine and Auto
  These provide the automated and interactive ways of theorem-
  proving. Essentially, Agda looks throughout the environment to
  find an inhabitant (a variable) that has the type of the hole.
  They are definitely not as powerfull as any functionality given by
  Coq of Isabelle, but it can save some typing.

1.4.4 Arguments

Agda introduces some syntax for various types of arguments you can provide to functions. As you probably saw, there is a difference in handling the polymorphic types (in the case of List) and the values given as arguments to type constructors (in the case of Vec).

In the declaration of Vec:

```
data Vec (A : Set) : ℕ → Set where
  nil : Vec A zero
  _::_ : ∀ {n : ℕ} → A → Vec A n → Vec A (suc n)
```

The first (A : Set) is the type argument for instantiating a polymorphic type, before the :, while the ℕ is the type of the value argument for the dependently typed instantiation.

Another think to notice here is the curly brackets ({n : ℕ}) in the declaration of the _::_ constructor. This is called an implicit argument. Agda will bind n to a value it sees fit in the scope. If there are more possibilities, it will take a guess.

1.4.5 Instance Arguments

Throughout this dissertation, we will be using some properties of certain types, for example of having a monoid operation associated with them. In Haskell, you would accomplish that with the use of type classes [**?**]

In order to mimic this behaviour we will use instance arguments. They are declared by using double square brackets, {{ }} or the unicode equivalent.

What Agda does in this case, it looks for a possible instantiation of that type in the current scope, following some predefined rules. [**?**]

It is important there is only one available possibility, otherwise it will fail to type check.

The use will become obvious in the Implementation section.

Chapter 2

Preparation

2.1 Programs as proofs

-refl
-proof that + 0 and nat form monoid
-proof that some operation is transitive (can't do leq because it's done in tutorial)

2.1.1 Curry-Howard Isomorphism

-theoretical underpinings of the things above

2.1.2 Equivalence-Reasoning

-show the syntax mostly

2.1.3 with, rewrite, inspect

-syntax
-unimplemented feature of with
-rewrite explained as a sequence of with statements
-reason behind inspect

2.1.4   instance arguments

-type classes in haskell
-the monoid record
-how we use it

2.2   Views

-introduction via wadler http://www.cs.tufts.edu/ nr/cs257/archive/phil-
wadler/views.pdf
-emphasise the point in section 10 induction
-show use in nested types

2.3   Termination checking

-correctness requires termination checking
-how termination checking works in agda - 'structurally smaller'
-how views with nested types fail

2.4   Well-foundedness

-what is well-founded induction
-tb to larry paulson's paper: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.
-agda's standard library provides the tools

Chapter 3

Implementation

3.1 FingerTrees in general

-what are they
-previous work: isabelle, coq, agda.

3.2 FingerTrees in Agda

-without measurement - quite boring
-with measurment
-view from the left is not working because of termination check
(pointer to coq paper)
-trying to get solve the issue by defining a well-founded induction
-use the measurement info, since it's there

3.3 Random Access Sequences as an easier example

-size and entry
-trying to prove the well-foundedness
-boom, we fail because we didn't follow a dependently typed implementation

3.4   Dependently typed FingerTrees

3.5   Verbatim text

Verbatim text can be included using \begin{verbatim} and \end{verbatim}. I normally use a slightly smaller font and often squeeze the lines a little closer together, as in:

```
GET "libhdr"

GLOBAL { count:200; all  }

LET try(ld, row, rd) BE TEST row=all
                        THEN count := count + 1
                        ELSE { LET poss = all & ~(ld | row | rd)
                               UNTIL poss=0 DO
                              { LET p = poss & -poss
                                poss := poss - p
                                try(ld+p << 1, row+p, rd+p >> 1)
                              }
                           }
LET start() = VALOF
{ all := 1
  FOR i = 1 TO 12 DO
  { count := 0
    try(0, 0, 0)
    writef("Number of solutions to %i2-queens is %i5*n", i, count)
    all := 2*all + 1
  }
  RESULTIS 0
}
```

3.6   Tables

Here is a simple example[1] of a table.

Left Justified	Centred	Right Justified
First	A	XXX
Second	AA	XX
Last	AAA	X

There is another example table in the proforma.

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[1]A footnote

3.7 Simple diagrams

Chapter 4

Evaluation

4.1 Printing and binding

If you have access to a laser printer that can print on two sides, you can use it to print two copies of your dissertation and then get them bound by the Computer Laboratory Bookshop. Otherwise, print your dissertation single sided and get the Bookshop to copy and bind it double sided.

Better printing quality can sometimes be obtained by giving the Bookshop an MSDOS 1.44 Mbyte 3.5" floppy disc containing the Postscript form of your dissertation. If the file is too large a compressed version with `zip` but not `gnuzip` nor `compress` is acceptable. However they prefer the uncompressed form if possible. From my experience I do not recommend this method.

4.1.1 Things to note

- Ensure that there are the correct number of blank pages inserted so that each double sided page has a front and a back. So, for example, the title page must be followed by an absolutely blank page (not even a page number).

- Submitted postscript introduces more potential problems. Therefore you must either allow two iterations of the binding process (once in a digital form, falling back to a second, paper, submission if necessary) or submit both paper and electronic versions.

- There may be unexpected problems with fonts.

4.2   Further information

See the Computer Lab's world wide web pages at URL:
    `http://www.cl.cam.ac.uk/TeXdoc/TeXdocs.html`

Chapter 5

Conclusion

I hope that this rough guide to writing a dissertation is LaTeX has been helpful and saved you time.