Medical Image Registration Advanced

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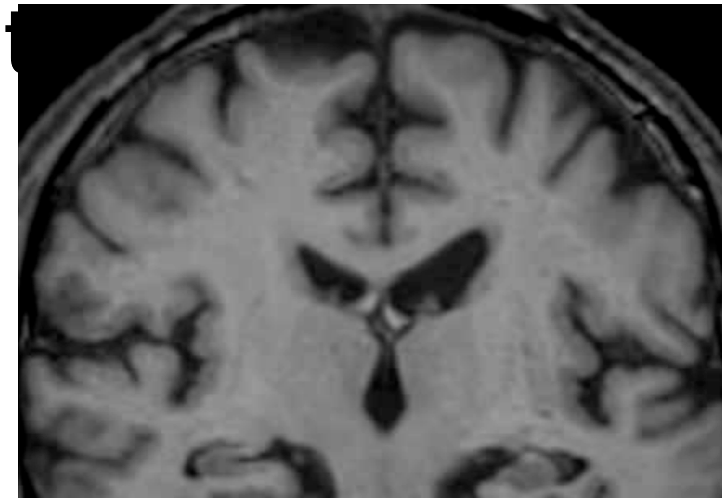


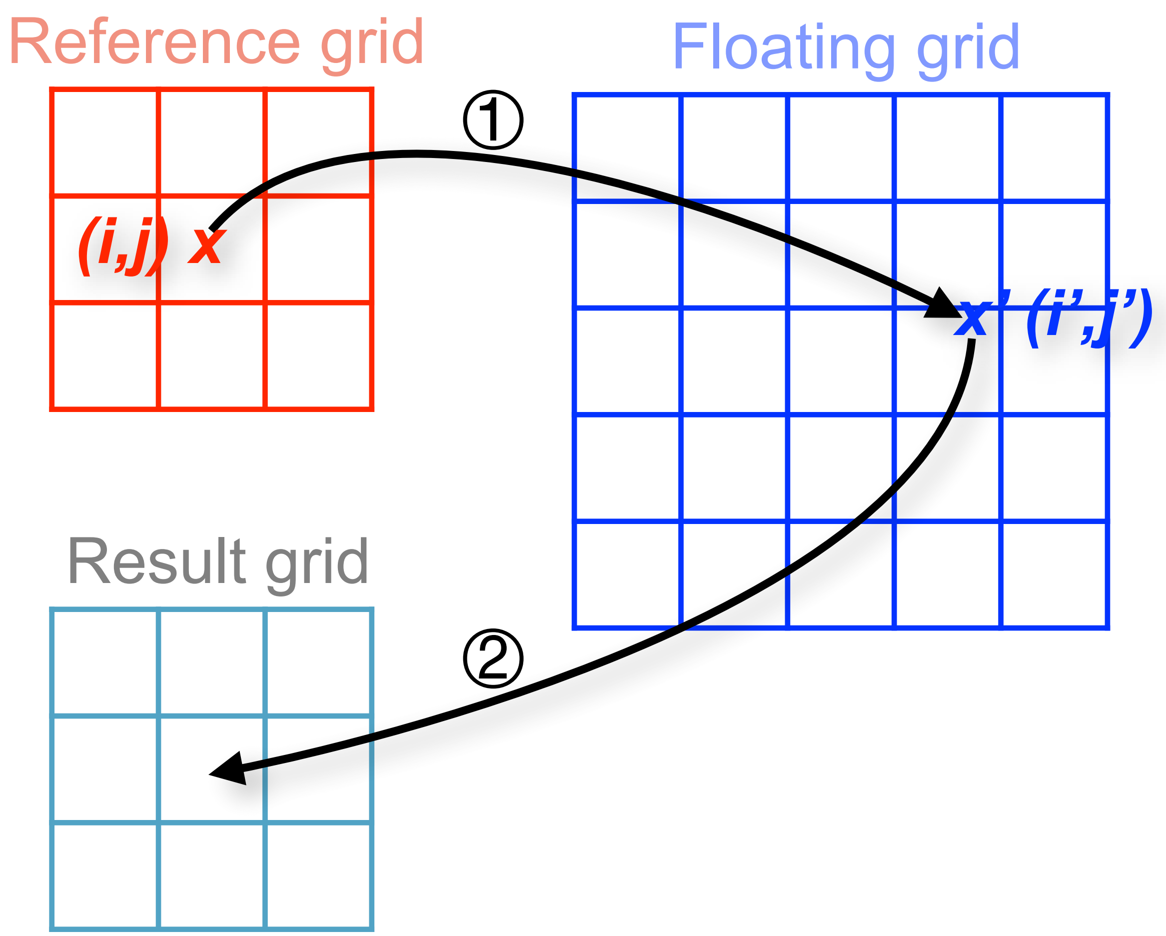
Outline

- Focus is on transformation parametrisation
  - Deformation field composition, application to:
    - The demons algorithm (1996)
    - Regridding
      - » The fluid algorithm (1996)
      - » The FFD algorithm (2006)
  - Velocity field parametrisation
    - Concept
      - » The Euler integration
      - » The scaling-and-squaring approach (2006)
    - Stationary velocity field
      - » The demons (2009)
      - » The Dartel (2007)
    - Non-stationary velocity field
      - » The LDDMM (2005)

Reminder on deformation field

① For each pixel with coordinate *(i,j)* in the reference image, we know the corresponding position *(i’,j’)* in the floating image.

② In order to deform the floating image, the intensity at position *(i’,j’)* is evaluated using resampling technique and displayed in the result image.

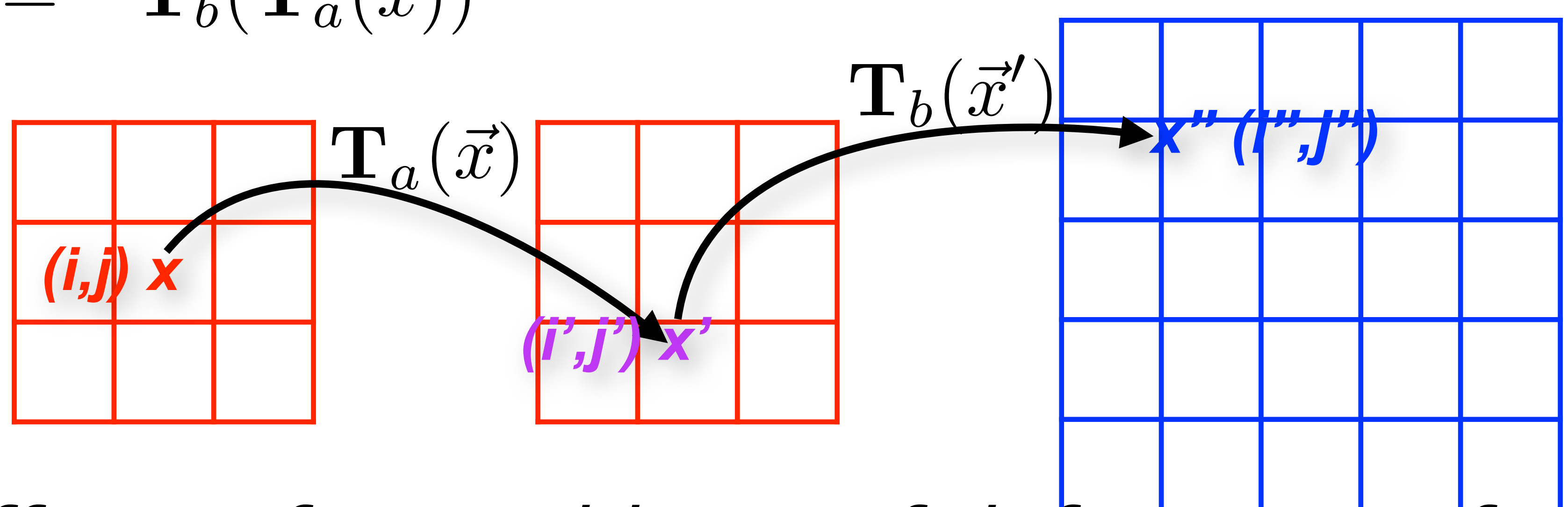


Def. field composition - Principle

- Simple transformation

  $\mathbf{T}(\vec{x}) = \vec{x}'$, where $\vec{x} = \{i, j\}$ and $\vec{x}' = \{i', j'\}$
- Composition of 2 transformations $\mathbf{T}_a$ and $\mathbf{T}_b$ to generate a third one $\mathbf{T}_c$

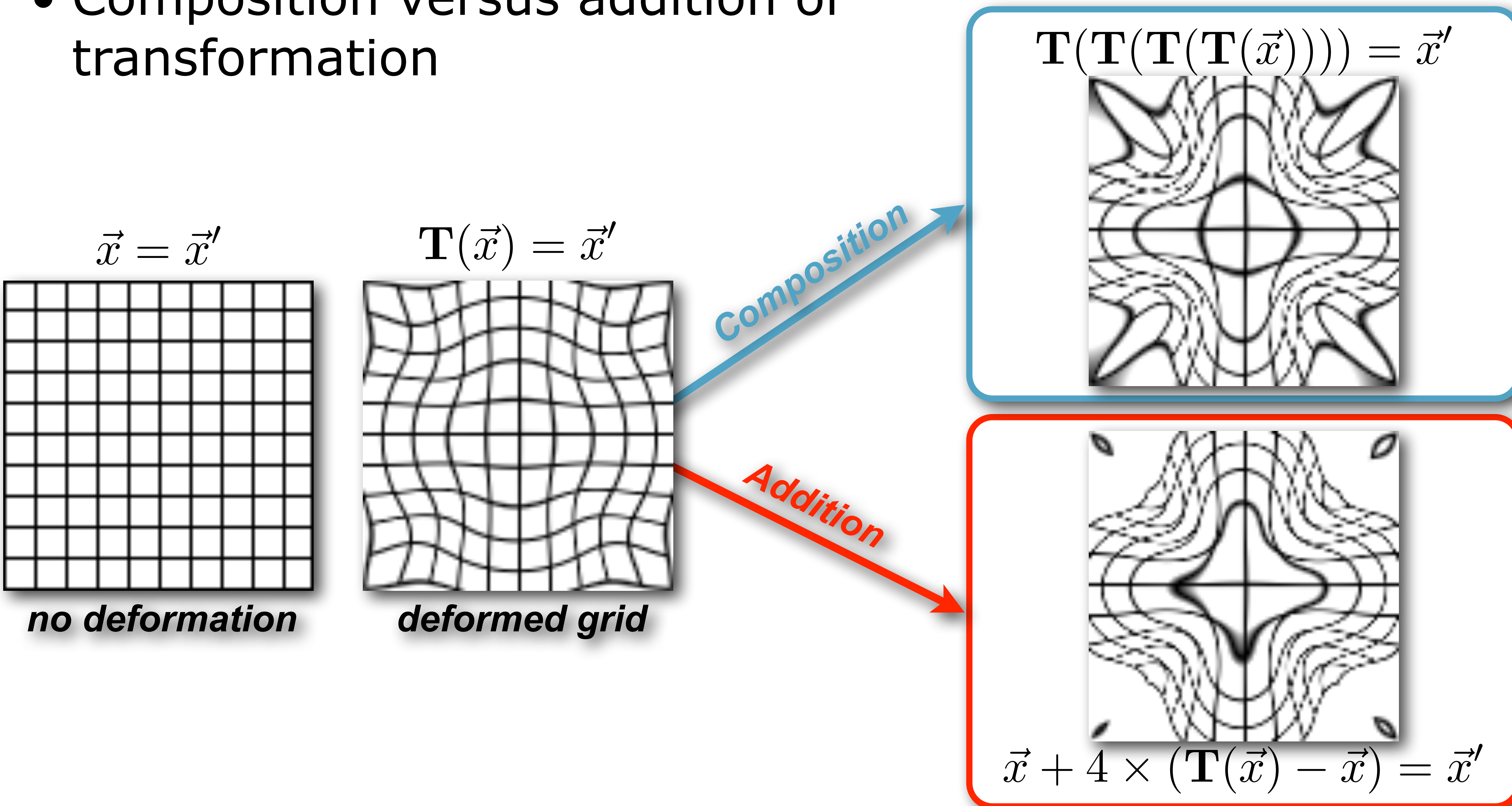
$$\begin{aligned} \mathbf{T}_a(\vec{x}) &= \vec{x}' \text{ and then } \mathbf{T}_b(\vec{x}') = \vec{x}\text{''}, \\ \mathbf{T}_c(\vec{x}) &= \mathbf{T}_b(\mathbf{T}_a(\vec{x})) \end{aligned}$$



- ***!!!!!!* Different from addition of deformation field**

Def. field composition - Illustration

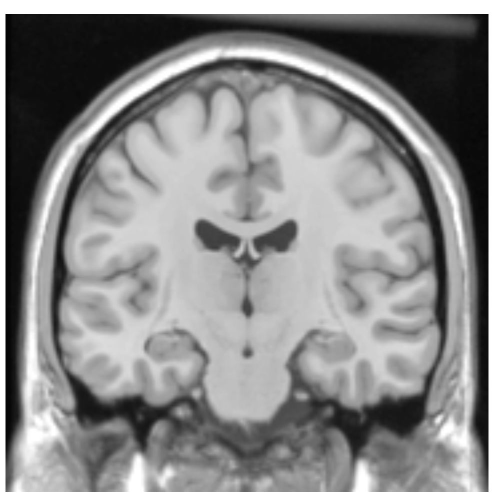
- Composition versus addition of transformation



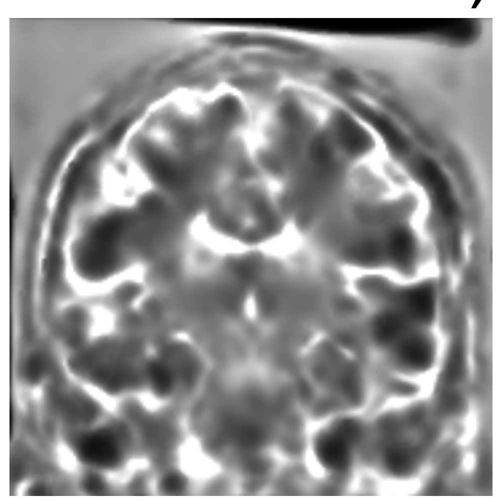
Def. field composition - Advantages

- The composition of two diffeomorphisms is a diffeomorphism
  - One-to-one mapping
  - The topology is conserved
  - The transformation is invertible

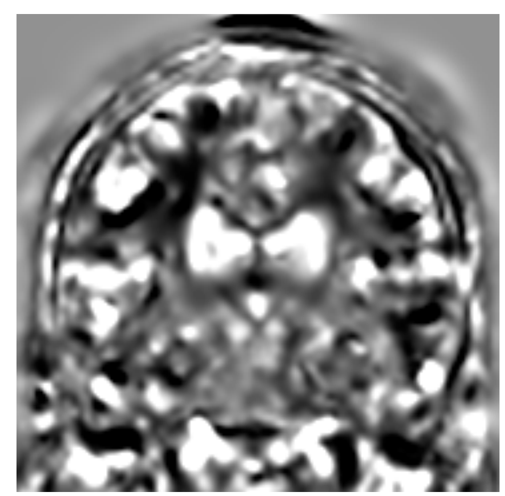
Jacobian determinant, visualisation scale: [0;2]



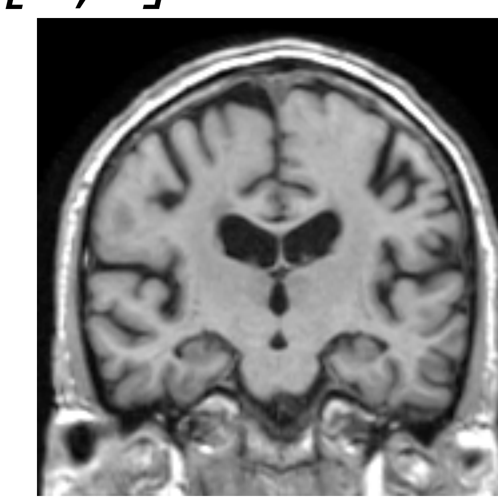
Reference



Composition
Robust range
[0.25;2.32]



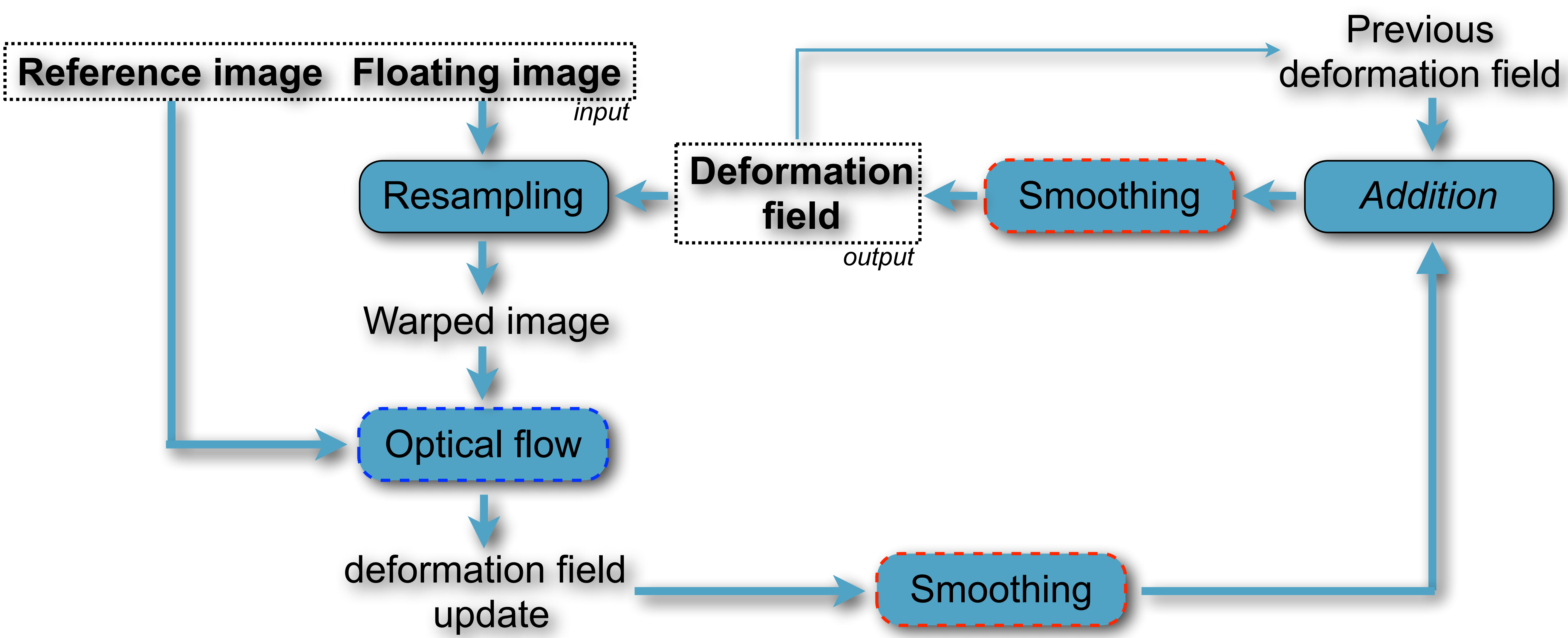
Addition
Robust range
[-0.01;2.95]



Floating

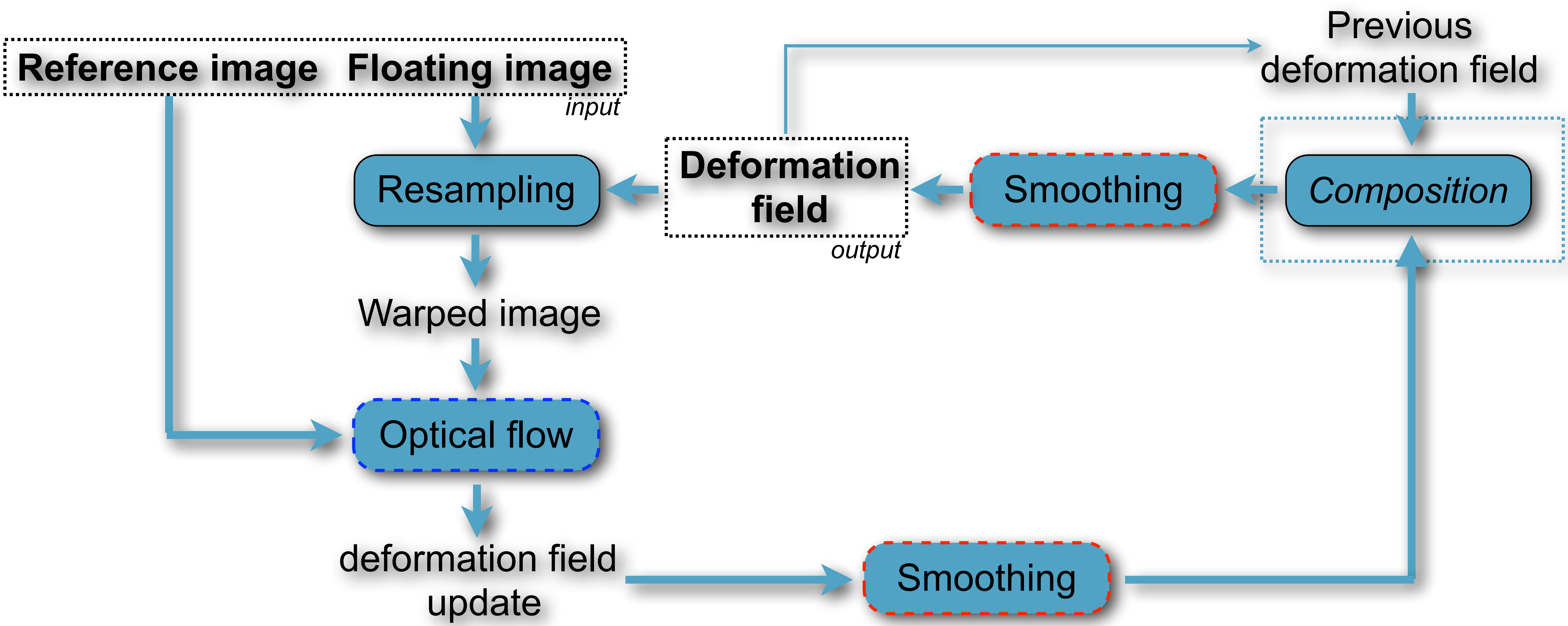
Demons algorithm

- From the additive demon algorithm to the diffeomorphic version
  - Additive version (introduced during the previous lecture):



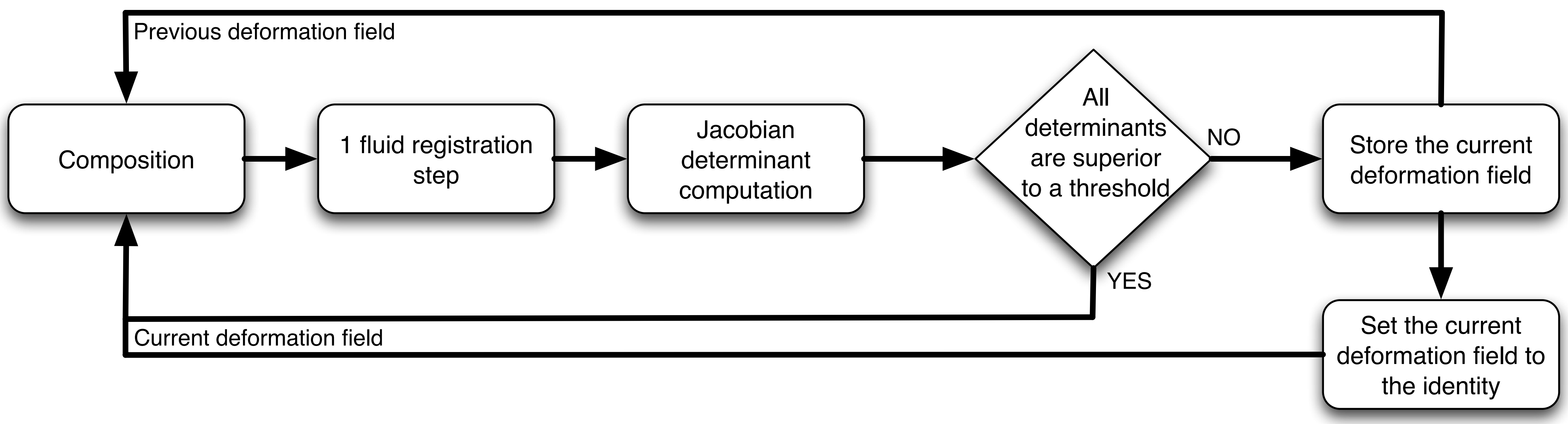
Demons algorithm

- From the additive demon algorithm to the diffeomorphic version
  - Diffeomorphic version (with constraint on the update):



Regridding (Jac. det. monitoring)

- Example of application: regridding applied to the fluid algorithm



  - Current grid is initialised to identity transformation
  - Regridding avoid negative Jacobian determinant using appropriate threshold value.

Regridding (Hard constraint)

- Example of application: regridding applied to a cubic B-Spline parametrisation

Theorem 1. *A FFD based on cubic B-splines is locally injective over all the domain if $\delta_x < \frac{1}{K}$, $\delta_y < \frac{1}{K}$ and $\delta_z < \frac{1}{K}$.*

Choi and Lee [21] have determined a value of $K \approx 2.48$ so that the maximum displacement of control points given by the bound $\frac{1}{K}$ is approximately 0.40. This means that the maximum displacement of control points is given by the spacing of control points in the lattice. For example, for a lattice with 20mm control point spacing the maximum control point displacement is 8mm while for a lattice with 2.5mm control point spacing the maximum control point displacement is 1mm.

From Rueckert *et al.*, MICCAI'06

Velocity field parametrisation

- Integration of a flow field over time to generate a deformation field

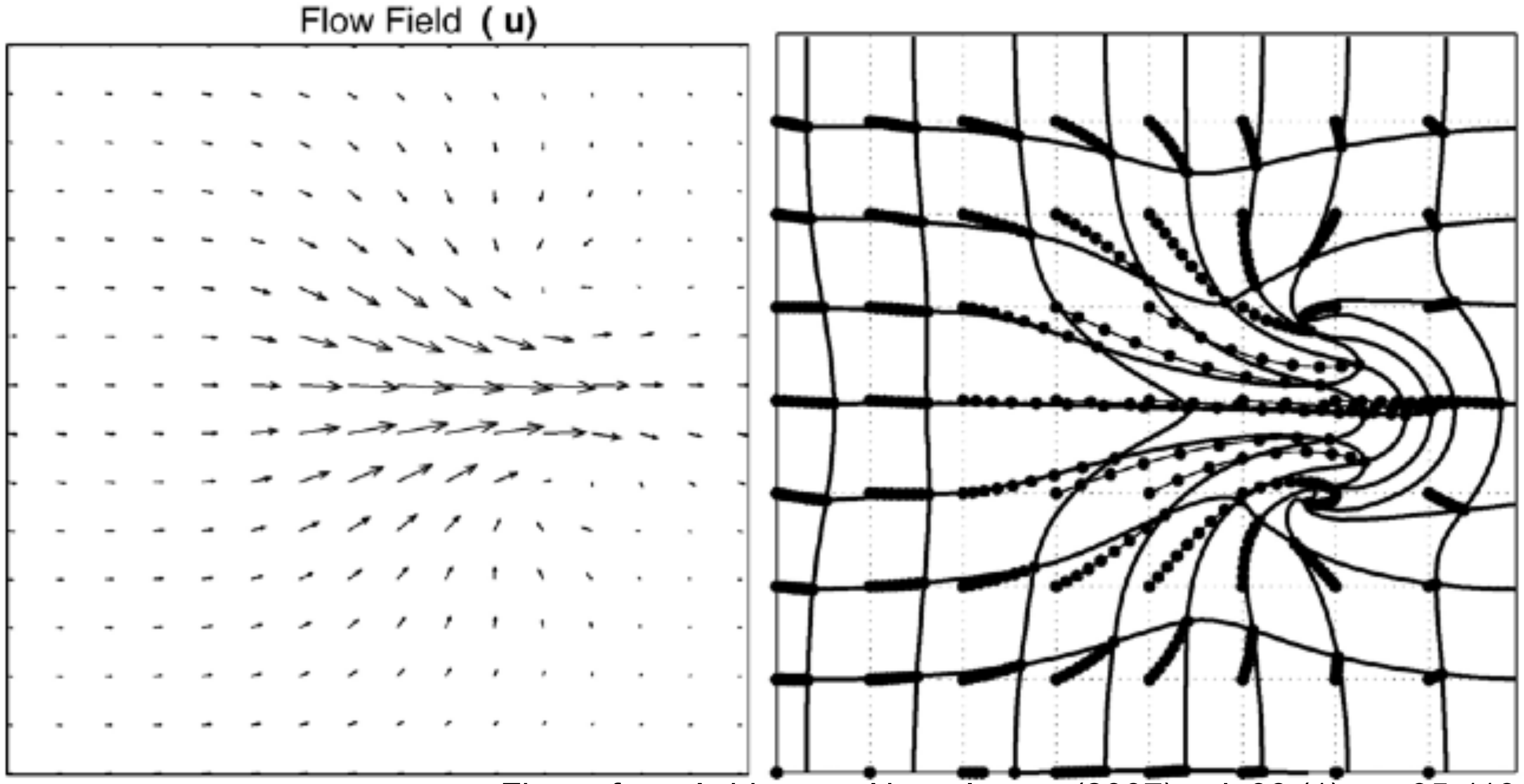


Figure from Ashburner, NeuroImage (2007) vol. 38 (1) pp. 95-113

$$\exp(\text{Velocity field}) = \text{Deformation Field}$$

Velocity field parametrisation

- How to integrate a velocity field parametrisation
  - Euler integration
    - Composition of infinitesimally small steps

$$\begin{aligned}\exp(a+b) &= \exp(a)\exp(b)\\ \exp(v) &= \exp(N\times\frac{v}{N}) = \prod_{0}^{N-1}\exp(\frac{v}{N})\end{aligned}$$

if a and b are close to identity: $\exp(a+b)\approx a\circ b$

$$\exp(v) = \frac{v}{N}\circ\frac{v}{N}\circ\frac{v}{N}\circ\frac{v}{N}\circ\ldots,\ \text{with } N\ggg 1$$

Velocity field parametrisation

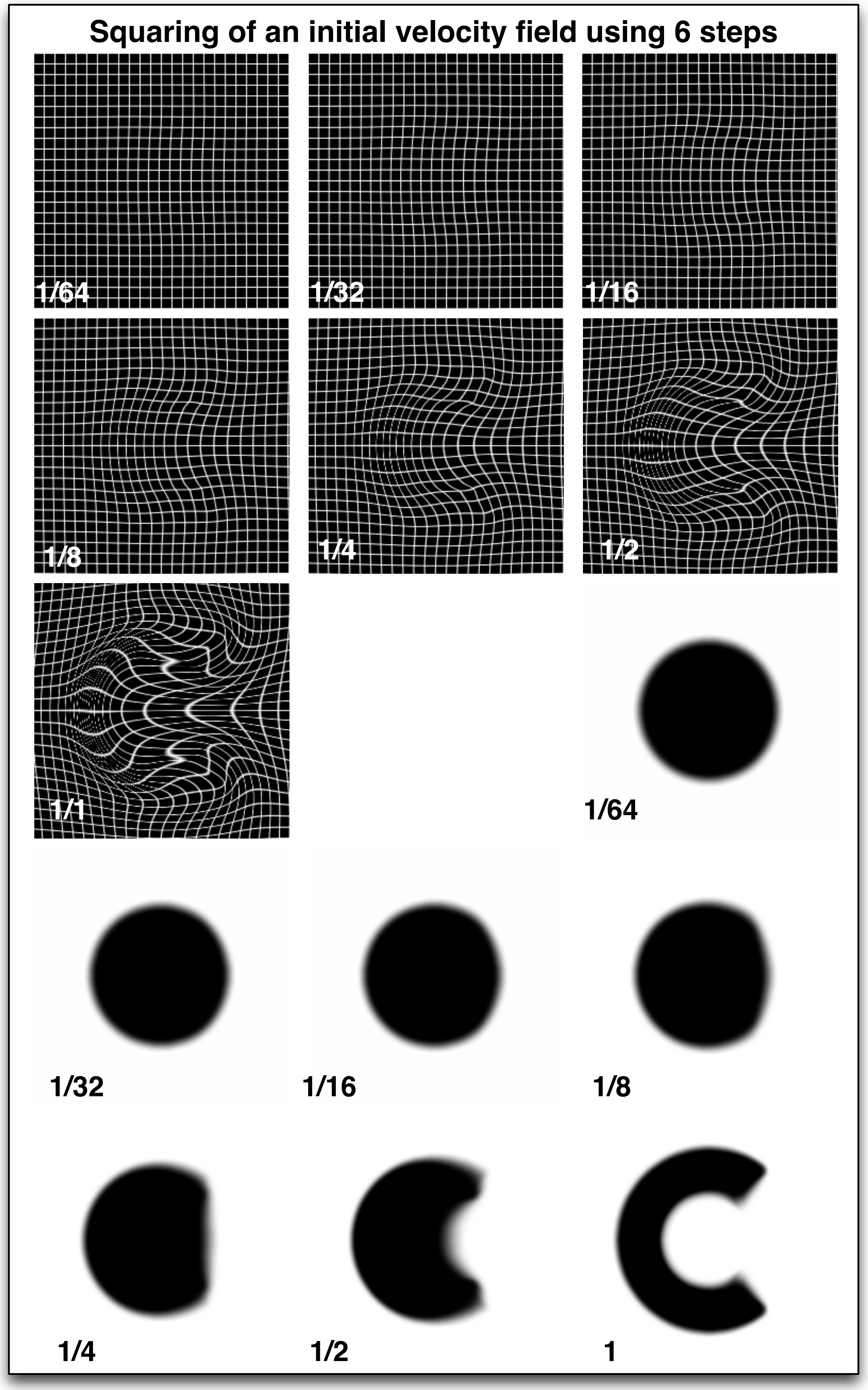
- How to integrate a velocity field parametrisation
  - Euler integration is computationally expensive

$$
\begin{aligned}
\phi^{1/64} &= x + u(x)/8 \\
\phi^{2/64} &= \phi^{1/64} \circ \phi^{1/64} \\
\phi^{3/64} &= \phi^{1/64} \circ \phi^{2/64} \\
\vdots &= \vdots \\
\phi^{64/64} &= \phi^{1/64} \circ \phi^{63/64}
\end{aligned}
$$

Velocity field parametrisation

- How to integrate a velocity field parametrisation
  - Euler integration is computationally expensive
  - Scaling and squaring for stationary velocity field

$$
\begin{aligned}
\phi^{1/64} &= x + u(x)/64 \\
\phi^{2/64} &= \phi^{1/64} \circ \phi^{1/64} \\
\phi^{4/64} &= \phi^{2/64} \circ \phi^{2/64} \\
\phi^{8/64} &= \phi^{4/64} \circ \phi^{4/64} \\
\phi^{16/64} &= \phi^{8/64} \circ \phi^{8/64} \\
\phi^{32/64} &= \phi^{16/64} \circ \phi^{16/64} \\
\phi^{64/64} &= \phi^{32/64} \circ \phi^{32/64}
\end{aligned}
$$



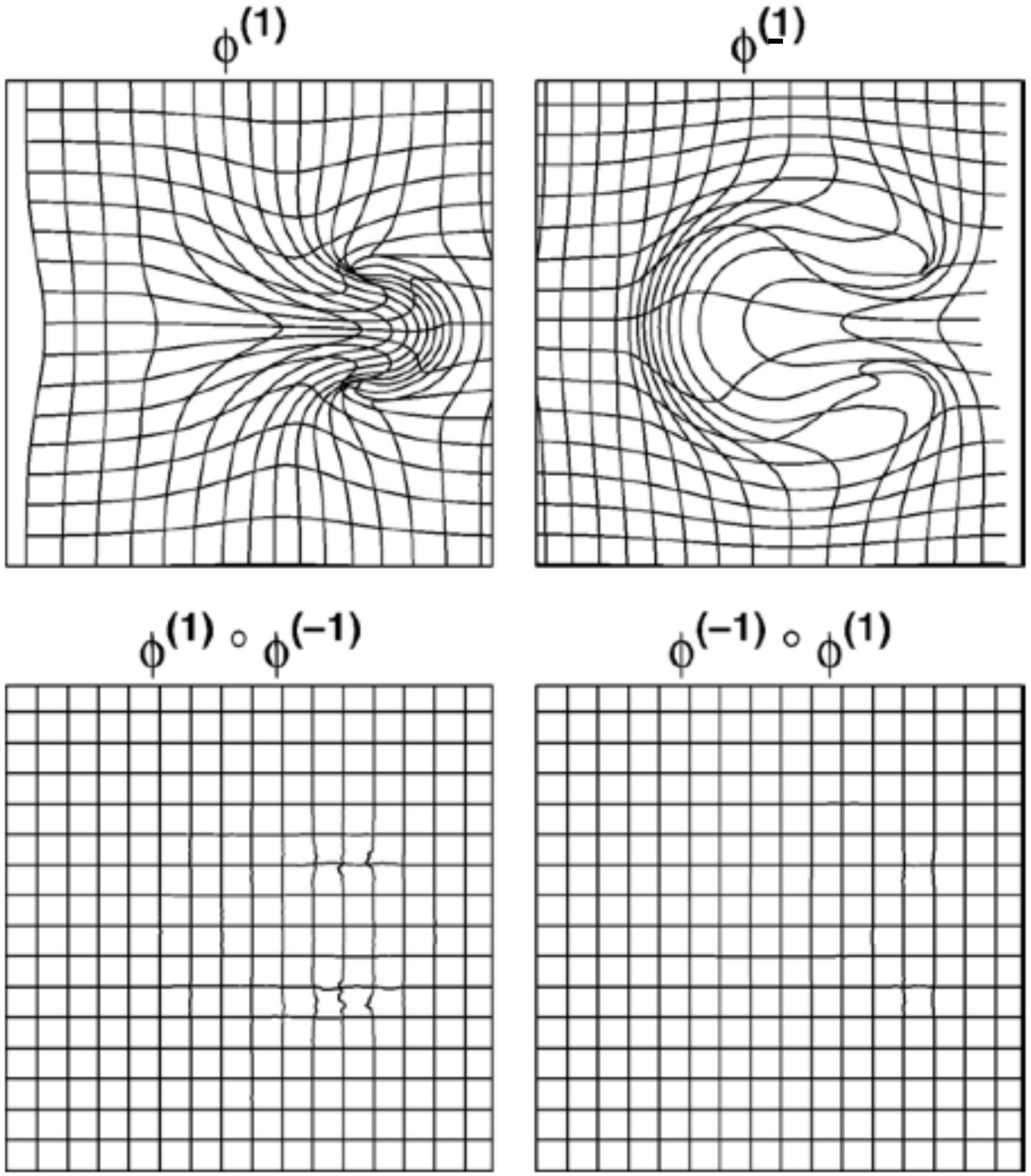
DARTEL

Diffeomorphic Anatomical Registration Through Exponential Lie algebra

- Takes advantage of the scaling-and-squaring

$$\exp(v)\exp(-v) = 0$$

$$\exp(v) = \exp(-v)^{-1}$$



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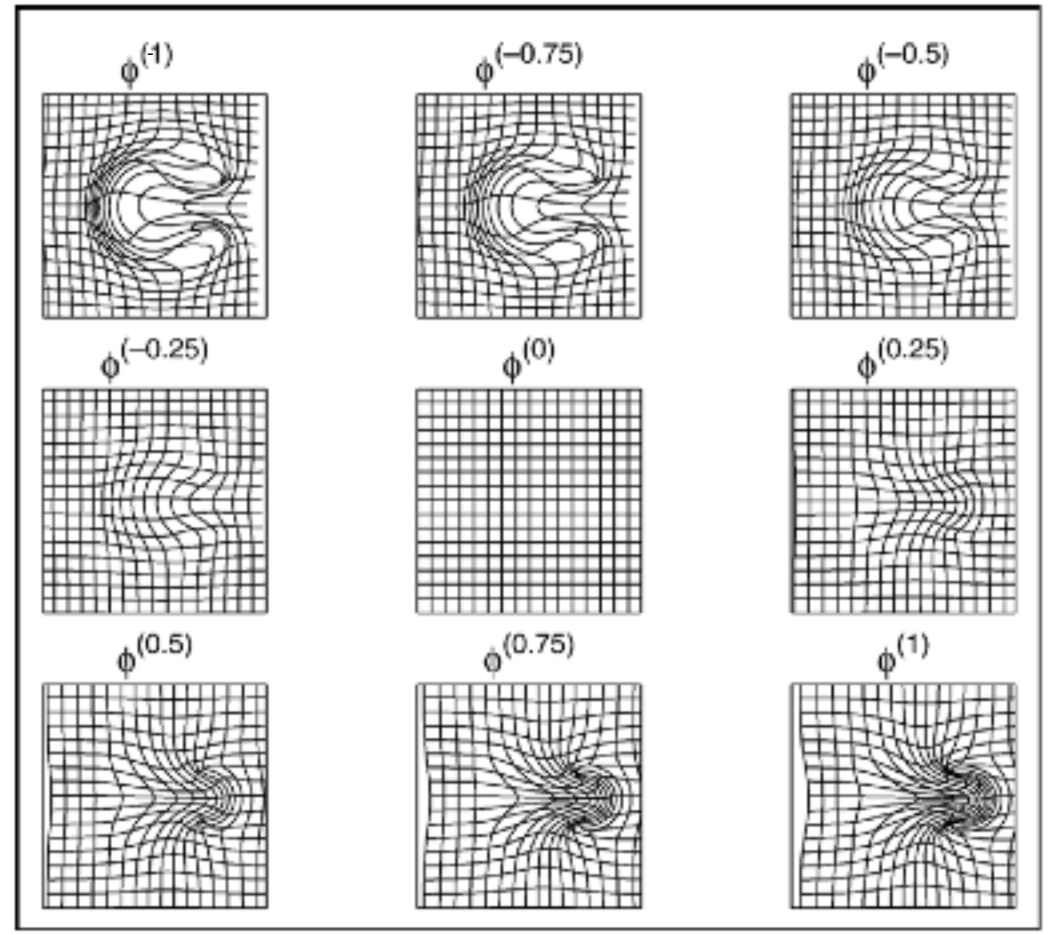

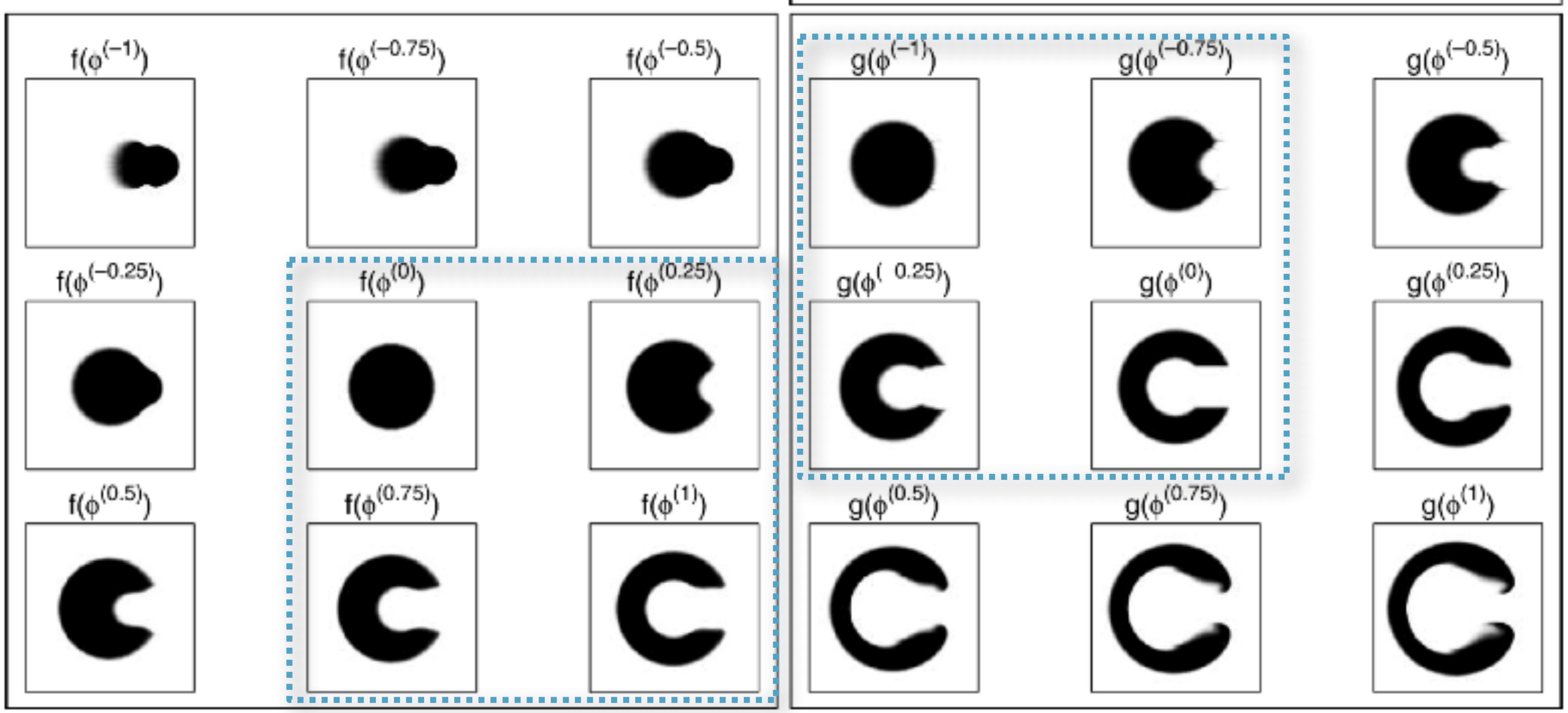

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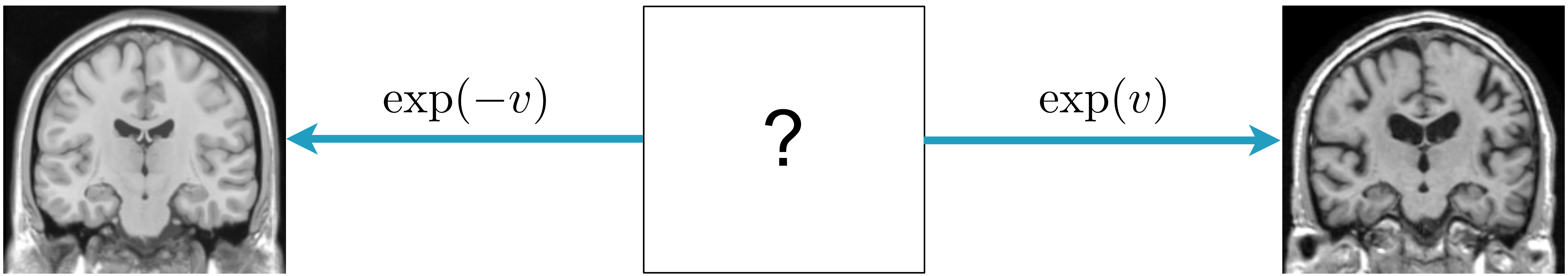
Symmetric demons

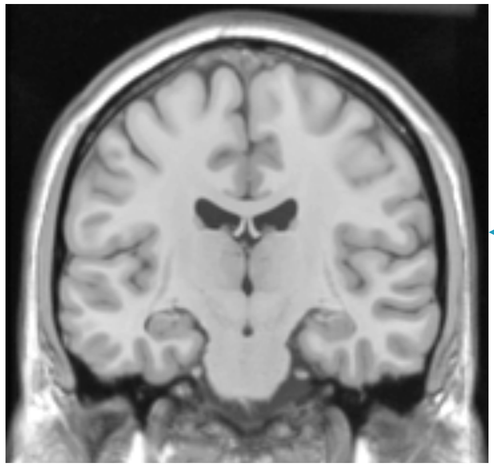
- Use of the stationary velocity field and an average space



Reference R *Floating F*

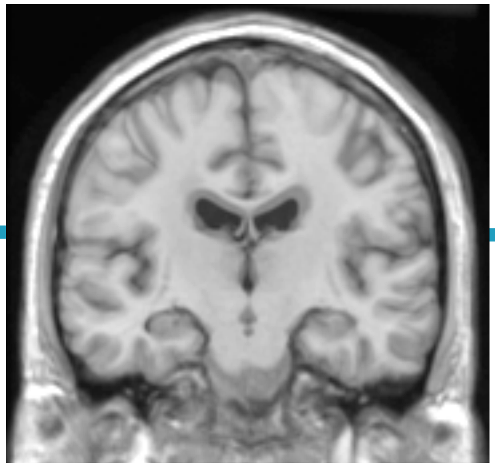
Symmetric demons

- Use of the stationary velocity field and an average space



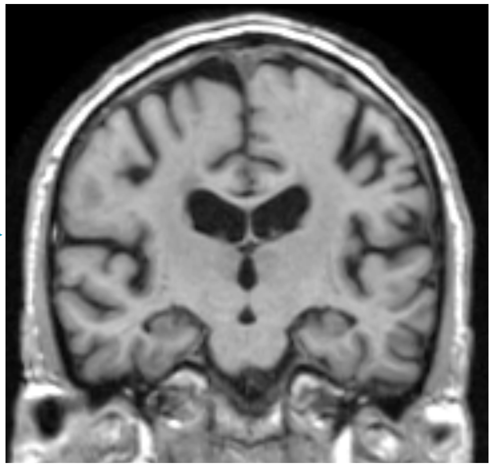
Reference R

$\exp(-v)$



$$\frac{R(\exp(-v)) + F(\exp(v))}{2}$$

$\exp(v)$



Floating F

$$\text{Starting with: } \exp(v) = \exp(-v) = \text{Identity}$$

Symmetric demons

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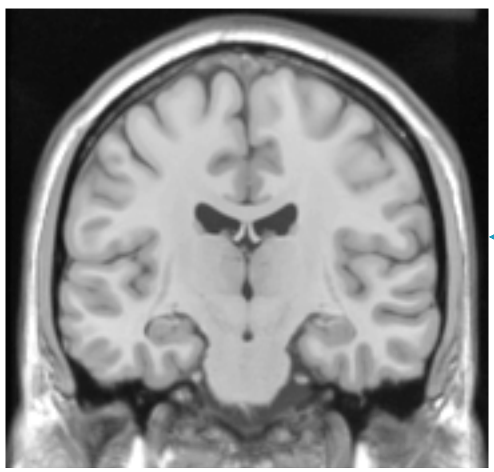
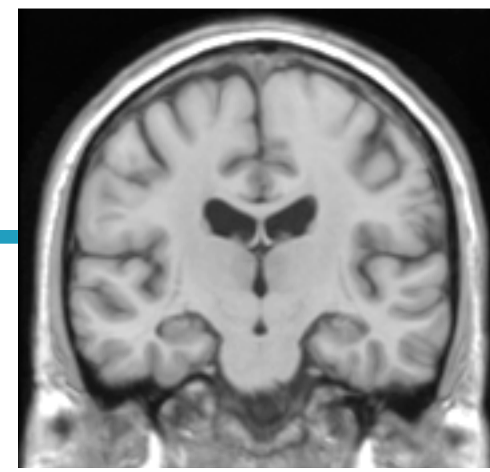


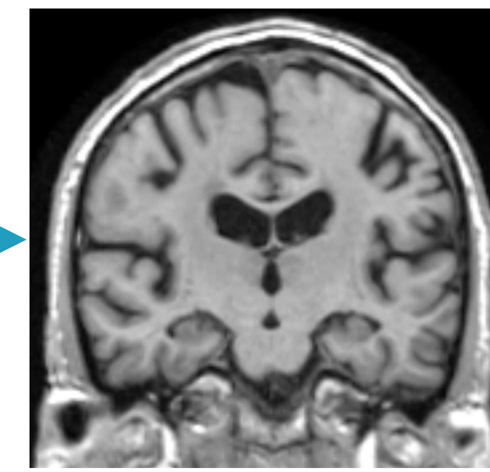
Reference R

$\exp(-v)$

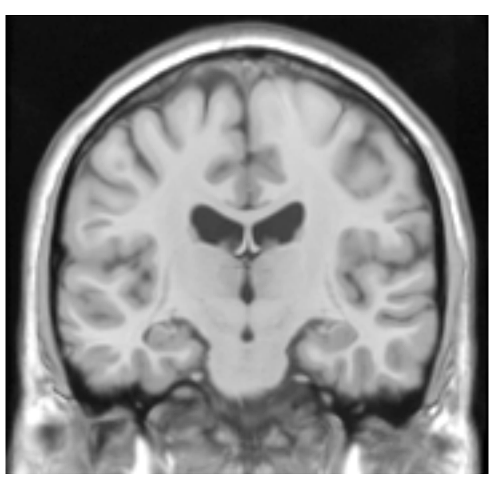


$$\frac{R(\exp(-v)) + F(\exp(v))}{2}$$

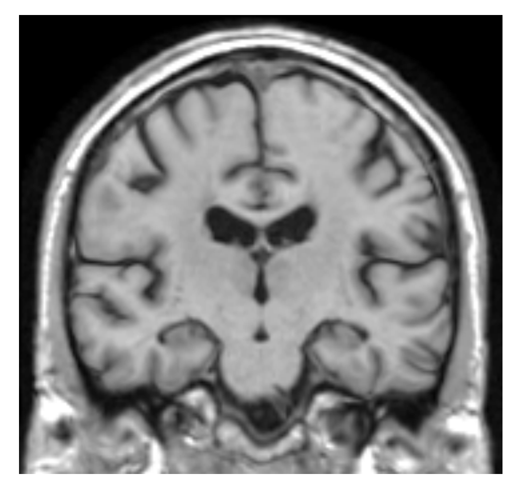
$\exp(v)$



Floating F



$R(\exp(-v))$



$F(\exp(v))$

Symmetric demons

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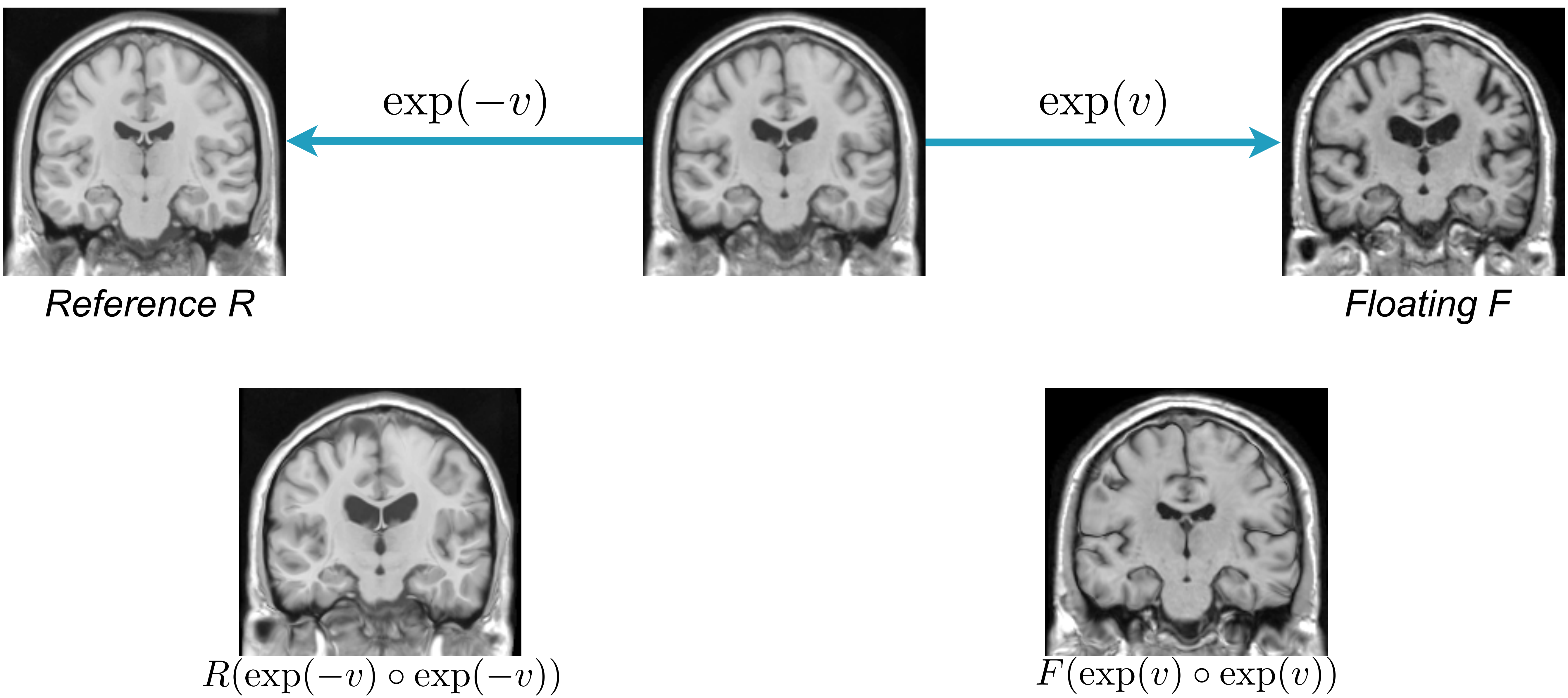
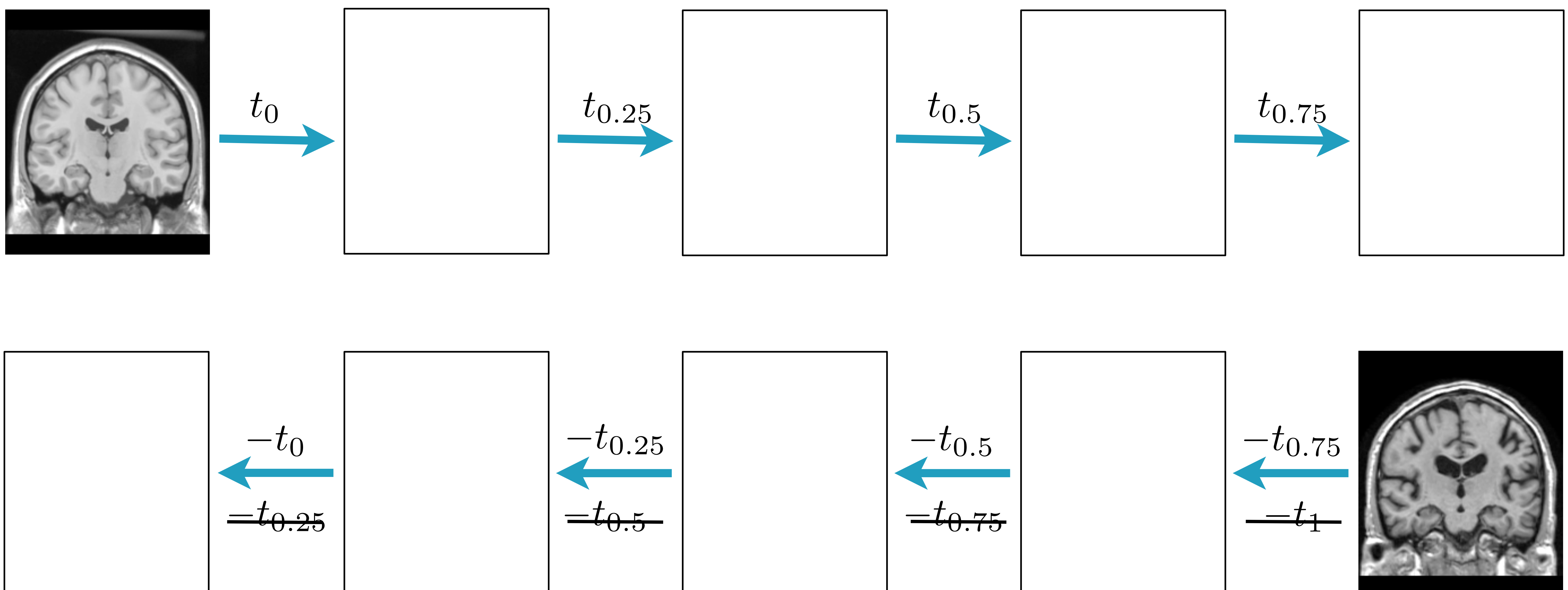


LDDMM

Large Deformations Diffeomorphic Metric Mapping

- Non-stationary velocity field optimisation (time-varying)
- Optimisation of geodesics

Reference



Floating

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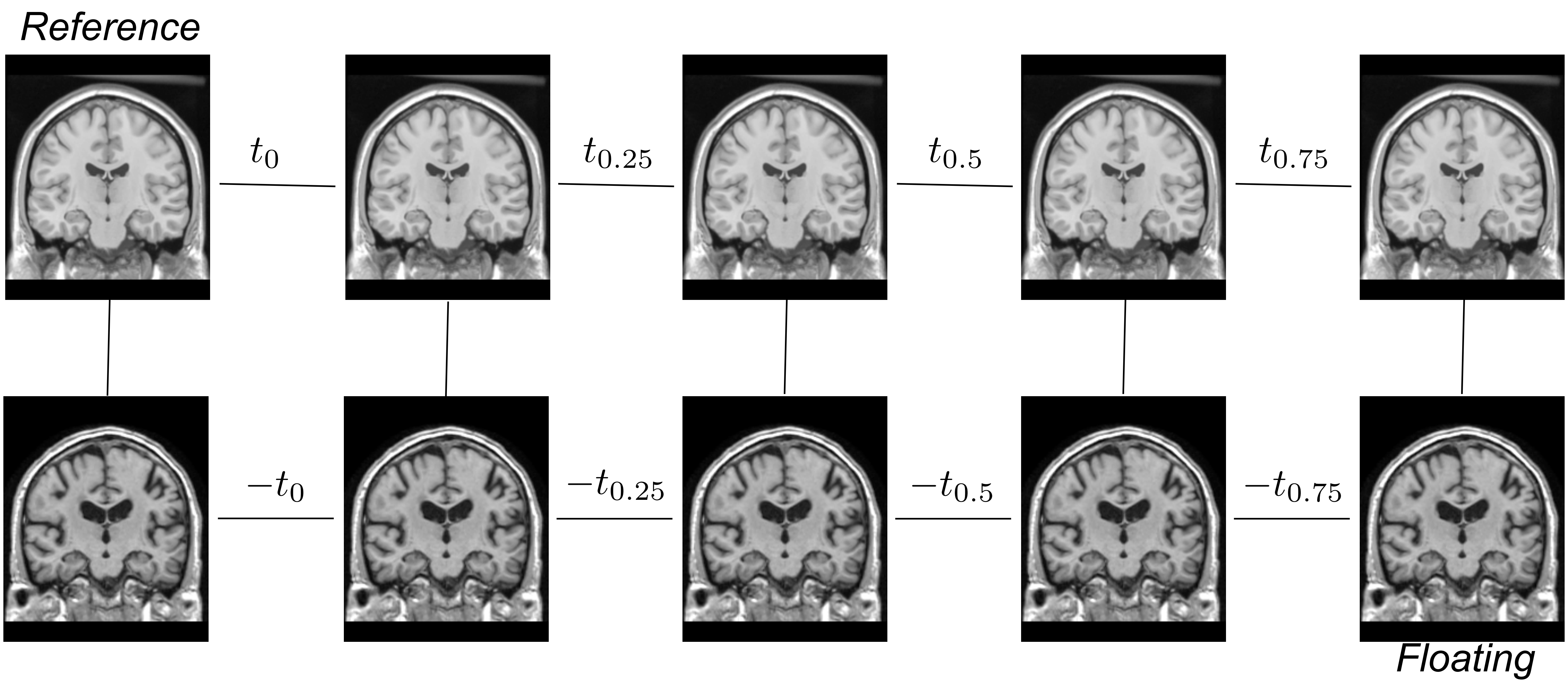

LDDMM

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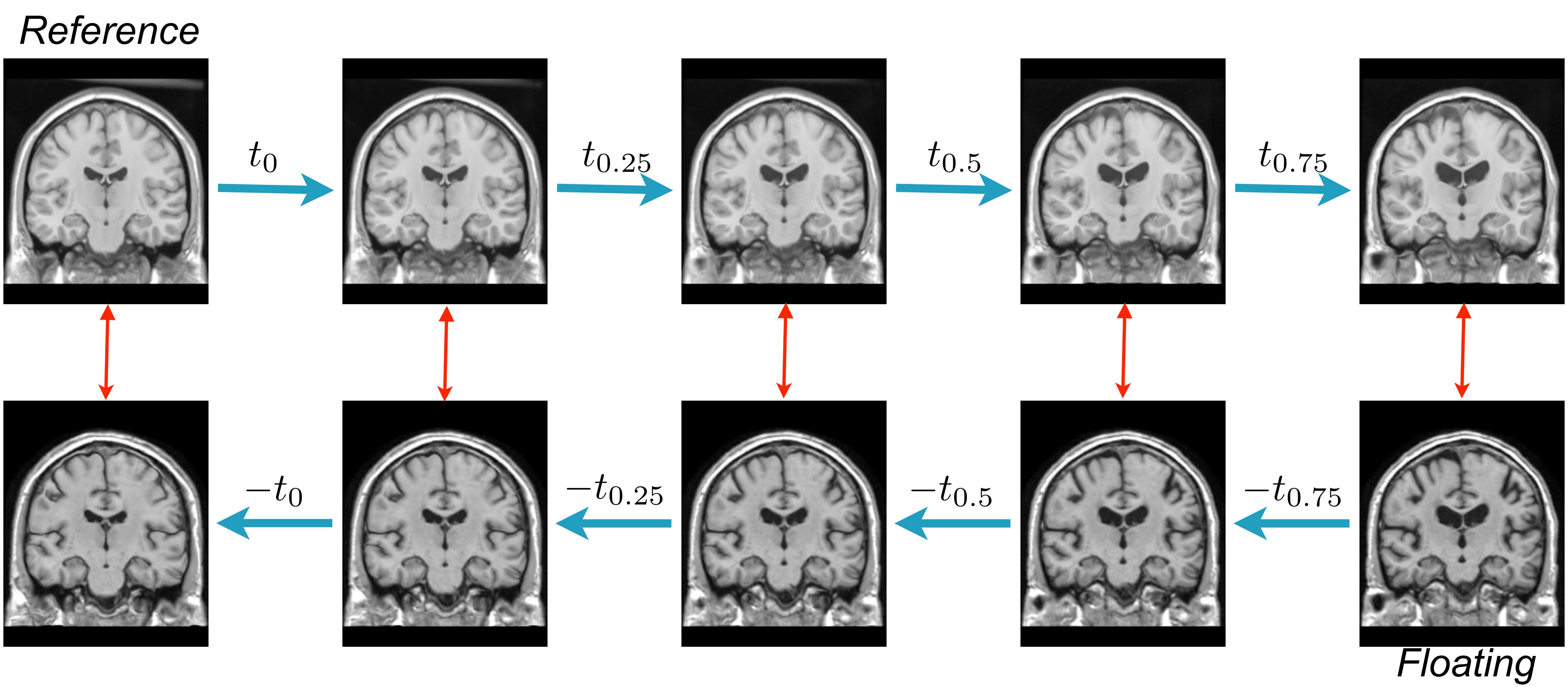
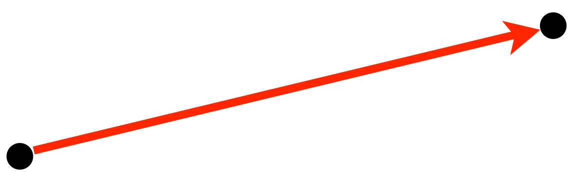
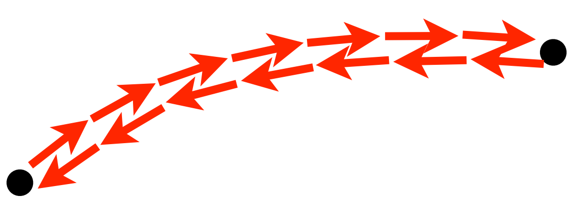
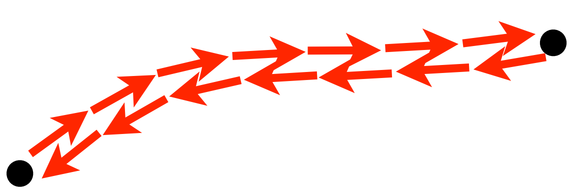


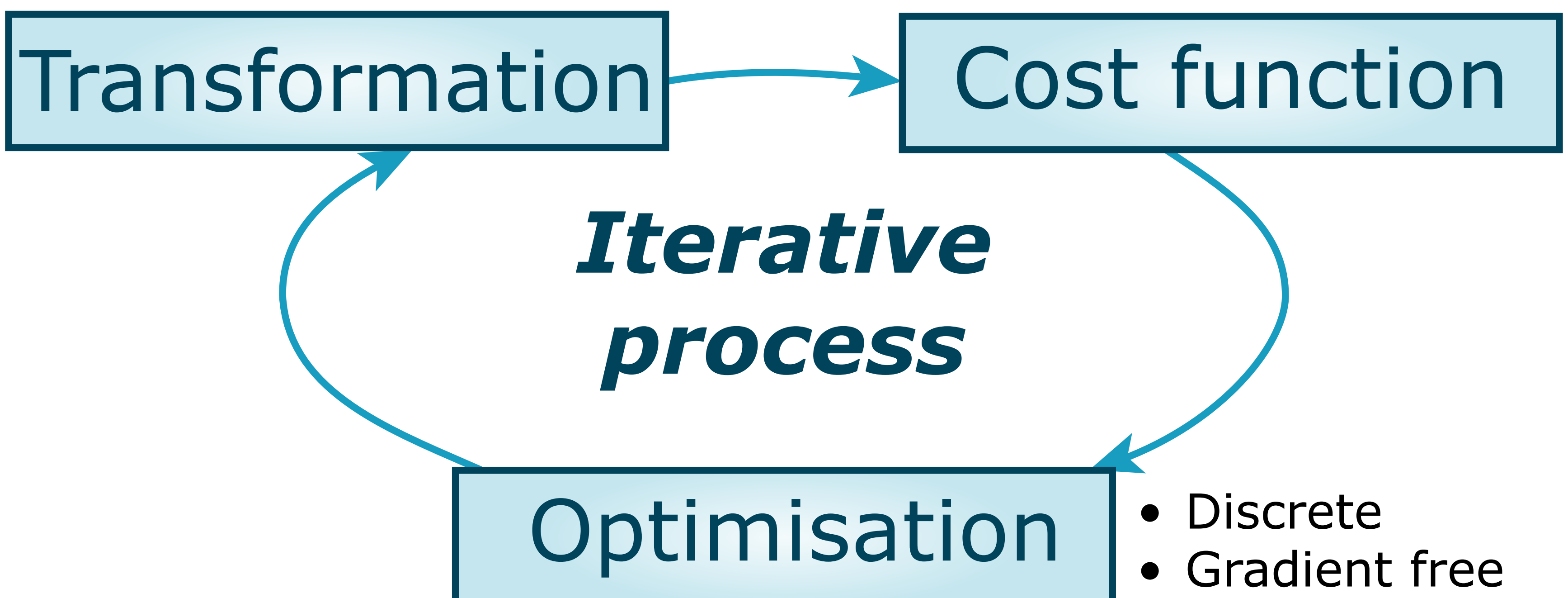
Comparison

	Advantages	Drawbacks
Soft or hard constraints on the transformation 	Simple optimisation as additive scheme can be used	Asymmetric transformation model Transformation paths are lost
Stationary velocity field 	Optimisation of forward and backward direction Possibility to average transformations (groupwise)	More complicated optimisation as additive scheme cannot be used anymore
Non-stationary velocity field 	Optimisation of geodesics (Useful to compute distances between images)	Computationally very expensive

Medical image registration

- Overall scheme

- Rigid
- Affine
- Elastic
- Fluid
- …



- Measure of similarity
- Penalisation term
- …

- Discrete
- Gradient free
- Gradient based
- …