MACHINE LEARNING AND PATTERN RECOGNITION
Assignment 1

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Examination number - B076165

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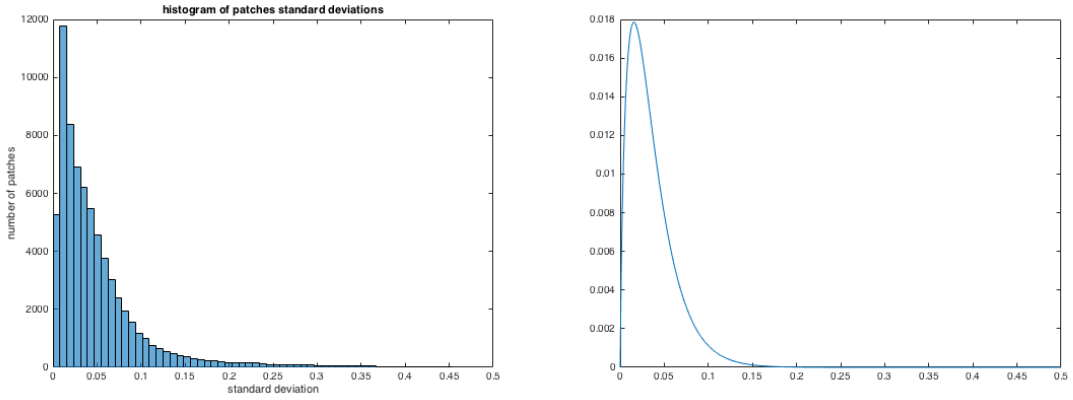
1 The Next Pixel Prediction Task

Note

For all code snippets in the part 1 it is assumed that I had loaded imgregdata.mat file via matlab terminal before I ran the scripts. For the tasks starting from 1.3 it is also assumed that I have loaded NetLab and welltrainedMLP.mat file. All examples were ran on the Mac OS X
I don't calculate RMSE upper and lower bounds for cross-validation because it is quite difficult to achieve this with default matlab crossval function. But in such cases I report RMSE with bounds on the test set.

1.1 Data preprocessing and visualization

Figure 1: standard deviations in the xtr dataset



(a) histogram of the xtr dataset

(b) approximate distribution $t^{0.78}exp(-50t)$

(a) From the figure 1a we can see that after the peak on the second bin (associate it with deviation around 1 discrete pixel value) the number of patches declines exponentially as standard deviation increases . We can conclude that most of patches have the standard deviation between 0 and 0.05. It can be confirmed from subtask c. 52739 patches which are more than 75% of our initial dataset have standard deviation within 0 and 0.0635 range, and 0.0635 is a quite small standard deviation. Therefore, most of the patches are flat ones.
The distribution is not symmetric and can be approximated by family of distributions $P(t) \propto t^{\alpha}exp(-\beta t)$ (from experience in physics). For example, if I put $\alpha = 0.78$ and $\beta = 50$ (fitted so we have the same maximum) then I will have the figure 1b
The maximum possible value of the standard deviation is $\frac{max.-min.}{2}$, so in our case after normalisation it is $\frac{1-0}{2} = 0.5$. Our threshold to distinguish discrete values of pixels is $\frac{1}{64} =\approx 0.0156$ so that values between 0 and 0.0156 will be regarded as discrete intensity of

zero and between 63 * 0.0156 and 1 will be discrete intensity of 63. Because most of the patches are flat ones we can approximately describe every such patch as Gaussian. That is why we can say that approximately 95% of pixels will be within two standard deviations from mean of the patch. Due to the fact that each bin corresponds to the standard deviation by the equation $bin - standard - deviation = bin - index * bin - width$ and, therefore, two standard deviations $2 * bin - standard - deviation = bin - index * 2 * bin - width$. So by choosing bin-width as half of our threshold 0.0156 we can associate each bin with discrete (original pixel value) value of 2 standard deviations. That is why it will be possible to say that for the patch in the first bin most of the pixels are different from the mean value only by 1 discrete value of intensity, for the second bin it will be 2 discrete value of intensity, etc until we reach non-flat patches. $bin - width = \frac{threshold}{2} = \frac{0.0156}{2} = 0.5/64 = 0.0078 = \frac{max.-min.}{64}$ so we should choose 64 as the number of our bins. Code snippet to plot histogram:

```
%launch via - tsk1_1_a(xtr)
function [] = tsk1_1_a(xtr)
    patches = xtr ./ 63;
    patches_std = std(patches,0,2);

    figure;
    h = histogram(patches_std,64);
    h.BinWidth = 0.5 / 64;
    title('histogram of patches standard deviations');
    xlabel('standard deviation');
    ylabel('number of patches');
end
```

(b) I would choose the mean of all pixels in the patch as a simple predictor for flat patches. Given the definition of flat patches the intensity of the target pixel (as any other pixel in the patch respectively) in the flat patch should be something like this $f(\boldsymbol{x}_{\textbf{all other pixels}}) = const_{this flat\ patch} + o(\boldsymbol{x}_{all\ other\ pixels})$ where $o(\boldsymbol{x}_{all\ other\ pixels})$ is small function in comparison to $const_{flat\ patch}$ and it is not completely deterministic so we can approximately replace it with Gaussian $f(\boldsymbol{x}_{\textbf{all other pixels}}) = const_{this flat\ patch} + \mathcal{N}(0, \sigma_{flat\,patch})$. For flat patches the following is true $\sigma_{flat\,patch} \leq \sigma_{flat\,pach\,max}$. So it is natural to propose $const_{this flat\ patch}$ as our prediction, which can be received by calculating mean over all pixels in the flat patch The performance of this simple predictor can be estimated by considering an extreme case when after normalisation (all pixel values between 0 and 1) most pixels are zeroes and small portion of pixels are ones (correspond to original intensity of 63). Let $N - m$ be the number of zeros and let m be the number of ones and I denote μ as the mean.

$$m < N - m$$

$$\mu = \frac{(N-m)*0 + m*1}{N} = \frac{m}{N}$$

$$\sigma^2 = \frac{1}{N}[(N-m)(0 - \frac{m}{N})^2 + m(1 - \frac{m}{N})^2]$$

$$= \frac{(N-m)m^2}{N^3} + \frac{m(N-m)^2}{N^3}$$

$$N^3\sigma^2 = m^2N - m^3 + mN^2 - 2m^2N + m^3$$

$$= mN^2 - m^2N$$

$$m^2 - mN + N^2\sigma^2 = 0$$

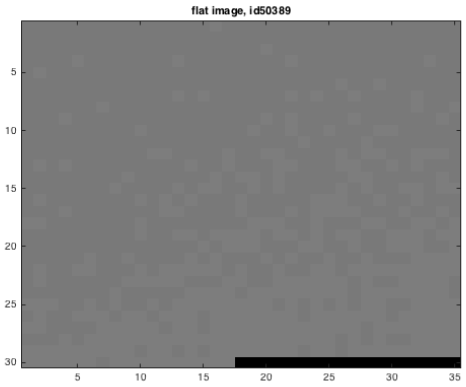
$$m = \frac{N}{2}(1 - \sqrt{1 - 4\sigma^2}) \quad \text{(minus because our case is } m < N - m \text{)}$$

putting $\sigma_{flat\,pach\,max} = \frac{4}{63} \approx 0.0635$ instead of σ and using $N = 1032$ we get

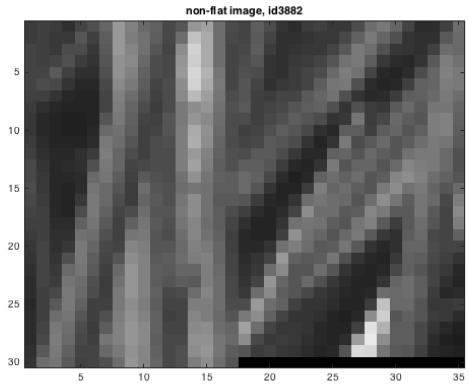
$$m = \frac{1032}{2}(1 - \sqrt{1 - 4 * 0.0635^2}) \approx 4.178$$

rounding m to the closest integer we receive $m = 4$.
Thus, in most extreme case of flat patch we can have 4 ones (correspond to original pixel intensity of 63) and 1028 zeros, so it is natural that we want to predict zero as discrete value of our target pixel. The mean gives us $\mu = \frac{1028*0+1*4}{1032} \approx 0.0038$. Dividing range between 0 and 1 by 64 we get 0.0156 as our threshold to distinguish between discrete pixel values. 0.0038 lies between 0 and 0.0156 so our mean value would correspond to 0 as the discrete value of our target pixel and that is what we wanted.

Figure 2: patch images



(a) flat patch image



(b) non-flat patch image

(c) Code snippet to show patch images on figure 2:

```
%launch via tsk1_1_c(xtr)
function [] = tsk1_1_c(xtr)
    %normalising
    patches = xtr ./ 63;
    patches_std = std(patches,0,2);

    %the threshold 4/63 is taken from the task
    flat_threshold = 4/63;
    flat_patches_ids     = patches_std <= flat_threshold;

    non_flat_patches_ids = patches_std >  flat_threshold;

    %split on flat and non-flat patches
    flat_patches     = patches(flat_patches_ids, :);
    non_flat_patches = patches(non_flat_patches_ids,:);

    function [rnd_image, rnd_image_id] = get_rnd_image(patches)
        rnd_image_id = randi(size(patches, 1), 1);
        patch = patches(rnd_image_id, :);
        %expanding patch to the full size
        patch(1050) = 0;
        %reshaping to image
        rnd_image = reshape(patch, [35, 30]);
        %transposing them to ensure right position on the plot
        rnd_image = rnd_image';
    end

    function [] = show_image(image, title_str, image_id)
        figure;
        imagesc(image, [0, 1]);
        title(strcat(title_str, num2str(image_id)));
        colormap gray;
    end

    [flat_image, flat_id] = get_rnd_image(flat_patches);
    [non_flat_image, non_flat_id] = get_rnd_image(non_flat_patches);

    show_image(flat_image, 'flat image, id ', flat_id);
    show_image(non_flat_image, 'non-flat image, id ', non_flat_id);
end
```

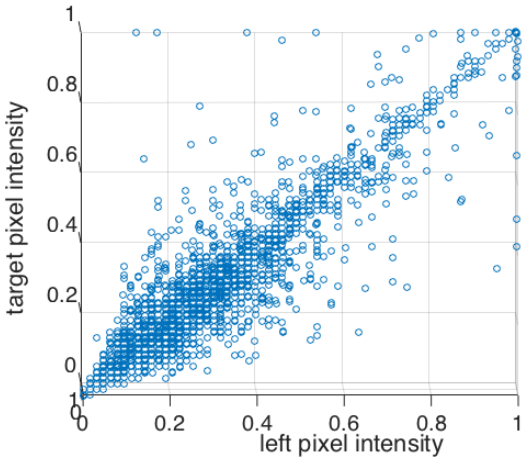
1.2 Linear regression with adjacent pixels

(a) I used 5000 training points from xtr_nf and ytr_nf to figure 3c. From it we can see that x(j, end), x(j, end - 34), y(j) are strongly positively correlated. However, there is some number of deviations from this trend. It seems that these deviations are normally distributed as the bigger distance from general trend is the fewer instances we have on the plot.

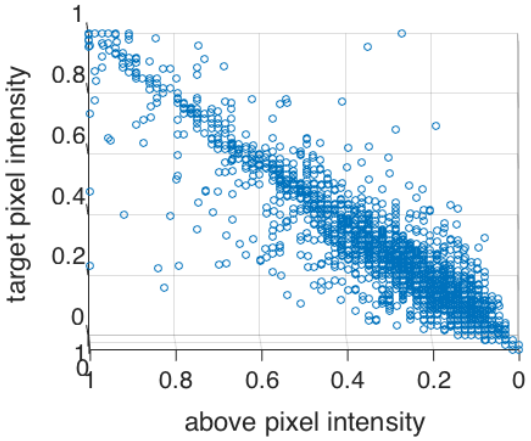
By looking at the figures 3a, 3a and the huge bulb like distribution in the beginning of the figure 3c it may seem that σ_η standard deviation of the noise η in this expression for linear regression model $y = \boldsymbol{w}^T\boldsymbol{x} + \eta \quad \eta \approx Gaussian(0, \sigma_\eta)$ in reality is not a constant and it depends heavily on the target pixel value.

But it happens due to the fact that it is much more likely to have darker target pixels, and

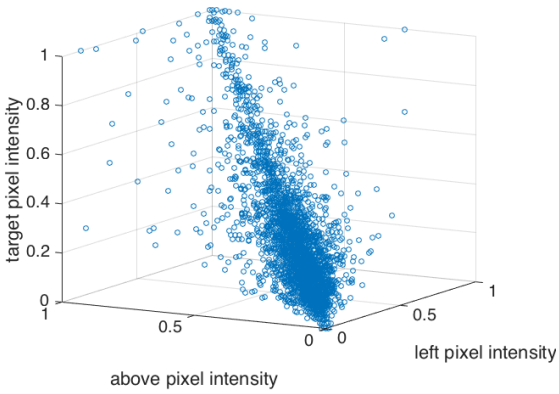
Figure 3



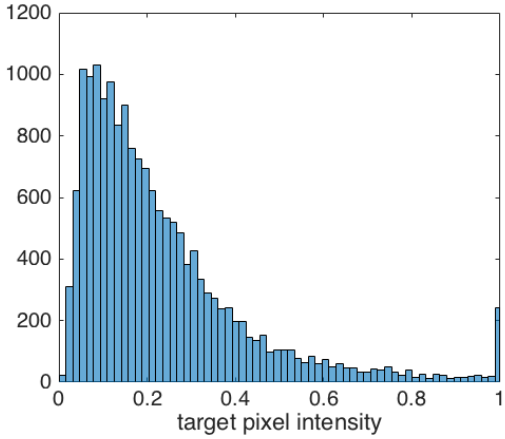
(a) 5000 data points from xtr_nf and ytr_nf



(b) 5000 data points from xtr_nf and ytr_nf



(c) 5000 data points from xtr_nf and ytr_nf



(d) histogram of target pixel intensity on the full training set

therefore we have more samples from our distribution in the darker area so it looks more dense. This can be seen on the histogram 3d, and I have slightly inclined figures 3a, 3a so it is easier to see the number of data points.

Code snippet for scatter plot:

```
%launch via - tsk1_2_a(xtr_nf, ytr_nf)
function [] = tsk1_2_a(xtr_nf, ytr_nf)
    %I choose 5000 so plot doesn't look cluttered
    num_data_points = 5000;
    x_left   = xtr_nf(1:num_data_points, end);
    x_above  = xtr_nf(1:num_data_points, end - 34);
    x_target = ytr_nf(1:num_data_points);

    figure;
    scatter3(x_left, x_above, x_target);
    xlabel('left pixel intensity');
    ylabel('above pixel intensity');
    zlabel('target pixel intensity');
    set(gca,'FontSize', 20);

    figure;
    histogram(ytr_nf,64);
```

```
    xlabel('target pixel intensity');
    set(gca,'FontSize', 20);
end
```

(b) Derivation of this solution can be taken from MLPR lecture 7 slides 8-11 link. The solution for weights from the lecture is:

$$\hat{\boldsymbol{w}} = (\Phi^T \Phi)^{-1} \Phi^T \boldsymbol{y}$$

In our notation matrix Φ will become:

$$\Phi = X = \begin{pmatrix} 1, x(1, end), x(1, end-34) \\ 1, x(2, end), x(2, end-34) \\ \dots \\ 1, x(N, end), x(N, end-34) \end{pmatrix}$$

$$\hat{\boldsymbol{w}} = (X^T X)^{-1} X^T \boldsymbol{y}$$

where N is the number of training data points and x is our dataset (it will be xtr_nf in the next task)
I asked professor Chris Williams and he said there is no need to repeat all derivations from MLPR lecture.

(c) After training the weights are:

bias	left pixel	above pixel
0.0026	0.4606	0.5241

the RMSE for test and training sets:

	Training set	Test set
RMSE	0.0506 ± 0.0010	0.0503 ± 0.0017

Taking bounds into consideration we can see that the performance on training and test is almost the same. That is why we can conclude that linear regression is not over-fitting the data in this problem. It can be seen from figure 4 that indeed linear regression describes the main trend very well.

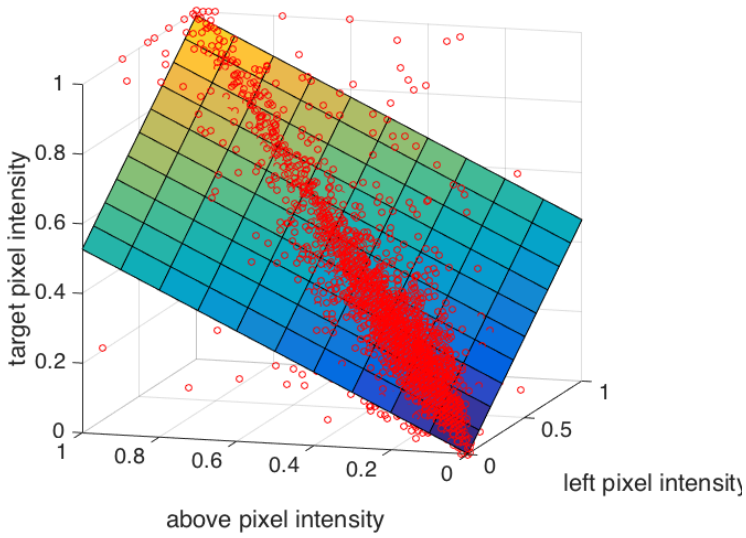


Figure 4: plot of the linear regression function after training along with test data points

Code snippet to get linear regression predictor (will also be used afterwards):

```
function [w, predictor] = cs_linear_regression(x_train, t_train)
    %custom linear regression
    %it inserts bias term automatically %Ruslan Burakov

    %adding bias term
    calc_Phi = @(x)[ones(size(x, 1), 1), x];

    %computing weights
    w = pinv(calc_Phi(x_train)) * t_train;

    predictor = @(x_test)calc_Phi(x_test) * w;
end
```

Code snippet to compute root mean square error (RMSE) with upper/lower bounds (implementation pointed out by professor Chris Williams), which corresponds to upper/lower bounds of mean square error(MSE) through standard error of MSE:

$$RMSE\ with\ bounds = sqrt(MSE \pm standard\ error\ of\ SE) \approx sqrt(MSE) \pm \frac{standard\ error\ of\ SE}{2 * sqrt(MSE)}$$
$$= RMSE \pm \frac{standard\ error\ of\ SE}{2 * sqrt(MSE)}$$

(further explanation in code comments) (will also be used afterwards):

```
function [rmse_est, std_err] = cs_rmse(t, y)
    %custom root mean square error
    diff = t - y;
    %mean square error
    sqr_diff = diff .* diff;
    mse_est = mean(sqr_diff);
    rmse_est = sqrt(mse_est);
    if nargout > 1
        %professor Chris Williams pointed out that
        %to produce some bounds on rmse:
        %mse = mse_est +/- mse_std_err
        %rmse = f(mse) = f(mse_est +/- mse_std_err) = sqrt(mse_est +/- mse_std_err)
        %f(u +/- h) ~= f(u) +/- f'(u)h = sqrt(u) +/- 0.5 * h/(sqrt(u)) %Taylor expansion
        S = numel(t);
        var_est = var(sqr_diff);
        mse_std_err = sqrt(var_est/S);
        std_err = 0.5 * mse_std_err / rmse_est;
    end
end
```

Code snippet to show rmse on training and test set with bounds (will also be used afterwards):

```
function [] = show_rmse(t_train, y_train, t_test, y_test)
    [rmse_train, std_err_train] = cs_rmse(t_train, y_train);
    [rmse_test,  std_err_test]  = cs_rmse(t_test,  y_test);

    fprintf('rmse on training set %5.4f +/- %5.4f\n', rmse_train, std_err_train);
    fprintf('rmse on test set %5.4f +/- %5.4f\n', rmse_test, std_err_test);
end
```

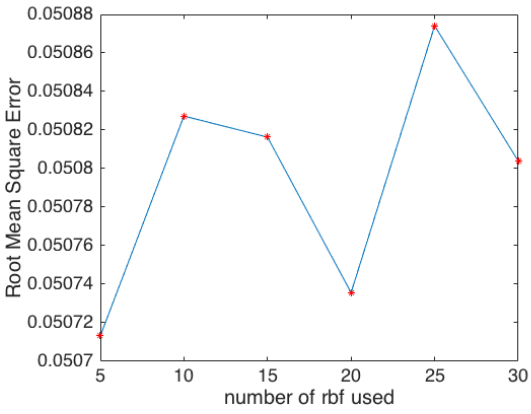
Code snippet for this task:

```
%launch via - tsk1_2_c(xtr_nf, ytr_nf, xte_nf, yte_nf)
function [] = tsk1_2_c(x_all_train, t_train, x_all_test, t_test)
    %t - means target values
    get_adjacent_pixels = @(x) [x(:, end), x(:, end - 34)];

    %prepare sets
    x_train = get_adjacent_pixels(x_all_train);
    x_test  = get_adjacent_pixels(x_all_test);

    %train (it will add bias term automatically)
    [w, predictor] = cs_linear_regression(x_train, t_train);
    display(w, 'weights for neighbours pixels features');

    show_rmse(t_train, predictor(x_train), t_test, predictor(x_test));
```

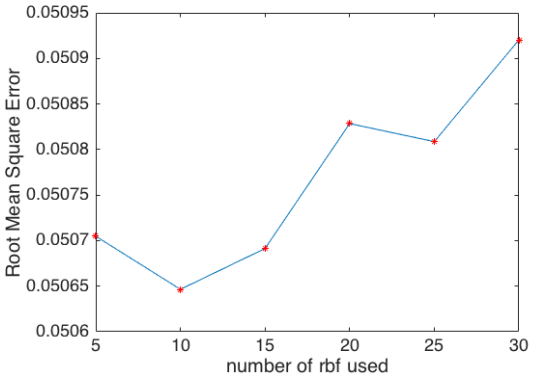
```
    %show surface
    figure,
    %I chose smaller step because our function is just a plane
    [dim1, dim2] = meshgrid(0:0.1:1, 0:0.1:1);
    %swapped ones from original snippet, because w(1) corresponds to bias
    %in my case
    ysurf = [ones(numel(dim1),1), [dim1(:), dim2(:)]] * w;
    surf(dim1, dim2, reshape(ysurf, size(dim1)));

    hold on;

    %show test set data points
    scatter3(x_test(:, 1), x_test(:, 2), t_test, 'red');
    xlabel('left pixel intensity');
    ylabel('above pixel intensity');
    zlabel('target pixel intensity');
    set(gca,'FontSize', 20);
end
```

1.3 RBF regression with adjacent pixels

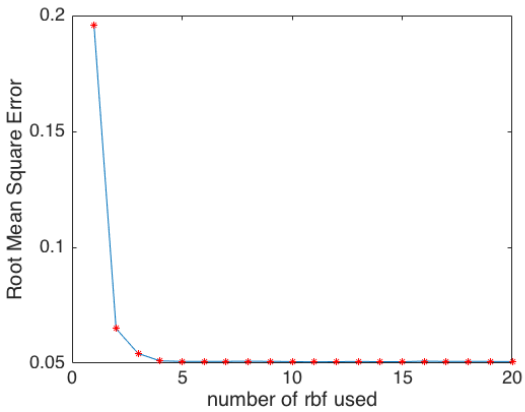
Figure 5: Root Mean Square Error against number of radial basis functions used



(a)



(b)



(c)

(a) When I ran cross validation procedure to determine which number of radial basis functions among { 5, 10, 15, 20, 25, 30} produces the best results, each time I received a different answer. The figure 5a suggests 5 as the best number of radial basis functions and 5b proposes 10 as the best choice. This happens probably due to the random numbers as matlab crossval uses them each time to divide input set on training and validation sets and rbf network initialises weights differently depending on the random numbers. After that I launched the procedure for number of radial basis functions between 1 and 20, and I have realised that it was just a matter of scale. The figure 5c demonstrates that we achieve almost no improvement if we use more than 5 radial bases functions in this task. That is why I have

chosen 5 as my number of radial basis functions because for the same efficiency it takes less time to compute.

Code snippet for this task:

```
%launch via:
%tsk1_3_a(xtr_nf, ytr_nf, 5:5:30)
%tsk1_3_a(xtr_nf, ytr_nf, 1:20)
function [] = tsk1_3_a(x_all, t, num_rbfs)
    %t - means target values
    get_adjacent_pixels = @(x) [x(:, end), x(:, end - 34)];
    x = get_adjacent_pixels(x_all);

    opt = foptions;
    opt(1)  = 1; % Display EM training
    opt(14) = 5; % number of iterations of EM
    dim     = 2; % left_pixel, above_pixel in our case

    function regf = create_rbf_regf(num_rbf)
        function y_test = rbf_reg(x_train, t_train, x_test)
            net = rbf(dim, num_rbf, 1, 'gaussian');
            net = rbftrain(net, opt, x_train, t_train);
            y_test = rbffwd(net, x_test);
        end
        regf = @rbf_reg;
    end

    rmse_records = zeros(1, length(num_rbfs));

    for i = 1:length(num_rbfs)
        num_rbf = num_rbfs(i);
        regf = create_rbf_regf(num_rbf);
        %default CV 10 folds
        cvMse = crossval('mse', x, t,'predfun',regf);
        rmse_records(i) = sqrt(cvMse);
    end

    plot(num_rbfs, rmse_records)
    hold on;
    plot(num_rbfs, rmse_records, 'r*');
    xlabel('number of rbf used');
    ylabel('Root Mean Square Error');
    set(gca,'FontSize', 18);
end
```

(b) the RMSE for test and training sets:

	Training set	Test set
RMSE	0.0506 ± 0.0010	0.0503 ± 0.0017

As it can be seen that radial basis functions don't give any noticeable improvement in comparison to linear regression when both of them are using only adjacent pixels as features. Given that test and training errors are very close, RBF should be an adequate model, as well, but if we have two models with the same performance it is natural to pick a more simple model (Ockham's razor). In our case it should be linear regression.
Code snippet for this task:

```
%launch via - tsk1_3_b(xtr_nf, ytr_nf, xte_nf, yte_nf)
function [] = tsk1_3_b(x_all_train, t_train, x_all_test, t_test)
    %t - means target values
    get_adjacent_pixels = @(x) [x(:, end), x(:, end - 34)];

    x_train = get_adjacent_pixels(x_all_train);
    x_test  = get_adjacent_pixels(x_all_test);

    opt = foptions;
    opt(1)  = 1; % Display EM training
    opt(14) = 5; % number of iterations of EM
```

```
    dim     = 2; % left_pixel, above_pixel in our case
    num_rbf = 5;% determined from previous task

    net = rbf(dim, num_rbf, 1, 'gaussian');
    net = rbftrain(net, opt, x_train, t_train);

    show_rmse(t_train, rbffwd(net, x_train), t_test, rbffwd(net, x_test));
end
```

1.4 Linear regression with all pixels

the RMSE for test and training sets:

	Training set	Test set
RMSE	0.0371 ± 0.0007	0.0456 ± 0.0018

The RMSE on the training set has dropped significantly in comparison to previous models with only adjacent pixels but the improvement on the test set is relatively small. Which may indicate that the complexety of the model has become larger than it is needed for our task. Overall, it is possible to conclude that whereas knowledge of all the pixels helps to predict the value of target pixel better, it seems that the most significant features are pixels adjacent to the target one. Code snippet for this task:

```
%launch via - tsk1_4(xtr_nf, ytr_nf, xte_nf, yte_nf)
function [] = tsk1_4(x_train, t_train, x_test, t_test)
    %t - means target values
    %train (it will add bias term automatically)
    [w, predictor] = cs_linear_regression(x_train, t_train);
    show_rmse(t_train, predictor(x_train), t_test, predictor(x_test));
end
```

1.5 Neural Network with all pixels

(a) the RMSE for test and training sets:

	Training set	Test set
RMSE	0.0333 ± 0.0006	0.0473 ± 0.0017

In comparison to linear regression with all pixels Neural Network (NN) slightly over-fits the data because its error is lower on training set but the error on test set is larger. In my opinion, this happens because NN with all pixels is a very sophisticated model for our task. Code snippet for this task (I loaded the good neural network, "net" variable, from well-trainedMLP.mat):

```
%launch via - tsk1_5_a(net, xtr_nf, ytr_nf, xte_nf, yte_nf)
function [] = tsk1_5_a(net, x_train, t_train, x_test, t_test)
    %t - means target values
    show_rmse(t_train, mlpfwd(net, x_train), t_test, mlpfwd(net, x_test));
end
```

(b) Neural Network RMSE for different random seeds:

Random seed	RMSE on full training set	RMSE on test set
2015	0.0500 ± 0.0009	0.0515 ± 0.0014
2016	0.0477 ± 0.0009	0.0504 ± 0.0014
2017	0.0485 ± 0.0009	0.0515 ± 0.0014
2018	0.0477 ± 0.0009	0.0516 ± 0.0014
2019	0.0489 ± 0.0009	0.0527 ± 0.0016

In comparison to previous task 1.5.a the difference between the errors on the full training set and test set is much smaller in all cases because we are using only 5000 instances from training set for learning and that is why NN is not able to fit all the data points from the full training set. At the same time, most likely due to the same reason, our performance on the full training and test sets has dropped.

Random seed determines the starting point from which it will try to converge (the initial weights) and therefore to what local minimum NN will try to converge. So depending on random seeds some runs will be converging faster than others or potentially they will be able to find better/worse local minimum. It explains why our results differ but taking bounds into consideration our performance on the test set is roughly the same for all runs.

Code snippet for this task:

```
%launched via - tsk1_5_b(xtr_nf, ytr_nf, xte_nf, yte_nf)
function [] = tsk1_5_b(x_train, t_train, x_test, t_test)
    %t - means target values
    nhid = 10; % number of hidden units

    % Set up vector of options for the optimiser.
    options = zeros(1,18);
    %options(1)  = 1; % This provides display of error values.
    %options(9)  = 1; % Check the gradient calculations.
    options(14) = 200; % Number of training cycles.

    function [] = launch_NN(seed)
        rng(seed,'twister');
        net = mlp(size(x_train,2), nhid, 1, 'linear');
        [net, tmp] = netopt(net, options, x_train(1:5000,:), t_train(1:5000,:), 'scg');

        fprintf('seed %4.0f:\n', seed);
        show_rmse(t_train, mlpfwd(net, x_train), t_test, mlpfwd(net, x_test));
    end

    seeds  = 2015:1:2019;

    for i = 1:length(seeds)
        seed = seeds(i)
        launch_NN(seed);
    end
end
```

1.6 Discussion

In previous tasks linear regression with all pixels showed the best performance. That is why I thought that it would be reasonable to try something else with linear regression. First of all there was noticeable gap between performance of linear regression with all pixels on training set and test set. So we should constraint it somehow. Although the models with all the pixels demonstrated the better performance than the models with adjacent pixels their performances on the test set were close. Judging by that It seemed to me that the closest pixels to the target one have the biggest impact on the intensity of the target pixel. But how many nearest pixels do we need? I wasn't sure so I evaluated linear regression performance depending on the number of pixels from the end of feature vector via cross-validation on the training set. The result can be seen on figure 6. **Note**: the plot step is not uniform, after 182 I used a much bigger step (100) because otherwise cross-validation would take too much time.
Figure suggests that the best value from the end is something like 120. Using that number of feature on the test set (xte_nf(:, 912:1032)) I received the following RMSE:

	Test set
RMSE	0.0419 ± 0.0017

This result shows improvement in comparison to the previous models. Also we can see that at the beginning figure 6 has form which is similar to the step function. I think it is so because of the fact that with period of 35 we receive pixels which are above the target one and they give a lot of information about target pixels. This leads to conclusion that because image is 2d-array of pixels where i, j are indexes on the X and Y axis respectively then we should select pixels/features depending on their euclidean distance from the target pixel $r = \sqrt{(18-i)^2 + (30-j)^2}$.
I have performed cross-validation on the training set to find the best distance r, the results are on figure 7. It points out that the best distance is 4.
After checking it on the test set:

	Test set
RMSE	0.0412 ± 0.0017

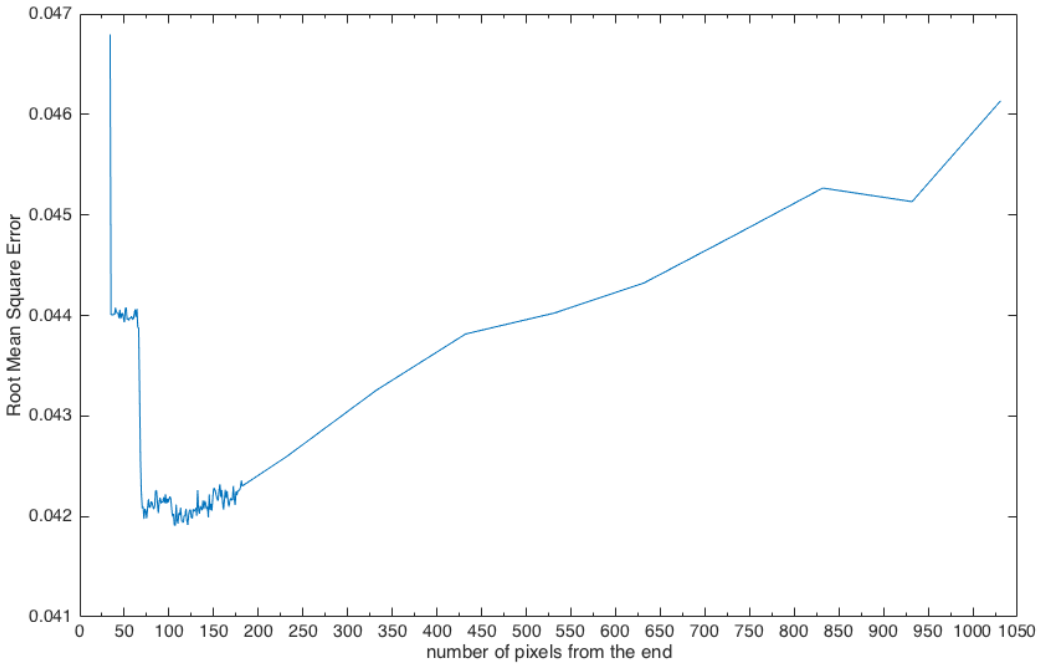


Figure 6: plot of linear regression RMSE depending on the number of pixels from the end used as features.

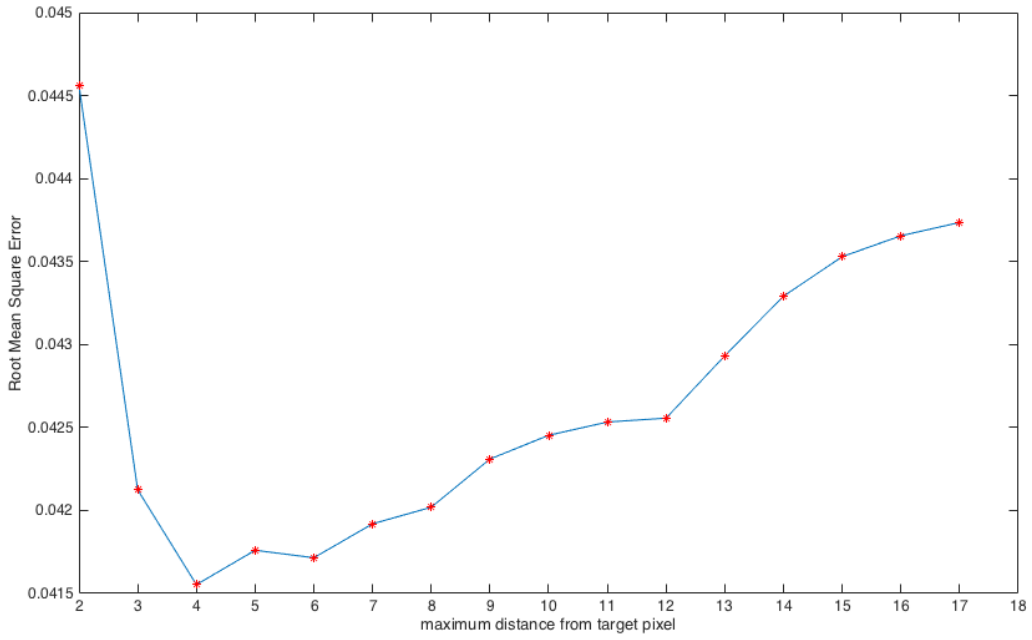


Figure 7: plot of linear regression RMSE depending on the maximum distances from the target pixels within which pixels were used as features.

This is a good result but the given bounds suggest that the performance is not significantly better. However this model is computationally much more efficient because it needs to take only 21 closest pixels in contrast to 120 in the previous one, or linear regression, or Neural Network with all the pixels.
Code snippet to plot figure 6:

```
%launch via - tsk1_6_1(xtr_nf, ytr_nf)
function [] = tsk1_6_1(x_all_train, t)
    %t - means target values
    function y_test = regf(x_train, t_train, x_test)
       [w, predictor] = cs_linear_regression(x_train, t_train);
       y_test = predictor(x_test);
    end

    offsets = (1032 - 34):-1:850;
    offsets = [offsets, 800, 700, 600, 500, 400, 300, 200, 100, 1];
    errors = zeros(length(offsets), 1);

    for i = 1:length(offsets)
        offset = offsets(i)
        x = x_all_train(:, offset:1032);
        %default 10 fold
        cvMse = crossval('mse', x, t,'predfun', @regf);
        errors(i) = cvMse ^ 0.5;
    end

    pixels_from_end = 1032 - offsets;
    plot(pixels_from_end, errors);
    xlabel('number of pixels from the end');
    ylabel('Root Mean Square Error');
end
```

Code snippet to get performance on the test set using 120 pixels from the end:

```
%launch via - tsk1_6_2(xtr_nf, ytr_nf, xte_nf, yte_nf)
function [] = tsk1_6_2(x_all_train, t_train, x_all_test, t_test)
    %t - means target values
    x_train = x_all_train(:, 912:end);
    x_test  = x_all_test (:, 912:end);

    [w, predictor] = cs_linear_regression(x_train, t_train);

    [rmse_test,  std_err_test]  = cs_rmse(t_test, predictor(x_test));
    fprintf('rmse on test set %5.4f +/- %5.4f\n', rmse_test, std_err_test);
end
```

Code snippet to get the closest pixels:

```
function x = get_closest_pixels(x_all, maximum_distance)
    function accepted = filter(vector_index)
        i = mod(vector_index, 35);
        j = vector_index / 35;
        r = sqrt((18 - i)^2 + (30 - j)^2);
        accepted = r < maximum_distance;
    end

    indexes = arrayfun(@filter, 1:1032);
    x = x_all(:, indexes);
end
```

Code snippet to plot figure 7:

```
%launch via - tsk1_6_3(xtr_nf, ytr_nf)
function [] = tsk1_6_3(x_all_train, t)
    %t - means target values

    function y_test = regf(x_train, t_train, x_test)
       [w, predictor] = cs_linear_regression(x_train, t_train);
       y_test = predictor(x_test);
    end

    max_distances = 2:17;
    rmse_records = zeros(length(max_distances), 1);
```

```
    for i = 1:length(max_distances)
        max_dist = max_distances(i)
        x = get_closest_pixels(x_all_train, max_dist);
        %to see complexity of the model
        size(x)
        %default 10 fold
        cvMse = crossval('mse', x, t,'predfun', @regf);
        rmse_records(i) = cvMse ^ 0.5;
    end

    plot(max_distances, rmse_records);
    hold on;
    plot(max_distances, rmse_records, 'r*');
    xlabel('maximum distance from target pixel');
    ylabel('Root Mean Square Error');
end
```

Code snippet to get performance on the test set with pixels within radius 4 from target one:

```
%launch via - tsk1_6_4(xtr_nf, ytr_nf, xte_nf, yte_nf)
function [] = tsk1_6_4(x_all_train, t_train, x_all_test, t_test)
    %t - means target values
    x_train = get_closest_pixels(x_all_train, 4);
    x_test  = get_closest_pixels(x_all_test, 4);

    [w, predictor] = cs_linear_regression(x_train, t_train);

    [rmse_test,  std_err_test]  = cs_rmse(t_test, predictor(x_test));
    fprintf('rmse on test set %5.4f +/- %5.4f\n', rmse_test, std_err_test);
end
```

2 Robust modelling

Note

It is assumed that I have loaded the code from part2_code_data.tar.gz in my matlab environment. I have also taken professor Iain Murray code from MLPR tutorial 5 to produce error bars - errobar_str.m. text_data.mat is assumed to be loaded as well.

2.1 Fitting the baseline model

(a) **Bias feature**
The code snippet to add bias term:

```
%adding bias term
add_bias = @(x)[x, ones(size(x, 1), 1)];

x_train = add_bias(x_train);
x_test  = add_bias(x_test);
```

The bias term is assumed to be added before launching any of the remaining tasks.

(b) **Maximizing the likelihood**
The code snippet to report logistic regression performance with error bars (will be used afterwards):

```
function [] = report_lr(ww, xx, yy, type_str)
    %report logistic regression
    sigmas = 1./(1 + exp(-yy.*(xx*ww))); %logistic regression function
    accuracy = sigmas > 0.5;
    log_sigmas = log(sigmas);

    fprintf('%s median of log prob = %5.4f\n', type_str, median(log_sigmas));
    fprintf('%s standard deviation of log prob = %5.4f\n', type_str, std(log_sigmas));
    fprintf('%s accuracy = %s \n',type_str ,errorbar_str(accuracy));
    fprintf('%s log probability = %s \n',type_str, errorbar_str(log_sigmas));
end
```

The code snippet for this and next task:

```
%launch via:
% b)   tsk2_1_bc(x_train, y_train, x_test, y_test)
% c)   tsk2_1_bc(x_train, y_train, x_test, y_test, 1:100)
% c.2) tsk2_1_bc(x_train, y_train, x_test, y_test, 2:100)
function [] = tsk2_1_bc(x_train, y_train, x_test, y_test, varargin)
    MAX_LIN_SEARCHES = 8000;
    if length(varargin) == 0
       train_limits = 1:size(x_train, 1);
    else
       train_limits = varargin{1};
    end

    function [Lp, dLp_dw] = target_fun(ww, xx, yy)
        [Lp, dLp_dw] = lr_loglike(ww, xx, yy);
        Lp = -1 * Lp;
        dLp_dw = -1 * dLp_dw;
    end

    function ww = train(xx, yy)
        initial_ww = zeros(size(xx, 2), 1);
        ww = minimize(initial_ww, @target_fun, MAX_LIN_SEARCHES, xx, yy);
    end

    weights = train(x_train(train_limits, :), y_train(train_limits, :))
    report_lr(weights, x_train(train_limits, :), y_train(train_limits, :), 'training set');
    report_lr(weights, x_test, y_test, 'test set');
end
```

I set initial weights to zeros as my starting point in minimisation procedure, because in that case linear regression is unbiased and will give equal probabilities to both labels for any data

point. In other words, it will start as a baseline predictor $P(y|\boldsymbol{x}) = 0.5$.
I received the following results:

	Training set	Test set
Accuracy	$83.35 \pm 0.46\%$	$90.71 \pm 0.72\%$
Mean log probability	-0.4398 ± 0.0081	-0.300 ± 0.012

Not a typo, my accuracy was better on the test set. Probably this indicates that training set is noisy (has some mislabelled records) but logistic regression is robust enough to capture general pattern instead of fitting the noise. And if we suggest that the test set was created more carefully then it will explain better accuracy on the test set.
The minimisation function converged by itself (I put very big value for maximum number of linear searches - 8000) on 4921 linear search. The log probability of the baseline line for each input is $log(P(y|\boldsymbol{x})) = log(0.5) = -0.6931$. The closer $mean(log(P(y|\boldsymbol{x})))$ is to zero the more confident predictor is in the right answers. Although we must remember the fact that we can have good predictor with good accuracy, but if it gives chance which is close to zero to the right answer even to the one data point then due to the nature of log function ($log(0) = -\infty$) our log mean quantity will be skewed heavily and much smaller than zero. So theoretically it is possible to have a predictor with 99.99% accuracy but with mean log probability far below zero.
As we can see our mean log probability in the test set is closer to 0 than those of a simple baseline $P(y|\boldsymbol{x}) = 0.5$ which together with our accuracy implies that our performance is better.

(c) **Limited training data**
As in the previous subtask I used zeros as my initial weights (the main code is from the previous subtask as well).
For the limited number of training examples, my results are:

	First 100 points from training set	Test set
Accuracy	$99.0 \pm 1.0\%$	$73.4 \pm 1.1\%$
Mean log probability	-0.0139 ± 0.0098	$-\infty$

From the table it can be seen that linear regression fitted the training data too well and it possibly fitted the noise as well. That is why it has extremely good accuracy and mean log probability close to zero on the training set (it is very confident in its predictions). On the other hand, the results on the test set is much worse in comparison to the previous task. The mean log probability is $-\infty$ which indicates that in some cases our predictor gives zero chances to the right labels from the test set. I checked the weights and they are very huge (something like 1000) so our sigmoid function turns into step function. The accuracy on the test is still better than those of a simple baseline $P(y|\boldsymbol{x}) = 0.5$ which means that linear regression is able to infer some useful pattern even from so limited and noisy data for learning,
The question is how is it possible that accuracy is not 100% on the limited training set. Dimensionality of our feature vector is 100 (101 with bias), so any binary classification task when we have no more than 100 instances should be perfectly linearly separable. The minimisation function converged on 186 iteration with value 1.386294, but in case of a perfectly linearly separable data it must be zero (see Kevin P. Murphy. Machine Learning A Probabilistic Perspective. paragraph 8.4.3 - Gaussian approximation for logistic regression). So I decided to investigate it further.
The 99.0% accuracy indicates that logistic regression was not able to classify one point correctly from the first 100 instances of the training set. Using the following script I found that due to the some wrong labels in the test data it is possible to have two data points which have the same $\boldsymbol{x}$ values but opposite labels:

```
%launch via - tsk2_1_c_investigation(x_train, y_train)
function [] = tsk2_1_c_investigation(x_train, y_train)
    y = y_train(1:100);
    x = x_train(1:100, :);

    [newmat,indexFirst] = unique(x,'rows','first');
    repeatedIndexFirst  = setdiff(1:size(x,1), indexFirst);
    [newmat,indexLast]  = unique(x,'rows','last');
    repeatedIndexLast   = setdiff(1:size(x,1), indexLast);
```

```
    repeated = [repeatedIndexFirst; repeatedIndexLast]

    is_x_1_equal_x_21  = isequal(x(1, :), x(21, :))
    is_y_1_equal_y_21  = isequal(y(1), y(21))
    is_x_10_equal_x_81 = isequal(x(10, :),x(81, :))
    is_y_10_equal_y_81 = isequal(y(10),y(81))

    y_1  = y(1)
    y_21 = y(21)
    y_10 = y(10)
    y_81 = y(81)
end
```

The data points with indices 1 and 21 in the training set are such example. After I threw away the first data point from training set I received the following results:

	points from 2 to 100 on training set	Test set
Accuracy	$100 \pm 0.0\%$	$76.3 \pm 1.1\%$
Mean log probability	0 ± 0.000	-14.70 ± 0.95

The minimisation function converged on 122 linear search with value 0. This and results in the table lead to the conclusion that our training set is perfectly linearly separable. It is interesting that our prediction accuracy on the test set dropped slightly, but we have the finite mean log probability on the test at least. But the mean log probability is still far below zero (in comparison to baseline $P(y|\boldsymbol{x}) = 0.5$, for example) which indicates that the predictor is too confident in some wrong answers and, therefore, sometimes it makes big mistakes.
Overall this small investigation helped me to understand what kind of noise we have in the training data.

2.2 Label noise model

(a) **Modifying the likelihood**
The events $s_n = 1$ and $s_n = 0$ are mutually exclusive and together they are collectively exhaustive events and also s_n is independent from $\boldsymbol{x}$, $\boldsymbol{w}$, y that is why:

$$\begin{aligned}
P(s_n = 1) &= 1 - \epsilon \quad P(s_n = 0) = \epsilon \\
P_{uniform}(y) &= \frac{1}{size(\{-1, +1\})} = \frac{1}{2} \\
P(y|\boldsymbol{x}, \boldsymbol{w}) &= \sigma(y\boldsymbol{w}^T\boldsymbol{x}) \\
P(y|\boldsymbol{x}, \boldsymbol{w}, \epsilon) &= P(s_n = 1)P(y|\boldsymbol{x}, \boldsymbol{w}) + P(s_n = 0)P_{uniform}(y) = \\
&= (1 - \epsilon)\sigma(y\boldsymbol{w}^T\boldsymbol{x}) + \frac{\epsilon}{2}
\end{aligned}$$

The derivatives of new model likelihood are:

$$v^{(n)} = y^{(n)}\boldsymbol{x}^{(n)T}\boldsymbol{w} \quad (\textit{swapped values because transpose of scalar is a scalar}) \tag{1}$$

$$L(\boldsymbol{w}, \epsilon) = \sum_{n=1}^{N} log(P(y^{(n)}|\boldsymbol{x^{(n)}}, \boldsymbol{w}, \epsilon)) = \sum_{n=1}^{N} log((1 - \epsilon)\sigma(v^{(n)}) + \frac{\epsilon}{2}) \tag{2}$$

$$\nabla_{\boldsymbol{w}}\sigma(v^{(n)}) = \frac{-1}{(1 + exp(-v^{(n)}))^2} exp(-v^{(n)})(-1)y^{(n)}\boldsymbol{x}^{(n)T} = \sigma(v^{(n)})(1 - \sigma(v^{(n)}))y^{(n)}\boldsymbol{x}^{(n)T} \tag{3}$$

$$\nabla_{\boldsymbol{w}}L(\boldsymbol{w}, \epsilon) = \sum_{n=1}^{N} \frac{(1 - \epsilon)\sigma(v^{(n)})(1 - \sigma(v^{(n)}))y^{(n)}\boldsymbol{x}^{(n)T}}{P(y^{(n)}|\boldsymbol{x^{(n)}}, \boldsymbol{w}, \epsilon)} \tag{4}$$

$$\frac{\partial L(\boldsymbol{w}, \epsilon)}{\partial \epsilon} = \sum_{n=1}^{N} \frac{-\sigma(v^{(n)}) + \frac{1}{2}}{P(y^{(n)}|\boldsymbol{x^{(n)}}, \boldsymbol{w}, \epsilon)} \tag{5}$$

I have chosen the following test case for checking gradients implementation:

$$\boldsymbol{x}^{(1)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad y^{(1)} = -1$$

$$\boldsymbol{x}^{(2)} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad y^{(2)} = 1$$

$$\boldsymbol{w} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \epsilon = 0.2$$

I have used step $h = 0.1$ which implies that the absolute value of error should not exceed $h^2 = 0.01$.
The results of checking "label noise model" likelihood derivatives via checkgrad.m are:

	derivative	finite difference	absolute error
w_1	0.0119	0.0120	0.0001
w_2	0.0119	0.0120	0.0001
w_{bias}	-0.4931	-0.4928	0.0003
ϵ	0.1818	0.1825	0.0008

As we can see in each case the absolute error doesn't exceed 0.01. So our gradient must work fine. As a sanity check I use the fact that test case is symmetric in terms of w_1 and w_2, so our derivatives and finite differences for w_1 and w_2 must be the same. The absolute error of ϵ is the biggest one possibly because the step h=0.1 is just too big for it.
Code snippet for "label noise model" log likelihood (will also be used in the next tasks):

```
function [Lp, dLp_dk] = nlm_loglike(kk, xx, yy)
    %nlm - noisy labels model
    %kk - ww + epsilon -> (D+1)x1
    %xx - NxD
    %yy - Nx1
    % Ensure labels are in {+1,-1}:
    yy  = (yy==1)*2 - 1;
    ww  = kk(1:end-1);
    eps = kk(end);

    sigmas = 1./(1 + exp(-yy.*(xx*ww))); % Nx1
    %probabilities
    pbs = (1-eps)*sigmas + 0.5*eps; % Nx1
    Lp = sum(log(pbs));

    if nargout > 1
        %inverse probabilities
        inv_pbs = 1./pbs; % Nx1
        dLp_dw   = (inv_pbs.*(1-eps).*(1-sigmas).*sigmas.*yy)' * xx; % 1xD
        dLp_deps = sum(inv_pbs.*(-sigmas + 0.5)); % 1x1
        dLp_dk   = [dLp_dw, dLp_deps]'; % (D+1)x1
    end
end
```

Code snippet for this task:

```
function [] = tsk2_2_a()
    %test case
    weights = [1; 1; 1]; % x1, x2, bias % Dx1 = 3x1
    epsilon = 0.2;
    k = [weights; epsilon]; % (D+1)x1 = 4x1
    x = [0, 0, 1; 2, 2, 1]; % x1, x2, bias % NxD = 2x3
    y = [-1; 1]; % Nx1 = 2x1
    h = 0.1; % so the error should be no more 0.001

    %show not perturbed values
    [Lp, dLp_dk] = nlm_loglike(k, x, y)

    fprintf('checkgrad output:\n');
    fprintf('derivative    finite difference\n');
```

```
    accuracy = checkgrad(@nlm_loglike, k, h, x, y)
end
```

(b) **Fitting a constrained parameter**

Slightly modifying formula (5) from the previous task we can adapt the code from the previous example:

$$\frac{\partial L(\boldsymbol{w},\epsilon)}{\partial a} = \frac{\partial L(\boldsymbol{w},\epsilon)}{\partial \epsilon}\frac{\partial \epsilon}{\partial a} = \sum_{n=1}^{N} \frac{-\sigma(v^{(n)}) + \frac{1}{2}}{P(y^{(n)}|\boldsymbol{x^{(n)}},\boldsymbol{w},\epsilon)} \frac{-1}{(1+exp(-a))^2} exp(-a)(-1)$$

$$= \sum_{n=1}^{N} \frac{-\sigma(v^{(n)}) + \frac{1}{2}}{P(y^{(n)}|\boldsymbol{x^{(n)}},\boldsymbol{w},\epsilon)} \sigma(a)(1-\sigma(a)) = \frac{\partial L(\boldsymbol{w},\epsilon)}{\partial \epsilon}\epsilon(1-\epsilon)$$

After training label noise model and applying final weights from it on the logistic model, the performance is:

	Training set	Test set
Accuracy	$85.11 \pm 0.44\%$	$91.20 \pm 0.70\%$
Mean Log Probability	-9.31 ± 0.46	-1.10 ± 0.18

And the value of the fitted noise:

$$\epsilon = 0.2144 \quad a = -1.2989$$

Here we can see that the accuracy has improved on both training and test sets in comparison to the results from task 2.1.b . Although considering the mean log probability for training and test set we can conclude that the predictor has become too confident in some wrong cases (the value below the result o af simple baseline $P(x|y) = 0.5$).
The better accuracy and thus, better classification performance points out that by using the model which reflects the condition of our data set better (mislabelled data) we can infer more useful information from the data set and make our predictor more robust to incorrect data points.
The code snippet for this task:

```
%launch via - tsk2_2_b(x_train, y_train, x_test, y_test)
function [] = tsk2_2_b(x_train, y_train, x_test, y_test)
    %default parameters
    MAX_LIN_SEARCHES = 8000;
    sigma = @(x) 1/(1 + exp(-x));

    function [Lp, dLp_dq] = target_fun(qq, xx, yy)
        a = qq(end);
        eps = sigma(a);
        kk = qq(:);%copy original array
        kk(end) = eps;

        [Lp, dLp_dk] = nlm_loglike(kk, xx, yy);
        Lp = -Lp;
        %augmenting derivative
        dLp_dq = -dLp_dk(:);%copy original array
        dLp_dq(end) = dLp_dq(end) * eps * (1 - eps);
    end

    function qq = train(xx, yy)
        initial_qq = zeros(size(xx, 2) + 1, 1);% plus one to account parameter a
        qq = minimize(initial_qq, @target_fun, MAX_LIN_SEARCHES, xx, yy);
    end

    qq = train(x_train, y_train);
    weights = qq(1:end-1)
    final_a = qq(end)
    epsilon = sigma(final_a) % report it for the task
    report_lr(weights, x_train, y_train, 'training set');
    report_lr(weights, x_test,  y_test,  'test set');
end
```

2.3 Hierarchical model and MCMCl

(a) Log is concave and monotonically increasing function (if base > 0) that is why the largest log-likelihood happens when likelihood attains the biggest possible value. Likelihood is multiplication of probabilities of each observation for given model and each probability is between 0 and 1, so likelihood maximum value is 1 which correspond to the case when our model is able to predict all training labels correctly and it is 100% sure about that.

$$max(\mathcal{L}) = max(log(likelihood)) = log(max(likelihood)) = log(1) = 0$$

From the equation 3 in the assignment in our model when all weights are zero is:

$$P(y|\boldsymbol{x}, \boldsymbol{w} = 0, \epsilon) = (1 - \epsilon)\sigma(0) + \frac{\epsilon}{2} = (1 - \epsilon)0.5 + 0.5\epsilon = 0.5 - 0.5\epsilon + 0.5\epsilon = 0.5$$

So it produces the same probability as the original logistic regression when all of the weights are zero or the baseline that predicts $P(y|x) = 0.5$ for every case. Putting it in the log-likelihood:

$$\mathcal{L}(\boldsymbol{w} = 0, \epsilon) = \sum_{n=1}^{N} log(P(y|\boldsymbol{x}, \boldsymbol{w} = 0, \epsilon)) = \sum_{n=1}^{N} log(0.5) = log(0.5)N = -0.6931N$$

Which is constant given the data (so N is fixed). Using this result, the log-posterior from the equation 6 in the assignment, when all weights are zeros, becomes:

$$logP(\epsilon, \boldsymbol{w} = 0, log(\lambda) = \infty|data) = -0.6931N - \lambda * 0 + \frac{D}{2}log(\lambda) + const = \frac{D}{2}log(\lambda) + const2$$
$$= \infty + const2 = \infty$$

But the posterior is a probability, therefore, its values lies between 0 and 1 and the maximum value of the log-posterior must 0. So the situation when λ has no bounds (the 'improper' limit) and it can potentially become infinite must be avoided.

(b) The table of initial parameters for slice sampling:

Parameter(s)	Value
burn-in phase	300 iterations
number of samples	700
initial weights	all zeros
ϵ	0
$log(\lambda)$	0
weights width	10
$log(\lambda)$ width	10
ϵ width	1.0

I think the most important parameter here is what part of algorithm's iterations will be devoted to the burn-in phase. If we start to sample too early it will affect our prediction performance as samples at the beginning may not correctly represent our distribution. On the other hand, given that in the assignment we should use exactly 1000 iterations if we set burn-in phase to something big then we probably won't have enough samples to predict confidently. And it is a bit hard to set this parameter via cross-validation because each run for 1000 iterations takes considerable time. So eventually I have chosen 300 iterations for burn-in phase and I take 700 samples.
I set initial weights to zero as it seems reasonable to start from a simple baseline that predicts $P(y|x) = 0.5$ for every observation. Initial epsilon is zero so I start from the simple logistic regression.
As for widths in David MacKay's text book (Information Theory, Inference, and Learning Algorithms) p377 it is said that algorithm takes linear time to expand interval but logarithmic time to shrink it. So I put weights width bigger than the values of weights I usually observed during debugging phase in previous tasks (they usually were between -1 and 1 but with some exceptions). ϵ changes between 0 and 1 so it is natural to put width 1.0 on it.
From the figure 8 we can see that there is some negative correlation between ϵ and λ. It

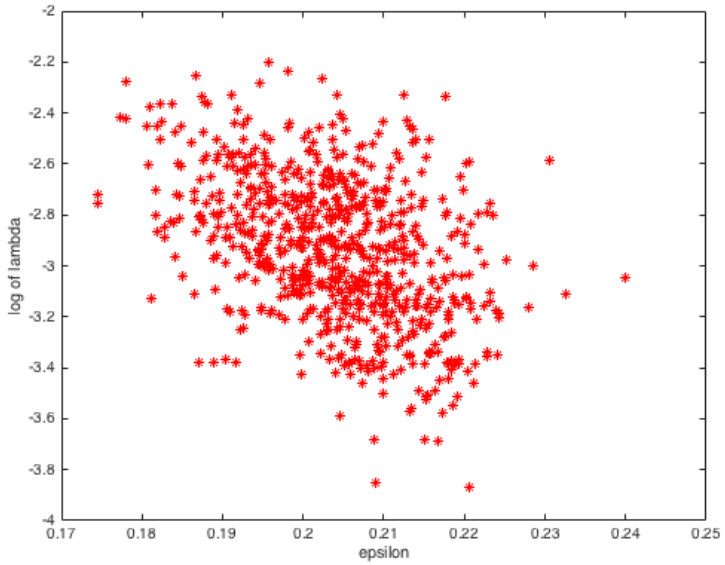


Figure 8: plot of $log(\lambda)$ against ϵ. Both drawn from sampling procedure

probably happens because both ϵ and λ ($log(\lambda)$) are connected via posterior (can be seen in assignment equation 6) and are used in the model to avoid noise fitting although in different ways. That is why when one of them is high then the other one can be smaller.
Most of the ϵ's are not far from our previous value 0.2144 and they lay within 0.17 and 0.25 range. So the value for ϵ from previous task seems quite reasonable. Although I note that the most likely values of ϵ are closer to approximately 0.205.

Code to retrieve samples:

```
%launch via - samples = tsk2_3_sampler(x_train, y_train)
function [samples] = tsk2_3_sampler(x_train, y_train)
    function [log_posterior] = target_fun(zz, xx, yy)
        ww  = zz(1:end-2); %Dx1
        eps = zz(end-1);
        log_lmb = zz(end); %log of lambda
        lmb = exp(log_lmb);

        D = numel(ww); %dimensionality of weights

        %give zero chances to impossible cases
        if not(0 <= eps && eps <= 1)
            log_posterior = -Inf;
            return;
        end

        kk = zz(1:end-1);
        nlm_L = nlm_loglike(kk, xx, yy);

        log_posterior = nlm_L - lmb*dot(ww,ww) + 0.5*D*log_lmb;
    end

    D = size(x_train, 2);
    S = 700; %number of samples
    burn = 300; %number of iterations to burn out
    %initial point
    %so initial result correspond to the posteriror of 0.5
    ww0 = zeros(D, 1); %weights
    log_lmb0 = 0; %log of lambda = 0 -> lambda = 1
    eps0 = 0; %epsilon

    zz = [ww0; eps0; log_lmb0;];
    %in David MacKay's text book p377 it is said that shrinkage has log complexity
    %in contrast to expansion which takes linear time
    %I take big width for all variables, except for eps because 1 is more
```

```
    %then enough for it
    width = ones(D + 2, 1) * 10;
    width(D + 1) = 1.0;%eps
    samples = slice_sample(S, burn, @target_fun, zz, width, true, x_train, y_train);
end
```

Code for plotting:

```
%launch via - tsk2_3_b(samples)
function [] = tsk2_3_b(samples)
    epsilons = samples(end - 1, :);
    log_lmbs = samples(end, :);

    figure;
    plot(epsilons, log_lmbs, 'r*');
    xlabel('epsilon');
    ylabel('log of lambda');
end
```

(c) In contrast to the previous tasks which used maximum likelihood methods here we are using Bayesian method. Given that for this task we must only use different weights drawn from our posterior distribution, I use logistic regression and average its predictions for each input over weights from different samples.
The results for the test set are presented in the table:

	Value
Accuracy	$91.51 \pm 0.69\%$
Log mean	-0.189 ± 0.013

Taking into account error bars, our performance is essentially the same as in the previous task. However our log mean has improved significantly and it is much better than those of simple baseline which assign $P(y|\boldsymbol{x}) = 0.5$. It indicates that even when we are doing wrong prediction we are very unconfident in them but we have much higher confidence level for right answers. So it is the best predictor among those we created in the part 2.
Code for this task

```
%launch via - tsk2_3_c(samples, x_test, y_test)
function [] = tsk2_3_c(samples, x_test, y_test)
    function [] = performance(wws, yy, xx)
        %D - dimensionality of data points
        %N - number of instances
        %S - number of samples
        %wws D x S
        %xx  N x D
        %yy  N x 1
        %repmat(yy, [1, S]) N x S
        %xx * wws  N x S
        S = size(wws, 2);
        sigmas = 1./(1 + exp(-repmat(yy, [1, S]) .* (xx*wws)));%N x S
        probs = mean(sigmas, 2); % assemble samples
        log_probs = log(probs);
        accuracy = probs > 0.5;

        fprintf('test accuracy = %s \n', errorbar_str(accuracy));
        fprintf('test log probability = %s \n', errorbar_str(log_probs));
    end

    weights  = samples(1:end - 2, :);
    performance(weights, y_test, x_test);
end
```