MACHINE LEARNING AND PATTERN RECOGNITION
Assignment 1

Matriculation number - s1569105
Examination number - B076165

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1 The Next Pixel Prediction Task

1.1 Data preprocessing and visualization

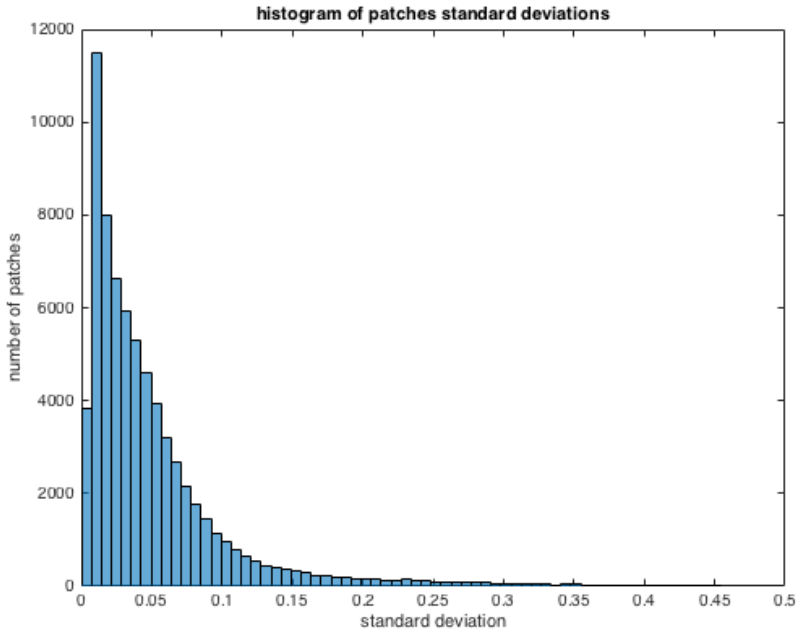


Figure 1: histogram of standard deviations in the xtr dataset after normalisation

(a) The maximum possible value of standard deviation is $\frac{max.-min.}{2}$, so in our case after normalisation it is $\frac{1-0}{2} = 0.5$. Our threshold to distinguish discrete values of pixels is $\frac{1}{64} = 0.5/32 \approx 0.0156$. If we use 32 bins on a range of possible values of standard deviation (between 0 and 0.5) then the width of one bin will be 0.0156 and standard deviations with values 0.0151 or 0.0021 will go to the same bin. But we usually would associate (after rounding using threshold) standard deviation 0.0151 with the discrete (original) pixel value of 1 and 0.0021 with 0 because $round(0.0151/0.0156) = round(0.968) = 1$ and $round(0.0021/0.0156) = round(0.135) = 0$. Therefore, we must choose minimum 64 bins in order to distinguish such cases because we will have bins width $\frac{0.5}{64} \approx \frac{0.0156}{2} = 0.0078$ and each bin will correspond to the specific discrete (original) pixel value.
From the figure 1 we can see that after the peak on the second bin the number of patches declines exponentially as standard deviation increases . We can conclude that most of patches have standard deviation within 0 and 0.05 range, and 0.05 is quite small standard deviation, therefore, most of the patches are flat ones.
Code snippet to plot histogram:

```
%load imgregdata.mat % I do it via terminal
xx = xtr ./ 63;
xx_std = std(xx,0,2);

%plot histogram
figure;
h = histogram(xx_std,64);
title('histogram of patches standard deviations');
xlabel('standard deviation');
ylabel('number of patches');
```

(b) I would choose mean of the all the pixels (1032) above and to the left of target pixel as a simplest predictor of the target pixel value for flat patches. Given definition of flat patches the value of the pixel in the flat patch will be something like this $f(x) = const_{flat\ patch} + o(0, \sigma_{flat\ patch})$ where o is random and small in comparison to $const_{flat\ patch}$, and it has 0 mean and as $\sigma_{flat\ patch}$ its standard deviation which follows $\sigma_{flat\,patch} \leq \sigma_{flat\,pach\,max}$. In general, I would prefer median because it is more robust to outliers if our dataset is noisy but in our case pixels can take only discrete values and I will show that mean suits us.
Consider extreme case where after normalisation (all pixel values between 0 and 1) in our flat patch most pixels are zeroes and small portion of pixels are ones (correspond to 63 intensity of original pixel). Let $N - m$ be number of zeros and let m be number of ones and I denote μ as mean.

$$m < N - m$$
$$\mu = \frac{(N-m)*0 + m*1}{N} = \frac{m}{N}$$
$$\sigma^2 = (N-m)(0 - \frac{m}{N})^2 + m(1 - \frac{m}{N})^2$$
$$= \frac{(N-m)m^2}{N^3} + \frac{m(N-m)^2}{N^3}$$
$$N^3\sigma^2 = Nm^2 - m^3 + mN^2 - 2m^2N + m^3$$
$$= mN^2 - m^2N$$
$$m^2 - mN + N^2\sigma^2 = 0$$
$$m = \frac{N}{2}(1 - \sqrt{1 - 4\sigma^2}) \quad \text{(minus because our case is } m < N - m \text{)}$$

putting $\sigma_{flat\,pach\,max} = \frac{4}{63} \approx 0.0635$ instead of σ and using $N = 1032$ we get

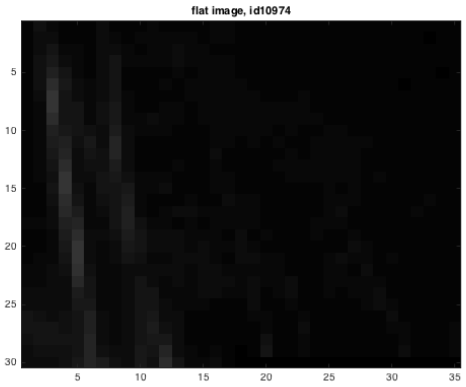
$$m = \frac{1032}{2}(1 - \sqrt{1 - 4 * 0.0635^2}) \approx 4.178$$

rounding m to the closest integer we receive $m = 4$.
Thus, in most extreme case of flat patch we can have 4 ones (correspond to original 63 pixel intensity) and 1028 zeros, so it is natural that we want to predict zero as discrete value of our target pixel. The mean gives us $\mu = \frac{1028*0+1*4}{1032} \approx 0.0038$. Dividing range between 0 and 1 by 64 we get 0.0156 as our threshold to distinguish discrete pixel values. $round(0.0038/0.0156) = round(0.244) = 0$ so our mean value will correspond to 0 as the discrete value of our target pixel and that is what we wanted.

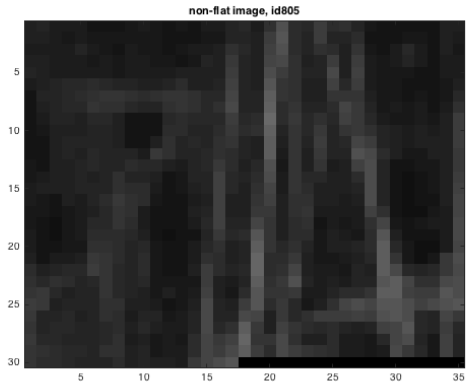
(c) Code snippet to show patch images on figure 2:

```
%load imgregdata.mat % I do it via terminal
xx = xtr ./ 63;
xx_std = std(xx,0,2);

%get indixes for flat and non-flat patches respectively
%the threshold 4/63 is taken from the task
xx_f_ids  = bsxfun(@le, xx_std, ones(size(xx_std)) .* 4/63);
xx_nf_ids = bsxfun(@gt, xx_std, ones(size(xx_std)) .* 4/63);

%slicing
xx_f  = xx(xx_f_ids, :);
xx_nf = xx(xx_nf_ids,:);
```

Figure 2: patch images



(a) flat patch image



(b) non-flat patch image

```
%pick one random example of flat patch and non-flat patch
get_rnd_row = @(X) randi(size(X, 1), 1);

%flat
rnd_flat_id = get_rnd_row(xx_f);
display(rnd_flat_id, 'random index of flat patch');
flat_patch = xx_f(rnd_flat_id, :);

%non-flat
rnd_non_flat_id = get_rnd_row(xx_nf);
display(rnd_non_flat_id, 'random index of non-flat patch');
non_flat_patch = xx_nf(rnd_non_flat_id, :);

%expanding patches to the full size
flat_patch(1050) = 0;
non_flat_patch(1050) = 0;

%creating images
flat_image = reshape(flat_patch, [35, 30]);
non_flat_image = reshape(non_flat_patch, [35, 30]);

%show images
%inverting them to ensure right position
%last index of the patch vector patch_vector(1050) == patch_image(30, 35)

%flat
figure;
imagesc(flat_image', [0, 1]);
title(strcat('flat image, id ', num2str(rnd_flat_id)));
colormap gray;

%non-flat
figure;
imagesc(non_flat_image', [0, 1]);
title(strcat('non-flat image, id ', num2str(rnd_non_flat_id)));
colormap gray;
```

1.2 Linear regression with adjacent pixels

(a) I used 5000 training points from xtr_nf and ytr_nf to plot figure 3. From it we can see that x(j, end), x(j, end - 34), y(j) are strongly positively correlated. However, there is some relatively small number of deviations from this trend. It seems that these deviations are normally distributed so linear regression should be reasonable model to describe such data.

(b) Derivation of this solution can be taken from MLPR lecture 7 slides 8-11 here. The solution for weights from there is:

$$\hat{\boldsymbol{w}} = (\Phi^T \Phi)^{-1} \Phi^T \boldsymbol{y}$$

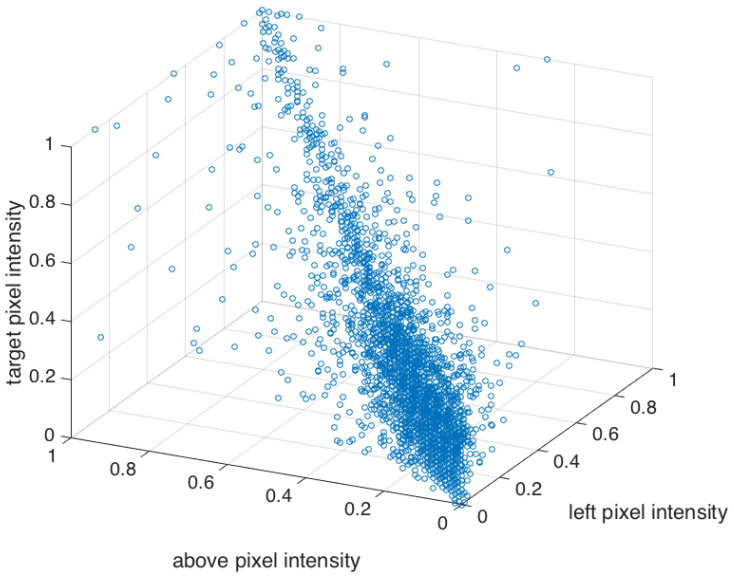


Figure 3: scatter plot of adjacent pixels. 5000 data points from xtr_nf and ytr_nf

In our notation matrix Φ will become:

$$\Phi = X = \begin{pmatrix} 1, x(1, end), x(1, end-34) \\ 1, x(2, end), x(2, end-34) \\ \dots \\ 1, x(N, end), x(N, end-34) \end{pmatrix}$$

$$\hat{\boldsymbol{w}} = (X^T X)^{-1} X^T \boldsymbol{y}$$

where N is a number of training data points and x is our dataset (it will be xtr_nf in the next task)

(c) code snippet which I will be using to get linear regression predictor:

```
function [w, predictor] = lr_predictor(x_tr, y_tr)
    %it incerts bias term automatically
    %assume y is column vector N x 1
    %assume x_tr is N x num_features

    %adding bias term
    calc_Phi = @(x)[ones(size(x, 1), 1), x];

    %computing weights
    w = pinv(calc_Phi(x_tr)) * y_tr;

    predictor = @(x)calc_Phi(x) * w;
end
```

code snippet to compute root mean square error (RMSE):

```
function result = rmse(t, y)
    %assume y and are column vectors
    diff = t - y;
    %mean square error
    mse = mean(diff .* diff);
    result = mse ^ 0.5;
end
```

code snippet for this particular task:

```
%load imgregdata.mat % I do it via terminal

%left and above neighbours
get_neighbours = @(x) [x(:, 1032), x(:, 1032 - 34)];
```

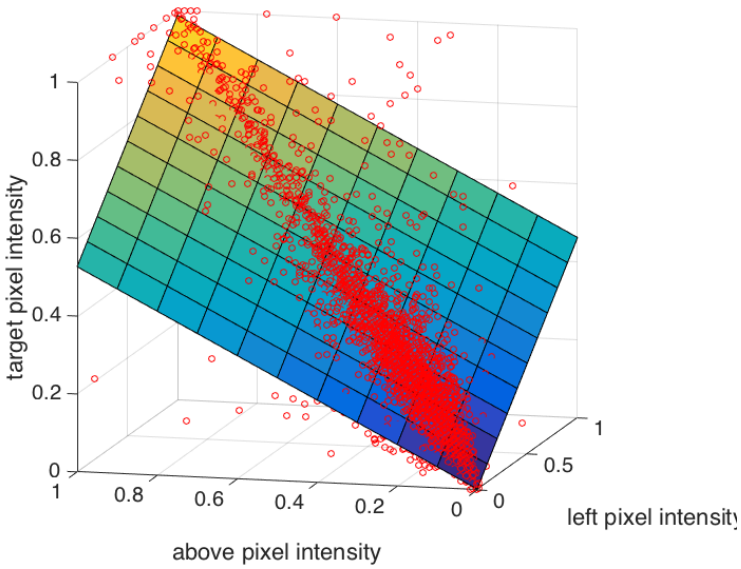


Figure 4: plot of the linear regression function after training along with test data points

```
%prepare training set
X_train = get_neighbours(xtr_nf);
%train (it will add bias term automatically)
[w, predictor] = lr_predictor(X_train, ytr_nf);
display(w, 'weights for neighbours pixels features');

%compute rmse for training set
Yp_train = predictor(X_train);
error_train = rmse(ytr_nf, Yp_train);
display(error_train, 'rmse on the training set');

%compute rmse for test set
X_test = get_neighbours(xte_nf);
Yp_test = predictor(X_test);
error_test = rmse(yte_nf, Yp_test);
display(error_test, 'rmse on the test set');

%show surface
figure,
%I chose smaller step because our function is just plane
[dim1, dim2] = meshgrid(0:0.1:1, 0:0.1:1);
%swapped ones from original snippet, because w(1) corresponds to bias
%in my case
ysurf = [ones(numel(dim1),1), [dim1(:), dim2(:)]] * w;
surf(dim1, dim2, reshape(ysurf, size(dim1)));
hold on;
scatter3(xte_nf(:, 1032), xte_nf(:, 1032 - 34), yte_nf, 'red');

xlabel('left pixel intensity');
ylabel('above pixel intensity');
zlabel('target pixel intensity');
set(gca,'FontSize', 20);
```

after training the weights are:

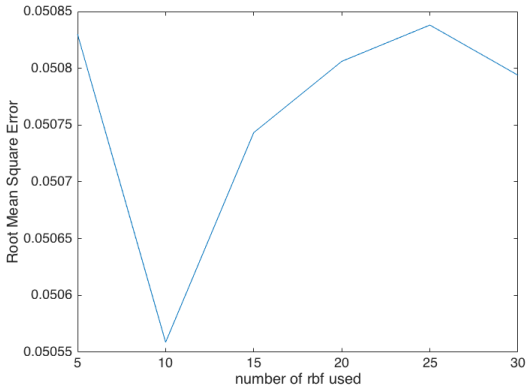
bias	left pixel	above pixel
0.0026	0.4606	0.5241

the RMSE for test and training set:s

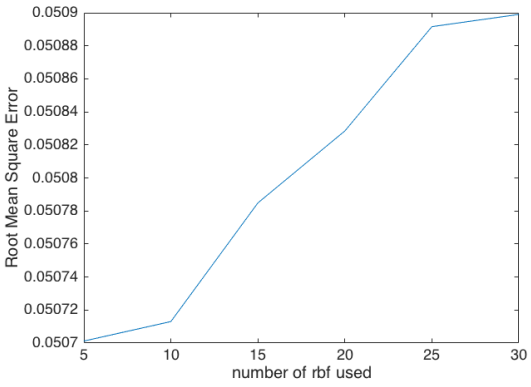
	Training set	Test set
RMSE	0.0506	0.0503

This is not a typo, surprisingly my performance is a bit better on the test set. That is why we can conclude that linear regression is not over-fitting the data in this problem. It can

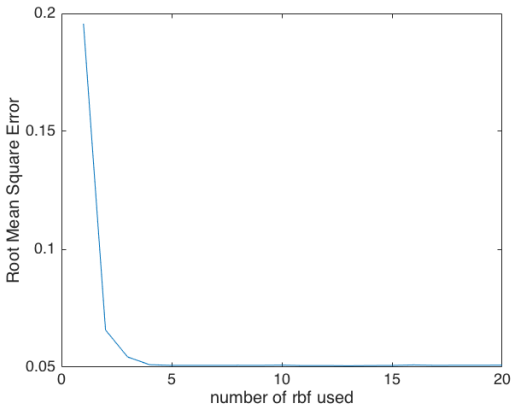
Figure 5: Root Mean Square Error against number of radial basis functions used



(a)



(b)



(c)

be seen from figure 4 that indeed there is strong positive correlation between adjacent pixels and target value pixel.

1.3 RBF regression with adjacent pixels

(a) When I ran cross validation procedure to determine which number of radial bases functions among { 5, 10, 15, 20, 25, 30} produces the best results, each time I received a different answer. The figure 5a suggests 10 as the best number of radial bases functions and 5b proposes 5 as the best choice. This happens because matlab crossval uses random numbers each time to divide input set on training and validation sets. After that I launched the procedure for number of radial basis functions between 1 and 20, and I have realised that it was just a matter of scale. The figure 5c demonstrates that we achieve almost no improvement if we use more than 5 radial bases functions in this task. That is why I have chosen 5 as my number of radial basis functions because for the same efficiency it takes less time to compute. Code snippet which I used in this task:

```
%load imgregdata.mat % I do it via terminal

%launched via
%tsk1_3_a(xtr_nf, ytr_nf, 5:5:30)
%and
%tsk1_3_a(xtr_nf, ytr_nf, 1:20)
function [] = tsk1_3_a(xtr_nf, ytr_nf, num_rbfs)
    X_train = [xtr_nf(:, 1032), xtr_nf(:, 1032 - 34)];
    opt = foptions;
    opt(1) = 1; % Display EM training
    opt(14) = 5; % number of iterations of EM
    dim = 2; % left_pixel, above_pixel in our case
```

```
    function regf = create_rbf_regf(num_rbf)
        function y_pred = rbf_reg(x_train, y_train, x_test)
            net = rbf(dim, num_rbf, 1, 'gaussian');
            net = rbftrain(net, opt, x_train, y_train);
            y_pred = rbffwd(net, x_test);
        end
        regf = @rbf_reg;
    end

    rbfs_rmse = zeros(1, length(num_rbfs));

    for i = 1:length(num_rbfs)
        num_rbf = num_rbfs(i);
        regf = create_rbf_regf(num_rbf);
        %default CV 10 folds
        cvMse = crossval('mse', X_train, ytr_nf,'predfun',regf);
        rmse = cvMse ^ 0.5;

        %logging
        display(num_rbf, 'current number of kernels')
        display(rmse, 'rmse')

        rbfs_rmse(i) = rmse;
    end

    display(num_rbfs, 'number of kernels tried');
    display(rbfs_rmse, 'rmse for different number of kernels');

    plot(num_rbfs, rbfs_rmse)
    xlabel('number of rbf used');
    ylabel('Root Mean Square Error');
    set(gca,'FontSize', 18);
end
```