MFE Economics
Problem set 5

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1. Technical: Galí composite shock

Although it is natural to look for equilibrium functions for the endogenous variables in terms of the fundamental shocks, such as

$$\begin{aligned}\pi_t &= \psi_{\pi,a}a_t + \psi_{\pi,z}z_t + \psi_{\pi,v}v_t \\ \tilde{y}_t &= \psi_{\tilde{y},a}a_t + \psi_{\tilde{y},z}z_t + \psi_{\tilde{y},v}v_t\end{aligned}$$

Galí looks for expressions in terms of the 'composite' shock, u_t (in Ch. 3, P. 65 onward)

$$\begin{aligned}\pi_t &= \psi_{\pi,u}u_t \\ \tilde{y}_t &= \psi_{\tilde{y},u}u_t \\ u_t &= -\psi_{yn,a}\left(\phi_y + \sigma(1-\rho_a)\right)a_t + (1-\rho_z)z_t - v_t\end{aligned}$$

He then makes the assumption that u_t follows an AR(1) and then derives $\psi_{\pi,u}$ and $\psi_{\tilde{y},u}$ - i.e. he solves the model treating u_t as the only state.

Without going into detail, this approach relies very heavily on the linearity of the model and an assumption that u_t follows an AR(1) (which it isn't), rather than acknowledging that it is the sum of different AR(1) processes. Nevertheless, if one calculates the response to an innovation to u_t then one can in fact use that response to derive the correct responses to innovations to a_t, z_t and v_t.

- What values of impulses to the various fundamental shocks (i.e. δ_a, δ_z and δ_v) will induce a unit impulse in u_t?
- What innovation simultaneously to v_t will cancel out an innovation to z_t? Give some intuition and what is the implication for monetary policy (HINT: Compare a fundamental desire to save with a response to an additional incentive to save].

2. First half technical - second half economic: DIS and NKPC

The New Keynesian Phillips curve (NKPC) is given by

$$\begin{aligned}y_t &= E_t[y_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t \\ \pi_t &= \beta E_t[\pi_{t+1}] + \kappa\tilde{y}_t\end{aligned}$$

- Sketch a plot of the NKPC with $\tilde{y}_t$ on the horizontal axis and π_t on the vertical axis.

- What is the slope?

Note that implicitly the location of the curve depends on assumed values for the structural parameters (β and recall κ is a function of various parameters) and expected inflation next period ($E_t[\pi_{t+1}]$).

- What happens to the curve in the following situations? Give intuition.
  - Suppose something shifts $E_t[\tilde{y}_{t+2}]$ up, holding all else equal.
  - Suppose something shifts $E_t[\tilde{y}_{t+3}]$ up, holding all else equal. Compare to your previous answer.
  - What if θ increases, or φ?
- Why is 'holding all else equal' an unnatural assumption in this context?

3. Technical: Autoregressive processes

[This is a continuation of the AR question in the previous problem set] In the models we consider we will often express the equilibrium values of endogenous variables as functions of a_t (and other shocks). Suppose a variable s_t is expressed

$$s_t = \psi_0 + \psi_1 a_t$$

- What is s_{t+1} in terms of a_{t+1}?
- What is s_{t+1} in terms of a_t and $\varepsilon_{a,t+1}$?
- What is the expected value of s_{t+1} given information available in t (i.e. given you know a_t)?
- What is the expected value of $\Delta s_{t+1} \equiv s_{t+1} - s_t$ given information available in t

4. Technical and a bit of intuition: Definitions of IRFs

Our equilibrium implies variables, say y_t, can be expressed in terms of three shocks

$$y_t = \psi_y + \psi_{y,a} a_t + \psi_{y,z} z_t + \psi_{y,v} v_t$$

$$\varepsilon_{a,t} = \varepsilon_{z,t} = \varepsilon_{v,t} = 0 \ \forall t$$

and the shocked path of the economy entails

$$\begin{aligned} \varepsilon_{a,1} &= \delta_a \\ \varepsilon_{a,t} &= 0 \ \forall t > 1 \\ \varepsilon_{z,t} = \varepsilon_{v,t} &= 0 \ \forall t \end{aligned}$$

As shown in class this yields the impulse response function

$$\Delta_{y,a}(t) = \psi_{y,a}\rho_a^{t-1}\delta_a$$

- What is the 'impact effect' (i.e. effect in $t = 1$ when the innovation hits)?
- Consider two sizes of impulse $\delta_a^{(B)} > \delta_a^{(S)} > 0$. Comment on the relationship between the impulse response to these two differently sized impulses - take their difference and their ratio. What does the ratio depend on (really, what does in *not* depend on)? *Hint: Try to distinguish magnitude from shape.*
- Consider two impulses, δ_a and $-\delta_a$. Comment on the relationship between the impulse response to these two differently signed impulses - take their ratio.
- In what way does the impulse response depend on the value of a_t prevailing in the period before the impulse?
- Do the properties you derived in the previous part of this question seem 'sensible'? Can you think of some simple examples/stories where the effect of an impulse might be dependent on the size/sign of the impulse and the state the economy is in on impact (in a more meaningful way than they do in our case)?

We defined an IRF by comparing two paths under a sequence of assumed future innovation realizations. Suppose *instead* someone (very reasonably) might want to think of an impulse response as how his/her **expectations** of the future might change, given an impulse today. In this case we will calculate the difference in expectations after the impulse vs without the impulse, acknowledging that future innovations are random variables.[1]

- What is the impulse response under this new approach of defining it as the difference in expected values of y_{t+j} conditional on $\varepsilon_{a,1} = \delta_a$ and conditional on $\varepsilon_{a,1} = 0$? NOTE: For this part of the question, eliminate z_t and v_t from the analysis, just to simplify the algebra slightly.

5. Economic awareness

- In a couple of paragraphs, explain the role of IRF analysis in assessing the effect of monetary policy on the economy. What is the consensus view? [HINT: See Walsh and Gali first chapters, some of the Ramey paper and if you can, early parts of Christiano, Eichenbaum and Evans]
- Explain how the New Keynesian Phillips curve relates to the original Phillips curve and expectations augmented Phillips curves. You should refer to the evolving attitude towards the existence of a tradeoff between inflation and output. [You will need to do some simple online research for this - keep your answers to a page max.]

[1] We continue to assume $\varepsilon_{z,1} = \varepsilon_{v,1} = 0$

- In one paragraph, discuss the current debate about the flattening of the Phillips curve - extra points if you look up AOC's exchange with Jerome Powell.[2] [HARDER] In an even shorter paragraph - explain what a flat Phillips curve implies for the impact of a monetary policy shock on inflation.

[2]There are no extra points.