Lecture 2
A Classical Monetary Model

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Disclaimer

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Roadmap

Roadmap

In this lecture we consider a (very) simple monetary economy

- The role of money will be limited to that of a 'unit of account'
- See textbook for alternative setup where money yields 'utility' through services as a 'means of exchange'

Essentially, this economy is an RBC model with a trivial monetary block

- The 'Classical dichotomy' holds in the long run **and** the short run
- All 'real' variables can be pinned down without knowing anything about monetary policy or the general price level
- **This is not a New Keynesian model**

So why bother?

- To introduce various concepts that are in common with the NK model
- But without the distractions of the NK model's other special features

Roadmap

The model features three sets of agents...

1. Households
   - Large number of households with identical tastes
   - Take prices as given (competitive)
   - 'Complete markets' and utility maximization $\Rightarrow$ all do the same thing
   - Allows us to work with a 'representative' household
2. Firms
   - Large number of firms with identical technology/production possibilities
   - Take prices as given (competitive)
   - Produce a single homogenous good (to be consumed by households)
   - Profit maximization $\Rightarrow$ all do the same thing
   - Allows us to work with a 'representative' firm
3. Monetary policymaker ('central bank')
   - Only matters for price level - not for consumption/production

Roadmap

We will solve for how the real variables behave in **'equilibrium'**

- Find out what households/firms want to do (at a given set of prices)
- Require that goods demanded (households) = goods supplied (firms)
- Require that labor demanded (firms) = labor supplied (households)
- Obtain (real) wage and (real) interest rate that achieves this market clearing (it won't happen under an arbitrary set of prices)

The final piece of the **'equilibrium'** is to assume the central bank's behavior and derive what that implies for the price level

- Money is a unit of account
- Aggregate price level, P_t is price of consumption good
- The 'real' part of the equilibrium is consistent with many P_t processes
- Properties of a policy 'interest rate rule' influence the price processes possible in equilibrium

Households

Households - Objective function

Objective function of a (representative) household

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t) \right]$$

- $U(C_t, N_t; Z_t)$ - Period utility function
- C_t and N_t - Consumption and 'hours worked' in time t
- Z_t - 'Preference shifter'
- β - Time discount factor where $\beta \in (0, 1)$
- $E_t[\cdot]$ - Expectation operator, conditional on time-t information

Households - Utility function

$$U\left(C_t, N_t; Z_t\right)$$

'Well behaved'

- $U_{c,t} \equiv \frac{\partial U_t}{\partial C_t} > 0$ - likes consumption
- $U_{cc,t} \equiv \frac{\partial^2 U_t}{\partial C_t^2} \leq 0$
- $U_{n,t} \equiv \frac{\partial U_t}{\partial N_t} \leq 0$ - dislikes work
- $U_{nn,t} \equiv \frac{\partial^2 U_t}{\partial N_t^2} \leq 0$

Preference shifter 'increases' $U_{c,t}$

- $U_{cz,t} \equiv \frac{\partial^2 U_t}{\partial C_t \partial Z_t} > 0$
- $Z_t \uparrow \Rightarrow$ consumption in t more highly valued on the margin

Households - Utility function

We will adopt convenient special cases, depending on σ:

$$U\left(C_t, N_t; Z_t\right) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t & : \sigma \neq 1 \\ \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t & : \sigma = 1 \end{cases}$$

and we assume that $z_t \equiv \log Z_t$ follows an AR(1)

$$\begin{aligned} z_t &= \rho_z z_{t-1} + \varepsilon_t^z \\ \varepsilon_t^z &\overset{iid}{\sim} N\left(0, \sigma_z^2\right) \end{aligned}$$

- σ controls attitudes to intertemporal substitution
- φ controls disutility of labor
- Effective time discount factor from perspective of t is $\frac{Z_{t+j}}{Z_t}\beta^j$

Households - Budget constraint(s)

Households choose dynamic plans for C_t and N_t subject to a sequence of 'flow' budget constraints

$$P_t C_t + Q_{n,t} B_t \leq B_{t-1} + W_t N_t + D_t$$

- P_t - Price of consumption good
- W_t - Nominal wage
- D_t - Dividends from firms owned by households
- $Q_{n,t}$ - Price of bond that pays unit of money in $t+1$ for certain
  - *Nominally* riskless 'discount' bond
  - Guaranteed \$1 tomorrow for each bond purchased today
  - 'Real' payoff tomorrow is unknown today (because P_{t+1} is unknown)

We also require a 'no-Ponzi' condition (discuss shortly)

$$\lim_{T \to \infty} E_t \left[\Lambda_{t,T} \frac{B_T}{P_T} \right] \geq 0$$

Households - Budget constraint(s)

What is the implied (nominal) return from t to $t+1$?

- Bond sold at t for $Q_{n,t}$
- Pays one unit (of money) at maturity in $t+1$
- Return on bond is therefore $Q_{n,t}^{-1}$

Why call it a 'discount' bond?

- Natural to think of $Q_{n,t} < 1$ so bond *'sold at a discount'* to generate a positive 'interest rate'

$$1 + r_{n,t} \equiv R_{n,t} \equiv Q_{n,t}^{-1} > 1$$

Households - Stochastic discount factor

$$\Lambda_{t,t+1} \equiv \beta \frac{U_{c,t+1}}{U_{c,t}} = \beta \frac{Z_{t+1}}{Z_t} \left(\frac{C_t}{C_{t+1}} \right)^{\sigma}$$

Encodes time discounting due to β

- When evaluating from t onward, $Z_t \uparrow$ acts like lower β (less patient)

Encodes how payoffs under different *contingencies* are valued

- Marginal payoff in $t+1$ will be valued using marginal utility in $t+1$
- 'Stochastic' from t perspective as C_{t+1} and Z_{t+1} not yet known
- Re-normalized by marginal utility in t

'Discounting' is really about relative value

- 'Dislike' (like) distant (immediate) payoffs
- 'Dislike' (like) payoffs when marginal utility is low (high)

Households - Stochastic discount factor

Temporarily ignore Z so SDF becomes...

$$\Lambda_{t,t+1} \equiv \beta \frac{U_{c,t+1}}{U_{c,t}} = \beta \left(\frac{C_t}{C_{t+1}} \right)^{\sigma}$$

Assume n possible outcomes for consumption in $t+1$

- Values: $\{C(i)\}_{i=1}^{n}$
- Probabilities (given info. in t): $\{p_i\}_{i=1}^{n}$

Let an asset pay $Y(i)$ in $t+1$ when $C_{t+1} = C(i)$

- Value of asset obtained by *discounting* random payoffs with $\Lambda_{t,t+1}$

$$V_t^y \equiv E_t[Y_{t+1}\Lambda_{t,t+1}] = \sum_{i=1}^{n} p_i Y(i)\Lambda(i) = \beta \sum_{i=1}^{n} p_i Y(i) \left(\frac{C_t}{C(i)} \right)^{\sigma}$$

Households - Stochastic discount factor

SDF for longer horizons are implicit given definition of 1-period SDF

$$\begin{aligned}
\Lambda_{t,t+2} &= \Lambda_{t+1,t+2}\Lambda_{t,t+1} \\
\Lambda_{t,t+3} &= \Lambda_{t+2,t+3}\Lambda_{t,t+2} = \Lambda_{t+2,t+3}\Lambda_{t+1,t+2}\Lambda_{t,t+1} \\
&\ldots
\end{aligned}$$

Use this recursion to see

$$\Lambda_{t,t+j} = \beta^j \frac{U_{c,t+1}}{U_{c,t}} \frac{U_{c,t+2}}{U_{c,t+1}} \ldots \frac{U_{c,t+j}}{U_{c,t+j-1}} = \beta^j \frac{U_{c,t+j}}{U_{c,t}}$$

Recall no-Ponzi condition (which holds for all t)

$$\lim_{T\to\infty} E_t \left[\Lambda_{t,T} \frac{B_T}{P_T}\right] \geq 0$$

Agent can't have positive debt in the infinite future (as valued from t)

Households - Stochastic discount factor

SDF for longer horizons are implicit given definition of 1-period SDF

$$
\begin{aligned}
\Lambda_{t,t+2} &= \Lambda_{t+1,t+2}\Lambda_{t,t+1} \\
\Lambda_{t,t+3} &= \Lambda_{t+2,t+3}\Lambda_{t,t+2} = \Lambda_{t+2,t+3}\Lambda_{t+1,t+2}\Lambda_{t,t+1} \\
&\dots
\end{aligned}
$$

Use this recursion to see

$$\Lambda_{t,t+j} = \beta^j \frac{U_{c,t+1}}{U_{c,t}} \frac{U_{c,t+2}}{U_{c,t+1}} \cdots \frac{U_{c,t+j}}{U_{c,t+j-1}} = \beta^j \frac{U_{c,t+j}}{U_{c,t}}$$

Recall no-Ponzi condition (which holds for all t)

$$\lim_{T\to\infty} E_t \left[\Lambda_{t,T} \frac{B_T}{P_T} \right] \geq 0$$

Agent can't have positive debt in the infinite future (as valued from t)

Households - Optimality conditions

Maximizing utility subject to the sequence of budget constraints implies:

- 'Intratemporal' optimality (labor-consumption tradeoff in t)

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

- 'Intertemporal' optimality or 'Euler equation' (tradeoff between C_t and C_{t+1} implicit in choice of B_t)

$$Q_{n,t} = E_t \left[\Lambda_{t,t+1} \frac{P_t}{P_{t+1}} \right]$$

Households - Optimality conditions

$$-U_{n,t} = \frac{W_t}{P_t} U_{c,t+1}$$

Consider marginally more N_t

- Cost: Foregone leisure on margin
- Benefit: Extra wages earned, convertible to additional consumption

$$\frac{Q_{n,t}}{P_t} U_{c,t} = \beta E_t \left[U_{c,t+1} \frac{1}{P_{t+1}} \right]$$

Consider marginally less C_t (more B_t)

- Cost: Forgoing utility from marginal unit of consumption today
- Benefit: Extra saving $\Rightarrow$ raises utility from consumption tomorrow

Households - Optimality conditions

We also have an additional optimality condition

$$\lim_{T\to\infty} E_t\left[\Lambda_{t,T}\frac{B_T}{P_T}\right] = 0$$

'Similar' to the no-Ponzi constraint

$$\lim_{T\to\infty} E_t\left[\Lambda_{t,T}\frac{B_T}{P_T}\right] \geq 0$$

Why $= 0$ and not ≥ 0?

- > 0 undesirable for agent
- Would be creditor 'in the limit'
- Saving/lending too much
- Can do better by increasing consumption path in some manner

Households - Optimality conditions

Useful to convert/approximate these conditions with a 'log-linear' form

- We will discuss linearizations in the first seminar
- See also the accompanying notes on this topic
- Notation: Lower case m for variable M means $m \equiv \log(M)$

Households - Optimality conditions

If we take logs of both sides of the intratemporal condition and re-arrange...

$$n_t = \frac{1}{\varphi}\left(w_t - p_t - \sigma c_t\right)$$

Given marginal utility of consumption (captured by σc_t) this yields a 'labor supply relation'

$$n_t = \tilde{n}^s\left(w_t, p_t; c_t\right)$$

It is perhaps more natural to think in terms of the (log) real wage

$$\begin{aligned} n_t &= n^s\left(w_t^r; c_t\right) \\ w_t^r &\equiv w_t - p_t \end{aligned}$$

The 'Frisch elasticity', φ^{-1}, controls sensitivity of n to w^r

Households - Optimality conditions

Using a log-linear approximation of the intertemporal condition we obtain

$$c_t = E_t\left[c_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) + \frac{1}{\sigma}\left(\rho + (1-\rho_z)\, z_t\right)$$

where $\rho \equiv -\log\beta$

The 'nominal interest rate' is defined as (recall discount bond discussion)

$$i_t \equiv -\log\left(Q_{n,t}\right)$$

The gross inflation rate is defined as

$$\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$$

and we refer to the 'inflation rate', $\pi_{t+1} \equiv \log\left(\Pi_{t+1}\right)$

Households - Optimality conditions

$$
\begin{aligned}
c_t &= E_t\left[c_{t+1}\right] - \frac{1}{\sigma}\left(\overbrace{i_t - E_t\left[\pi_{t+1}\right]}^{\text{Real Interest Rate}}\right) + \frac{1}{\sigma}\left(\underbrace{\rho + \left(1-\rho_z\right) z_t}_{\text{Time Discount}}\right) \\
&= E_t\left[c_{t+1}\right] - \underbrace{\frac{1}{\sigma}}_{EIS}\left(r_t - \zeta\left(z_t\right)\right)
\end{aligned}
$$

where we define the real interest rate as

$$r_t \equiv i_t - E_t\left[\pi_{t+1}\right]$$

and a ‘composite’ discount term

$$\zeta\left(z_t\right) \equiv \rho + \left(1-\rho_z\right) z_t$$

Households - Optimality conditions

$$E_t\left[\Delta c_{t+1}\right] = \frac{1}{\sigma}\left(r_t - \zeta\left(z_t\right)\right)$$

Shape of consumption path is influenced by

- Time discounting (β and path of z_t)
- Terms of trade for tilting consumption path (r_t)

Elasticity of intertermporal substitution (EIS) $= \sigma^{-1}$

- Controls willingness to reallocate consumption over time
- Example: Suppose σ is big (so EIS is low)
  - Consider an increase in r_t keeping all else equal
  - More incentive for an increasing consumption profile
  - But planned increase in C_{t+1} relative to C_t will be 'small'

Firms

Firms

There is a large number of identical firms producing a homogenous consumption good using same production function

$$Y_t = A_t N_t^{1-\alpha}$$

The 'technology' process, $a_t \equiv \log A_t$, follows an AR(1)

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_t^a \\ \varepsilon_t^a &\overset{iid}{\sim} N\left(0, \sigma_a^2\right) \end{aligned}$$

Maximize profits in each period, taking price and wage as given

$$P_t Y_t - W_t N_t$$

Firms - Optimality condition

Firms choose how much labor to hire, leading to the optimality condition

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

In log linear form,

$$w_t - p_t = a_t - \alpha n_t + \log(1-\alpha)$$

Given technology, this defines labor demand relations in terms of the (log) nominal wage and consumption good price or, alternatively, the real wage

$$\begin{aligned} n &= \tilde{n}^d (w, p; a_t) \\ n &= n^d (w^r; a_t) \end{aligned}$$

Firms - Optimality condition (MC interpretation)

Note that we can rewrite the labor demand condition as follows

$$P_t = \frac{W_t}{(1-\alpha) A_t N_t^{-\alpha}}$$

making explicit that optimality requires price = marginal cost

- Labor is the only input
- To produce a marginal unit of output one needs $\frac{1}{MPL}$ of labor
- That additional labor is paid W_t
- Hence $W_t \times \frac{1}{MPL}$ is the cost of the marginal output

Equilibrium

Competitive equilibrium

Equilibrium requires that optimality conditions are respected but also. . .

- Decisions are consistent and feasible in aggregate
- Prices ensure that market clearing occurs

Initially we will only derive the equilibrium values of ‘real’ variables

- Consumption
- Output
- Employment
- Real wage
- Real savings
- Real interest rate

We will then model the nominal variables separately

- Separation of real and nominal is a *very* special aspect of this model
- Does not hold in NK model (that’s the point. . .)

Equilibrium conditions - real block

1. Final good market clearing (demand = supply)
$$c_t = y_t^s$$
2. Labor market clearing (demand = supply)
$$n_t^d = n_t^s$$
3. Aggregate final goods supply (technical constraint)
$$y_t^s = a_t + (1-\alpha)\, n_t^d$$
4. Labor supply (optimality)
$$w_t^r = \sigma c_t + \varphi n_t^s$$
5. Labor demand (optimality)
$$w_t^r = \log(1-\alpha) + a_t - \alpha n_t^d$$
6. (Real) savings (accounting)
$$S_t^r = \exp(y_t^s) - \exp(c_t)$$
7. Euler equation (optimality)
$$r_t = \rho + (1-\rho_z)\, z_t + \sigma E_t[\Delta c_{t+1}]$$

Equilibrium conditions - real block

In the previous slide we had 7 equations relating 7 unknowns

- c_t, y_t^s, n_t^d, n_t^s, w_t^r, S_t^r, r_t

Additionally, the equations involved...

- 'Deep' or 'structural' parameters
  - Explicitly: α, σ, φ, $\rho \Leftrightarrow \beta$, ρ_z
  - Implicitly (in the expectation): ρ_a, σ_a and σ_z
- Exogenous driving processes / shocks
  - z_t and a_t

Solving for the model's equilibrium - real block

When economists speak of 'solving' for the equilibrium of a model, they mean...

- Find an expression for endogenous variables in terms of the 'state'
- Loose definition of 'the state':
  - What needs to be known to give a full description of the current situation of the economy
- Simply put, in our model, the state is z_t and a_t

Remember 'high school' math

- Asked to solve...

$$\begin{aligned} \zeta_{1,1}x_1 + \zeta_{1,2}x_2 &= \delta_1 \\ \zeta_{2,1}x_1 + \zeta_{2,2}x_2 &= \delta_2 \end{aligned}$$

- Think of x_1 and x_2 as n_t and y_t etc.
- Think of $\zeta_{i,j}$ and δ_i as some mix of structural parameters, z_t and a_t

Solving for the model's equilibrium - real block

First we note that we can use market clearing to say

$$n_t^s = n_t^d = \mathbf{n_t}$$
$$c_t = y_t^s = \mathbf{y_t}$$

so we can just talk in terms of employment (n_t) and output (y_t)

Then we have

$$\begin{aligned} y_t &= a_t + (1-\alpha)\, n_t && (1) \\ w_t &= \log(1-\alpha) + a_t - \alpha n_t && (2) \\ w_t &= \sigma y_t + \varphi n_t && (3) \end{aligned}$$

Solving for the model's equilibrium - real block

Using equations (2) and (3) we obtain

$$y_t = \frac{1}{\sigma}\left(\log\left(1-\alpha\right) + a_t - \left(\alpha+\varphi\right) n_t\right)$$

Combining with equation (1) $\Rightarrow$ n_t in terms of exogenous variables

$$\begin{aligned} n_t &= \psi_n + \psi_{n,a} a_t \qquad (4) \\ \psi_n &\equiv \frac{\log\left(1-\alpha\right)}{\left(1-\alpha\right)\sigma + \alpha + \varphi} \\ \psi_{n,a} &\equiv \frac{1-\sigma}{\left(1-\alpha\right)\sigma + \alpha + \varphi} \end{aligned}$$

Substituting back into equation (1) yields

$$\begin{aligned} y_t &= \psi_y + \psi_{y,a} a_t \qquad (5) \\ \psi_y &\equiv \left(1-\alpha\right)\psi_n \\ \psi_{y,a} &\equiv \frac{1+\varphi}{\left(1-\alpha\right)\sigma + \alpha + \varphi} \end{aligned}$$

Solving for the model's equilibrium - real block

It is straightforward to derive similar expressions the other variables

- w_t^r: Use the labor supply condition and expression for n_t
- c_t: We have $c_t = y_t$ so same as for y_t
- Savings follow from simple accounting - note that $c_t = y_t$

Note: We didn't need to solve for S_t^r to solve for the other variables

- Generally, when solving models there will be a core of variables that 'must' be solved for
- After that, one can define and solve for whatever other variables may be of interest

Solving for the model's equilibrium - real block

Note that $S_t^r \equiv \frac{Q_{n,t} B_t}{P_t} = 0$

- Real value of nominal bond holdings is zero
- Assuming positive prices, this implies $B_t = 0$
- Identical households means no one holds bonds **in equilibrium**
  - All households have same optimality conditions
  - If *anybody* wants to save, then *everybody* wants to save!
  - Not feasible because no one takes the other side of the trade

Sort of obvious from the start - why?

- No way to transfer resources to future *in aggregate*
- Other models: possible via investment in 'capital' or foreign borrowing

Equilibrium requires optimality *and* feasibility

Solving for the model's equilibrium - real block

- Households don't know/care about equilibrium
  - They just see prices and choose what's best for them individually
- It is our *assumption of equilibrium* (which must feature zero savings in this case) that pins down the necessary prices
  - How do these prices come about in a competitive model (where everyone is a price taker)?
  - Deep questions (see tatonnement, bargaining, the 'core', ...)
  - In NK model, (goods) price setting is determined explicitly by firms' decisions
- *In equilibrium* prices must induce zero savings as optimal choice
  - z_t affects the relative desirability of consumption today vs. tomorrow
  - $z_t \uparrow \Rightarrow$ people want to save less *for a given* r_t
  - So r_t must be a function of z_t in equilibrium

Solving for the model's equilibrium - real block

Finally, we derive the equilibrium real interest rate using...

- Intertemporal optimality (Euler equation)
- $c_t = y_t$ (where zero savings most obviously gets incorporated)
- The 'solution' for y_t
- AR(1) process for a_t

$$\begin{aligned} r_t &= \rho + (1-\rho_z)\, z_t + \sigma E_t\left[\Delta y_{t+1}\right] \\ &= \rho + (1-\rho_z)\, z_t + \sigma\,(1-\rho_a)\,\psi_{y,a} a_t \\ &\equiv \psi_r + \psi_{r,a} a_t + \psi_{r,z} z_t \end{aligned}$$

In a 'perfect foresight' steady state where $z_t = a_t = 0$ we have

$$r = \rho \equiv -\log(\beta)$$

The steady state real interest rate is deteremined by time preference

Solving for the model's equilibrium - real block

At this point we have...

- Expressions of the form

$$var_t = \psi_{var} + \psi_{var,a} a_t + \psi_{var,z} z_t$$

  for $var \in \{c_t, y_t, n_t, w_t^r, r_t\}$
- Exact expressions for savings $(= 0)$
- The AR(1) specifications for a_t and z_t

These components constitute the 'solution' of the real side of the model for all the real variables of interest

Equilibrium - Nominal Block

Solving for the model's equilibrium - nominal block

- The nominal aspects of the model have no implications for the behavior of real variables
- As such, they are irrelevant for welfare and imply no role for policy
- Nevertheless it is useful to consider how the behavior of P_t depends on monetary policy
- Policy is captured by a rule for the *nominal* interest rate i_t

Solving for the model's equilibrium - nominal block

Recall the Fisher equation

$$i_t = r_t + E_t[\pi_{t+1}]$$

In this model, r_t is already pinned down on the real side

- $\Rightarrow$ conditional on r_t, inflation expectations and i_t move 1:1
- This applies in the short run (not simply *long run* Classical dichotomy)

For a given steady state rate of inflation, π we can thus obtain the associated steady state value of i_t

$$i = \rho + \pi$$

since (recall) $r_t = \rho$ in steady state

Solving for the model's equilibrium - nominal block

Let us consider a simple policy rule for i_t

$$\begin{aligned} i_t &= \rho + \pi + \phi_\pi(\pi_t - \pi) + v_t \\ &= r + \pi + \phi_\pi(\pi_t - \pi) + v_t \end{aligned}$$

where $\phi_\pi > 0$ determines the strength of the response of policy to deviations in inflation from π

v_t is a policy 'shock' that follows an AR(1)

$$\begin{aligned} v_t &= \rho_v v_{t-1} + \varepsilon_t^v \\ \varepsilon_t^v &\overset{iid}{\sim} N\left(0, \sigma_v^2\right) \end{aligned}$$

What is v_t?

- A random, temporary deviation from 'usual' policy conduct
- Note: In recent years v_t likely have become 'smaller' (Ramey 2016)

Solving for the model's equilibrium - nominal block

If we subtract the policy rule from the Fisher equation we obtain (where we define $\hat{\pi} \equiv \pi_t - \pi$ and $\hat{r} \equiv r_t - r$)

$$\hat{\pi}_t = \frac{1}{\phi_\pi} E_t[\hat{\pi}_{t+1}] + \frac{1}{\phi_\pi}(\hat{r}_t - v_t)$$

If $\phi_\pi > 1$ then (under reasonable assumptions) we can solve for

$$\hat{\pi}_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t\left[\hat{r}_{t+k} - v_{t+k}\right]$$

Given our earlier solution for r_t in equilibrium and the assumed processes for a_t, z_t and v_t we can rewrite this as

$$\hat{\pi}_t = \frac{\sigma(1-\rho_a)\psi_{ya}}{\phi_\pi - \rho_a} a_t + \frac{1-\rho_z}{\phi_\pi - \rho_z} z_t - \frac{1}{\phi_\pi - \rho_v} v_t$$

Solving for the model's equilibrium - nominal block

With a solution for (an) equilibrium process for π_t (equivalently, $\hat{\pi}_t$) the process for prices arises from

$$p_t \equiv p_{t-1} + \pi_t$$

which allows the conversion of real wages to nominal

Under $\phi_\pi > 1$ this is the *only* solution for the nominal elements of equilibrium

- Under $\phi_\pi < 1$ we have 'equilibrium indeterminacy'
- Multiple equilibria are possible with the same 'real' block but with different processes for inflation
- Hints that may be desirable to keep $\phi_\pi > 1$ (the 'Taylor principle')
    - But in *this* economy *all* the equilibria are welfare-equivalent since consumption and employment are the same

Summary

Summary

- The model we have described has various unrealistic implications
  - In particular there is no role for monetary policy to influence the economy as empirical evidence suggests it should
  - Price setting is left undefined - in contrast to real world experience of sticky prices etc.
  - There is also no reason for policy to intervene, given efficiency of allocations
- However, it has allowed us to introduce various concepts
  - General equilibrium
  - Multiple sectors (households and firms)
  - Monetary policy 'rule' for i_t
- We now move to the New Keynesian framework which is conceptually richer and empirically more successful