The Basic New Keynesian Model

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The Basic New Keynesian Model

The New Keynesian model shares many common features with the classical model but. . .

1. The classical model had fully competitive firms, whereas the NK model has monopolistically competitive firms
    - Firms have pricing power
    - Face a downward sloping demand curve
2. There are no nominal rigidities in the classical model, whereas the NK model features price stickiness
    - Various ways to model this
    - Common assumption: A fraction of firms are randomly 'allowed' to change prices in each period
3. (Less important) Households buy multiple consumption goods
    - Household consumes a bundle of different consumption goods
    - Imperfect willingness to substitute between these goods is the source of firms' market power

These features go hand in hand

- Consider fully competitive model
- Suppose a firm cannot change its price
- If the market clearing price declines below that firm's price they would lose all demand

Pricing power justifies sticky prices and the decision of what price to set when firms can adjust

- The (minor) adjustment to household preferences underpins the pricing power
- Note that there are alternative ways of motivating the pricing power

Households

Households - Preferences

Objective function of a household

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U \left(C_t, N_t; Z_t \right) \right]$$

Looks same as before but now C_t is a bundle of different goods

$$C_t \equiv \left(\int_0^1 C_t \left(i \right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1}$$

Sometimes this is referred to as a 'Dixit-Stiglitz' aggregator

- It aggregates (adds up) contributions from different types of goods
- Each good is indexed by $i \in [0, 1]$
- $C_t(i)$ is a particular type of consumption
- C_t is 'overall' consumption

Households - Preferences

Objective function of a household

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t; Z_t\right) \right]$$

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$$C_t \equiv \left(\int_0^1 C_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

Sometimes this is referred to as a ‘Dixit-Stiglitz’ aggregator

- It aggregates (adds up) contributions from different types of goods
- Each good is indexed by $i \in [0, 1]$
- $C_t(i)$ is a particular type of consumption
- C_t is ‘overall’ consumption

Households - Multiple goods

The budget constraint is

$$\underbrace{\int_0^1 P_t(i)C_t(i)di}_{Total\ Expenditure} + \underbrace{Q_{n,t}B_t}_{Savings} \leq \overbrace{B_{t-1}}^{Payoff\ from\ Bonds} + \overbrace{W_tN_t}^{Earnings} + \overbrace{D_t}^{Dividends}$$

where we note that each consumption good has its own price, $P_t(i)$

At any optimum, the household must maximize C_t for any amount spent on consumption goods

- $C_t(i)$ chosen appropriately, given prices
- Optimal allocation across goods (see Galí Ch. 3 appendix)...

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$$

Households - Multiple goods

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$$\underbrace{\int_0^1 P_t(i)C_t(i)di}_{Total\ Expenditure} + \underbrace{Q_{n,t}B_t}_{Bond\ Purchases} \leq \overbrace{B_{t-1}}^{Payoff\ from\ Bonds} + \overbrace{W_t N_t}^{Earnings} + \overbrace{D_t}^{Dividends}$$

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$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$$

Households - Multiple goods

Optimal demand across goods

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$$

This is associated with a price index *defined* as

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$

Why is this an appropriate definition of a price index?

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t$$

Households - Multiple goods

Relative to overall demand (C_t), demand for good i decreases in its relative price

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$$

The elasticity of demand, ε, controls strength of this effect

- Large $\varepsilon \Rightarrow$ large decline in demand for good i
- Looking ahead - this will affect pricing power of firms
- High (low) elasticity $\Rightarrow$ low (high) market power
- $\varepsilon \to \infty$ represents price taking / perfect competition

Households - Multiple goods

Equation (10) means that the budget constraint can be re-written as

$$P_t C_t + Q_{n,t} B_t \leq B_{t-1} + W_t N_t + D_t$$

Consequently, we have the same intratemporal and intertemporal optimality conditions as in the classical model

$$\begin{aligned} -\frac{U_{n,t}}{U_{c,t}} &= \frac{W_t}{P_t} \\ Q_t &= \beta E_t \left[\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right] \end{aligned}$$

Firms

Firms - Monopolistic Competition

There is a continuum of firms indexed by $i \in [0,1]$

- They each produce a different good
- Identical production function
- Common technology

$$Y_t = A_t N_t(i)^{1-\alpha}$$

The technology process, $a_t \equiv \log A_t$, follows an AR(1)

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_t^a \\ \varepsilon_t^a &\overset{iid}{\sim} N\left(0, \sigma_a^2\right) \end{aligned}$$

Firms - Monopolistic Competition

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$$

Implies a demand curve for firm i's good, given C_t and P_t

- Note: **Aggregate** price level and consumption are taken as given
- Firm can choose **its** price and (thus) **its** quantity

The firms thus operate in a monopolistically competitive environment

- 'Monopolistic' - they are the only producer of their good i and can set their price
- 'Competitive' - goods partly substitutable (limits market power)
- Only producer of energy drink x but if 'too expensive' people will shift to energy drink y

Firms - Monopolistic Competition

In a 'standard' monopolistically competitive situation (see hwk.)

- Firm sets its price as a markup over marginal cost
- Given demand curve $\Rightarrow$ picking output and employment
- $\varepsilon \to \infty$ shows no markup in perfectly competitive limit

$$
\begin{aligned}
P_t(i)^* &= \frac{\varepsilon}{\varepsilon - 1} MC_t \\
MC_t &= \frac{W_t}{(1-\alpha) A_t N_t(i)^{-\alpha}}
\end{aligned}
$$

But in the NK model the firm may be unable to set their price as they wish. . .

Firms - Price Stickiness

Each firm may only reset its price with probability $1 - \theta$

- See Calvo (1983) - and hwk.
- Same independent probability across firms in each period
- Independent of time since the firm last was able to reset its price

This means that in each period a fraction θ of firms keep prices unchanged

- Continuum of firms $i \in [0, 1]$
- Law of large numbers
- Toss a fair coin a billion times and fraction heads will be ≈ 0.5, toss it $N \to \infty$ and fraction $\to 0.5$
- We have infinitely many ('small') firms with prob. θ - thus fraction θ keep price fixed

Average duration of a given price $= \frac{1}{1-\theta}$

- Natural to interpret θ as an index of price rigidity or 'stickiness'

Firms - Price Stickiness

In every period the distribution of prices across firms is a mixture of

1. The price of the $1-\theta$ of firms who get to reoptimize
   - All set the same price since they face the same optimization problem
2. The prices of the θ fraction of firms whose prices were reoptimized before t but are now fixed
   - Among these prices, a fraction $1-\theta$ were reoptimized in $t-1$ and a fraction θ were reoptimized before $t-1$
   - Among those prices reoptimized before $t-1$, a fraction $1-\theta$ were reoptimized in $t-2$ and a fraction θ were reoptimized before $t-2$
   - Continue the logic...

Firms - Optimal Pricing

In t we have prices prevailing that were reoptimized in the current period *and all previous periods*

- The fraction of prices in t that were set in $t-j$ is declining (to zero) as $j \to \infty$

When firms set their prices they do so acknowledging that...

- There is a distribution of prevailing prices now and in the future
- Their own price will prevail for a random length of time into the future

The problem is thus very different from standard 'static' monopolistic competition that implied

$$P_t(i)^* = \frac{\varepsilon}{\varepsilon - 1} MC_t$$

Firms - Optimal Pricing

Since a firm's price will prevail (with some probability) for several periods after it is set, the firms must consider the implications of that price for *future* profits in those contingencies

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(P_t^* Y_{t+k|t} - W_{t+k} N_{t+k|t} \right) \right]$$

This looks (and is quite) complicated but we will go through it carefully and see that it is very intuitive after all. . .

Firms - Optimal Pricing

Contribution to the value of a firm, in t, of profits *in periods and contingencies in which P_t^* prevails:*

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(P_t^* Y_{t+k|t} - W_{t+k} N_{t+k|t} \right) \right]$$

- θ_k is the probability of P_t^* still prevailing k periods from t
- $\Lambda_{t,t+k}$ values the stream of real profits
  - k-step household SDF because households are the shareholders
- $Y_{t+k|t}$ and $N_{t+k|t}$ are the output and associated employment in $t+k$ for firms who last reset their price in t
- $P_t^* Y_{t+k|t} - W_{t+k} N_{t+k|t}$ are nominal profits
- Dividing by P_{t+k} converts nominal profits to real

Firms - Optimal Pricing

Useful to clarify components of the maximand

$$
\begin{aligned}
W(P_t^*) &\equiv \sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(\mathcal{R}(Y_{t+k|t}, P_t^*) - \mathcal{C}(Y_{t+k|t}) \right) \right] \\
\mathcal{R}(Y_{t+k|t}, P_t^*) &\equiv P_t^* Y_{t+k|t} \qquad\qquad (Revenue) \\
\mathcal{C}(Y_{t+k|t}) &\equiv W_{t+k} \mathcal{N}(Y_{t+k|t}) \;\; (Cost)
\end{aligned}
$$

where (using the production function) we define the employment level induced by $Y_{t+k|t}$ as

$$
\mathcal{N}(Y_{t+k|t}) \equiv \left(\frac{Y_{t+k|t}}{A_{t+k}} \right)^{\frac{1}{1-\alpha}}
$$

Note: We only consider revenue and cost in the contingencies in which the price set in t is still prevailing

- Recall assumption: firms produce whatever is demanded at the prevailing price

Firms - Optimal Pricing

Using the demand curve implied by optimal allocation across goods

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k}$$

It is useful to define nominal marginal cost

$$\Psi_{t+k|t} \equiv \frac{d\mathcal{C}(Y_{t+k|t})}{dY_{t+k|t}}$$

Firms - Optimal Pricing

As stated in Galí (p. 56) the FOC for the choice of P_t^* can be rearranged to be

$$\sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+k} Y_{t+k|t} \frac{1}{P_{t+k}} \left(P_t^* - \mathcal{M} \Psi_{t+k|t} \right) \right] = 0 \tag{3}$$

If $\theta = 0$ then we are back in the static monopolistic competitive case

- Convention is $0^0 \equiv 1$
- Recover static optimal markup of $P_t^* = \mathcal{M}\Psi_t$
- Call $\mathcal{M}$ the 'desired' or 'natural' or 'flex-price' markup

Firms - Optimal Pricing

If $\theta \in (0, 1)$ then the static condition will (generically) not hold

$$P_t^* \neq \mathcal{M}\Psi_t$$

However, firms are setting prices to try to keep the deviations from this condition 'small' in all periods

- The optimality condition is a weighted sum of deviations of the firm's price in $t + k$ from $\mathcal{M}\Psi_{t+k|t}$
- Intuition for weights...
  - $\theta^k \Rightarrow$ particularly care about near future
  - $\Lambda_{t,t+k} \Rightarrow$ particularly care about reduced profits when MU is high
  - $Y_{t+k|t} \Rightarrow$ static suboptimality more concerning if producing a lot

Firms - Optimal Pricing

Approximating around the zero inflation steady state we obtain (lower case means logs)

$$\begin{aligned} p_t^* &= \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\psi_{t+k|t}\right] \qquad (4) \\ \mu &\equiv \log \mathcal{M} \end{aligned}$$

- Firms markup by $\mathcal{M}$ but not over current marginal cost
- Instead they markup over a weighted average of current and future marginal costs
- Weights $\propto$ time discount (β^k) and probability of price prevailing (θ^k)

Thus, firms set prices in a **forward looking** manner

Firms - Aggregate Price Dynamics

As shown in the text...

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

where, recall, $\Pi_t \equiv P_t / P_{t-1}$ is the gross inflation rate

If we take a log-linear approximation and rearrange we obtain

$$p_t = \theta p_{t-1} + (1-\theta) p_t^*$$

Thus the current price level is a weighted average of last period's price level and the new reset price

- Price level evolves as p_t^* typically $\neq p_{t-1}$
- Weights are intuitively connected to θ

Equilibrium - Non-policy Block

Equilibrium - Non-policy bloc

Household optimality conditions

- Labor supply
- Intertemporal optimality and the definition of the SDF
- Optimal allocation of consumption across goods (implies demand curves for firms)
- SDF will be used by firm (owned by households)

Firm optimality conditions

- Optimal price setting
- Labor demand and supply of good *i* are implied by the price decision
  - Production function taken as given
  - Assume firms supply what is demanded at a given price

We use these and market clearing/feasibility assumptions to solve for equilibrium (we will also need a monetary policy block) Extra derivations

Equilibrium - Non-policy bloc

Recall (from previous lectures) that firm marginal cost is wage over marginal product of labor (need 1/MPL for marginal unit of output, and pay that unit W):

$$\psi_t(i) = w_t - (a_t - \alpha n_t(i) + \log(1-\alpha))$$

Then, using the approximation $n_t = \int_0^1 n_t(i)di$, we can show

$$\psi_{t+k|t} = \psi_{t+k} + \alpha(n_{t+k|t} - n_{t+k})$$

If $\alpha > 0$, firms who haven't reset since t have MC higher than the average if their employment levels are relatively high (except for $\alpha = 0$ - why?)

- $\Leftrightarrow$ their output is relatively high (why? use the prod. fn.)
- $\Leftrightarrow$ their price is relatively low (why? use the dem. curve)

$$\psi_{t+k|t} = \psi_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(p_t^* - p_{t+k})$$

Equilibrium - Non-policy bloc

We derived an expression for p_t^* earlier in terms of expected $\psi_{t+k|t}$ - combining that with the expression for $\psi_{t+k|t}\dots$

$$\pi_t = \beta E_t[\pi_{t+1}] - \lambda\hat{\mu}_t$$

with

$$\begin{aligned}\lambda &\equiv \theta^{-1}(1-\theta)(1-\beta\theta)\Theta > 0 \\ \Theta &\equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\end{aligned}$$

and where

$$\begin{aligned}\mu_t &\equiv p_t - \psi_t && \textit{Markup over marginal cost} \\ \hat{\mu}_t &\equiv \mu_t - \mu && \textit{Deviation from desired markup}\end{aligned}$$

Equilibrium - Non-policy bloc

Inflation reflects expected path of markup 'gaps'

$$\pi_t = -\lambda \sum_{k=0}^{\infty} \beta^k E_t[\hat{\mu}_{t+k}]$$

- Markups expected to be below desired $\Rightarrow \pi_t > 0$
- Markups expected to be at desired level $\Rightarrow \pi_t = 0$
- Markups expected to be above desired level $\Rightarrow \pi_t < 0$

$\Rightarrow$ pricing decisions of reoptimizing firms tends to restore the desired markup and **these adjustments induce non-zero inflation**

Equilibrium - Non-policy bloc

We want to connect the markup to the level of activity (y_t) in the economy

- Note that $\mu_t = -mc_t$ where mc_t is **real** marginal cost
- Use this to derive an expression for μ_t in terms of y_t and a_t

$$\begin{aligned}
\mu_t &= p_t - \psi_t \\
&= -(w_t - p_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&= -(\sigma y_t + \varphi n_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&= -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)
\end{aligned}$$

Equilibrium - Non-policy bloc

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$$
\begin{aligned}
\mu_t &= p_t - \psi_t \\
&\overset{\psi_t}{=} -(w_t - p_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&\overset{HHOLD}{=} -(\sigma c_t + \varphi n_t^S) + (a_t - \alpha n_t^D + \log(1-\alpha)) \\
&= -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)
\end{aligned}
$$

Equilibrium - Non-policy bloc

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\end{aligned}$$

Equilibrium - Non-policy bloc

A **very important concept** to grasp is the ‘natural’ value of a (real) variable

- It is the value that prevails in the ‘flexible price’ form of this model
- We obtain this by setting $\theta = 0$
  - There is no price stickiness
  - All firms can reset price in each period
  - Markups are always = desired ($P_t = \mathcal{M}\Psi_t$)
  - Firms are always operating at desired scale

Not equivalent to the ‘steady state’

- Fluctuations in technology will shift the natural rate over time
- **It isn’t constant**

Equilibrium - Non-policy bloc

Since $\theta = 0$ is just a special case of the economy we have been discussing, all our equations still apply

$$\mu_t = -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t + \left(\frac{1 + \varphi}{1 - \alpha}\right) a_t + \log(1 - \alpha)$$

Thus by imposing that the markup is constantly at the desired level, we can *define* the natural rate of output

$$\begin{aligned} y_t^n &= \psi_y + \psi_{y,a} a_t \\ \psi_y &\equiv -\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha) + \varphi + \alpha} \\ \psi_{y,a} &\equiv \frac{1+\varphi}{\sigma(1-\alpha) + \varphi + \alpha} \end{aligned}$$

Equilibrium - Non-policy bloc

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Equilibrium - Non-policy bloc

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Thus by imposing that the markup is constantly at the desired level, we can *define* the natural rate of output

$$\begin{aligned} y_t^n &= \psi_{yn} + \psi_{yn,a} a_t \\ \psi_{yn} &\equiv -\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha) + \varphi + \alpha} \\ \psi_{yn,a} &\equiv \frac{1+\varphi}{\sigma(1-\alpha) + \varphi + \alpha} \end{aligned}$$

Equilibrium - Non-policy bloc

Compare natural rate of output...

$$
\begin{aligned}
y_t^n &= \psi_{yn} + \psi_{yn,a} a_t \\
\psi_{yn} &\equiv -\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} \\
\psi_{ny,a} &\equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}
\end{aligned}
$$

...with output in Classical model (see lecture 2)

$$
\begin{aligned}
y_t^c &= \psi_{yc} + \psi_{yc,a} a_t \\
\psi_{yc} &\equiv \frac{(1-\alpha)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} \\
\psi_{yc,a} &\equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}
\end{aligned}
$$

Equilibrium - Non-policy bloc

Monopolistic competition even suppresses output **in steady state**

- To see this, put a_t to its steady state / unconditional mean of zero

This means that even eliminating price stickiness won't get us back to the (efficient) Classical model

- Emphasizes that NK model differs in price stickiness **and** in having competitive distortions

See homeworks for extensive discussion of monopolistic competition and Pigouvian subsidy

Equilibrium - Non-policy bloc

Returning to the expression for the natural rate of output...

$$y_t^n = \psi_{yn} + \psi_{yn,a} a_t$$

The natural rate of output **does not depend on**

- z_t
- Monetary policy

Gaps between natural and actual versions of variables reflect price stickiness

- We will see price stickiness implies real effects of monetary policy
- Monetary policy can influence these gaps

Equilibrium - Non-policy bloc

We define the '*output gap*'

$$\tilde{y}_t \equiv y_t - y_t^n$$

The phrase 'output gap' is used by various people with various meanings

- In the NK model it means something *very* specific
- y_t^n is a particular theoretical object and not some smooth 'trend' or 'moving average'
- y_t^n emerges from an imaginary world with flexible prices and a very particular structure

An empirical question whether commonly used 'trends' are similar to y_t^n

- Remember, our model is only a simplification of reality
- Using a statistical rather than model-based gap may still be useful

Equilibrium - Non-policy bloc

Using (for y_t **and** y_t^n)...

$$\mu_t = -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t + \left(\frac{1 + \varphi}{1 - \alpha}\right) a_t + \log(1 - \alpha)$$

combined with...

$$\pi_t = \beta E_t[\pi_{t+1}] - \lambda \hat{\mu}_t$$

we obtain

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t \tag{5}$$

where

$$\kappa \equiv \lambda \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t$$

Equilibrium - Non-policy bloc

This relation is called the 'New Keynesian Phillips Curve' (NKPC)

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t$$

- Phillips (1958) observed an (apparent) relationship between (wage) inflation and a measure of unemployment
- Investigating the relationship between (wage and/or price) inflation and activity (employment, growth, output gap . . .) has been a central macro question since. . . forever!
- For a long time a relationship was asserted (particularly by Keynesians) but with little or no theoretical foundations
- The New Keynesian model proposes microeconomic foundations that are internally consistent, within a General Equilibrium framework

May not be an ideal model - **but a huge intellectual achievement!**

Equilibrium - Non-policy bloc

To obtain an 'Dynamic IS' relationship we use the household intertemporal optimality condition (with $y_t = c_t$ implicit)

$$y_t = E_t\left[y_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) + \frac{1}{\sigma}\left(\rho + \left(1 - \rho_z\right) z_t\right)$$

Setting $y_t = y_t^n$, and using our solution for y_t^n, we define

$$\begin{aligned} r_t^n &\equiv -\sigma(1-\rho_a)\psi_{y,a}a_t + \rho + (1-\rho_z)z_t \qquad (6) \\ &\equiv \psi_{rn} + \psi_{rn,a}a_t + \psi_{rn,z}z_t \end{aligned}$$

This 'natural' real interest rate would prevail in a flexible price economy and we can show

$$\begin{aligned} \tilde{y}_t &= E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right] - r_t^n\right) \qquad (7) \\ &= -\frac{1}{\sigma}\sum_{k=0}^{\infty} E_t[r_{t+k} - r_{t+k}^n] \end{aligned}$$

$$\tilde{y}_t = E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right] - r_t^n\right)$$

Equilibrium - Non-policy bloc

At this point we have

$$\tilde{y}_t \equiv y_t - y_t^n \tag{8}$$
$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t \tag{9}$$
$$\tilde{y}_t = E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right] - r_t^n\right) \tag{10}$$
$$y_t^n = \psi_{yn} + \psi_{yn,a} a_t \tag{11}$$
$$r_t^n = \psi_{rn} + \psi_{rn,a} a_t + \psi_{rn,z} z_t \tag{12}$$

These equations constitute the 'non-policy' block of the NK model

- NKPC determines inflation given an expected path for the output gap
- DIS determines the output gap given an expected path for the natural and actual real interest rates
- The real interest rate reflects expected inflation and the nominal interest rate, which is set by policy

Equilibrium - Non-policy block

At this point we have

$$
\begin{aligned}
\tilde{y}_t &\equiv y_t - y_t^n && (13)\\
\pi_t &= \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t && (14)\\
\tilde{y}_t &= E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right] - r_t^n\right) && (15)\\
y_t^n &= \psi_{yn} + \psi_{yn,a} a_t && (16)\\
r_t^n &= \psi_{rn} + \psi_{rn,a} a_t + \psi_{rn,z} z_t && (17)
\end{aligned}
$$

These equations constitute the 'non-policy' block of the NK model

- NKPC determines inflation given an expected path for the output gap
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Equilibrium - Introducing Policy

Equilibrium - Introducing policy

We can't solve for the real variables without specifying monetary policy

- Price stickiness $\Rightarrow$ expected inflation does not move 1:1 with i_t
- $r_t = i_t - E_t[\pi_{t+1}] \Rightarrow r_t$ is affected by policy actions
- Agents' actions influenced by r_t (intertemporal terms of trade)

We imagine i_t being set according to a policy 'rule'...

$$\begin{aligned} i_t &= \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t \\ \hat{y}_t &= y_t - y \\ v_t &= \rho_v v_{t-1} + \varepsilon_t^v \end{aligned}$$

These sort of rules are called 'Taylor Rules' (after Taylor 1993 and 1999)

- Assume $\phi_\pi > 1$ and it is standard to assume $\phi_y > 0$
- v_t is a policy 'shock'

Equilibrium - Introducing policy

The system we must solve is

$$\begin{aligned}
i_t &= \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t \\
&= \rho + \phi_\pi \pi_t + + \phi_y \tilde{y}_t + \phi_y \hat{y}_t^n + v_t \\
r_t^n &= \rho - \sigma(1-\rho_a)\psi_{yn,a} a_t + (1-\rho_z) z_t \\
\tilde{y}_t &= -\frac{1}{\sigma}\left(i_t - E_t[\pi_{t+1}] - r_t^n\right) + E_t[\tilde{y}_{t+1}])
\end{aligned}$$

The solution will be expressions for $\tilde{y}_t$ and π_t in terms of some combination of shocks (a_t, z_t and v_t)

- Recall, we already know r_t^n and y_t^n in terms of shocks

Equilibrium - Introducing policy

Although it is natural to look for equilibrium functions

$$\begin{aligned}\pi_t &= \psi_{\pi,a}a_t + \psi_{\pi,z}z_t + \psi_{\pi,v}v_t \\ \tilde{y}_t &= \psi_{\tilde{y},a}a_t + \psi_{\tilde{y},z}z_t + \psi_{\tilde{y},v}v_t\end{aligned}$$

Galí looks for expressions in terms of the 'composite' shock, u_t

$$\begin{aligned}\pi_t &= \psi_{\pi,u}u_t \\ \tilde{y}_t &= \psi_{\tilde{y},u}u_t \\ u_t &= -\psi_{yn,a}\left(\phi_y + \sigma(1-\rho_a)\right)a_t + (1-\rho_z)z_t - v_t\end{aligned}$$

He then makes the assumption that u_t follows an AR(1)

- Given this, he derives $\psi_{\pi,u}$ and $\psi_{\tilde{y},u}$
- Only correct if one shock hits at any one time
- It *works* but it makes things unclear

Response to Shocks

Response to Shocks

We have solved for the equilibrium of the model...

- All endogenous variables have been expressed as functions of the state
- Here, the state comprises the shocks (a_t, z_t and v_t)
  - a_t is the 'productivity shock' in the production function
  - z_t is the 'time preference shock' in the utility function
  - v_t is the 'policy shock' in the Taylor rule
- We know how these evolve (as AR(1) processes) so we can simulate the economy

Response to Shocks

Thinking of variables as being functions of shocks in equilibrium leads us to ask

- What is the impact of shocks on the economy?
- How important are the various shocks in explaining the movement of variables in the economy?

We will focus on the former

- Traditional to use 'impulse response functions' to answer this question

Impulse responses

Our equilibrium implies variables, say y_t, can be expressed in terms of three shocks

$$y_t = \psi_y + \psi_{y,a} a_t + \psi_{y,z} z_t + \psi_{y,v} v_t$$

Suppose we are asked: What is the effect of $\varepsilon_{a,t}$ being δ higher than expected in $t = 1$

- We are being asked to consider the path of the economy with a particular value of $\varepsilon_{a,t}$ in the first period, relative to the path if $\varepsilon_{a,t}$ were to take its default value
- In general models, we haven't been given enough information (e.g. what is the 'default' value of $\varepsilon_{a,t}$, what happens to other shocks in the future,...), but in linear models we have - see homework

Impulse responses

Suppose the baseline path of the economy entails

$$\varepsilon_{a,t} = \varepsilon_{z,t} = \varepsilon_{v,t} = 0 \ \forall t$$

Suppose the shocked path of the economy entails

$$\begin{aligned} \varepsilon_{a,1} &= \delta_a \\ \varepsilon_{a,t} &= 0 \ \forall t > 1 \\ \varepsilon_{z,t} = \varepsilon_{v,t} &= 0 \ \forall t \end{aligned}$$

We are going to *define* the impulse response as being the difference in y_t in each period under these two paths

Impulse responses

Given the AR(1) structure of the shocks we have

- Under the baseline case (for all shocks $s \in \{a, z, v\}$ and $\forall t$)

$$s_t = \rho_s^t s_0$$

- Under the shocked case (for shocks $s \in \{z, v\}$ and $\forall t$)

$$s_t = \rho_s^t s_0$$

- Under the shocked case (for the technology shock)

$$\begin{aligned} a_1 &= \rho_a a_0 + \delta_a \\ a_t &= \rho_a^{t-1} a_1 \ \forall t > 1 \end{aligned}$$

Impulse responses

Given the AR(1) structure of the shocks we have

- Under the baseline case (for all shocks $s \in \{a, z, v\}$ and $\forall t$)

$$s_t = \rho_s^t s_0$$

- Under the shocked case (for shocks $s \in \{z, v\}$ and $\forall t$)

$$s_t = \rho_s^t s_0$$

- Under the shocked case (for the technology shock)

$$a_t = \rho_a^t a_0 + \rho_a^{t-1} \delta_a \ \forall t$$

Impulse responses

Under the baseline case

$$y_t^B = \psi_y + \psi_{y,a}\rho_a^t a_0 + \psi_{y,z}\rho_z^t z_0 + \psi_{y,v}\rho_v^t v_0$$

Under the shocked case

$$y_t^S = \psi_y + \psi_{y,a}\rho_a^t a_0 + \psi_{y,z}\rho_z^t z_0 + \psi_{y,v}\rho_v^t v_0 + \psi_{y,a}\rho_a^{t-1}\delta_a$$

Defining the impulse response in t as $\Delta_{y,a}(t) \equiv y_t^S - y_t^B$

$$\Delta_{y,a}(t) = \psi_{y,a}\rho_a^{t-1}\delta_a$$

Impulse responses - NK Application

Galí constructs composite shock u_t and imagines the impact on variable var_t of shocking each of its component in turn

$$
\begin{aligned}
var_t &= \psi_{var} + \psi_{var,u} u_t \\
u_t &= -\psi_{yn,a} \left(\phi_y + \sigma(1-\rho_a)\right) a_t + (1-\rho_z) z_t - v_t
\end{aligned}
$$

Since only one component (a_t, z_t or v_t) are shocked in turn it will be like u_t is an AR(1)

- We still talk about an innovation to ε_s for $s \in \{a, z, v\}$
- Each one will correspond to an appropriately scaled innovation to u_t

Impulse responses - NK Application

Parameter	Value	Interpretation/Justification
β	0.99	Steady state (annualized) $r = 4\%$
σ	1	Log utility
φ	5	Frisch L^s elasticity $= 0.2$
α	0.2	Hmmm...
ε	9	12.5% markup
θ	0.75	Expected price duration of 4 quarters
ϕ_π	1.5	$\approx$ Original Taylor rule
ϕ_y	0.125	$\approx$ Original Taylor rule
ρ_a	0.9	Technology shock persistence
ρ_z	0.5	Preference shock persistence
ρ_v	0.5	Policy shock persistence

Impulse responses - Monetary policy shock

Let us consider a monetary policy shock of $\varepsilon_{v,1} = 0.25$

- All else equal i_t will be higher
- But (we will see) $\pi_t \downarrow$ and $\hat{y}_t \downarrow$
- Puts downward pressure on i_t (via assumed rule)
- $\implies$ i_t rises by 'less' (but under our parameterization still rises)
- r_t rises unambiguously

This reflects a monetary 'tightening' or a 'contractionary shock'

- $\varepsilon_{v,1} < 0$ would be a 'loosening' or 'expansionary shock'

Impulse responses - Monetary policy shock

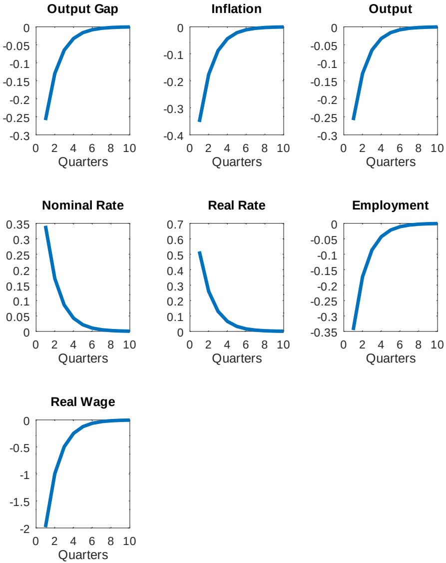
After impact the impulse response dies off smoothly and monotonically at rate ρ_v so we can just talk about the impact effect of our policy shock for the following variables

Variable	Impact Effect
Output Gap	$-$
Output	$-$
Employment	$-$
Real Wage	$-$
Real Rate	$+$
Nominal Rate	$+$
Inflation	$-$

IRFs $\rightarrow 0$ for the variables above, the LR impact on the price level is < 0

- Recall, the impact on inflation is < 0 and then remains ≤ 0
- Effect on the price level is sum of these negative numbers

Impulse responses - Monetary policy shock



Impulse responses - Monetary policy shock

- The (persistent) increase in r_t due to $i_t \uparrow$ (and $\pi_t \downarrow$) deters current expenditure
- Demand for final goods is reduced, and thus output and employment
- Recall that y_t^n does not depend on v_t so output gap declines too
- Reflecting this reduction in scale (and in marginal cost), firms who can initially reset prices set them lower, which underpins the initial $\pi_t \downarrow$ as $P_t \downarrow$
- Reduced labor demand puts downward pressure on real wage (and thus marginal cost) and this lower wage is consistent with household optimality since $-U_{n,t}/U_{c,t}$ declines due to the lower C_t and N_t that results from the shock
- Note that since $P_t \downarrow$ the real wage decline means W_t must also $\downarrow\downarrow$
- Lower wage income also contributes to lower consumption demand

Impulse responses - Time preference shock

Set δ_z to induce a 25bp decline in r_t^n

- Implies $z_t \downarrow$ on impact - as if households become more 'patient'
- Recall: In SDF Z_{t+1}/Z_t enters like β
- Like a contractionary 'demand' shock - but here not induced by policy

Variable	**Impact Effect**
Output Gap	—
Output	—
Employment	—
Real Wage	—
Real Rate	—
Nominal Rate	—
Inflation	—

Similar effects as policy shock - except in the interest rate variables

- Can set δ_z to replicate effects on non-interest rate variables *exactly*

Impulse responses - Time preference shock

Impulse responses - Time preference shock

Why do policy and preference shocks have (after appropriate scaling) the 'same effect' on economy?

- Intuitive connection between extra patience induced by preferences $\approx$ extra patience induced by prices - both act through the Euler equation
- Recall definition of u_t

$$u_t = -\psi_{yn,a}\left(\phi_y + \sigma(1-\rho_a)\right) a_t + (1-\rho_z) z_t - v_t$$

- Can scale shocks to have same impact on u_t and thus π_t and $\tilde{y}_t$
  - If $\rho_v \neq \rho_z$ then slightly more complicated but still the same message
- Why can't we say the same about a_t (next slide)?
  - v_t and z_t leave y_t^n unchanged so knowing movement in $\tilde{y}_t(\equiv y_t - y_t^n)$ is sufficient to know movement of y_t (and thus n_t, w_t^r etc.)
  - a_t affects y_t^n and this will break the equivalence (though holds for π_t)

Impulse responses - Technology shock

Consider a positive technology shock ($\delta_a > 0$)

Variable	**Impact Effect**
Output Gap	$-$
Output	$+$
Employment	$-$
Real Wage	$-$
Real Rate	$-$
Nominal Rate	$-$
Inflation	$-$

Note that some of these signs depend on our choice of parameterization

- $\sigma \geq 1$ and ψ_y sufficiently large $\Rightarrow$ a positive shock induces $n_t \downarrow$
- Accords with empirical evidence on effects of technology shocks

How can we reconcile this with procyclicality of n_t in data?

- Suggests shocks *other than technology* may be driving business cycle

Impulse responses - Technology shock

Summary

Summary

- We have described and discussed the implications of a basic New Keynesian model
  - Specified the primitives (firm and household problems and a policy rule)
  - Solved for equilibrium
  - Analyzed the effects of shocks (under a particular parameterization)
- Although the model is extremely simple. . .
  - Qualitatively, the effects of shocks are consistent with much empirical evidence
  - The model implies an important impact of nominal rigidities and monetary policy

Extra Slides

Equilibrium - Non-policy bloc

Goods-level market clearing

$$Y_t(i) = C_t(i)$$

Given the definition of C_t (recall equation (2)) it is useful to *define* 'aggregate output' as

$$Y_t \equiv \left(\int_0^1 Y_t\left(i\right)^{\frac{\varepsilon - 1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon - 1}}$$

Thus, we can connect C_t in the Euler equation to the outputs of the various goods i, yielding

$$\begin{aligned} y_t &= E_t[y_{t+1}] - \frac{1}{\sigma}\left(i_t - E_t[\pi_{t+1}] - \zeta\left(z_t\right)\right) \\ \zeta\left(z_t\right) &\equiv \rho + \left(1 - \rho_z\right) z_t \end{aligned}$$

Equilibrium - Non-policy bloc

Aggregate labor demand can be obtained by aggregating over all the firms' individual labor demands (and using $C_t = Y_t$)

$$N_t^D \equiv \int_0^1 N_t^D(i)di = \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di$$

We again will be seeking log-linearity so, taking logs and rearranging

$$n_t^D = \frac{1}{1-\alpha}(y_t - a_t)$$

where we also use

$$d_t \equiv (1-\alpha)\log\left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di\right) \overset{FO}{\approx} 0$$

Equilibrium - Non-policy bloc

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Back