DSGE and RBC

Rhys Bidder

Federal Reserve Bank of San Francisco

Michaelmas Term 2019

Disclaimer

The views expressed in this presentation, and all errors and omissions, should be regarded as those solely of the authors, and are not necessarily those of the Federal Reserve Bank of San Francisco, the Federal Reserve Board of Governors or the Federal Reserve System.

GE in an Endowment Economy

Simple consumption-savings problem of household

We begin with an optimization of a price-taking household

- Utility is intertemporally separable
- Period 'felicity' function $U(c_t^i)$
- Household i receives known endowment $\left\{y_t^i\right\}_{t=0}^{\infty}$
- Savings $\left\{a_t^i\right\}_{t=0}^{\infty}$ earn known interest rate $\{R_t = (1 + r_t)\}_0^{\infty}$
- **No uncertainty ('DGE' for now)**
- Household chooses savings and consumption

Ultimately, the problem of the household (or households) will be one part of the equilibrium

- Will also eventually need firm optimality, market clearing and feasibilty conditions...

Household budget constraint

Flow budget constraint (note timing of return on wealth in this model without uncertainty)

$$a^i_{t+s+1} = R_{t+s} a^i_{t+s} + y^i_{t+s} - c^i_{t+s} \text{ for } \forall s \geq 0$$

Iterate forward

$$\prod_{t=0}^{T} R_t^{-1} a^i_{T+1} = a^i_0 + \sum_{t=0}^{T} \left(\prod_{s=0}^{t} R_s^{-1} \right) \left(y^i_t - c^i_t \right)$$

Present value budget constraint

$$\frac{a^i_{T+1}}{\tilde{R}_T} = a^i_0 + \sum_{t=0}^{T} \frac{y^i_t - c^i_t}{\tilde{R}_t}$$

where $\tilde{R}_t \equiv R_0 R_1 R_2 \ldots R_t$

Household budget constraint

Present value budget constraint

$$\underbrace{\frac{a^i_{T+1}}{\tilde{R}_T} + \sum_{t=0}^{T} \frac{c^i_t}{\tilde{R}_t}}_{Uses} = \underbrace{a^i_0 + \sum_{t=0}^{T} \frac{y^i_t}{\tilde{R}_t}}_{Sources}$$

You only get utility from c_t sequence and suppose your 'sources' are fixed

- How to make PV of consumption $>$ than 'sources', for $T < \infty$?
- Offset with negative $\frac{a^i_{T+1}}{\tilde{R}_T}$ (interpretation?)

Let $T \to \infty$, does this seem plausible?

- Assuming $R_t > 1$ this is going to require *explosive* debt
- Standard to rule that out
- a_T can be negative in limit - it's only the PV that must be ≥ 0

Transversality condition (TVC)

No Ponzi condition to rule out explosive borrowing

- PV of terminal saving 'cannot' be strictly negative
- No-one is going to give you a free lunch

$$\lim_{T\to\infty} \frac{a^i_{T+1}}{\tilde{R}_T} \geq 0$$

PV of terminal saving 'won't' be > 0 as would be individually suboptimal

- Note this is a distinct issue from No Ponzi
- Can weakly increase c_t in all periods and strictly in at least one
- *Feasible* improvement contradicts optimality requirement

Thus, condition will in fact hold with equality in equilibrium (TVC)

- Present value BC $\Leftrightarrow$ PV of consumption $=$ PV of resources

$$\sum_{t=0}^{\infty} \frac{c^i_t}{\tilde{R}_t} = a^i_0 + \sum_{t=0}^{\infty} \frac{y^i_t}{\tilde{R}_t}$$

Household problem

$$\max_{\{c^i_{t+s}, a^i_{t+s+1}\}} \sum_{s=0}^{\infty} \beta^s U(c^i_{t+s})$$

s.t.

$$a^i_{t+s+1} = R_{t+s} a^i_{t+s} + y^i_{t+s} - c^i_{t+s} \text{ for } \forall s \geq 0$$

$$a^i_t \text{ given,}$$

$$\lim_{T \to \infty} \frac{a^i_{T+1}}{\tilde{R}_T} \geq 0$$

Solution Method #1: Direct substitution

Substitute for c^i_{t+s} in utility function using flow budget constraint

$$\max_{\{a^i_{t+s+1}\}} \sum_{s=0}^{\infty} \beta^s U(R_{t+s} a^i_{t+s} + y^i_{t+s} - a^i_{t+s+1})$$

First order condition with respect to a^i_{t+1}

$$-U_{c^i,t} + \beta U_{c^i,t+1} R_{t+1} = 0$$

Intertemporal **Euler equation** for consumption

$$\beta R_{t+1} \frac{U_{c^i,t+1}}{U_{c^i,t}} - 1 = 0$$

Method #2: Graphical

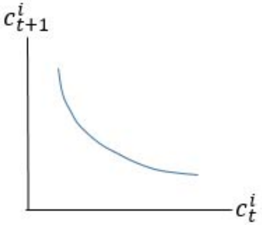
Expand utility function

$$\sum_{s=0}^{\infty} \beta^s U(c^i_{t+s}) = U(c^i_t) + \beta U(c^i_{t+1}) + \ldots = \bar{U}$$

Total differentiation taking $\bar{U}$ and c^i_{t+s} as given $\forall s \geq 2$

$$\frac{dc^i_{t+1}}{dc^i_t} = -\frac{1}{\beta}\frac{U_{c^i,t}}{U_{c^i,t+1}} = \text{MRS}$$

Indifference curve in (c^i_t, c^i_{t+1}) space



Method #2: Graphical

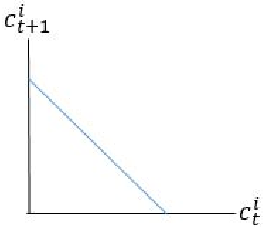
Expand budget constraint

$$a^i_{t+2} = R_{t+1}\left(R_t a^i_t + y^i_t - c^i_t\right) + y^i_{t+1} - c^i_{t+1}$$

Total differentiation taking a^i_t, a^i_{t+2}, y^i_t and y^i_{t+1} as given

$$\frac{dc^i_{t+1}}{dc^i_t} = -R_{t+1} = \mathrm{MRT}$$

Budget constraint in (c^i_t, c^i_{t+1}) space

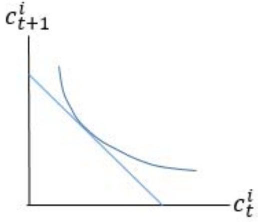


Method #2: Graphical

Optimising household sets MRS=MRT

$$\beta R_{t+1} \frac{U_{c^i,t+1}}{U_{c^i,t}} - 1 = 0$$

Optimality in (c_t^i, c_{t+1}^i) space



Method #3: Value function

Value function (this is a very brief illustration of this advanced technique)

$$V(a_t^i) = \max_{a_{t+1}^i} \left[U(R_t a_t^i + y_t^i - a_{t+1}^i) + \beta V(a_{t+1}^i) \right]$$

First order condition still involves V, which is not yet explicitly, defined

$$U_{c^i,t} = \beta V'(a_{t+1}^i)$$

Differentiate $V(a_t^i)$ w.r.t. a_t^i (an envelope condition is being used) here)

$$V'(a_t^i) = U_{c^i,t} R_t$$

Then just roll forward one period and substitute for $V'(a_{t+1}^i)$

$$\beta R_{t+1} \frac{U_{c^i,t+1}}{U_{c^i,t}} - 1 = 0$$

Method #3: Value function

Define (present value) Lagrangian

$$\mathcal{L}^i = \sum_{s=0}^{\infty} \beta^s U(c^i_{t+s}) + \sum_{s=0}^{\infty} \lambda^i_{t+s} \beta^s \left(R_{t+s} a^i_{t+s} + y^i_{t+s} - c^i_{t+s} - a^i_{t+s+1} \right)$$

First order conditions

$$\begin{array}{lcr} c^i_t & : & U_{c^i,t} = \lambda^i_t \\ c^i_{t+1} & : & U_{c^i,t+1} = \lambda^i_{t+1} \\ a^i_{t+1} & : & \lambda^i_{t+1} \beta R_{t+1} - \lambda^i_t = 0 \end{array}$$

Combine to obtain the now familiar...

$$\beta R_{t+1} \frac{U_{c^i,t+1}}{U_{c^i,t}} - 1 = 0$$

Quick check...

Q: Does the following condition pin down the sequence of consumption that solves the household's problem?

$$\beta R_{t+1} \frac{U_{c^i,t+1}}{U_{c^i,t}} - 1 = 0$$

A: No, remember we need to satisfy the budget constraint

$$\frac{a^i_{T+1}}{\tilde{R}_T} = a^i_0 + \sum_{t=0}^{T} \frac{y^i_t - c^i_t}{\tilde{R}_t}$$

The 'Euler equation' shows how the slopes of the $U_{c,t}$ profile relate to the prevailing interest rate

- Many $c_t{}_{t=0}^{\infty}$ sequences have the same slope, but are higher or lower
- Only one will exhaust the household's resources exactly

Interpretation of Euler equation for consumption

$$\beta R_{t+1} \frac{U_{c^i,t+1}}{U_{c^i,t}} - 1 = 0$$

Consider marginally less c_t

- Cost: Foregoing utility from marginal unit of c_t
- Benefit: Extra saving $\Rightarrow$ utility $\uparrow$ from c_{t+1} (traded at market rate, R)
- Marginal cost and marginal benefit should be equal at optimum

Re-arrange to see even more clearly

$$\underbrace{\beta U_{c^i,t+1} \times R_{t+1}}_{MB} = \underbrace{U_{c^i,t}}_{MC}$$

Interpretation of Euler equation for consumption

$$\beta R_{t+1} \frac{U_{c^i,t+1}}{U_{c^i,t}} - 1 = 0$$

From the perspective of the price taking household...

- Growth in $U_{c^i,t}$ is determined by R_t
- Change R_t and the household will re-optimize
- Given a functional form for $U(\cdot)$ we can recover growth in c_t

Consider the following cases:

- $\beta R_{t+1} = 1 \rightarrow U_{c^i,t+1} = U_{c^i,t} \rightarrow c^i_{t+1} = c^i_t$
- $\beta R_{t+1} > 1 \rightarrow U_{c^i,t+1} < U_{c^i,t} \rightarrow c^i_{t+1} > c^i_t$
- $\beta R_{t+1} < 1 \rightarrow U_{c^i,t+1} > U_{c^i,t} \rightarrow c^i_{t+1} < c^i_t$

Interpretation of Euler equation for consumption

$$\beta R_{t+1} \frac{U_{c^i,t+1}}{U_{c^i,t}} - 1 = 0$$

From the perspective of the price taking household...

- Growth in $U_{c^i,t}$ is determined by R_t
- Change R_t and the household will re-optimize
- Given a functional form for $U(\cdot)$ we can recover *growth in* c_t

Consider the following cases:

- $\beta R_{t+1} = 1 \rightarrow U_{c^i,t+1} = U_{c^i,t} \rightarrow c^i_{t+1} = c^i_t$
- $\beta R_{t+1} > 1 \rightarrow U_{c^i,t+1} < U_{c^i,t} \rightarrow c^i_{t+1} > c^i_t$
- $\beta R_{t+1} < 1 \rightarrow U_{c^i,t+1} > U_{c^i,t} \rightarrow c^i_{t+1} < c^i_t$

General equilibrium

So far, we haven't really made explicit the other parts of the economy

- Suppose this is an endowment economy
- No firms
- No labor market
- No *aggregate* saving technology (e.g. CA, government or capital)

Additionally, assume the households all have 'log' utility

$$U(c_t^i) = \log \left(c_t^i\right.$$

General equilibrium

Aggregate household Euler equations *with log utility*

- Add up both sides of household (i) Euler equations

$$\sum_i c_{t+1}^i = \beta R_{t+1} \sum_i c_t^i$$

Market clearing with no aggregate savings

- **Equilibrium** $\Rightarrow$ agents' atomistic decisions are consistent in aggregate

$$\sum_i y_t^i = \sum_i c_t^i \quad \text{for } \forall t$$

Market interest rate

- Solve for (endogenous) R_t
- H'holds don't care about market clearing, they optimize given prices
- **Prices must be such that the aggregate endowment is equal to aggregate consumption**

$$\beta R_{t+1} = \frac{\sum_i y_{t+1}^i}{\sum_i y_t^i}$$

General equilibrium - Representative agent

Let us define 'aggregate' consumption:

$$c_t \equiv \sum_i c_t^i$$

and 'aggregate' output y_t

$$y_t \equiv \sum_i y_t^i$$

Clearly, by our previous discussion, $c_t = y_t$

General equilibrium - Representative agent

Aggregate household Euler equations *with log utility*

- Add up both sides of household (i) Euler equations

$$\sum_i c_{t+1}^i = \beta R_{t+1} \sum_i c_t^i$$

Market clearing with no aggregate savings

- **Equilibrium** $\Rightarrow$ agents' atomistic decisions are consistent in aggregate

$$\sum_i y_t^i = \sum_i c_t^i \quad \text{for } \forall t$$

Market interest rate

- Solve for (endogenous) R_t
- H'holds don't care about market clearing, they optimize given prices
- **Prices must be such that the aggregate endowment is equal to aggregate consumption**

$$\beta R_{t+1} = \frac{\sum_i y_{t+1}^i}{\sum_i y_t^i}$$

General equilibrium - Representative agent

Aggregate household Euler equations *with log utility*

- Add up both sides of household (i) Euler equations

$$\sum_i c_{t+1}^i = \beta R_{t+1} \sum_i c_t^i$$

Market clearing with no aggregate savings

- **Equilibrium** $\Rightarrow$ agents' atomistic decisions are consistent in aggregate

$$\sum_i y_t^i = \sum_i c_t^i \quad \text{for } \forall t$$

Market interest rate

- Solve for (endogenous) R_t
- H'holds don't care about market clearing, they optimize given prices
- **Prices must be such that the aggregate endowment is equal to aggregate consumption**

$$\beta R_{t+1} = \frac{\sum_i y_{t+1}^i}{\sum_i y_t^i}$$

General equilibrium - Representative agent

Aggregate household Euler equations *with log utility*

- Add up both sides of household (i) Euler equations

$$c_{t+1} = \beta R_{t+1} \sum_i c_t$$

Market clearing with no aggregate savings

- **Equilibrium** $\Rightarrow$ agents' atomistic decisions are consistent in aggregate

$$y_t = c_t \quad \text{for } \forall t$$

Market interest rate

- Solve for (endogenous) R_t
- H'holds don't care about market clearing, they optimize given prices
- **Prices must be such that the aggregate endowment is equal to aggregate consumption**

$$\beta R_{t+1} = \frac{y_{t+1}}{y_t}$$

General equilibrium - Representative agent

The same conditions we had before are satisfied under the aggregate (and under the 'average' - divide by N or integrate over a mass of households)

- This shows that this economy admits a 'representative' agent
- Equilibrium prices and aggregate/average quantities can be obtained as if there were one 'representative' (competitive) household with the aggregate endowment process as its 'income'

Note, we haven't said all households have the same $c_{i,t}$ or $y_{i,t}$

- Doesn't matter for aggregates or solving for R_t in this case
- To obtain them, simply use R_t, Euler equation and budget constraint and whatever y_t^i is relevant to your problem/case
- Any split of the y_t pie into individual y_t^i will be consistent with the aggregate endogenous variables found

Note, also, the importance that they were facing the same prices

General equilibrium - Some subtle points

Recall the condition

$$\beta R_{t+1} = \frac{y_{t+1}}{y_t}$$

and compare with the Euler equation of a given household, i

$$\beta R_{t+1} = \frac{c^i_{t+1}}{c^i_t}$$

In the latter case, we could talk about the interest rate *causing* consumption growth for the household

- The household is a price taker and does not individually affect R

In the former, our equilibrium assumption $\Rightarrow$ the *exogenously specified* y_t process necessitates a particular R_t

- In richer models (with labor, capital accumulation etc.) 'output growth' will also be endogenous
- Causal relations between endogenous variables are tricky in an equilibrium and may involve feedback

Ramsey Growth Model

Ramsey model

Production economy, rather than endowment

- Log utility
- Labor supply exogenous (constant and normalized to unity)
- Constant returns to scale in production function
- 100% depreciation of capital ('investment' in $t = K_{t+1}$)
- Technology, θ_t, grows at rate g
- Infinitely-lived *representative* agent

Household problem in Ramsey

Representative household maximizes lifetime utility

$$\max_{\{C_{t+s}, K_{t+s+1}\}} \sum_{s=0}^{\infty} \beta^s \log C_{t+s}$$

s.t.

$$K_{t+s+1} = r_{t+s} K_{t+s} + w_{t+s} - C_{t+s} \; \forall s \geq 0$$

K_t given,

$$\lim_{T \to \infty} \frac{K_{T+1}}{\tilde{R}_T} \geq 0$$

Euler equation for consumption as before

$$\frac{C_{t+1}}{\beta C_t} = r_{t+1}$$

Household problem in Ramsey

Representative household maximizes lifetime utility

$$\max_{\{C_{t+s},K_{t+s+1}\}} \sum_{s=0}^{\infty} \beta^s \log C_{t+s}$$

s.t.

$$\overbrace{K_{t+s+1}}^{\textit{Investment}} = \overbrace{\underbrace{r_{t+s}K_{t+s} + w_{t+s}}_{\textit{Earnings}} - C_{t+s}}^{\textit{Savings}} \quad \forall s \geq 0$$

$$K_t \text{ given,}$$

$$\lim_{T\to\infty} \frac{K_{T+1}}{\tilde{R}_T} \geq 0$$

Euler equation for consumption as before

$$\frac{C_{t+1}}{\beta C_t} = r_{t+1}$$

Household problem in Ramsey

Representative household maximizes lifetime utility

$$\max_{\{C_{t+s}, K_{t+s+1}\}} \sum_{s=0}^{\infty} \beta^s \log C_{t+s}$$

s.t.

$$K_{t+s+1} = r_{t+s} K_{t+s} + w_{t+s} \underbrace{L_t^S}_{\equiv 1} - C_{t+s} \ \forall s \geq 0$$

$$K_t \text{ given,}$$

$$\lim_{T \to \infty} \frac{K_{T+1}}{\tilde{R}_T} \geq 0$$

Euler equation for consumption as before

$$\frac{C_{t+1}}{\beta C_t} = r_{t+1}$$

Firm problem in Ramsey

Firm maximizes profits

$$\max_{K_t, L_t^D} \left(K_t^{\alpha} \left(\theta_t L_t^D \right)^{1-\alpha} - w_t L_t^D - r_t K_t \right)$$

First order conditions define factor prices as before

$$\begin{aligned} r_t &= \alpha K_t^{\alpha-1} \left(\theta_t L_t^D \right)^{1-\alpha} \\ w_t &= (1-\alpha) K_t^{\alpha} \theta_t \left(\theta_t L_t^D \right)^{-\alpha} \end{aligned}$$

Firm problem in Ramsey

Why have I written L_t^D and L_t^S rather than L_t?

- $L_t^S \equiv 1$ isn't important
- What *is* important is that outside equilibrium (without appropriate 'prices') there's no reason to think $L_t^S = L_t^D$
- We need w_t and r_t to (among other things) induce firms to set L_t^D - thus clearing markets

$L_t^S = L_t^D$ is an equilibrium condition

- Incorporating it allows us to use L_t as the amount of labor (supplied and demanded) in equilibrium
- I should really be doing this for K_t^S and K_t^D but life's too short and typos too numerous...

In a price taking world, who sets the prices to the 'necessary' values?

- Hmmmm,... ponder...
- Anyway, regardless, these prices induce an equilibrium

Equilibrium in Ramsey

A competitive equilibrium is a sequence

$$\{r_t, w_t, C_t, K_t, L_t\}_{t=0}^{\infty}$$

s.t.

1. $\{C_t, K_{t+1}\}$ solves household problem given $\{r_t, w_t\}$
2. $\{K_t, L_t\}$ solves firm problem given $\{r_t, w_t\}$
3. Markets clear $L_t = 1$ and $C_t + K_{t+1} = Y_t = K_t^{\alpha} (\theta_t L_t)^{1-\alpha}$
4. K_t given
5. TVC

Equilibrium in Ramsey

Euler equation for consumption and return to capital imply

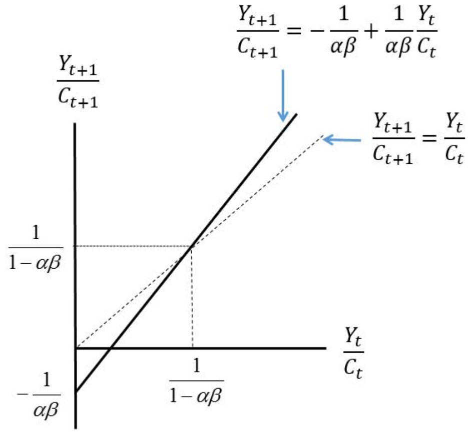
$$\frac{1}{C_t} = \alpha\beta \frac{1}{C_{t+1}} K_{t+1}^{\alpha-1} \left(\theta_{t+1} L_{t+1}\right)^{1-\alpha}$$

Substitute in for market clearing conditions and rearrange

$$\frac{Y_{t+1}}{C_{t+1}} = -\frac{1}{\alpha\beta} + \frac{1}{\alpha\beta} \frac{Y_t}{C_t}$$

Only stable solution is $\frac{Y_t}{C_t} = \frac{1}{1-\alpha\beta}\ \forall t \geq 0$

Equilibrium in Ramsey



Only stable solution is $\frac{Y_t}{C_t} = \frac{1}{1-\alpha\beta}\ \forall t \geq 0$

Equilibrium in Ramsey with technological change

- $Y_t/C_t,\ Y_t/K_{t+1}$ constant
- θ_t, C_t, K_t, Y_t grow at rate g
- $r_t = \alpha K_t^{\alpha-1} (\theta_t L_t)^{1-\alpha} = \alpha \tilde{K}_t^{\alpha-1}$ constant
- $w_t = (1-\alpha) K_t^{\alpha} \theta_t (\theta_t L_t)^{-\alpha} = (1-\alpha) \theta_t \tilde{K}_t^{\alpha}$ grows at rate g

Check Kaldor (1957) facts

1. Output per worker grows at a roughly constant rate YUP
2. Capital per worker grows over time YUP
3. Capital/output ratio is roughly constant YUP
4. Rate of return to capital is constant YUP
5. Shares of capital and labor in net income are nearly constant YUP
6. Real wage grows over time YUP
7. Ratios of consumption and investment to GDP are constant YUP

Endogenous growth models

Growth is exogenous in this model

- θ_t an exogenous process

Endogenous growth models seek to explain growth

- Not covering in depth - but may be in a problem set, say

Extended accumulation models

- Overcome diminishing returns to capital by adding externalities in capital accumulation (learning-by-doing and AK model)
- Alternatively, additional factors of production (human capital)

Innovation models

- Explain technological progress as a function of endogenous variables
- E.g. Investment in R&D

Growth accounting

Take logs (see lect. 1) and differentiate $Y_t = K_t^{\alpha} (\theta_t L_t)^{1-\alpha}$ to decompose into weighted percentage growth rates

$$\frac{1}{Y_t}\frac{dY_t}{dt} = \alpha\frac{1}{K_t}\frac{dK_t}{dt} + (1-\alpha)\frac{1}{\theta_t}\frac{d\theta_t}{dt} + (1-\alpha)\frac{1}{L_t}\frac{dL_t}{dt}$$

Capital share of income in equilibrium$= \alpha$, so set $\approx 1/3$ (as in historical data)

$$\begin{aligned}\text{Growth in } Y_t \quad &= \quad \frac{1}{3} \times \text{Growth in } K_t \\ &\quad + \frac{2}{3} \times (\text{Growth in } \theta_t + \text{Growth in } L_t)\end{aligned}$$

Growth accounting in the 20^{th} century, Crafts (2000)

1913-50	Output growth	Contribution of TFP	Contribution of capital	Contribution of labour
Japan	2.2%	0.7%	1.2%	0.3%
UK	1.3%	0.4%	0.8%	0.1%
US	2.8%	1.3%	0.9%	0.6%
Germany	1.3%	0.3%	0.6%	0.4%

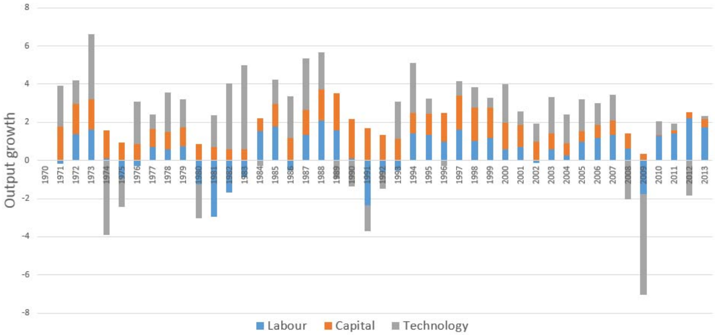
1950-73	Output growth	Contribution of TFP	Contribution of capital	Contribution of labour
Japan	9.2%	3.6%	3.1%	2.5%
UK	3.0%	1.2%	1.6%	0.2%
US	3.9%	1.6%	1.0%	1.3%
Germany	6.0%	3.3%	2.2%	0.5%

1973-92	Output growth	Contribution of TFP	Contribution of capital	Contribution of labour
Japan	3.8%	1.0%	2.0%	0.8%
UK	1.6%	0.7%	0.9%	0.0%
US	2.4%	0.2%	0.9%	1.3%
Germany	2.3%	1.5%	0.9%	-0.1%

Growth accounting in emerging markets 1960-1994

	Output growth	Contribution of TFP	Contribution of capital	Contribution of labour
Hong Kong	7.3%	2.4%	2.8%	2.1%
Indonesia	5.6%	0.8%	2.9%	1.9%
Korea	8.3%	1.5%	4.3%	2.5%
Philippines	3.8%	-0.4%	2.1%	2.1%
Singapore	8.1%	1.5%	4.4%	2.2%
South Asia	4.2%	0.8%	1.8%	1.6%
Latin America	4.2%	0.2%	1.8%	2.2%
Africa	2.9%	-0.6%	1.7%	1.8%
Middle East	4.5%	-0.3%	2.5%	2.3%

Growth accounting for the UK 1970-2013, ONS (2015)



Average UK growth of 2.1%

- 0.4% from technology
- 0.2% from capital
- 0.5% from labor

Monetary Models - Background

Quantity theory of money

Much discussion (though not in recent years) of the effects of monetary policy began with some form of the following equation

$$M_t V_t \equiv P_t Y_t \tag{1}$$

where M_t is the quantity of money, V_t the velocity of its circulation, P_t the general price level and Y_t real output.

Equation (1) is an *identity* (always true by definition)

- Holds regardless of CB targeting interest rate, i_t, or M directly
- Uninteresting unless a theory restricts behavior of at least one variable

(Long run) 'classical dichotomy' / 'quantity theory of money'

- In the long run, Y and V are determined by non-monetary factors
- Thus, M and P move 1:1 in the long run

Quantity theory of money

Economists agree about little but...

- The *long run* classical dichotomy is arguably an exception
- Long run growth determined by 'supply' factors - not monetary
- In LR, monetary factors only influence *nominal*, but not *real* variables
- Long run correlations between M and $P \approx 1$ in the data

Almost complete consensus that *'there is no long-run trade-off between the rate of inflation and the rate of unemployment'* - Taylor (1996)

Fisher equation

Its rare nowadays for central banks to use M_t as an explicit policy tool

- Typically now set a short term nominal interest rate, i_t
- Given 'money demand', the central bank adapts money supply so market clears at desired i_t

In this context, the 'quantity equation' is less intuitive - instead the 'Fisher equation' is useful to aid understanding

$$i_t = r_t + E_t[\pi_{t+1}] \tag{2}$$

where i_t/r_t is the nominal/real interest rate and π_t is (net) inflation.

The (long run) classical dichotomy implies that r_t is unrelated to monetary factors

- i_t and inflation move 1:1, conditional on r_t
- If r_t changes without a change in i_t inflation adjusts

Interaction of real and nominal factors

In the shorter run the classical dichotomy is not broadly accepted

- Money/interest rates and output (or other measures of activity, such as unemployment) appear to co-move
- Central banks' activities are predicated on the assumption that changing i_t induces a change in r_t

Co-movements are suggestive that there is a connection between real and nominal variables (see Ch. 1 Walsh)

- Lead-lag correlations $\Rightarrow$ high M_t typically precedes high Y_t
- Cyclical movements in money track those of GDP growth fairly well until early 80s
- Short term nominal rates generally track - and somewhat precede - cyclical movements in GDP

Interaction of real and nominal factors

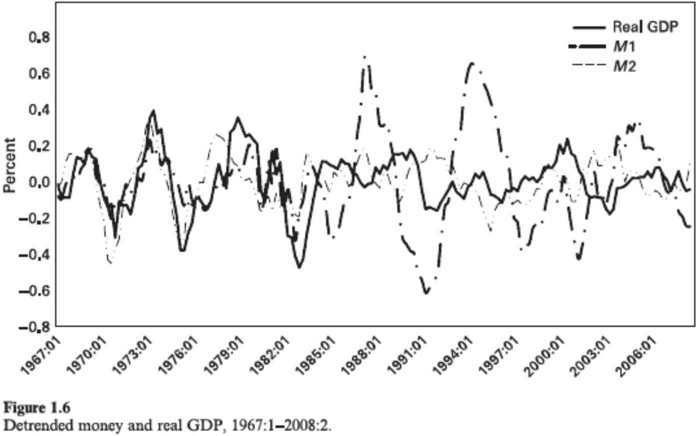


Figure 1: De-trended money and output (from Walsh Ch.1)

Interaction of real and nominal factors

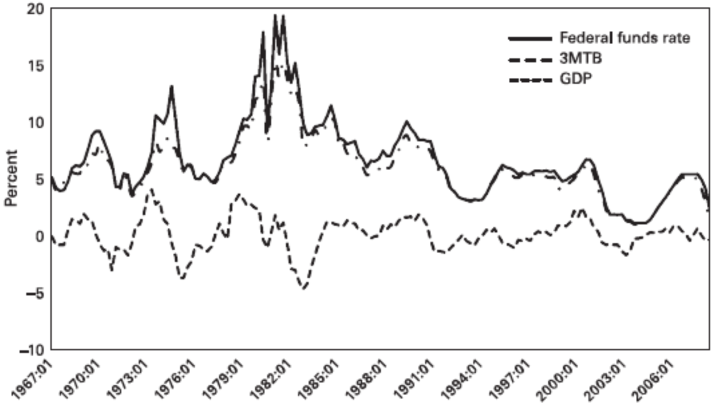


Figure 1.7
Interest rates and detrended real GDP, 1967:1–2008:2.

Figure 2: Short rate and de-trended output (from Walsh Ch.1)

Interaction of real and nominal factors

Difficult to disentangle direction of causality

- Are these movements *induced* by monetary policy or is policy *responding* to the economy?
- Even if policy variable appears to lead activity, we face the *post hoc ergo propter hoc* fallacy

Want to examine periods after an unanticipated 'shock' from policymakers

- Derives purely from policymaker - not economy - and thus closer to a 'natural experiment'
- Then estimate propagation of shock through economy
  - If M_t is the tool, does it affect M/P and real activity, rather than passing 1:1 into prices? - Recall 'quantity theory'
  - If i_t is the tool, does it affect r_t and real activity, rather than passing 1:1 into $E_t[\pi_{t+1}]$? - Recall Fisher equation

Identifying monetary policy shocks and their effects

Friedman and Schwarz (1963) and narrative approaches

- Seminal work on influence of monetary policy in the U.S. (most notably in the Great Depression)
    - Documentary evidence to isolate ΔM unrelated to economic conditions
    - Suggests that fluctuations in money supply led to those in real activity
    - Related to case study analyses of disinflationary policy (Sargent (1986))
- Influential - but problematic elements in empirical approach
    - Later support from Romer and Romer (1989) in a more modern form
    - Further strengthened by Romer and Romer (2004)

Other recent work on obtaining measures of policy surprises

- Nakamura and Steinsson (2013) and Gertler Karadi (2015) use 'high frequency information'
- Look at asset price movements in short intervals around FOMC announcements to identify 'surprises'

Identifying monetary policy shocks and their effects

On three occasions the System deliberately took policy steps of major magnitude which cannot be regarded as necessary or inevitable economic consequences of contemporary changes in money income and prices. Like the crucial experiments of the physical scientist, the results are so consistent and sharp as to leave little doubt about their interpretation. The dates are January-June 1920, October 1931, and June 1936-January 1937

- Friedman and Schwarz, 1963, p.688

Identifying monetary policy shocks and their effects

There was another major anti-inflationary shock to monetary policy on October 6, 1979. In effect, the Federal Reserve decided that its measures over the previous year had been unsuccessful in reducing inflation and that much stronger measures were needed. Although the shift in policy was to some extent presented as a technical change, the fact that it was intended to lead to considerably higher interest rates and lower money growth was clear. For example, "the Committee anticipated that the shift . . . would result in ... a prompt increase ... in the federal funds rate"

- Romer and Romer, 1989, p.142

Identifying monetary policy shocks and their effects

Friedman and Schwarz (1963) and narrative approaches

- Seminal work on influence of monetary policy in the U.S. (most notably in the Great Depression)
  - Documentary evidence to isolate ΔM unrelated to economic conditions
  - Suggests that fluctuations in money supply led to those in real activity
  - Related to case study analyses of disinflationary policy (Sargent (1986))
- Influential - but problematic elements in empirical approach
  - Later support from Romer and Romer (1989) in a more modern form
  - Further strengthened by Romer and Romer (2004)

Other recent work on obtaining measures of policy surprises

- Nakamura and Steinsson (2013) and Gertler Karadi (2015) use 'high frequency information'
- Look at asset price movements in short intervals around FOMC announcements to identify 'surprises'

Vector Autoregressions

Another powerful approach to assessing the impact of monetary policy shocks is Vector Autoregression (VAR) analysis (see Walsh Ch. 1 for this example)

> *While researchers have disagreed on the best means of identifying policy shocks, there has been a surprising consensus on the general nature of the economic responses to monetary policy shocks. A variety of VARs estimated for a number of countries all indicate that, in response to a policy shocks, output follows a hump-shaped pattern in which the peak impact occurs several quarters after the initial shock.*

- Walsh, 1998, p.31

See discussion in lecture 1

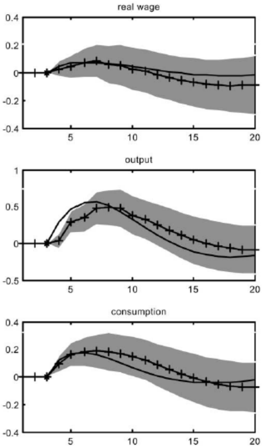


Figure 3: Subset of IRFs from CEE (2005)

Vector Autoregressions

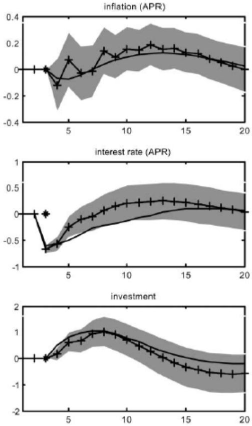


Figure 4: Subset of IRFs from CEE (2005)

Vector Autoregressions

VAR approach useful but not without flaws/critics...

- Monetary policy shocks are now likely rare/small (Ramey (2016))
  - Makes it more difficult to use *policy* shocks to estimate impact of policy
  - Can still estimate role of policy but need *other* shocks **and** a model
- Changes in underlying parameters (Lucas critique and Rational Expectations)
  - Response to shocks depends on parameters describing policymakers' approach **and** all other parameters in the economy (e.g. regulation, preferences...)
  - If they change, previous estimates may become obsolete
- Responses may not be accurately recovered even from data generated by a model (Chari *et al* (2008))
  - Can still use, but in a 'moment matching' exercise *involving a model*
  - VARs on actual data and on data from model - minimize discrepancy

More problems with VAR analysis...

- Unhelpful for welfare analysis
  - Knowing that policy affects activity is one thing...
  - ...but knowing *how it should try to affect it* is another
  - Agent's optimization problems must be explicit for micro-founded welfare analysis
  - VARs are silent on this
- Story telling / incorporation of microeconomic evidence
  - Policymakers like to understand/explain transmission mechanism
  - Elements of models (such as, say, household risk aversion) can be pinned down by evidence from experiments/more granular research
  - Not possible with VARs (or very difficult)

A lot of these 'problems' can be addressed by using a model...

Role of monetary policy - DSGE models

DSGE models initially associated with the 'Real Business Cycle' literature

- Seminal work of Kydland and Prescott (1982) and Prescott (1986)
- No (or minor) distortions ⇒ despite fluctuations (the 'business cycle') the economy is always efficient
- Limited role of monetary policy - main shocks were 'real' (technology)

Beautiful models - but unsatisfactory in various dimensions

- If monetary policy was included, optimal policy looked nothing like real world practice
- Effects of policy shocks often counterfactual (recall evidence discussed above)
- Hard to reconcile dominant role of technology shocks with. . .
  - *Unconditional* positive comovement of employment and output in data
  - Empirical studies ⇒ *technology* shocks move them in opposite direction

Role of monetary policy - DSGE models

The challenge:

- Keep the 'good' aspects of RBC models. . .
- . . . while enhancing ability to explain and justify monetary policy

Role of monetary policy - DSGE models

New Keynesian modeling was a response to this challenge

- Interpret as a **micro-founded** formalization of 'Keynesian' ideas
  - IS-LM analysis 'much less careful' - rather *ad hoc*
  - 'We are an equation short' (Keynes) - price setting not modeled
- Emphasis on **nominal rigidities** as a reason for fluctuations
  - Empirical evidence of significant price (incl. wage) stickiness
  - Taylor (1999), Bewley (1999), Dhyne *et al* (2006), Nakamura and Steinsson (2008)...
- (Very) loosely - sticky prices $\Rightarrow$ **Classical Dichotomy broken**
  - ΔM_t shock doesn't *immediately* pass 1:1 to prices
  - Δi_t shock doesn't *immediately* pass 1:1 to $E_t[\pi_{t+1}]$ - thus r_t changes
- Explicit distortions from **imperfect competition** and sticky prices implies **economy not fully efficient**
  - Scope for policy to be welfare-enhancing