MFE Economics
Problem set 4

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1. New Keynesian Model - Price setting and Intuition

As discussed in class and shown in the textbook (P. 60-63) we have

$$\pi_t = \beta E_t[\pi_{t+1}] - \lambda\hat{\mu}_t \quad (1)$$
$$= \beta E_t[\pi_{t+1}] + \kappa\tilde{y}_t \quad (2)$$

where

$$\lambda \equiv \theta^{-1}(1-\theta)(1-\beta\theta)\Theta > 0$$
$$\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$$
$$\kappa \equiv \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$$

and where

$$\hat{\mu}_t \equiv \mu_t - \mu$$
$$\mu_t \equiv p_t - \psi_t$$
$$\tilde{y}_t \equiv y_t - y_t^n$$

- Use equation (1) to show that

  $$\pi_t = -\lambda\sum_{k=0}^{\infty}\beta^k E_t[\hat{\mu}_{t+k}]$$

  HINT: Use equation (1) in t+1 to get an expression for π_{t+1} and substitute it into the (1) on the RHS in the expectation, then use (1) in t+2 in a similar way. Keep going and you'll see the pattern emerging. You'll have terms like $E_t[E_{t+1}[E_{t+2}\ldots]]]$. For this you will need to use the 'law of iterated expectations' which means that $E_t[E_{t+j}[X]] = E_t[X]$ for a random variable X and $j \geq 0$. Basically it means your expectation of 'X' now (with your limited information available at t) of your expectation in the future of 'X' (with greater information at $t+j$) is simply equal to your expectation now - which is intuitive. So $E_t[E_{t+1}[E_{t+2}\ldots]]]$ is just $E_t[\ldots]$

- Show that λ is decreasing in the 'Calvo parameter', θ and briefly discuss how a greater degree of price stickiness influences the value of inflation associated with a given expected path of markup deviations?[1]

- As discussed in class and shown in the textbook (P. 62) we have

$$\begin{aligned}
\mu_t &= p_t - \psi_t \\
&= -(w_t - p_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&= -(\sigma y_t + \varphi n_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&= -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)
\end{aligned} \tag{3}$$

  [HARDER - don't spend too long] Briefly give intuition for the dependence of the coefficient on y_t in equation (3) on σ, φ and α. Hint: Note that the negative of the log markup is log real marginal cost. The intuition may be easier if you discuss the coefficient in terms of real marginal cost and its association with the output level...

$$mc_t \equiv \psi_t - p_t = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t - \left(\frac{1+\varphi}{1-\alpha}\right) a_t - \log(1-\alpha)$$

2. DIS and NKPC

The dynamic IS curve (DIS) and the New Keynesian Phillips curve (NKPC) are given by

$$\begin{aligned}
y_t &= E_t[y_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - \rho) + \frac{1}{\sigma}(1-\rho_z) z_t \\
\pi_t &= \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t
\end{aligned}$$

First we will consider the DIS curve and go through a few thought experiments to investigate some of its properties.

- Sketch a plot of the DIS with y_t on the horizontal axis and i_t on the vertical axis (rearrange the DIS to have i_t on the LHS and then sketch - don't worry about exact numbers, but generally people think σ is a positive number greater than 1 and β is something like 0.99.)
- What is the slope? Give intuition in a sentence or two.

Note that implicitly the location of the curve depends on assumed values for the structural parameters (σ, ρ_z and $\rho \equiv -\log(\beta)$), the other endogenous variables that feature in the DIS ($E_t[y_{t+1}]$, $E_t[\pi_{t+1}]$) and the preference shock (z_t). Recall, also, that in our simple model $y_t = c_t$ (in richer models it might also feature investment and government expenditure).

- What happens to the curve in the following situations? Give intuition.

[1] For this you will need to recall that

$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + g'(x)f(x) \text{ and } \frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$$

- Suppose something shifts $E_t[y_{t+1}]$ up (and increase in confidence about the future, say), holding all else equal.
- Suppose something shifts $E_t[\pi_{t+1}]$ up (some inflationary expectations pick up), holding all else equal.
- What if z_t increases, holding all else equal.
- What if β increases, or σ?

- In a few sentences, why is 'holding all else equal' an unnatural assumption in this context?
- Redraw the DIS but now with the real interest rate on the vertical axis
  - Suppose, again, something shifts $E_t[\pi_{t+1}]$ up, holding all else equal. What movement is associated with this, now that we have re-drawn the DIS? Is it a curve shift? Comment briefly.

Now we turn to the NKPC and consider some more thought experiments. Think about plotting it in $(\tilde{y}_t, \pi_t)$ space (sketch it if you want)

- What happens to the curve in the following situations? Give intuition.
  - What happens if something shifts $E_t[\tilde{y}_{t+2}]$ up, holding all else equal (again, an unnatural assumption). Give intuition in a couple of sentences.
  - What if θ increases, or φ? Give intuition in a couple of sentences.

3. Useful prep for next lecture (try it and then come back after the lecture): Autoregressive processes

In the models we consider, there are random 'shock' or 'driving' processes that are exogenous to the model.[2] In our case these shocks (technology, time preference and monetary policy) will constitute the 'state' of the economy in the sense that all the endogenous variables (consumption, output, wages, interest rates) will in equilibrium be expressible as functions of these three variables (or subsets thereof, depending on the model). All the shocks we consider, when logged, follow an autoregressive process of order 1 or, for short, an $AR(1)$. Consider the technology shock A_t from the production function $Y_t = A_t N_t^{1-\alpha}$. When expressed in logs it follows this process[3]

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_{a,t} \\ \varepsilon_{a,t} &\overset{iid}{\sim} N(0, \sigma_a^2) \\ a_t &\equiv \log(A_t) \end{aligned} \tag{4}$$

[2]You can brush up on random variables here, on Normal variables here and on expected value (or 'the mean') here.

[3]Note that is convenient to model it this way as it means that A_t, while random, will always be positive (as makes sense for a technology term that multiplies - some function of - labor to produce non-negative output). Frequently in economics or finance we use the exponential of a random variable to obtain a transformed random variable that is positive. In fact, exponentials have other nice properties when working with Normal distributions.

The random variable, $\varepsilon_{a,t}$ (what I will often refer to as an 'innovation'), is a Gaussian or 'Normal' variable with zero mean ($E_t[\varepsilon_{a,t+1}] = 0$) and variance, σ_a^2 ($E_t[(\varepsilon_{a,t+1} - 0)^2] = \sigma_a^2$). It is also independently and identically distributed (iid) which means that there is no dependence between its draws in different periods or between its draw and any other variables and the distribution from which it is drawn ($N(0, \sigma_a^2)$) is constant over time. The parameter ρ_a will be referred to as a 'persistence' parameter. We will always (in this course) consider cases where $|\rho_a| \in (0, 1)$ - in the language of stochastic processes, this ensures that it is a 'stationary' process.

- Show that[4]

$$a_t = \sum_{j=0}^{J-1} \rho_a^j \varepsilon_{a,t-j} + \rho_a^J a_{t-J} \tag{5}$$

Suppose data started in $t = 0$, so that a_0 is just given to us, then we just set $J = t$ in the above expression. If there is no explicit starting point then we can use the assumption on $|\rho_a| \in (0, 1)$ to state

$$a_t = \sum_{j=0}^{\infty} \rho_a^j \varepsilon_{a,t-j}$$

since $\rho_a^J a_{t-J} \to 0$ as $J \to \infty$. Intuitively, if the effects of shocks dies of to zero in the limit - and given that our shocks are well behaved - we can ignore the last term in expression (5) you just derived because it gets arbitrarily small.

- What is the effect of an innovation j periods ago on a_t (i.e. the effect of $\varepsilon_{a,t-j}$)?
- What is the effect of an innovation in t on a_{t+1}? On a_{t+2}? On a_{t+j}?

Any effect an innovation in t has on future values of a_t, say a_{t+j}, may be flooded by the effects of future innovations in later periods before period $t + j$. But it is still useful to talk about the effect the innovation has as this nevertheless does *contribute* to a_{t+j} (look back at the expression you derived above - a_t is made up of a weighted sum of all current and previous innovations, with those weights declining as the innovation period recedes into the distant past). Given the process we are considering and the iid assumptions made on $\varepsilon_{a,t}$, an innovation today does affect the expected value - from the perspective of today - of future values of the technology shock.[5]

- What is the expected value of a_{t+1} given information available in t (i.e. given you know a_t)?
- What is the expected value of a_{t+2} given information available in t

[4]HINT: Use equation (4) but for earlier periods (a_{t-1}, a_{t-2} etc.) to repeatedly replace the lagged values of a_t on the right hand side of equation (4). This is a bit like the first part of the first question above - but you're now going back in time rather than forwards.

[5]To answer questions involving expectations below, recall that the expectations operator is linear (in particular, that means $E_t[X + Y] = E_t[X] + E_t[Y]$), the expectation of a constant (or something already known when the expectation is being formed) is the constant itself and the expectation of a scalar constant times a random variable is the scalar constant times the expectation of the random variable.

- What is the expected value of a_{t+j} given information available in t
- What is the expected value of $\Delta a_{t+1} \equiv a_{t+1} - a_t$ given information available in t
- How does today's (t) innovation affect your expectation of a_{t+j} relative to the expectation you held in $t-1$ before you knew $\varepsilon_{a,t}$?

4. Optional/Hard: Optimal price setting and practice linearization

Use a first order approximation of

$$\sum_{k=0}^{\infty} \theta^k E_t \left[\Lambda_{t,t+1} Y_{t+k|t} \frac{1}{P_{t+k}} \left(P_t^* - \mathcal{M} \Psi_{t+k|t} \right) \right] = 0 \tag{6}$$

around the zero inflation steady state, to obtain (as claimed in the text p. 57 2nd ed.)

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t[\psi_{t+k|t}]$$