A Classical Monetary Model

Rhys Bidder

Federal Reserve Bank of San Francisco

Michaelmas Term 2019

Disclaimer

The views expressed in this presentation, and all errors and omissions, should be regarded as those solely of the authors, and are not necessarily those of the Federal Reserve Bank of San Francisco, the Federal Reserve Board of Governors or the Federal Reserve System.

Roadmap

Roadmap

In this lecture we consider a (very) simple monetary economy

- The role of money will be limited to that of a 'unit of account'
- See textbook for alternative setup where money yields 'utility' through services as a 'means of exchange'

Essentially, this economy is an RBC model with a trivial monetary block

- The 'Classical dichotomy' holds in the long run **and** the short run
- All 'real' variables can be pinned down without knowing anything about monetary policy or the general price level
- **This is not a New Keynesian model**

So why bother?

- To introduce various concepts that are in common with the NK model
- But without the distractions of the NK model's other special features

Roadmap

The model features three sets of agents...

1. Households
   - Large number of households with identical tastes
   - Take prices as given (competitive)
   - ‘Complete markets’ and utility maximization $\Rightarrow$ all do the same thing
   - Allows us to work with a *‘representative’ household*
2. Firms
   - Large number of firms with identical technology/production possibilities
   - Take prices as given (competitive)
   - Produce a single homogenous good (to be consumed by households)
   - Profit maximization $\Rightarrow$ all do the same thing
   - Allows us to work with a *‘representative’ firm*
3. Monetary policymaker (‘central bank’)
   - Only matters for price level - not for consumption/production

Roadmap

We will solve for how the real variables behave in **'equilibrium'**

- Find out what households/firms want to do (at a given set of prices)
- Require that goods demanded (households) = goods supplied (firms)
- Require that labor demanded (firms) = labor supplied (households)
- Obtain (real) wage and (real) interest rate that achieves this market clearing (it won't happen under an arbitrary set of prices)

The final piece of the **'equilibrium'** is to assume the central bank's behavior and derive what that implies for the price level

- Money is a unit of account
- Aggregate price level, P_t, is price of consumption good
- The 'real' part of the equilibrium is consistent with many P_t processes
- Properties of a policy 'interest rate rule' influence the price processes possible in equilibrium

Households

Households - Objective function

Objective function of a (representative) household

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t; Z_t\right) \right]$$

- $U\left(C_t, N_t; Z_t\right)$ - Period felicity function
- C_t and N_t - Consumption and 'hours worked' in time t
- Z_t - 'Preference shifter'
- β - Time discount factor where $\beta \in (0, 1)$
- $E_t\left[\cdot\right]$ - Expectation operator, conditional on time-t information

$$U\left(C_t, N_t; Z_t\right)$$

'Well behaved'

- $U_{c,t} \equiv \frac{\partial U_t}{\partial C_t} > 0$ - likes consumption
- $U_{cc,t} \equiv \frac{\partial^2 U_t}{\partial C_t^2} \leq 0$
- $U_{n,t} \equiv \frac{\partial U_t}{\partial N_t} \leq 0$ - dislikes work
- $U_{nn,t} \equiv \frac{\partial^2 U_t}{\partial N_t^2} \leq 0$

Preference shifter 'increases' $U_{c,t}$

- $U_{cz,t} \equiv \frac{\partial^2 U_t}{\partial C_t \partial Z_t} > 0$
- $Z_t \uparrow \Rightarrow$ consumption in t more highly valued on the margin

Households - Utility function

We will adopt convenient special cases, depending on σ:

$$U\left(C_t, N_t; Z_t\right) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t & : \sigma \neq 1 \\ \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right) Z_t & : \sigma = 1 \end{cases}$$

and we assume that $z_t \equiv \log Z_t$ follows an AR(1)

$$\begin{aligned} z_t &= \rho_z z_{t-1} + \varepsilon_t^z \\ \varepsilon_t^z &\overset{iid}{\sim} N\left(0, \sigma_z^2\right) \end{aligned}$$

- σ controls attitudes to intertemporal substitution
- φ controls disutility of labor
- Effective time discount factor from perspective of t is $\frac{Z_{t+j}}{Z_t}\beta^j$

Households - Budget constraint(s)

Households choose dynamic plans for C_t and N_t subject to a sequence of 'flow' budget constraints

$$P_t C_t + Q_{n,t} B_t \leq B_{t-1} + W_t N_t + D_t$$

- P_t - Price of consumption good
- W_t - Nominal wage
- D_t - Dividends from firms owned by households
- $Q_{n,t}$ - Price of bond that pays unit of money in $t+1$ for certain
  - *Nominally* riskless 'discount' bond
  - Guaranteed \$1 tomorrow for each bond purchased today
  - 'Real' payoff tomorrow is unknown today (because P_{t+1} is unknown)

We also require a 'no-Ponzi' condition (discussed last week and in ex. 2)

$$\lim_{T\to\infty} E_t\left[\Lambda_{t,T}\frac{B_T}{P_T}\right] \geq 0$$

Households - Budget constraint(s)

What is the implied (nominal) return from t to $t+1$?

- Bond sold at t for $Q_{n,t}$
- Pays one unit (of money) at maturity in $t+1$
- Return on bond is therefore $Q_{n,t}^{-1}$

Why call it a 'discount' bond?

- Natural to think of $Q_{n,t} < 1$ so bond *'sold at a discount'* to generate a positive 'interest rate'

$$1 + r_{n,t} \equiv R_{n,t} \equiv Q_{n,t}^{-1} > 1$$

Households - Stochastic discount factor

$$\Lambda_{t,t+1} \equiv \beta \frac{U_{c,t+1}}{U_{c,t}} = \beta \frac{Z_{t+1}}{Z_t} \left(\frac{C_t}{C_{t+1}} \right)^{\sigma}$$

Encodes time discounting due to β

- When evaluating from t onward, $Z_t \uparrow$ acts like lower β (less patient)

Encodes how payoffs under different *contingencies* are valued

- Marginal payoff in $t+1$ will be valued using marginal utility in $t+1$
- 'Stochastic' from t perspective as C_{t+1} and Z_{t+1} not yet known
- Re-normalized by marginal utility in t

'Discounting' is really about relative value

- 'Dislike' (like) distant (immediate) payoffs
- 'Dislike' (like) payoffs when marginal utility is low (high)

Households - Stochastic discount factor

Temporarily ignore Z so SDF becomes...

$$\Lambda_{t,t+1} \equiv \beta \frac{U_{c,t+1}}{U_{c,t}} = \beta \left(\frac{C_t}{C_{t+1}} \right)^{\sigma}$$

Assume n possible outcomes for consumption in $t+1$

- Values: $\{C(i)\}_{i=1}^{n}$
- Probabilities (given info. in t): $\{p_i\}_{i=1}^{n}$

Let an asset pay $Y(i)$ in $t+1$ when $C_{t+1} = C(i)$

- Value of asset obtained by *discounting* random payoffs with $\Lambda_{t,t+1}$

$$V_t^y \equiv E_t[Y_{t+1}\Lambda_{t,t+1}] = \sum_{i=1}^{n} p_i Y(i)\Lambda(i) = \beta \sum_{i=1}^{n} p_i Y(i) \left(\frac{C_t}{C(i)} \right)^{\sigma}$$

Households - Stochastic discount factor

SDF for longer horizons are implicit given definition of 1-period SDF

$$
\begin{aligned}
\Lambda_{t,t+2} &= \Lambda_{t+1,t+2}\Lambda_{t,t+1} \\
\Lambda_{t,t+3} &= \Lambda_{t+2,t+3}\Lambda_{t,t+2} = \Lambda_{t+2,t+3}\Lambda_{t+1,t+2}\Lambda_{t,t+1} \\
&\ldots
\end{aligned}
$$

Use this recursion to see

$$\Lambda_{t,t+j} = \beta^j \frac{U_{c,t+1}}{U_{c,t}} \frac{U_{c,t+2}}{U_{c,t+1}} \ldots \frac{U_{c,t+j}}{U_{c,t+j-1}} = \beta^j \frac{U_{c,t+j}}{U_{c,t}}$$

Recall no-Ponzi condition (which holds for all t)

$$\lim_{T \to \infty} E_t \left[\Lambda_{t,T} \frac{B_T}{P_T} \right] \geq 0$$

Agent can't have positive debt in the infinite future (as valued from t)

Households - Stochastic discount factor

SDF for longer horizons are implicit given definition of 1-period SDF

$$
\begin{aligned}
\Lambda_{t,t+2} &= \Lambda_{t+1,t+2}\Lambda_{t,t+1} \\
\Lambda_{t,t+3} &= \Lambda_{t+2,t+3}\Lambda_{t,t+2} = \Lambda_{t+2,t+3}\Lambda_{t+1,t+2}\Lambda_{t,t+1} \\
&\dots
\end{aligned}
$$

Use this recursion to see

$$\Lambda_{t,t+j} = \beta^j \frac{U_{c,t+1}}{U_{c,t}} \frac{U_{c,t+2}}{U_{c,t+1}} \dots \frac{U_{c,t+j}}{U_{c,t+j-1}} = \beta^j \frac{U_{c,t+j}}{U_{c,t}}$$

Recall no-Ponzi condition (which holds for all t)

$$\lim_{T\to\infty} E_t \left[\Lambda_{t,T} \frac{B_T}{P_T}\right] \geq 0$$

Agent can't have positive debt in the infinite future (as valued from t)

Households - Optimality conditions

Maximizing utility subject to the sequence of budget constraints implies:

- 'Intratemporal' optimality (labor-consumption tradeoff in t)

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

- 'Intertemporal' optimality or 'Euler equation' (tradeoff between C_t and C_{t+1} implicit in choice of B_t)

$$Q_{n,t} = E_t\left[\Lambda_{t,t+1}\frac{P_t}{P_{t+1}}\right]$$

Households - Optimality conditions

$$-U_{n,t} = \frac{W_t}{P_t} U_{c,t}$$

Consider marginally more N_t

- Cost: Foregone leisure on margin
- Benefit: Extra wages earned, convertible to additional consumption

$$\frac{Q_{n,t}}{P_t} U_{c,t} = \beta E_t \left[U_{c,t+1} \frac{1}{P_{t+1}} \right]$$

Consider marginally less C_t (more B_t)

- Cost: Forgoing utility from marginal unit of consumption today
- Benefit: Extra saving $\Rightarrow$ raises utility from consumption tomorrow
- Similar intuition from last lecture - but here expectations and a *stochastic* discount factor are involved

Households - Optimality conditions

We also have an additional optimality condition

$$\lim_{T\to\infty} E_t\left[\Lambda_{t,T}\frac{B_T}{P_T}\right] = 0$$

'Similar' to the no-Ponzi constraint

$$\lim_{T\to\infty} E_t\left[\Lambda_{t,T}\frac{B_T}{P_T}\right] \geq 0$$

Why $= 0$ and not ≥ 0? We discussed this last lecture...

- > 0 undesirable for agent
- Would be creditor 'in the limit'
- Saving/lending too much
- Can do better by increasing consumption path in some manner

Households - Optimality conditions

Useful to convert/approximate these conditions with a ‘log-linear’ form

- We covered linearizations in the first exercise (will be in third, too)
- See also the accompanying notes on this topic online
- Notation: Lower case m for variable M means $m \equiv \log(M)$

Households - Optimality conditions

If we take logs of both sides of the intratemporal condition and re-arrange...

$$n_t = \frac{1}{\varphi}\left(w_t - p_t - \sigma c_t\right)$$

Given marginal utility of consumption (captured by σc_t) this yields a 'labor supply relation'

$$n_t = \tilde{n}^s\left(w_t, p_t; c_t\right)$$

It is perhaps more natural to think in terms of the (log) real wage

$$\begin{aligned} n_t &= n^s\left(w_t^r; c_t\right) \\ w_t^r &\equiv w_t - p_t \end{aligned}$$

The 'Frisch elasticity', φ^{-1}, controls sensitivity of n to w^r

Households - Optimality conditions

Using a log-linear approximation of the intertemporal condition we obtain

$$c_t = E_t\left[c_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) + \frac{1}{\sigma}\left(\rho + \left(1 - \rho_z\right) z_t\right)$$

where $\rho \equiv -\log \beta$

The 'nominal interest rate' is defined as (recall discount bond discussion)

$$i_t \equiv -\log\left(Q_{n,t}\right)$$

The gross inflation rate is defined as

$$\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$$

and we refer to the 'inflation rate', $\pi_{t+1} \equiv \log\left(\Pi_{t+1}\right)$

Households - Optimality conditions

$$
\begin{aligned}
c_t &= E_t\left[c_{t+1}\right] - \frac{1}{\sigma}\left(\overbrace{i_t - E_t\left[\pi_{t+1}\right]}^{\text{Real Interest Rate}}\right) + \frac{1}{\sigma}\left(\underbrace{\rho + (1-\rho_z)\, z_t}_{\text{Time Discount}}\right) \\
&= E_t\left[c_{t+1}\right] - \underbrace{\frac{1}{\sigma}}_{EIS}\left(r_t - \zeta\left(z_t\right)\right)
\end{aligned}
$$

where we define the real interest rate as (market terms of trade)

$$r_t \equiv i_t - E_t\left[\pi_{t+1}\right]$$

and a ‘composite’ discount term (reflecting preferences)

$$\zeta\left(z_t\right) \equiv \rho + (1-\rho_z)\, z_t$$

Households - Optimality conditions

$$
\begin{aligned}
c_t &= E_t\left[c_{t+1}\right] - \frac{1}{\sigma}\left(\overbrace{i_t - E_t\left[\pi_{t+1}\right]}^{\text{Real Interest Rate}}\right) + \frac{1}{\sigma}\left(\underbrace{\rho + (1-\rho_z)\, z_t}_{\text{Time Discount}}\right) \\
&= E_t\left[c_{t+1}\right] - \underbrace{\frac{1}{\sigma}}_{EIS}\left(\underbrace{r_t}_{Market} - \underbrace{\zeta\left(z_t\right)}_{Preferences}\right)
\end{aligned}
$$

where we define the real interest rate as (market terms of trade)

$$
r_t \equiv i_t - E_t\left[\pi_{t+1}\right]
$$

and a ‘composite’ discount term (preferences)

$$
\zeta\left(z_t\right) \equiv \underbrace{\rho}_{\text{'Average' Impatience } (-\log\beta)} + \underbrace{(1-\rho_z)\, z_t}_{\text{Impatience Shock}}
$$

Households - Optimality conditions

$$E_t\left[\Delta c_{t+1}\right] = \frac{1}{\sigma}\left(r_t - \zeta\left(z_t\right)\right)$$

Shape of consumption path is influenced by

- Time discounting (β and path of z_t)
- Terms of trade for tilting consumption path (r_t)

Elasticity of intertermporal substitution (EIS) $= \sigma^{-1}$

- Controls willingness to reallocate consumption over time
- Example: Suppose σ is big (so EIS is low)
  - Consider an increase in r_t keeping all else equal
  - More incentive for an increasing consumption profile
  - But planned increase in C_{t+1} relative to C_t will be 'small'
- Recall homework 2 - calculating EIS

Firms

Firms

There is a large number of identical firms producing a homogenous consumption good using same production function

$$Y_t = A_t N_t^{1-\alpha}$$

The 'technology' process, $a_t \equiv \log A_t$, follows an AR(1)

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_t^a \\ \varepsilon_t^a &\overset{iid}{\sim} N\left(0, \sigma_a^2\right) \end{aligned}$$

Maximize profits in each period, taking price and wage as given

$$P_t Y_t - W_t N_t$$

Firms - Optimality condition

Firms choose how much labor to hire, leading to the optimality condition

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

In log linear form,

$$w_t - p_t = a_t - \alpha n_t + \log(1-\alpha)$$

Given technology, this defines labor demand relations in terms of the (log) nominal wage and consumption good price or, alternatively, the real wage

$$\begin{aligned} n &= \tilde{n}^d(w, p; a_t) \\ n &= n^d(w^r; a_t) \end{aligned}$$

Firms - Optimality condition (MC interpretation)

Note that we can rewrite the labor demand condition as follows

$$P_t = \frac{W_t}{(1-\alpha) A_t N_t^{-\alpha}}$$

making explicit that optimality requires price $=$ marginal cost

- Labor is the only input
- To produce a marginal unit of output one needs $\frac{1}{MPL}$ of labor
- That additional labor is paid W_t
- Hence $W_t \times \frac{1}{MPL}$ is the cost of the marginal output

Equilibrium

Equilibrium conditions - real block

In the previous slide we had 7 equations relating 7 unknowns

- c_t, y^s_t, n^d_t, n^s_t, w^r_t, S^r_t, r_t

Additionally, the equations involved...

- 'Deep' or 'structural' parameters
  - Explicitly: α, σ, φ, $\rho \Leftrightarrow \beta$, ρ_z
  - Implicitly (in the expectation): ρ_a, σ_a and σ_z
- Exogenous driving processes / shocks
  - z_t and a_t

When economists speak of 'solving' for a model's equilibrium they mean...

- Find an expression for endogenous variables in terms of the 'state'
- In our case, the state, s_t, is (a_t, z_t)

Where's the 'curve shifting'?

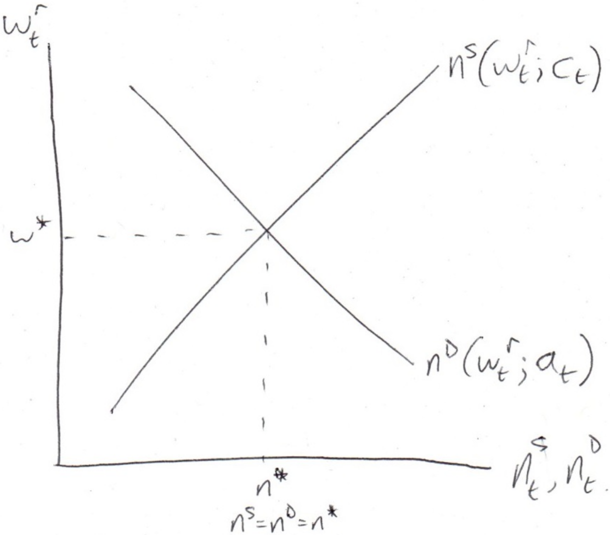
We just derived a labor demand and a labor supply curve

- $n^d(w_t^r; a_t)$ from firm optimality
- $n^s(w_t^r; c_t)$ from household optimality

Draw a diagram, shift the curves around $\Leftrightarrow$ Economics

- I apologize for what you are about to see. . .

Where's the 'curve shifting'?



Where's the 'curve shifting'?

In *modern* macro, we must consider G.E. effects

- Spillovers and interdependence
- Multiple simultaneous directions of causality
- Connections between variables required by aggregate/market clearing constraints

Need to think *very* carefully before shifting curves

- Especially if we imagine shifting one in isolation
- Why is it shifting?
- What part of the state has changed?

Where's the 'curve shifting'?

In equilibrium, all endogenous variables in t are functions of the state in t

- Though we haven't yet solved for it, $c_t = c(s_t)$ in equilibrium
- Note that the firm's labor demand only depends on a_t

Since household optimality $\Rightarrow$ desired labor supply depends on a_t *and* c_t we can re-write the labor supply curve as a function of w_t^r where its location in (n_t, w_t^r) space depends on s_t

$$n_t^s = \hat{n}^s(w_t^r; s_t) \equiv n^s(w_t^r; c(s_t))$$

Where's the 'curve shifting'?

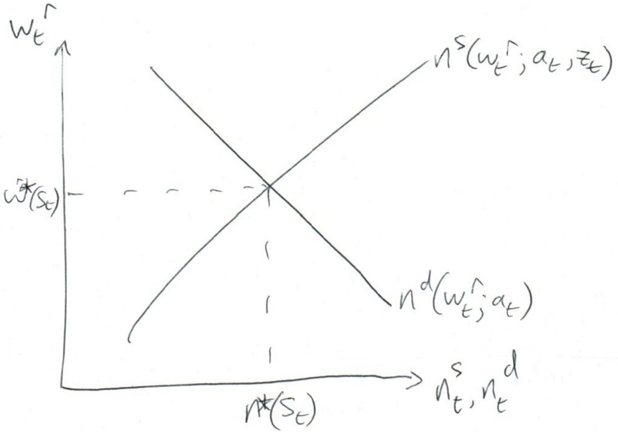


Figure 2: Labor demand and supply

Where's the 'curve shifting'?

In this simple model (keeping preference parameters etc. fixed) can you shift one curve and not the other?

- No if it is a_t that is changing
- (Maybe) yes if it is z_t that is changing (only n^s shifts - or does it. . . ?)

Finally, note that w_t^r (the real wage) is also an equilibrium object

- To find the function by which equilibrium c_t depends on s_t we will need to use other equations (mkt. clearing, production function etc.)
- Without that we can't find the intersection with the n^d curve as the location of the labor supply curve is unknown
- You now see why I talk about solving a system of equations. . .

Bachmann *et al* (2013)

Competitive equilibrium

Equilibrium requires that optimality conditions are respected but also. . .

- Decisions are consistent and feasible in aggregate
- Prices ensure that market clearing occurs

Initially we will only derive the equilibrium values of ‘real’ variables

- Consumption
- Output
- Employment
- Real wage
- Real savings
- Real interest rate

We will then model the nominal variables separately

- Separation of real and nominal is a *very* special aspect of this model
- Does not hold in NK model (that’s the point. . .)

Equilibrium conditions - real block

1. Final good market clearing (demand = supply)
$$c_t = y_t^s$$
2. Labor market clearing (demand = supply)
$$n_t^d = n_t^s$$
3. Aggregate final goods supply (technical constraint)
$$y_t^s = a_t + (1-\alpha)\, n_t^d$$
4. Labor supply (optimality)
$$w_t^r = \sigma c_t + \varphi n_t^s$$
5. Labor demand (optimality)
$$w_t^r = \log(1-\alpha) + a_t - \alpha n_t^d$$
6. (Real) savings (accounting)
$$S_t^r = \exp(y_t^s) - \exp(c_t)$$
7. Euler equation (optimality)
$$r_t = \rho + (1-\rho_z)\, z_t + \sigma E_t [\Delta c_{t+1}]$$

Solving for the model's equilibrium - real block

First we note that we can use market clearing to say

$$n_t^s = n_t^d = \mathbf{n_t}$$
$$c_t = y_t^s = \mathbf{y_t}$$

so we can just talk in terms of employment (n_t) and output (y_t)

Then we have

$$\begin{aligned} y_t &= a_t + (1-\alpha)\, n_t & (1) \\ w_t &= \log(1-\alpha) + a_t - \alpha n_t & (2) \\ w_t &= \sigma y_t + \varphi n_t & (3) \end{aligned}$$

Using equations (2) and (3) we obtain

$$y_t = \frac{1}{\sigma}\left(\log\left(1-\alpha\right) + a_t - \left(\alpha+\varphi\right) n_t\right)$$

Combining with equation (1) $\Rightarrow n_t$ in terms of exogenous variables

$$\begin{aligned} n_t &= \psi_n + \psi_{n,a} a_t \qquad (4) \\ \psi_n &\equiv \frac{\log\left(1-\alpha\right)}{\left(1-\alpha\right)\sigma + \alpha + \varphi} \\ \psi_{n,a} &\equiv \frac{1-\sigma}{\left(1-\alpha\right)\sigma + \alpha + \varphi} \end{aligned}$$

Substituting back into equation (1) yields

$$\begin{aligned} y_t &= \psi_y + \psi_{y,a} a_t \qquad (5) \\ \psi_y &\equiv \left(1-\alpha\right)\psi_n \\ \psi_{y,a} &\equiv \frac{1+\varphi}{\left(1-\alpha\right)\sigma + \alpha + \varphi} \end{aligned}$$

Solving for the model's equilibrium - real block

We can use the labor supply condition, goods market clearing and the equilbrium expression for n_t to obtain

$$\begin{aligned} w_t &= \sigma y_t + \varphi n_t \\ &= \sigma(\psi_y + \psi_{y,a} a_t) + \varphi(\psi_n + \psi_{n,a} a_t) \end{aligned}$$

So we obtain...

$$\begin{aligned} w_t &= \psi_w + \psi_{w,a} a_t \\ \psi_w &\equiv \frac{(\sigma(1-\alpha)+\varphi)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} \\ \psi_{w,a} &\equiv \frac{\sigma+\varphi}{(1-\alpha)\sigma+\alpha+\varphi} > 0 \end{aligned} \tag{6}$$

Solving for the model's equilibrium - real block

$$\psi_{n,a} \equiv \frac{1-\sigma}{(1-\alpha)\,\sigma+\alpha+\varphi}$$

Income effect (through $c\uparrow$) vs. substitution effect (through $w\uparrow$)

- $\sigma \to 1$ (log utility) then $\psi_{n,a} \to 0$
- $\sigma > 1 \Leftrightarrow EIS < 1 \Leftrightarrow \psi_{n,a} < 0$
- $\sigma < 1 \Leftrightarrow EIS > 1 \Leftrightarrow \psi_{n,a} > 0$
- Income (substitution) effect dominates with $EIS < (>)1$
- Strength of response attenuated as $\varphi\uparrow$

$$\psi_{y,a} \equiv \frac{1+\varphi}{(1-\alpha)\,\sigma+\alpha+\varphi} > 0$$

If $\sigma \to 1$ (log utility) then $\psi_{y,a} \to 1$

- Only change in output must come from a_t without any $n_t\uparrow$

Solving for the model's equilibrium - real block

$$\psi_{w,a} \equiv \frac{\sigma + \varphi}{(1-\alpha)\sigma + \alpha + \varphi} > 0$$

Wages are pro-cyclical under technology-induced fluctuations

- Both $\psi_{y,a}$ and $\psi_{w,a}$ are positive

It is immediate to obtain expressions for other variables (setting aside r_t)

- c_t: We have $c_t = y_t$ so same as for y_t
- Savings follow from simple accounting

Solving for the model's equilibrium - real block

Note that $S_t^r \equiv \frac{Q_{n,t} B_t}{P_t} = 0$

- Real value of nominal bond holdings is zero
- Assuming positive prices, this implies $B_t = 0$
- Identical households means no one holds bonds **in equilibrium**
  - All households have same optimality conditions
  - If *anybody* wants to save, then *everybody* wants to save!
  - Not feasible because no one takes the other side of the trade

Sort of obvious from the start - why?

- No way to transfer resources to future *in aggregate*
- Other models: possible via investment in 'capital' or foreign borrowing

Equilibrium requires optimality *and* feasibility

Solving for the model's equilibrium - real block

This is one way of seeing that, despite appearances, we could have guessed earlier that the aggregate n^s curve would not depend on z_t

- Imposing $c_t = y_t$ and rearranging the labor supply condition (to get the supply curve)

$$
\begin{aligned}
n_t^s &= \frac{1}{\varphi} w_t - \frac{\sigma}{\varphi} c_t \\
&= \frac{1}{\varphi} w_t - \frac{\sigma}{\varphi} y_t
\end{aligned}
$$

- From firm optimality, labor demand is only going to depend on a_t
- But, in equilibrium, labor demand pins down output, y_t, through the production function

So the curves can't be shifted independently, even with a z_t impulse

Where's the 'curve shifting'? - Revisited

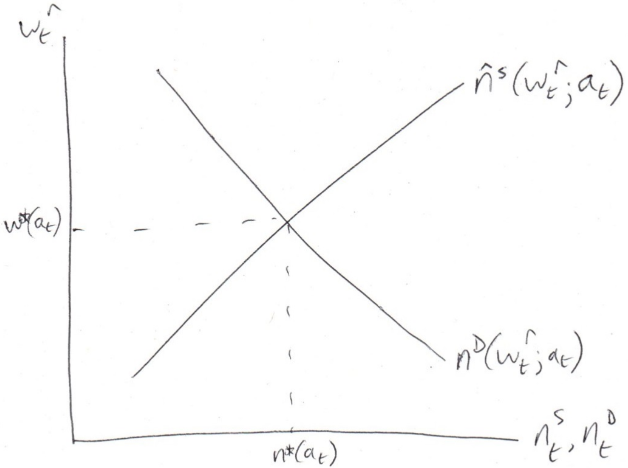


Figure 3: Labor demand and supply

Solving for the model's equilibrium - real block

- Households don't know/care about equilibrium
  - They just see prices and choose what's best for them individually
- It is our *assumption of equilibrium* (which must feature zero savings in this case) that pins down the necessary prices
  - How do these prices come about in a competitive model (where everyone is a price taker)?
  - Deep questions (see tatonnement, bargaining, the 'core', ...)
  - In NK model, (goods) price setting is determined explicitly by firms' decisions
- *In equilibrium* prices must induce zero savings as optimal choice
  - z_t affects the relative desirability of consumption today vs. tomorrow
  - $z_t \uparrow \Rightarrow$ people want to save less *for a given* r_t
  - So r_t must be a function of z_t in equilibrium

Finally, we derive the equilibrium real interest rate using...

- Intertemporal optimality (Euler equation)
- $c_t = y_t$ (where zero savings most obviously gets incorporated)
- The 'solution' for y_t
- AR(1) process for a_t

$$\begin{aligned} r_t &= \rho + (1 - \rho_z) z_t + \sigma E_t [\Delta y_{t+1}] \\ &= \rho + (1 - \rho_z) z_t - \sigma (1 - \rho_a) \psi_{y,a} a_t \\ &\equiv \psi_r + \psi_{r,a} a_t + \psi_{r,z} z_t \end{aligned}$$

In a 'perfect foresight' steady state where $z_t = a_t = 0$ we have

$$r = \rho \equiv -\log(\beta)$$

The steady state real interest rate is determined by time preference

Solving for the model's equilibrium - real block

At this point we have...

- Expressions of the form

$$var_t = \psi_{var} + \psi_{var,a} a_t + \psi_{var,z} z_t$$

  for $var \in \{c_t, y_t, n_t, w_t^r, r_t\}$
- Exact expressions for savings ($= 0$)
- The AR(1) specifications for a_t and z_t

These components constitute the 'solution' of the real side of the model for all the real variables of interest

Equilibrium - Nominal Block

Solving for the model's equilibrium - nominal block

- The nominal aspects of the model have no implications for the behavior of real variables
- As such, they are irrelevant for welfare and imply no role for policy
- Nevertheless it is useful to consider how the behavior of P_t depends on monetary policy
- Policy is captured by a rule for the *nominal* interest rate i_t

Solving for the model's equilibrium - nominal block

Recall the Fisher equation

$$i_t = r_t + E_t[\pi_{t+1}]$$

In this model, r_t is already pinned down on the real side

- $\Rightarrow$ conditional on r_t, inflation expectations and i_t move 1:1
- This applies in the short run (not simply *long run* Classical dichotomy)

For a given steady state rate of inflation, π we can thus obtain the associated steady state value of i_t

$$i = \rho + \pi$$

since (recall) $r_t = \rho$ in steady state

Solving for the model's equilibrium - nominal block

Let us consider a simple policy rule for i_t

$$\begin{aligned} i_t &= \rho + \pi + \phi_\pi(\pi_t - \pi) + v_t \\ &= r + \pi + \phi_\pi(\pi_t - \pi) + v_t \end{aligned}$$

where $\phi_\pi > 0$ determines the strength of the response of policy to deviations in inflation from π

v_t is a policy 'shock' that follows an AR(1)

$$\begin{aligned} v_t &= \rho_v v_{t-1} + \varepsilon_t^v \\ \varepsilon_t^v &\overset{iid}{\sim} N\left(0, \sigma_v^2\right) \end{aligned}$$

What is v_t?

- A random, temporary deviation from 'usual' policy conduct
- Note: In recent years v_t likely have become 'smaller' (Ramey 2016)

Solving for the model's equilibrium - nominal block

If we subtract the policy rule from the Fisher equation we obtain (where we define $\hat{\pi} \equiv \pi_t - \pi$ and $\hat{r} \equiv r_t - r$)

$$\hat{\pi}_t = \frac{1}{\phi_\pi} E_t[\hat{\pi}_{t+1}] + \frac{1}{\phi_\pi} (\hat{r}_t - v_t)$$

If $\phi_\pi > 1$ then (under reasonable assumptions) we can solve for

$$\hat{\pi}_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t \left[\hat{r}_{t+k} - v_{t+k}\right]$$

Given our earlier solution for r_t in equilibrium and the assumed processes for a_t, z_t and v_t we can rewrite this as

$$\hat{\pi}_t = \frac{\sigma(1-\rho_a)\psi_{ya}}{\phi_\pi - \rho_a} a_t + \frac{1-\rho_z}{\phi_\pi - \rho_z} z_t - \frac{1}{\phi_\pi - \rho_v} v_t$$

Solving for the model's equilibrium - nominal block

With a solution for (an) equilibrium process for π_t (equivalently, $\hat{\pi}_t$) the process for prices arises from

$$p_t \equiv p_{t-1} + \pi_t$$

which allows the conversion of real wages to nominal

Under $\phi_\pi > 1$ this is the *only* solution for the nominal elements of equilibrium

- Under $\phi_\pi < 1$ we have 'equilibrium indeterminacy'
- Multiple equilibria are possible with the same 'real' block but with different processes for inflation
- Hints that may be desirable to keep $\phi_\pi > 1$ (the 'Taylor principle')
    - But in *this* economy *all* the equilibria are welfare-equivalent since consumption and employment are the same

Summary

Summary

- The model we have described has various unrealistic implications
  - In particular there is no role for monetary policy to influence the economy as empirical evidence suggests it should
  - Price setting is left undefined - in contrast to real world experience of sticky prices etc.
  - There is also no reason for policy to intervene, given efficiency of allocations
- However, it has allowed us to introduce various concepts
  - General equilibrium
  - Multiple sectors (households and firms)
  - Monetary policy 'rule' for i_t
- We now move to the New Keynesian framework which is conceptually richer and empirically more successful

Escape Slides

Bachmann, Cabellero and Engel (2013)

While reading papers for my own work, I found this nice statement in Bachmann *et al* (2013)...

> *Our calibration begins by noting that the objective in any dynamic macroeconomic model is to trace the impact of aggregate shocks on aggregate endogenous variables*

Bachmann, Cabellero and Engel (2013)

This reflects the standard attitude in modern macro

- The state typically will not only involve exogenous shocks
- The state may involve accumulated capital (or complicated objects like the distribution of wealth over households) that themselves are endogenous, but predetermined in t
- But we typically talk about economies responding to ‘innovations’ or ‘impulses’ to the shock process
- These innovations are - in the context of the model - the only true ‘source’ of curve shifts
- If you know how *all* the ‘curves’ shift, then you can calculate fluctuations in the endogenous variable in equilibrium

Bachmann, Cabellero and Engel (2013)

Think in terms of...

- ...variables being solved to satisfy equilibrium requirements
- ...dependence on state
- ...innovations to shocks

(Shifting curves $\rightarrow$ easy pictures, difficult (maybe wrong) concepts)

C. *Recursive Equilibrium*

A *recursive competitive equilibrium* is a set of functions

$$(\omega, p, V^1, N, K, C, N^h, \Gamma),$$

that satisfy

(i) *Production unit optimality*: Taking ω, p, and Γ as given, $V^1(\epsilon_S, \epsilon_I, k; z, \mu)$ solves (10) and the corresponding policy functions are $N(\epsilon_S, \epsilon_I, k; z, \mu)$ and $K(\epsilon_S, \epsilon_I, k, \xi; z, \mu)$.

(ii) *Household optimality*: Taking ω and p as given, the household's consumption and labor supply satisfy (13) and (14).

(iii) *Commodity market clearing*:

$$C(z,\mu) = \int z\epsilon_S\epsilon_I k^\theta N(\epsilon_S, \epsilon_I, k; z, \mu)^\nu \, d\mu - \int\int_0^{\bar{\xi}} [\gamma K(\epsilon_S, \epsilon_I, k, \xi; z, \mu) - (1-\delta)k] \, dG d\mu.$$

(iv) *Labor market clearing*:

$$N^h(z,\mu) = \int N(\epsilon_S, \epsilon_I, k; z, \mu) \, d\mu + \int\int_0^{\bar{\xi}} \xi \mathcal{J}(\gamma K(\epsilon_S, \epsilon_I, k, \xi; z, \mu) - \psi(1-\delta)k) \, dG d\mu,$$

where $\mathcal{J}(x) = 0$, if $x = 0$ and 1, otherwise.

(v) *Model consistent dynamics*: The evolution of the cross section that characterizes the economy, $\mu' = \Gamma(z, \mu)$, is induced by $K(\epsilon_S, \epsilon_I, k, \xi; z, \mu)$ and the exogenous processes for z, ϵ_S, and ϵ_I.

Conditions (i), (ii), (iii), and (iv) define an equilibrium given Γ, while step (v) specifies the equilibrium condition for Γ.

Figure 4: Equilibrium definition in Bachmann *et al* (2013)

Back