Lab 1 – The Apollo Missions

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I. Introduction

The Apollo mission strives to send humans on the Saturn V rocket to the Moon. For this goal to be accomplished, we must effectively determine that the rocket will be able to escape the gravitational pull of the Earth and make it to the Moon safely. Such determination can be made by calculations of physics concepts and simple determinations of the change of speed and altitude reached of the rocket. In this report, the gravitational potential and force of the Earth and Moon system will be determined, and the burn time and altitude reached will be calculated and compared to actual values.

II. The Gravitational Potential of the Earth-Moon System

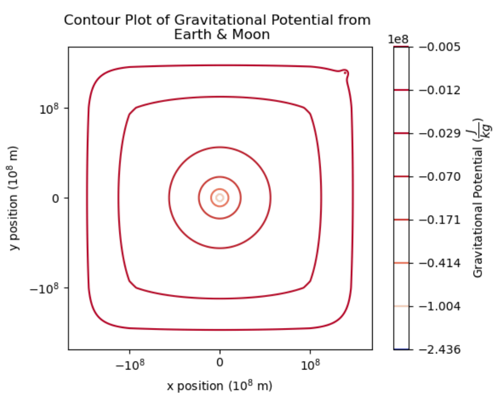
Before examining the performance of the Saturn V system itself, examining the physics is instrumental to understanding how to interpret it. One indicator of how strong an object is pulling (or exerting a force) on an object is via the gravitational potential, which is the measurement of the strength of a gravitational field at a certain point. The equation to determine the gravitational potential, Φ, is as follows:

$$\Phi = -\frac{GM}{r}$$

where G is the Newtonian gravitational constant, $6.67 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$, M is the mass of the object that is pulling on the object in question, and r is the distance from the pulling object to the object in question. In this case, the gravitational potential of this system of the Earth and Moon is:

$$\Phi_{total} = -\frac{GM_{Earth}}{(x - x_{Earth})} + \frac{GM_{Moon}}{(x - x_{Moon})}$$

where $M_{Earth} = 5.9 \times 10^{24} kg$, $M_{Moon} = 7.3 \times 10^{22} kg$, $x_{Earth} = 0\ m$ (setting it as the center of the system), and $x_{Moon} = (\frac{3.8 \times 10^8}{\sqrt{2}})m$. With this, the gravitational potential of the Earth-Moon system radially from the Earth to 1.5 times the distance from the Earth to the Moon is shown in Figure 1:



where the contour lines indicate the potential at a certain point. As seen, the potential decreases the further away the object is from Earth, and the Moon does not exert much of a change in the potential.

III. The Gravitational Force of the Earth-Moon System

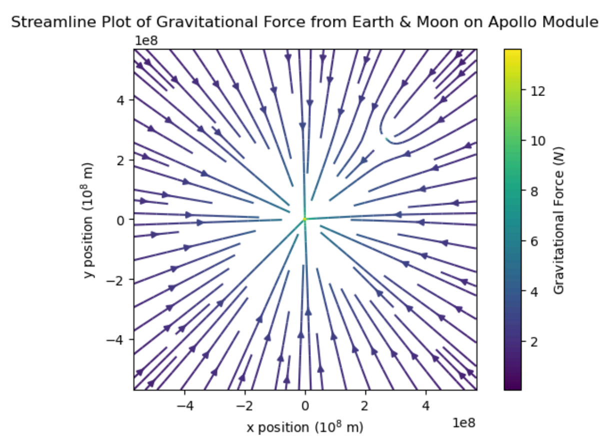
Another indicator of how strong the Earth pulls on objects is a plot of the gravitational force exerted on an object. The gravitational force at a certain point can be calculated via the following equation:

$$\overline{F}_{21} =- G\frac{M_1 m_2}{\left|\overline{r_{21}}\right|^2}\hat{r}_{21}$$

where M_1 is the mass of the larger object, m_2 is the mass of the smaller object, and r_{21} is the vector of the larger object to the smaller object where r = (position m - position M), and $\hat{r}_{21}$ is the magnitude of the vector $\sqrt{(x_m - x_M)^2 + (x_m - x_M)^2}$. In the Earth-Moon system's force on the Saturn V , The equation of the gravitational force is as follows:

$$\overline{F}_{total} =- G\frac{M_E m}{\left|\overline{r_{Em}}\right|^2}\hat{r}_{Em} +- G\frac{M_M m}{\left|\overline{r_{Mm}}\right|^2}\hat{r}_{Mm}$$

Where m is the mass of the Saturn V rocket, 5500 kg, r_{Em} is the position vector from the Earth to Saturn V, and r_{Mm} is the position vector from the Moon to Saturn V. With this, the gravitational force of the Earth-Moon system radially from the Earth to 1.5 times the distance from the Earth to the Moon can be demonstrated in a stream plot as shown in Figure 2:



where the lines indicate the direction of gravitational force and the color indicates the magnitude of the force (scaled logarithmically). As seen above, most of the gravitational force points towards the Earth, with a constant but decreasing force being exerted on the Saturn V rocket. Therefore, more acceleration is needed closer to the Earth and can decrease throughout its journey to the moon.

IV. Projected Performance of the Saturn V Stage 1

With this information, an examination of the Saturn V rocket and its change in velocity can be made. The equation to determine the change of velocity for a given time when in this case is as follows:

$$\Delta v(t) = v_e ln(\frac{m_0}{(m_0 - m_{dot} t)}) - gt$$

where v_e is the exhaust velocity of Saturn V Stage 1, $2.4 \times 10^3 \frac{m}{s}$, m_0 is the wet mass of the rocket, $2.8 \times 10^6 kg$, m_{dot} is the burn rate of the rocket, $1.3 \times 10^4 \frac{kg}{s}$, and g is the gravitational acceleration, $9.81 \frac{m}{s^2}$. To determine the total burn time of the rocket, the following equation can be used:

$$T = \frac{m_0 - m_f}{m_{dot}}$$

where m_f is the dry mass of the rocket, $7.5 \times 10^5 kg$. Plugging in the following values returns the burn time of Saturn V as $T_{theoretical} = 157.7s$. To determine the altitude the rocket will reach, the following equation can be used:

$$h = \int_0^T \Delta v(t)dt$$

With all of the values plugged in and the equation integrated, the altitude that will be reached by Saturn V is around 65.9 km.

V. Discussion and Future Work

Throughout the calculations of the values within this report, assumptions and approximations were made to simplify the results. The Earth and Moon were treated as point masses instead of imperfect spheres with non-uniform gravitational fields. The orbit of the moon around the Earth is treated as a sphere rather than its true elliptical orbit. No air drag through the Earth's atmosphere is accounted for and the Newtonian model of gravity is used over the more accurate gravitational rules of general relativity. It is for these reasons that it comes with no surprise that the experimental values of $T_{actual} = 160s$ and $h_{actual} = 70km$ are greater than the experimental values, mostly due to the differences in the Newtonian and general relativity models of gravity and neglecting drag. Therefore, a further examination and testing of the rocket in real life, along with an update on calculating the gravitational potential and force along with the burn time and altitude reached in, is needed prior to moving forward along this mission.