Lab 2 – Mine Crafting

PHYS 265 Section 0101 – April 10h, 2025

I. Introduction

This report examines the physics of an object falling in a 4 km mineshaft, considering gravitational variation, air resistance, and the Coriolis effect. We calculate the fall time of a 1 kg mass and assess free fall as a method for measuring shaft depth. We also analyze how assumptions about Earth's density profile influence fall dynamics and compare to similar conditions Moon.

II. Calculation of Fall Time

To determine how long the 1 kg test mass will take to reach the bottom of this mine, an equation that approximates the physics can be made. For a falling object, an equation that determines an object's acceleration over some time is as follows:

$$a = \frac{d^2y}{dt^2} =- g(r) + \alpha\left|\frac{dy}{dt}\right|^{\gamma}$$

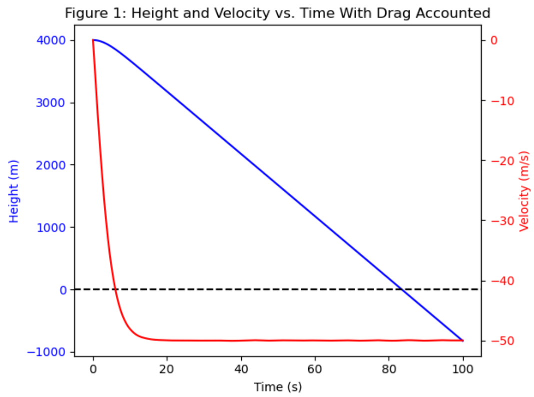
$$v = \frac{dy}{dt}$$

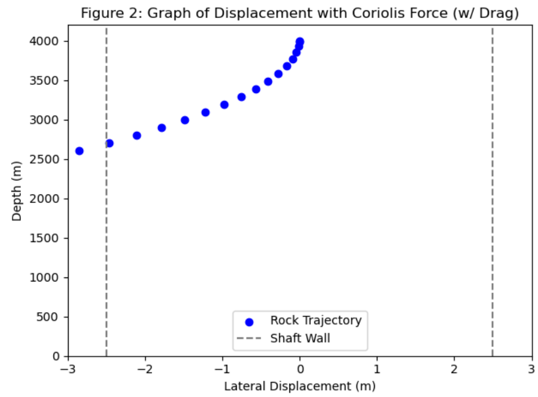
$$g(r) = g_o(\frac{r}{R_\oplus})$$

where t is the time elapsed (in seconds), y is the height of the object, α is the drag coefficient, and γ is the speed dependence of the drag. In this equation, the acceleration a is defined as the second time derivative of y ($\frac{d^2y}{dt^2}$). Gravity also changes when going down the shaft, so there is a variable gravity equation as well, where g_o is the gravitational acceleration of the Earth at the surface (~9.81 $\frac{m}{s^2}$), $R_\oplus$ is the radius of the Earth ($6.3781 \times 10^6 m$), and r is defined as:

$$r = 4000 - R_\oplus + y$$

where 4000 is the height of the mineshaft (4000m = 4 km), $R_\oplus$ is the radius of the Earth, and y is the height from the previous set of equations. The other derivative term is the velocity v, which is the time derivative of y ($\frac{dy}{dt}$). These equations can be integrated over time, which gives the instantaneous velocity and height at a certain time. In this case, the drag coefficient is $\alpha = 0.00392$, the speed dependence of the drag $\gamma = 2$, as the drag force grows proportionally to the speed squared. With this, a plot of an object's height and velocity falling in the mineshaft over time can be graphed, as shown in Figure 1:





As the above graph shows, the test mass will reach a terminal velocity of -50 m/s and reach the bottom of the shaft 83.5 seconds after dropping.

III. Feasibility of Depth Measurement Approach

One problem that arises from the previous set of calculations is that the effects of the Coriolis force were not accounted for. This is a force that occurs due to the rotation of the Earth, and can be defined by the following set of equations:

$$F_{c_x} = \ + 2m\Omega v_y$$

$$F_{c_y} = \ - 2m\Omega v_x$$

Where m is the mass of the object (in this case, 1 kg) and Ω is the rate of Earth's rotation ($7.272 \times 10^{-5} m/s$); v_x and v_y are the x and y-direction components of velocity for the test mass. When accounting for this and adding them to the equations of the falling object, a graph of the object y vs x position can be made (shown in Figure 2). The mine is only 5 meters wide, which is displayed as dashed lines in Figure 2. With these parameters, the test mass will not reach the bottom before hitting one of the shaft walls first. Given the model above, accounting for both drag and the Coriolis forces on the mass, the test mass would hit the wall 29.6 seconds after dropping it at a depth of 2697.2 meters. This means that simply dropping the test mass down won't effectively measure the depth of the actual mine, as it would hit a wall first. Simulating such a trajectory to the bottom of the mine given these conditions is nearly impossible given the variation that could occur in reality and therefore is not recommended.

IV. Crossing a Homogenous vs. Non-Homogenous Earth (and Moon)

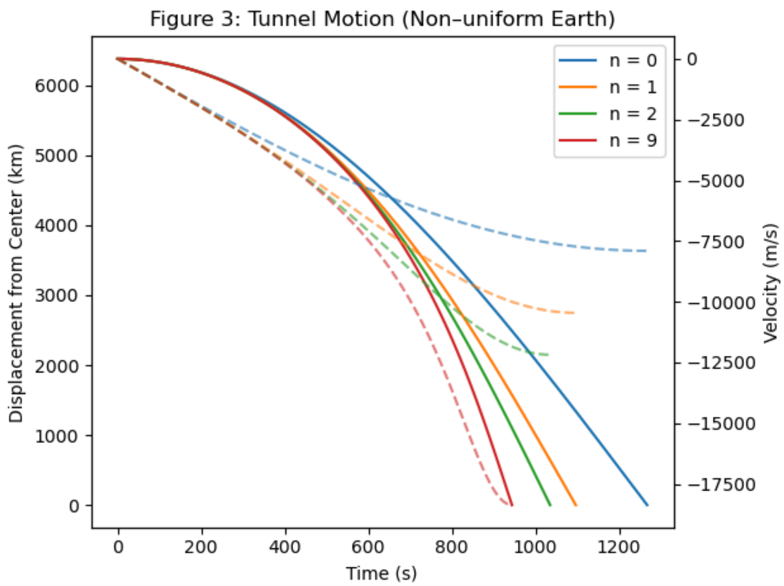
Another approximation has been made throughout these calculations as well: approximating the density of the Earth to be homogenous, or constant, throughout. In reality, the Earth changes density throughout its radius, with it increasing towards the center of the planet. A straightforward model for the density ρ as a function of distance from the center of the Earth r is the following equation:

$$\rho(r) = \rho_n(1 - \frac{r^2}{R_{\oplus}^2})^n$$

where ρ_n is the normalizing constant and n is some exponent. This is such that n=0 would mean an Earth of constant density. With this function, the mass of the Earth can also be calculated as such:

$$M = 4\pi \int_0^{R_\oplus} \rho(r) r^2 dr$$

With this adjustment, the time it would take for the object to reach the bottom of the mineshaft is affected. To show a more extreme example, let's consider a hole that went through the whole of the Earth. In this case, an object would fall at different rates depending on the exponential term n, which defines how differing the density of the Earth is between the upper and lower layers. To show this difference, Figure 3 displays the Height and Velocity vs time of objects falling in density profiles of n = 0,1,2, and 9.



As shown here, having a differing density throughout the Earth can lead to drastic changes in the time it would take for an object to hypothetically reach the center of the Earth. With n=0, the time it would take for the test mass to reach the center would be 1,267.3 seconds, while it would take only 943.8 seconds if n=9. Overall, the time it takes for an object to cross the center of the planet is scaled as $\frac{1}{\sqrt{\rho}}$.The same holds for the Moon as well, which at a density ratio ($\frac{\rho_{Moon}}{\rho_{Earth}}$) is around 0.61.

V. Discussion and Future Work

Throughout the calculations of the values within this report, assumptions and approximations were made to simplify the results. The Earth was treated as a perfect sphere with no imperfections, which is far from what the Earth is shaped like. Temperature and the ability to dig throughout the Earth were negated, as attempting to do so would prove impossible. The test mass did not have a defined shape, as certain shapes might prove to slow the object down more (a less aerodynamic shape would not as effectively reach terminal velocity). Given all of these assumptions, a future assessment should account for the fact that the Earth is non-spherical, account for increases in temperature as the further down in the crust the object gets, and consider the dimensions of the test mass itself. A reevaluation of how to measure the actual depth of the mine should also be considered, as dropping a mass down is shown to be inefficient, hitting the shaft walls before ever reaching the bottom.