Statistics with Spa  ows

Lecture 10

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Outline

- Hypothesis testing in linear models
- Interpretation (a bit)
- Standardizing
- Reporting

Hypothesis testing in linear models

- What do we actually test?

$$y_i = b_0 + b_1 x_i + \varepsilon_i$$

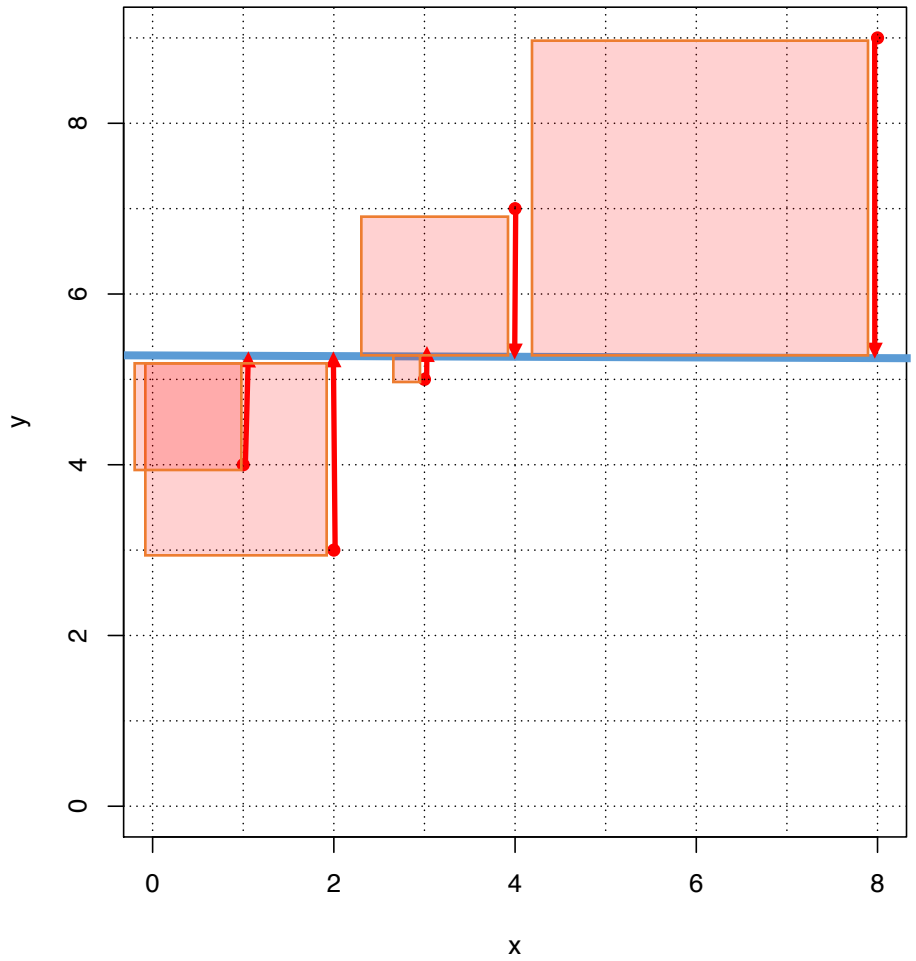
y	i
5	1
3	2
6	3
10	4
4	5

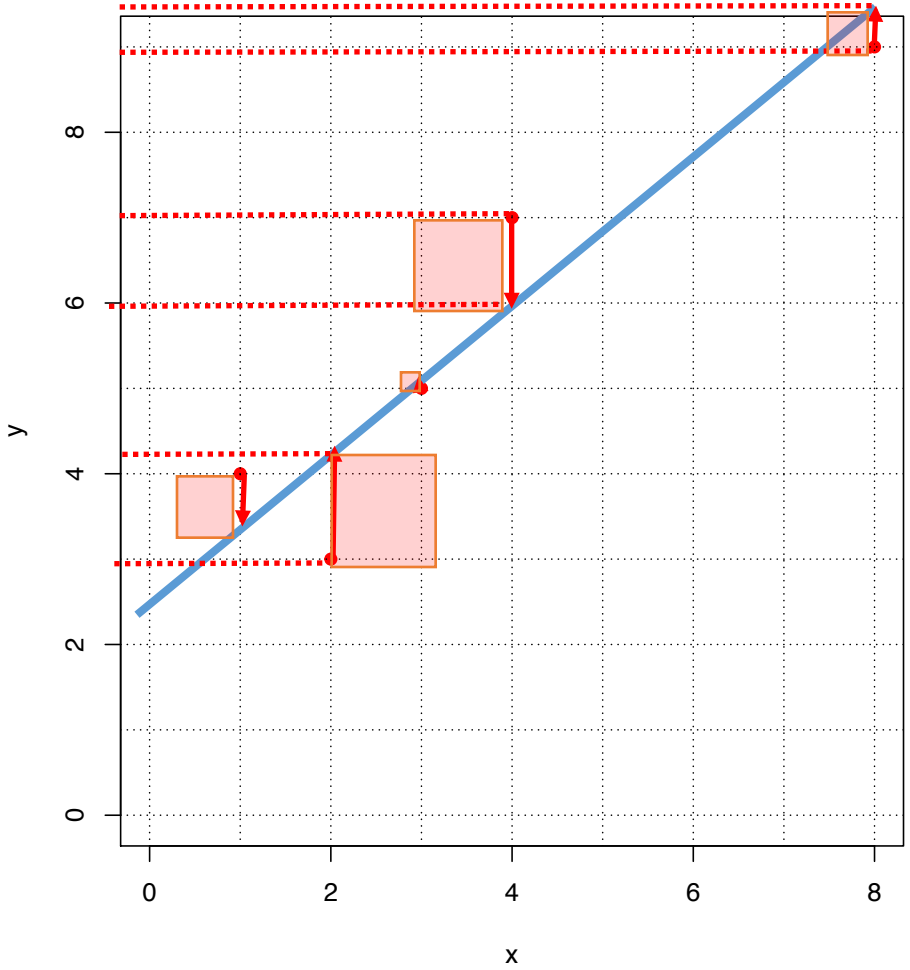
$b_0 = ?$

$b_1 = ?$

x	i
3	1
1	2
4	3
8	4
2	5

ε	i
?	1
?	2
?	3
?	4
?	5





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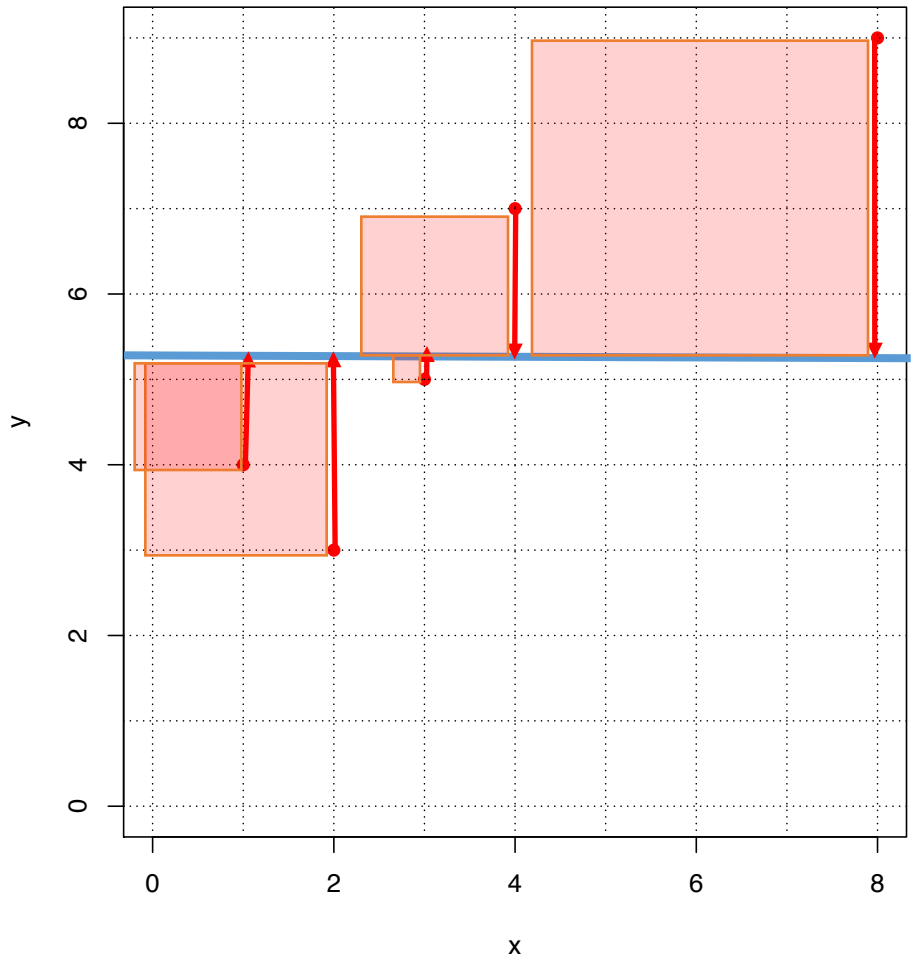

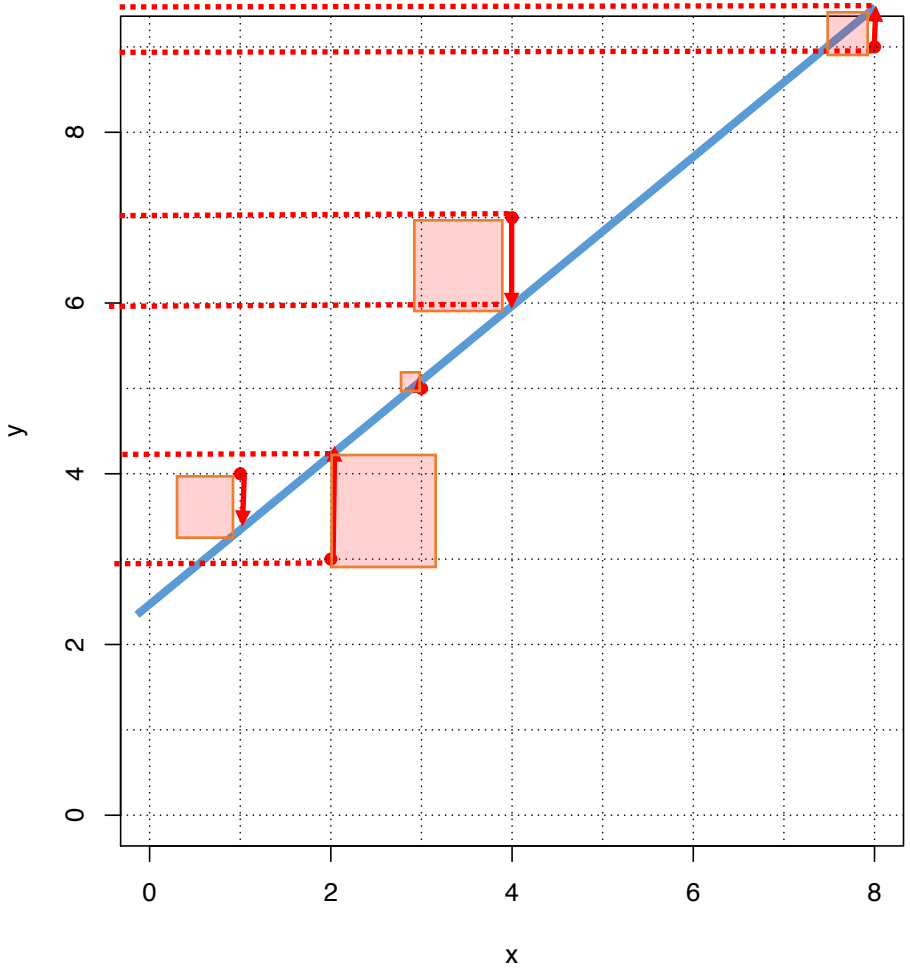

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→ Test if slope (b_1) estimate is different from 0
→ T-test!

Hypothesis testing:

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Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  5.83246    0.98195    5.94 3.48e-09 ***
Tarsus       1.18466    0.05295   22.37  < 2e-16 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.841 on 1642 degrees of freedom
Multiple R-squared:  0.2336,    Adjusted R-squared:  0.2332
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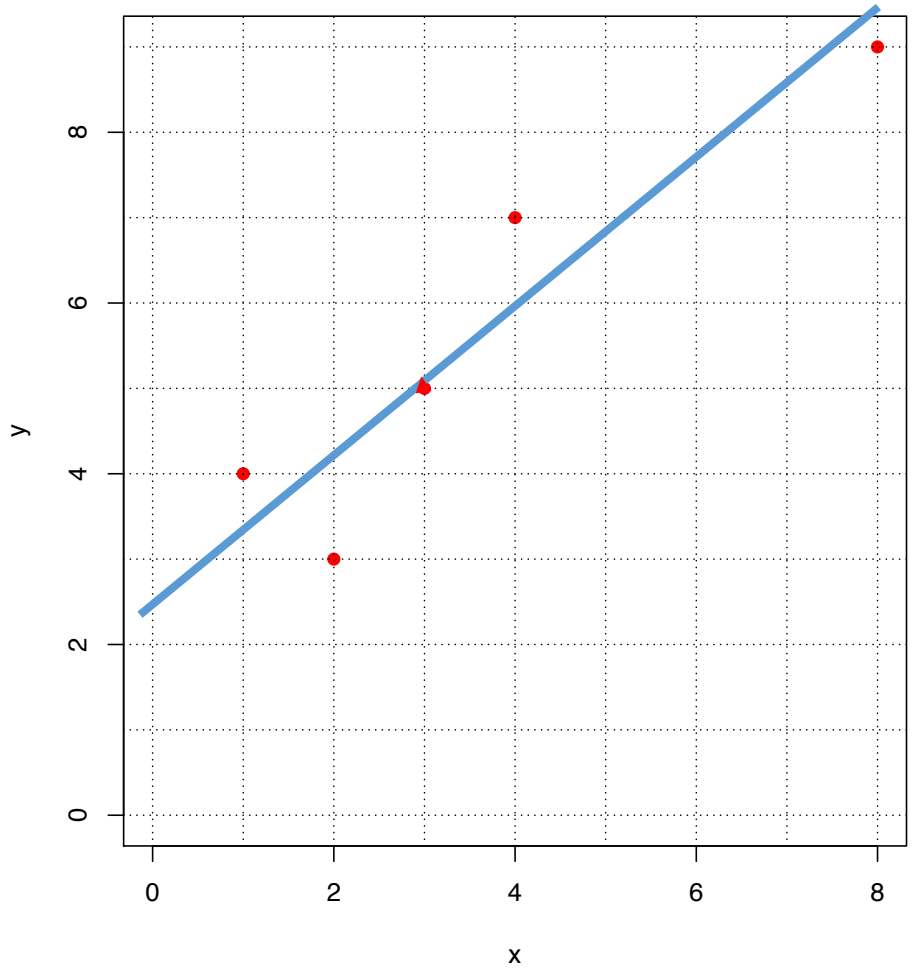

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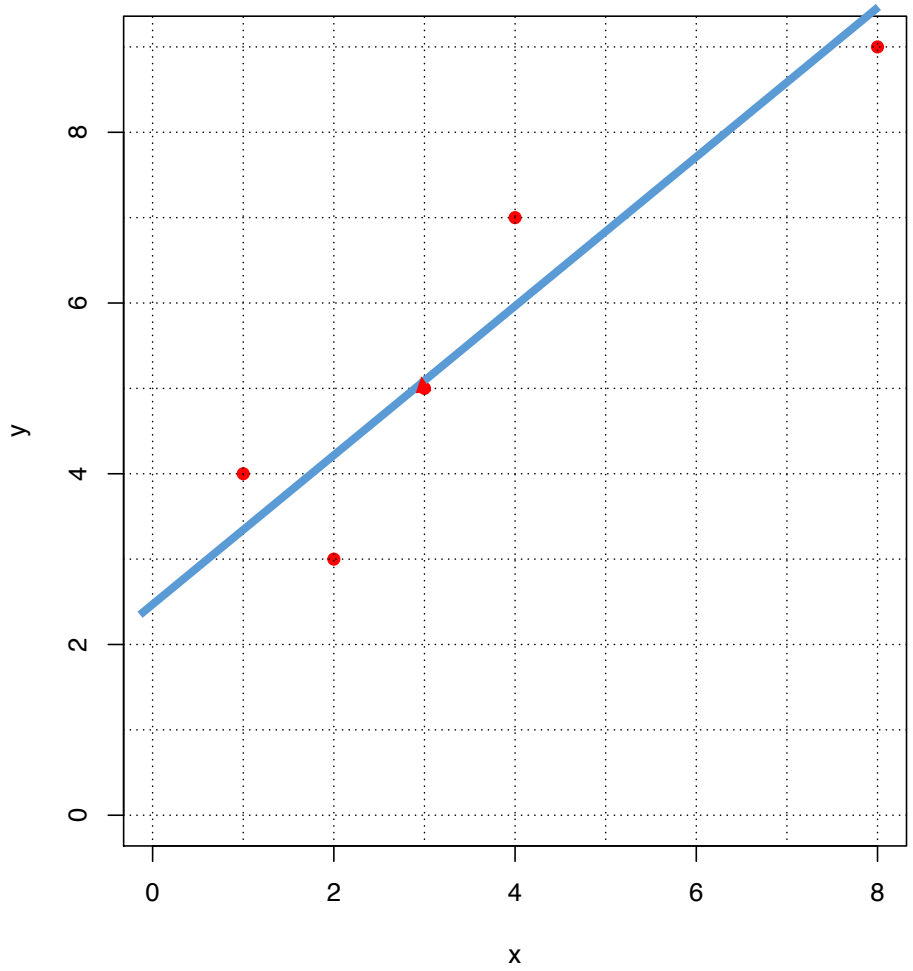
Usually not much for b_0



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The slope is 5.8
For each mm longer tarsus, a sparrow weights 1.18g more.

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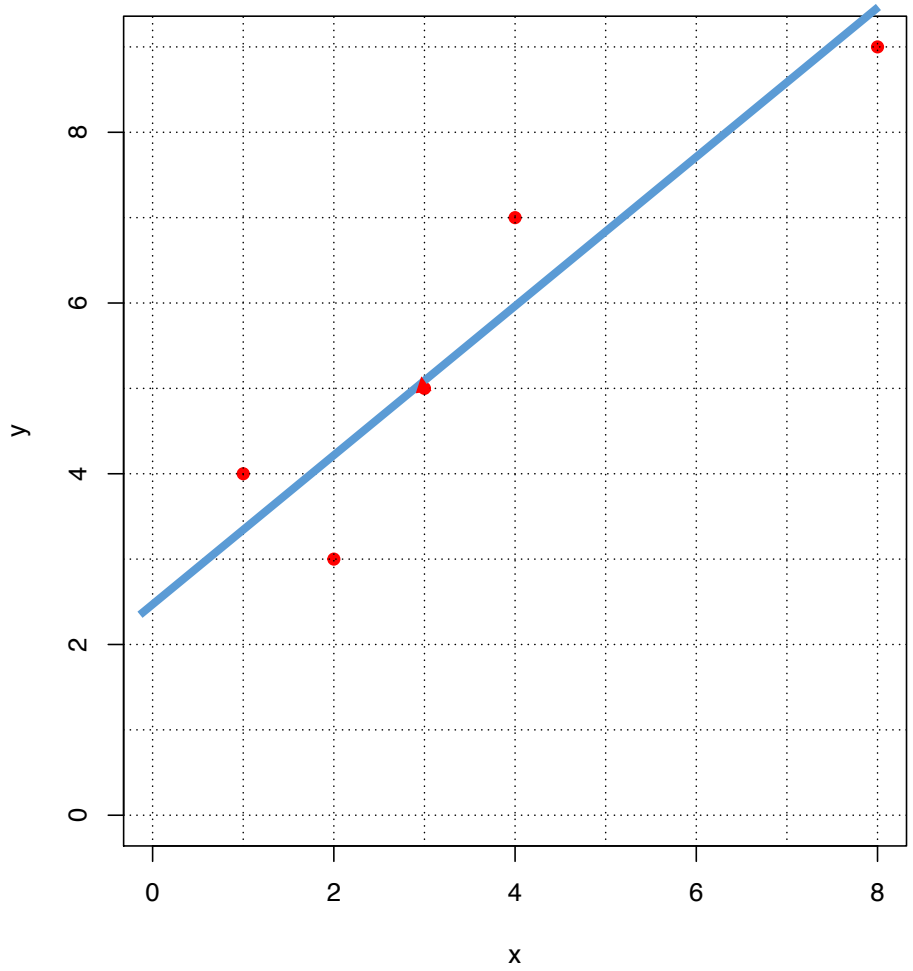
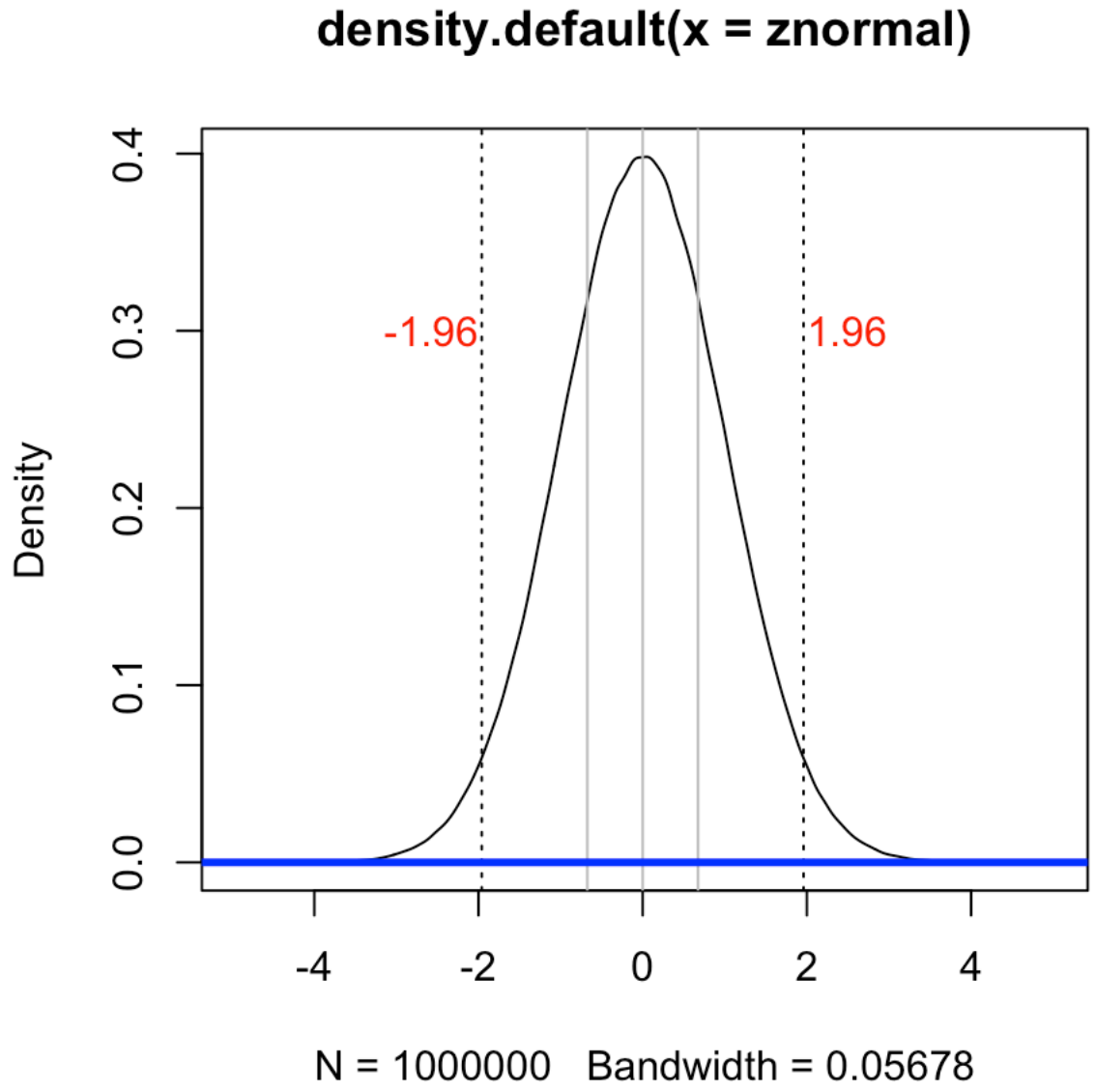
What does it mean if the null hypothesis is rejected?

Usually not much for b_0
b_1 is more interesting biologically speaking



Why standardize data?

- z-scores:
- Normal distributed
- Mean of 0
- Sd of 1



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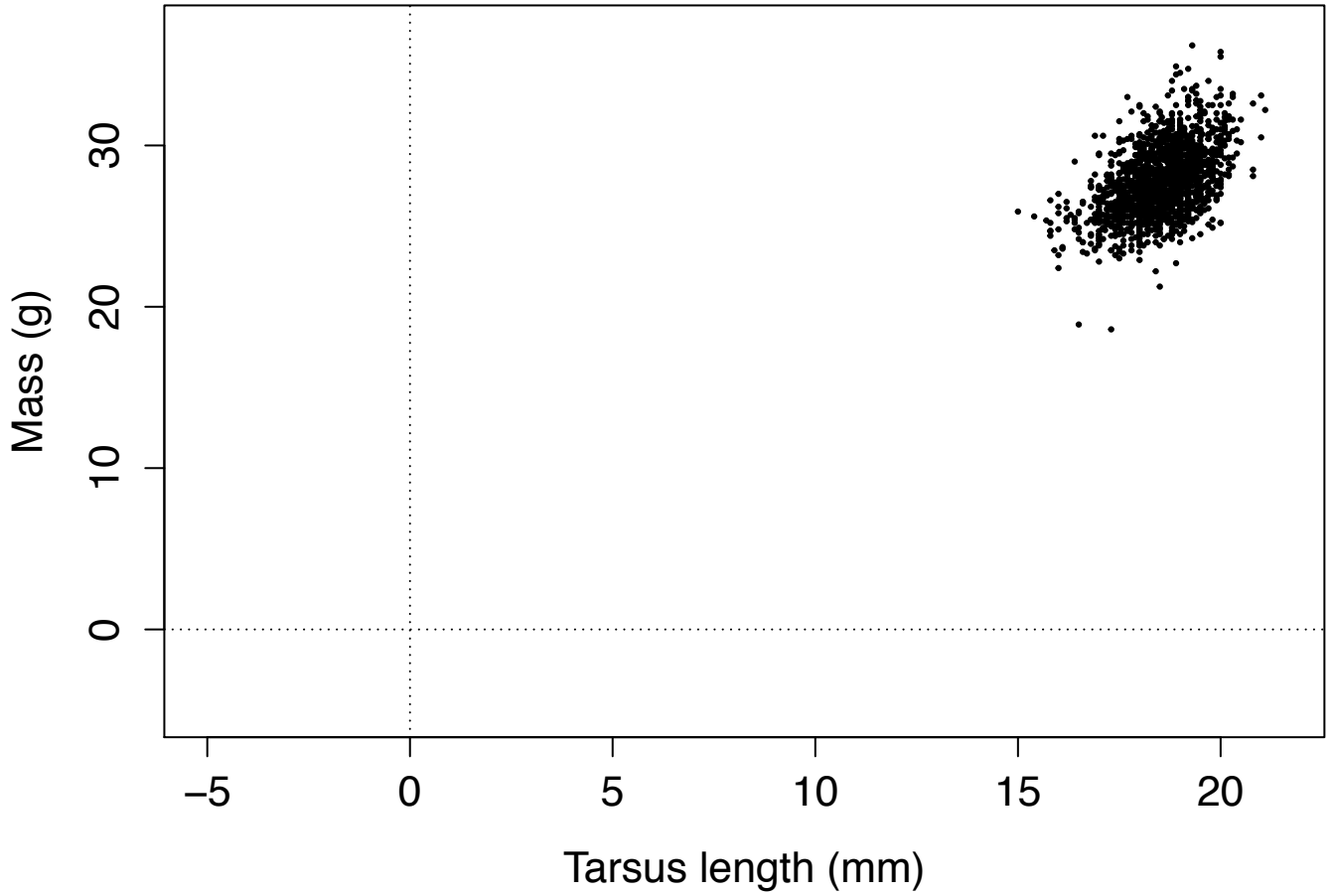

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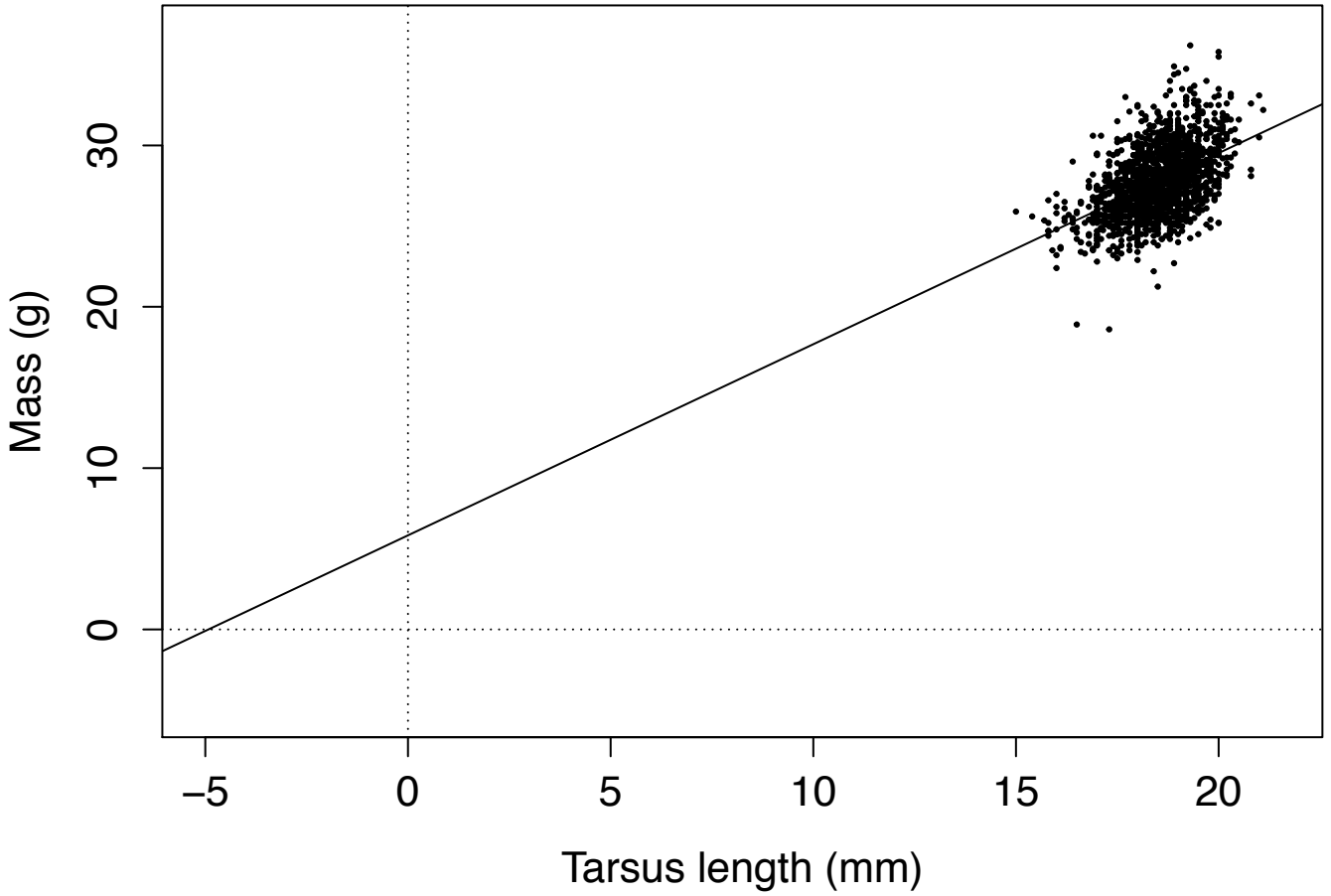

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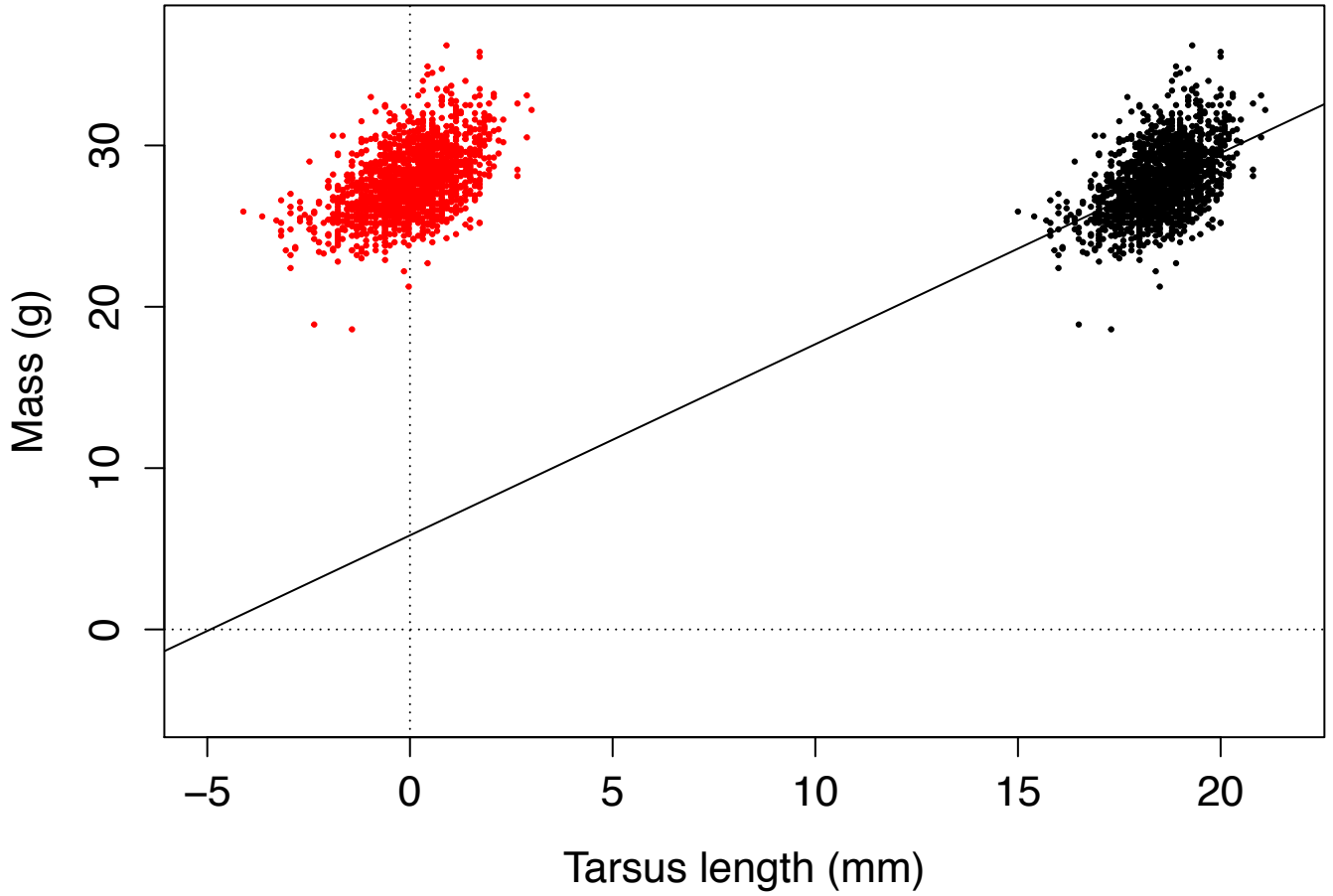

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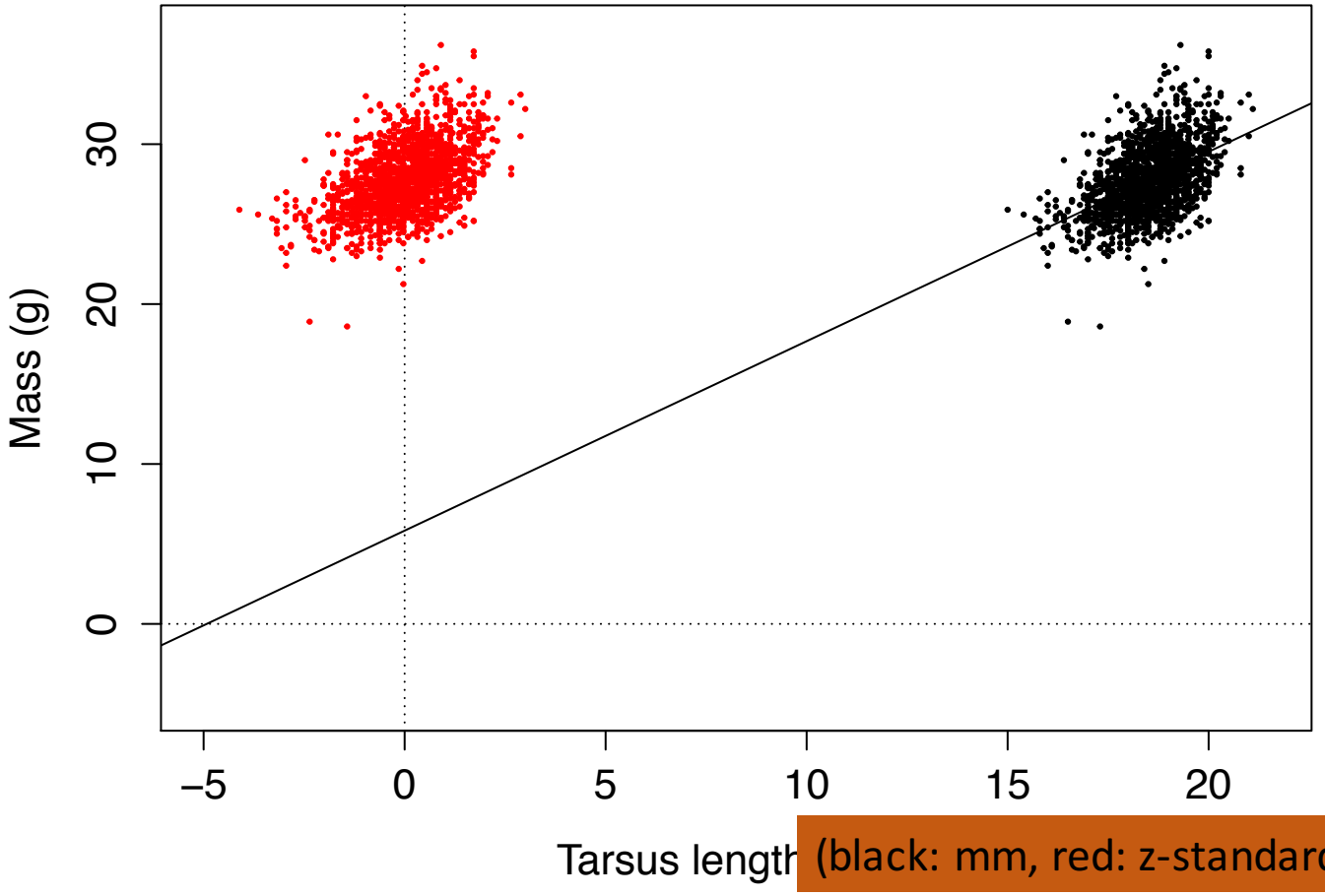

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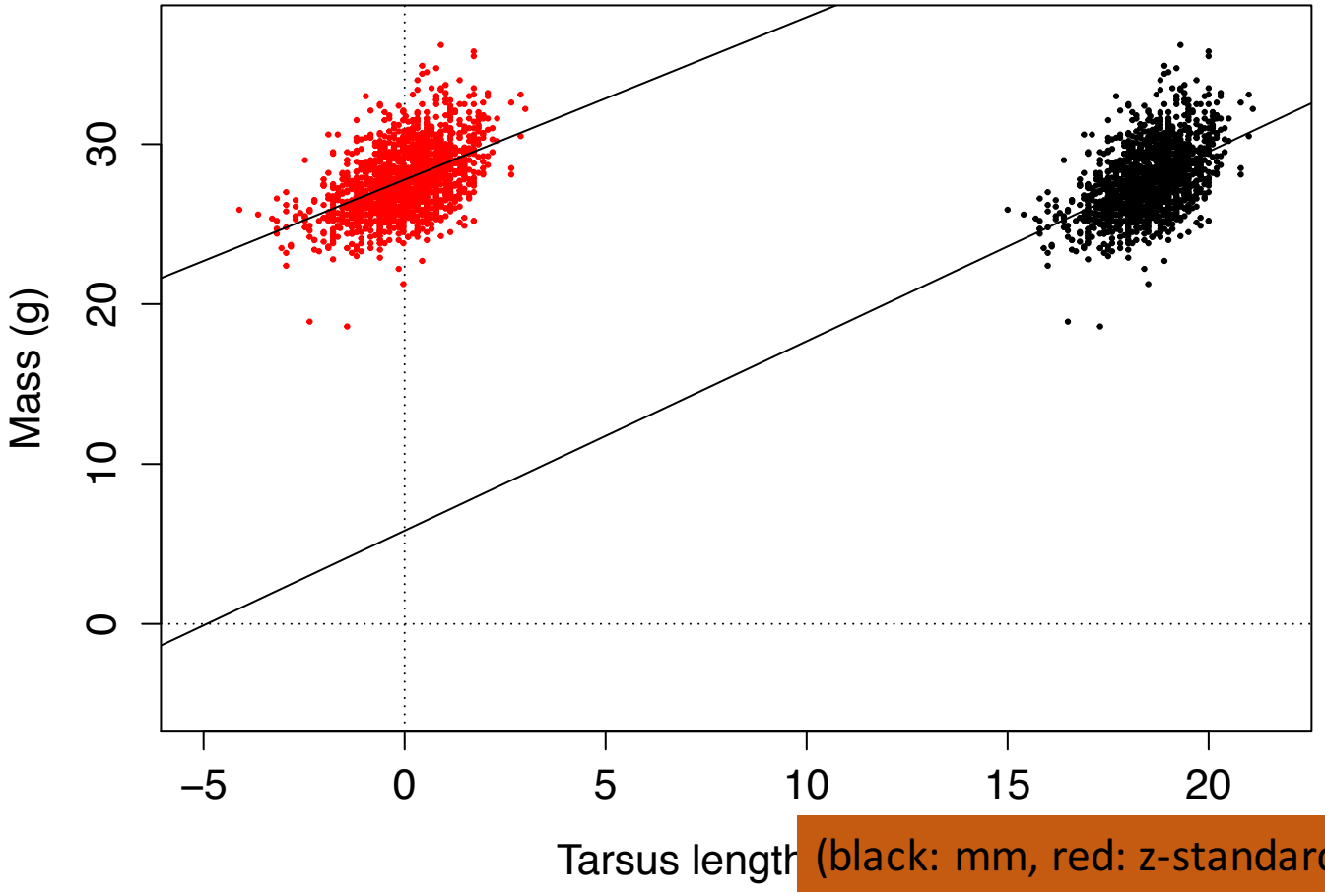

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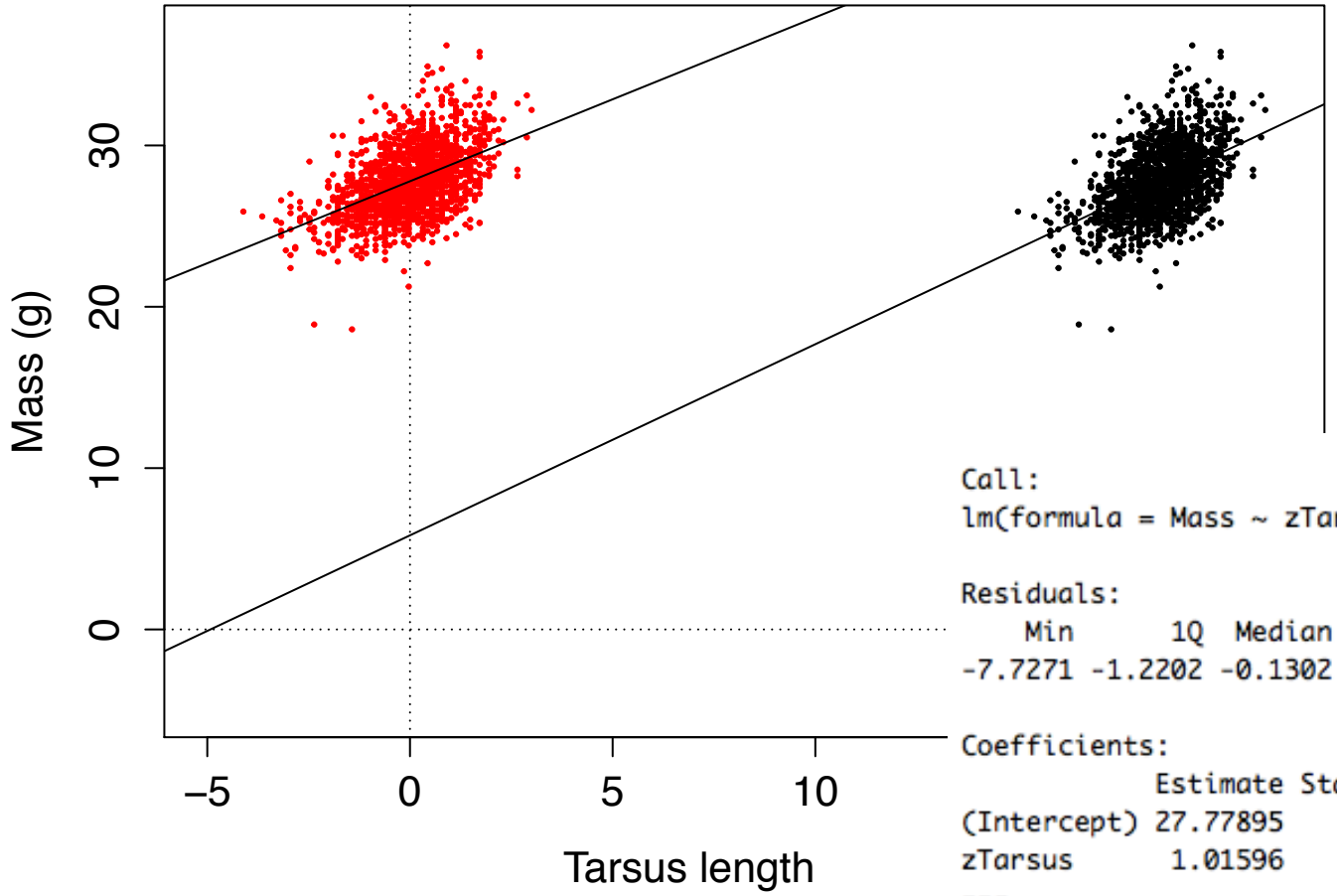

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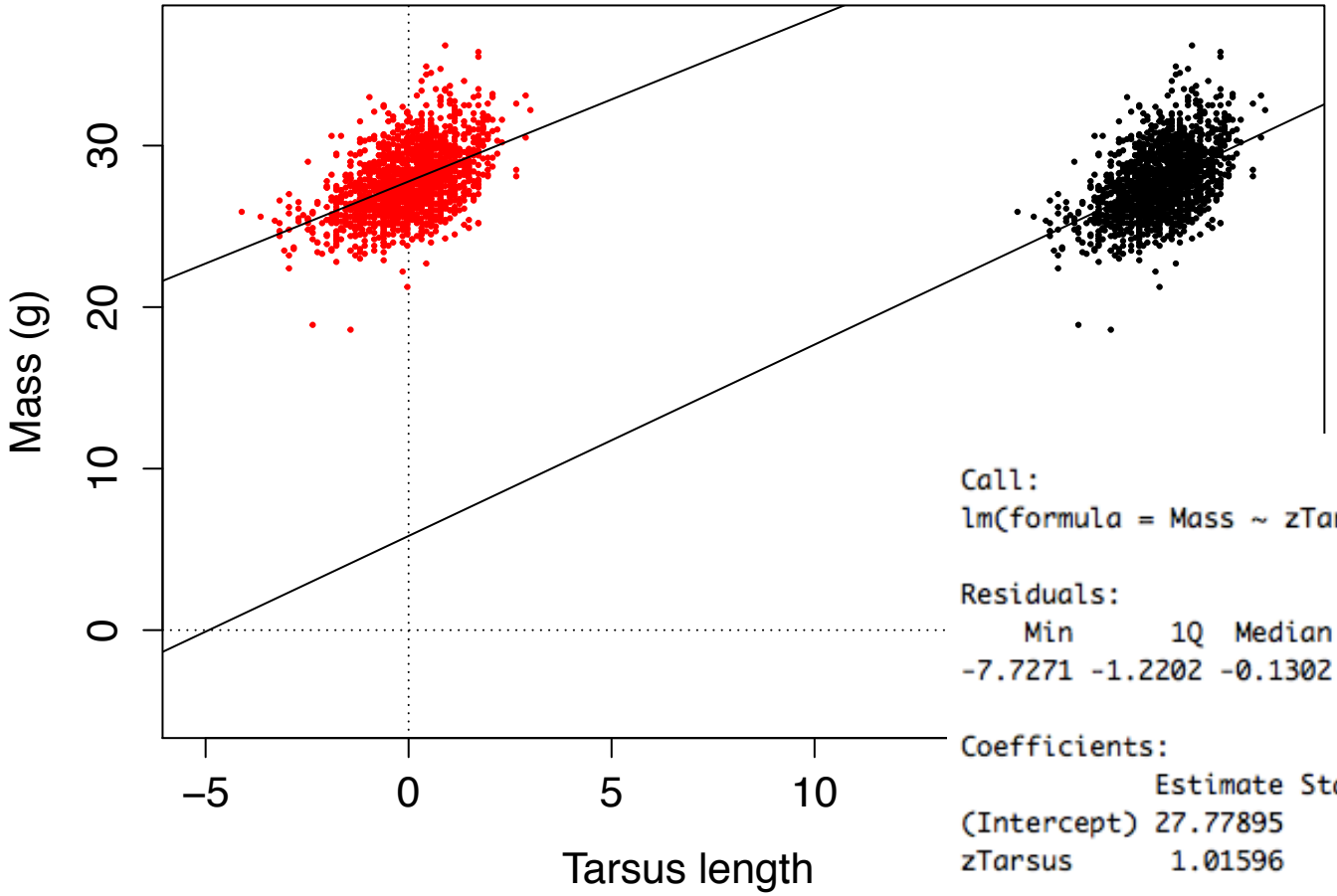

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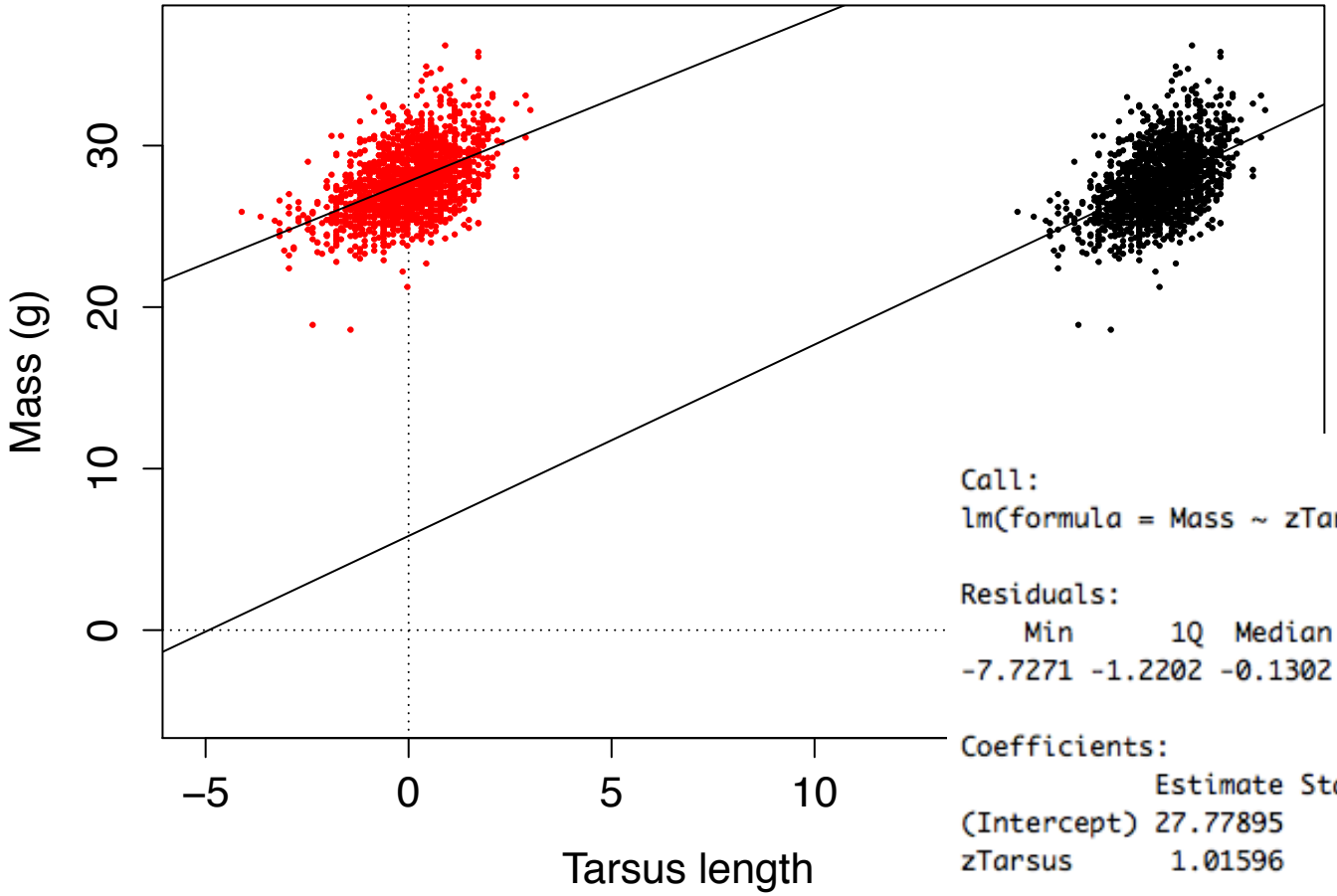

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Interpretation of statistics – take home

- Always consider units!
- Standardize to make intercept meaningful
- T-tests test for null hypothesis that parameter estimates equal –

- Always think of the biological meaning in units!

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- Table legend should be self-explanatory without refering to text!

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Variable	*b*	*SE*	*t*	*p*
Intercept	27.78	0.05	611.94	<0.001
Tarsus	1.02	0.05	22.37	<0.001

How to report – take home

- Methods:
- Always describe *all analyses you present in results. Be precise and specific. Describe what's the response variable. Say what is the explanatory variable, and say WHY you used those. Give the units. Say when you standardize, and WHY.
- Justify, justify, justify.

How to report – take home

- Results:
- Start with describing the dataset.
- Sample size, mean, range, missing values ect.
- Explain results of each analysis
- Make nice tables and think before copy/pasting values from R!
- Legends need be self-explanatory!

HO 10– DO IT NOW!

- Do HO 10! Do the excersises on HO 10!
- Run diagnostics for a model with sex as explanatory variable. Interpret the plots.
- Run a linear model, where you test the hypothesis that sparrows with bigger bills can eat more. The prediction is that the larger the bill, the heavier the sparrow.
- Write a short (1A4) report on methods and results on this last model.
- Before you go into the linear model, you should first describe your data, say how many sparrows, how many females and males, **whether there is a difference in your response between the sexes. If that difference is meaningful, you should test the sexes separately**. Write this section as you would write it for a scientific article.