

# Statistics with Spa OWS

## Lecture 11

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# Other ways than OLS to estimate $b$ 's...

- Maximum likelihood
- Bayesian inference
- Bootstrap
- Jack-knifing
- Sensitivity analysis

# Maximum Likelihood Estimation

- Another way to estimate parameters
- Maximizes the likelihood that the data you have comes from the model you assume

# Maximum Likelihood Estimation

- Example: we throw a coin:
- Heads, head, tail, heads, tail
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- Example: we throw a coin:
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- Null model: chance of heads/tails is 0.5
- Heads:  $p = 0.5$ , tails  $1-p = 0.5$
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- $(\text{heads})^3 * (\text{tails})^2$

# Maximum Likelihood Estimation

- Example: we throw a coin:
- Assuming  $p = 0.5$ , the likelihood is 0.031
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- Assuming  $p = 0.5$ , the likelihood is 0.031
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- Assumption:  $p = 0.6 \rightarrow q = 0.4$
- Likelihood:  $0.6^3 * 0.4^2$

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- Likelihood: 0.026 – NOES!

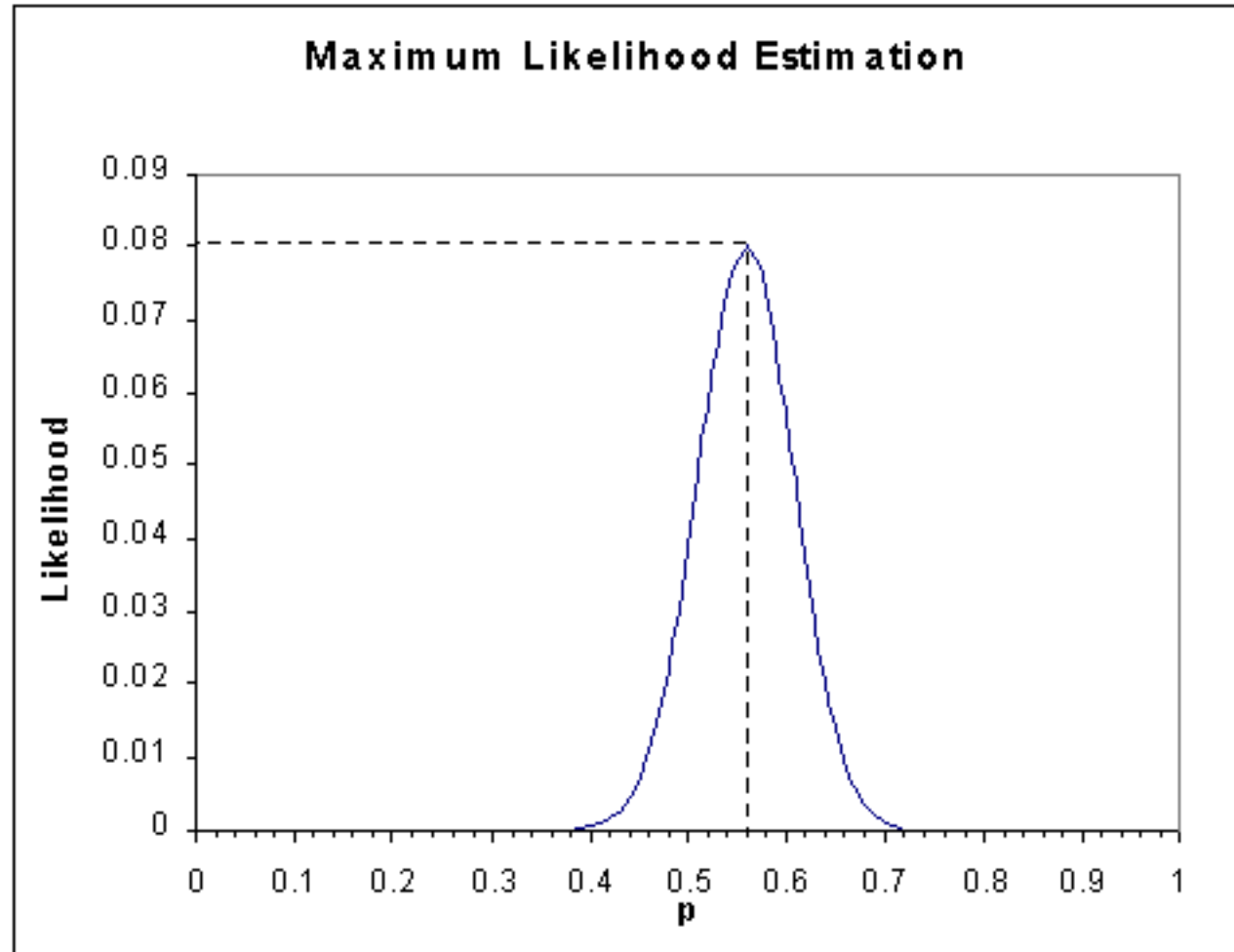
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- Likelihood:

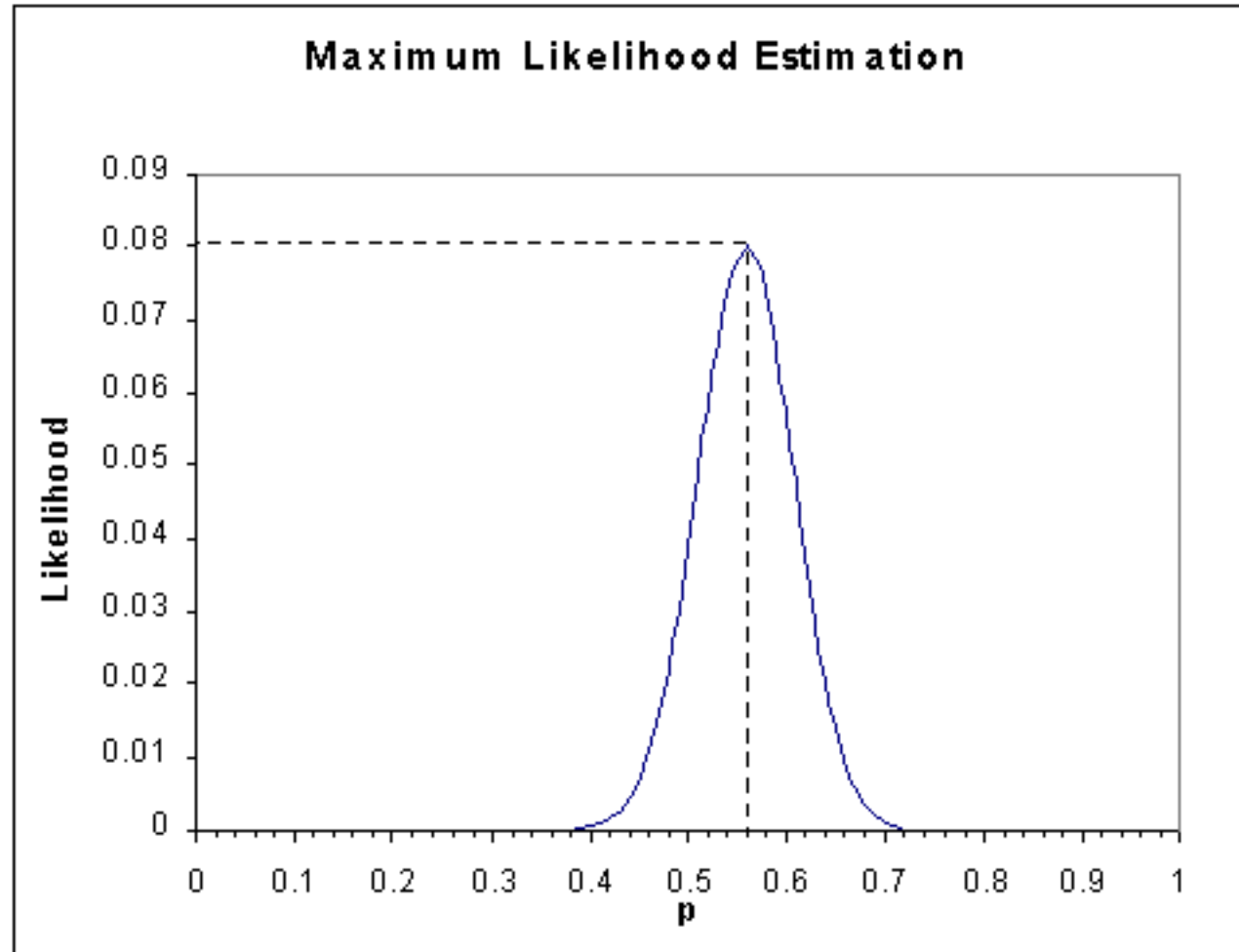
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- Likelihood: 0.33

# Maximum Likelihood Estimation



# Maximum Likelihood Estimation





# Ronald Fisher

Your new idol

- Father of
- Statistics
- Modern evolutionary synthesis
- Quantitative genetic



"a genius who almost single-handedly created the foundations for modern statistical science" (said someone I don't know)

"the greatest biologist since Darwin" (said R. Dawkins)

also an eugenicist and thus not always nice guy (says I) – but I am in awe of his genius!

# ML estimation

- Usually arithmetic with some iterations (going back and forth, “trying” things)
- We want to MAXIMIZE ML
- Due to some arithmetic magical things, it’s often easier to maximise the LogLikelihood
- That’s ok, too.

# ML vs OLS

- Both most often used
- Similarly precise
- OLS inappropriate when assumptions are violated (response and/or residuals not normally distributed)

# Bayesian inference

- Thomas Bayes, 1701–1760
- Uses likelihood and previous knowledge
- Solution is not uniform
- Many solutions with different probabilities (degrees-of-belief)



# Bootstrapping

- Re-sampling sample to get estimates

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# Jack-knife

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- $N-2$
- ...

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# Sensitivity analysis

- Systematically exclude datasets