A simple Stochastic SIR model for COVID-19 Infection Dynamics for Karnataka – Learning from Europe

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***Abstract*—In this short note we model the region-wise trends of the evolution to COVID-19 infections using a stochastic SIR model. The SIR dynamics are expressed using *Itô-stochastic differential equations*. We first derive the parameters of the model from the available daily data from European regions based on a 24-day history of infections, recoveries and deaths. The derived parameters have been aggregated to project future trends for the Indian subcontinent, which is currently at an early stage in the infection cycle. The projections are meant to serve as a guideline for strategizing the socio-political counter measures to mitigate COVID-19.**

I. Introduction

COVID-19 has brought misery to many regions in the world, especially in Europe. However, it has just taken off in other parts of the world. The remaining regions, including India hosts huge population approximately more than 50% of the world population. There is an eminent danger and most of these population are under the poverty-line or the countries do not have huge resources for medical interventions for affected patients. The options are limited – either go for herd immunity which can backfire and it is too risky or lockdown and restrict the population from moving around. Thus we try to factor in the percentage of population allowed to move around (some are essential services and some not obeying the orders) and assuming the mobility factors appropriately[1], we try to predict what can happen during the lockdown. We specifically take Italian case for Karnataka because of the population resemblance.

The dynamics governing the evolution of the COVID-19 infections have been modeled using a stochastic differential equation SIR model [1]. The parameters of this model have been initially optimized for a set of European regions, individually. An aggregate of these parameters have then been used for projecting the future trends for the Indian region, specifically for Karnataka state. Multiple projections have been generated by varying the exposure factor that influences the growth rate of infections. This is done in order to provide insights for selective regional quarantining and lock-downs.

[1]For example, we assume that a person is mobile only around (60 x 20) km^2 area, which is approximately 10% of the area of Karnataka. Such assumption which on the surface looks admissible

II. Stochastic SIR model

The evolution of COVID-19 infections in each region, has been modeled via the stochastic susceptible-infected-recovered (SIR) model [1] which is given as,

$$\begin{aligned} dS(t) &= -\beta S(t)C(t)dt \\ dC(t) &= \big(\beta S(t)C(t) - \gamma C(t)\big)dt + \sigma C(t)dW_t \\ dR(t) &= \gamma C(t)dt - \sigma C(t)dW_t, \end{aligned} \tag{1}$$

$$\begin{aligned} C(0) &= C_0,\ S(0) = S_0 \\ S(0) &= P_{total} - R(0) - C(0). \end{aligned} \tag{2}$$

Model states

1) t is a daily-time parameter.
2) $C(t)$ denotes the number of active infections at time t.
3) $S(t)$ denotes the total susceptible population at time t.
4) $R(t)$ denotes the total number of recoveries and deaths at time t.
5) $dC(t), dS(t), dR(t)$ denotes the change in the states at time t.
6) dW_t is an incremental Weiner process (Brownian motion), which models the randomness in the evolution.

Model parameters

- Growth rate: the constant β denotes the growth rate, which factors the rise in the number of infections, due to interactions between susceptible and infected population. This parameter is a lumped constants which is meant to account for: (a) the population size, (b) reproduction number R_0 of COVID-19, and (c) exposure-factor (which depends on mobility, precautionary measures, etc.).
- γ is the rate of outcomes, i.e., the rate at which the infections are neutralized, which may be due to recovery or death. It is assumed that recovered persons would not spread the infections again (at least for a window of a month).
- σ is a parameter used to model the stochasticity or randomness in the evolution, which may cause local deviations from the typical (exponential) trends.

TABLE I: Model parameters assumed. These numbers are slightly pessimistic.

Region	P_{total}	β	γ	σ
Germany	8.28×10^7	4.6×10^{-9}	0.005	0
Spain	4.67×10^7	7.5×10^{-9}	0.06	0
France	6.7×10^7	4.4×10^{-9}	0.03	0.02
Italy	10^7	2.8×10^{-8}	0.08	0.02
India	133.92×10^7	1.5×10^{-10}	0.015	0

- P_{total} is the population of the region, C_0 and S_0 are initial number of infections and susceptible individuals.

III. Parameters based on European trends

The parameters of the SIR model were optimized based on the data obtained for different European regions and India. The criterion for optimization was to simultaneously minimize the square integral error, terminal error and terminal rate error, between the actual data and daily samples of the simulated data. Further, because we have data for more number of days for European countries, we try to use the parameters from those countries and appropriate it on the Indian data which is for lesser number of days. The assumption is that India may be in the catch game (which, of course, we do not want) if the behaviour of people is taken to be similar. We have to base our predictions based on some gross assumptions under the given circumstances.

Note: We have taken the Italian and German data 4 days earlier than France and Spain in order to reflect the earlier trend before their respective lockdown conditions. This will enable us to simulate for various levels of lockdown percentages (exposure factor).

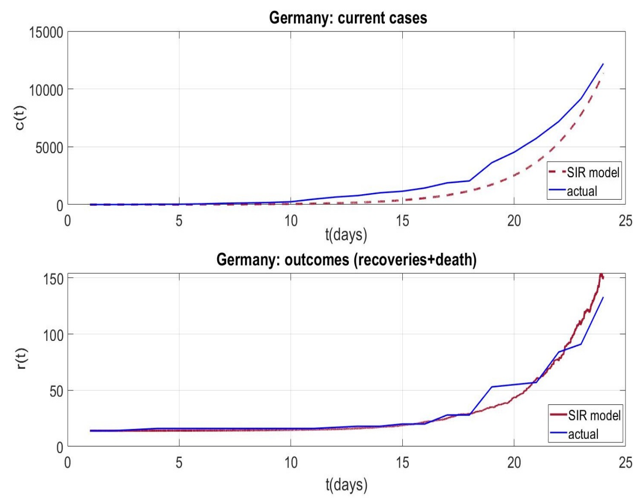

Fig. 1: SIR model for Germany. Day $1 = 24^{th}$ Feb 2020.

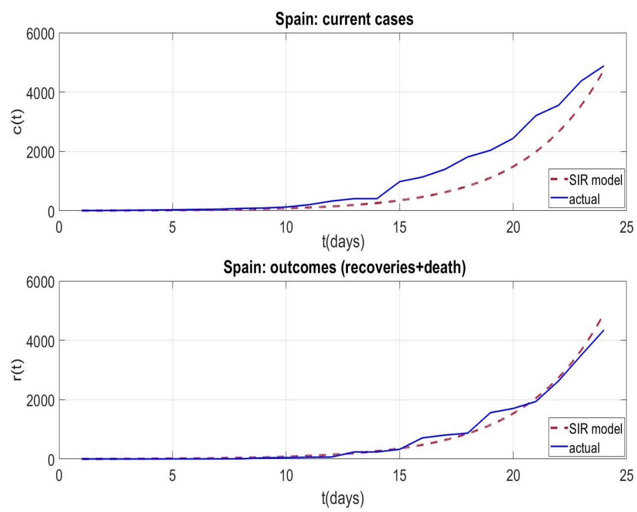

Fig. 2: SIR model for Spain. Day $1 = 28^{th}$ Feb 2020.

IV. Simulation results

The SIR model has been simulated and the parameters have been optimized based on the infection trends obtained for European countries for 24 days, and India for 8 days (after 15^{th} March, when the infections started to show an exponential trend). The stochastic differential equations have been simulated using the Euler-Maruyama numerical integration method [2]. We present below how our model is with respect to various European countries.

V. Projections for India

A. Nationwide projection

The SIR model has been simulated beyond the 8 days of considered data for India, with parameters averaged over the European regions considered above, as well as with the parameters obtained for the Indian region. This is done in order to demonstrate the possible growth of infections in India, based on the current trend so far (which is an optimistic case), or based on the European trend (which is the case to be prepared for). An exposure factor has been introduced to scale down the rate of infections β. This factor illustrates the additional reduction in mobility due to state-wide lock-down and quarantining measures, which are essential for maintaining the infections within reasonable limits. For India we have two sets, (i) using the available data only from India and (ii) taking the model parameters from considered European countries and projecting on Indian case.

Note: We have four graphs in each set. For example, when we use 100% exposure, that means there is no lockdown. When we use 50% exposure half of the country is lockdown. This is to show the gravity of the situation.

B. Karnataka Projections

The Italy and average European parameters have been used to project the future trends for the state of Karnataka. The Italy parameters have been chosen to predict the worst-case

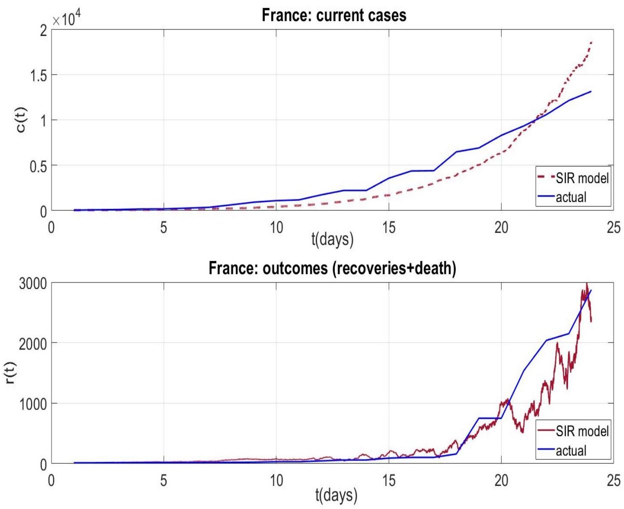

Fig. 3: SIR model for France. Day $1 = 28^{th}$ Feb 2020.

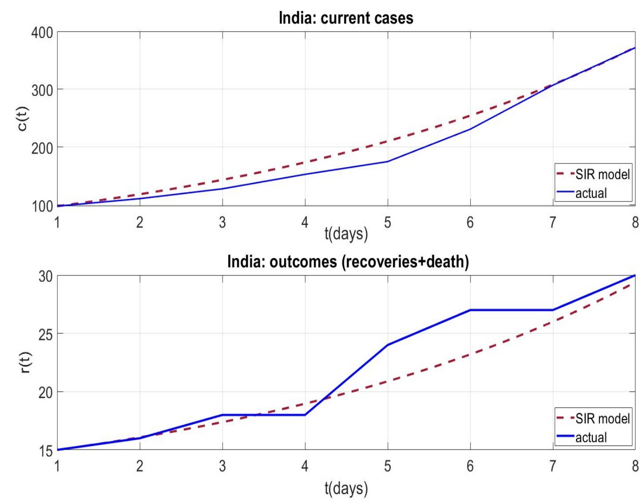

Fig. 5: SIR model for India. Day $1 = 15^{th}$ March 2020. Without projection, to fit model parameters.

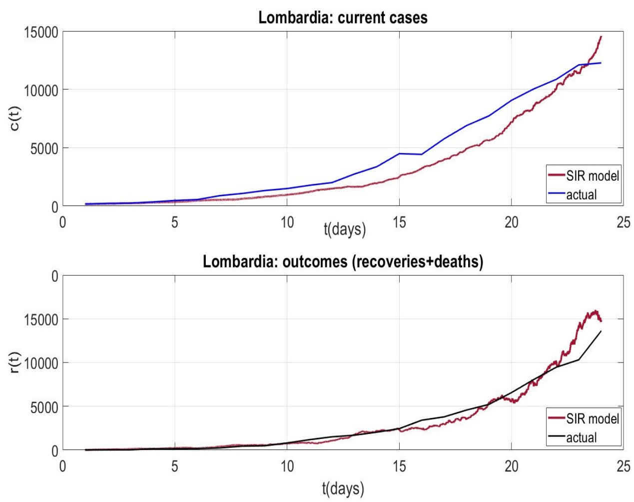

Fig. 4: SIR model for Italy (Lombardy region). Day $1 = 24^{th}$ Feb 2020. Last 7 days will be simulated further for comparison.

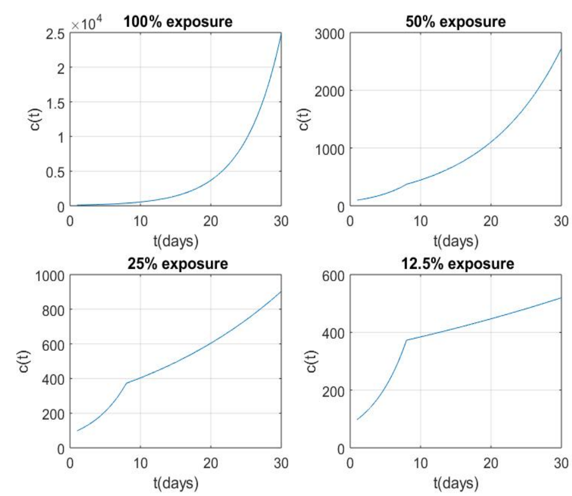

Fig. 6: Projections for India with varying rates of exposure: Indian parameters. Day $1 = 15^{th}$ March. Projections done after Day 8. The knee point is because of intervention that is percentage of lockdown. Till 8th day the curve follows the data as in Fig.5

scenario, and the European average parameters. The initial number of infections is assumed to be 100 (as of today) to accounted for additional unreported or undetected infections, over reported ones. We have made the assumption here that lockdown is in place and we have taken varied percentage of people obeying the rules. We only consider 20%-5% of the population mobile.

Let us summerize very briefly what we observe.

- Below 10% exposure required for avoiding exponential growth, with European average trend.
- Below 5% exposure required for avoiding exponential growth, with Italian trend. This is a very significant aspect in the whole of this work because, Karnataka has less infrastructure and similar population. Only advantage is that the mobility is impeded by lack of high speed transportation, which is a blessing in disguise compared to Italy.
- Below 6% exposure required for decline in infections, with European average trend.
- Below 3% exposure required for decline in infections, with Italian trend. That means we need to have less than 10 lakh people to be on the street to be well within the limit because of heavy assumptions.
- Fig.10 suggests the inevitable need for an **Extended lockdow period for several months** for Karnataka, or until a vaccine is available.
- By 'lockdown' we mean restricted mixing i.e. fewer susceptible individuals $s(t)$ interact with fewer infected individuals $c(t)$, thereby introducing a factor in the prod-

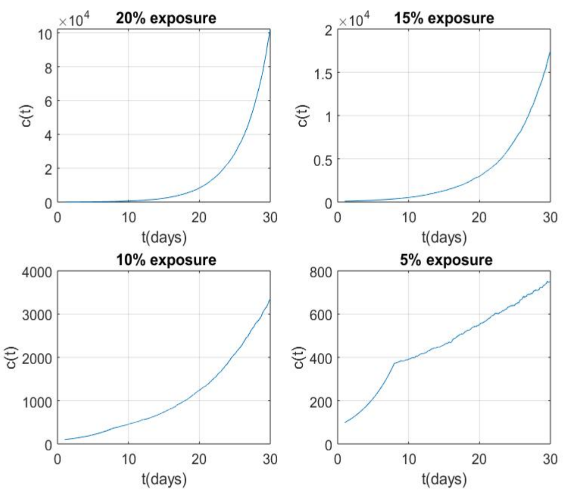

Fig. 7: Projections for India with varying rates of exposure: Average European parameters. Day $1 = 15^{th}$ March. Projections done after Day 8.

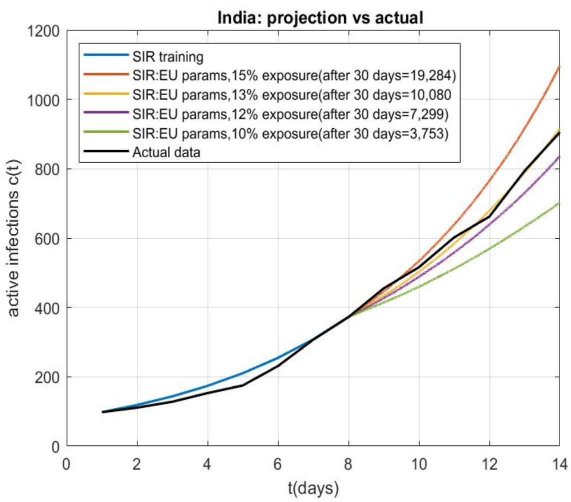

Fig. 8: Comparison of SIR model with European average parameters, and actual data until 28th March 2020. Day 1 is 15th march. Projection after 8 days. Current trend follows 13% exposure simulation. *Source: covid19india.org*

uct term $s(t)c(t)$ in the dynamics.

VI. Conclusions

This is a continuous work in which we are trying to find the model parameters everyday and project the possible scenarios, by varying the exposure factor for the rate of infection, as a result of evolving levels of quarantining. While this is not a completely verifiable projection, the model parameters look quite consistent, however they may reflect an over estimation of the projected number of infections in order to compensate for the unreported or undetected infections. Thus, we may take the numbers in this note to be a guide for further action by the law enforcing authorities.

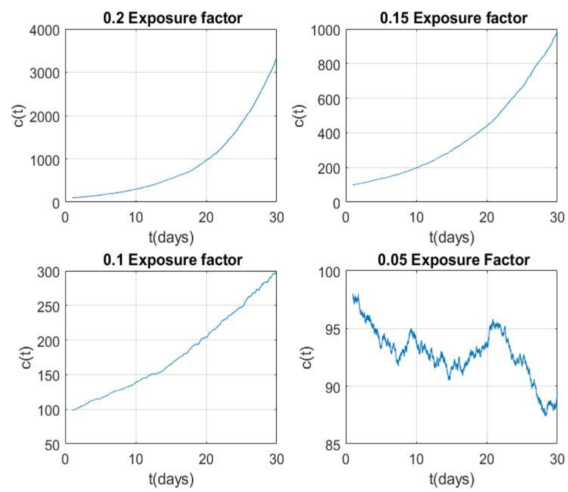

Fig. 9: Projections for Karnataka with varying rates of exposure: European Average parameters. Projections done for 30 days with 100 initial infections assumed on 25^{th} March. Last graph is because of the Randomness inherent in the model, for example, mobility assumed to be Brownian.

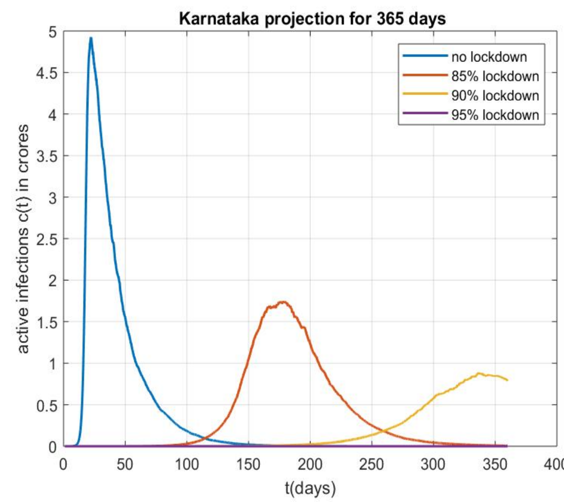

Fig. 10: 'Flatenning the curve': Projections for Karnataka 365 days without lockdown and 85% lockdown, using European average parameters.

References

[1] Maki, Yoshihiro, and Hideo Hirose. *"Infectious disease spread analysis using stochastic differential equations for SIR model."* 2013 4th International Conference on Intelligent Systems, Modelling and Simulation. IEEE, 2013.

[2] Kloeden, P. E., and Pearson, R. A. (1977). *The numerical solution of stochastic differential equations.* The ANZIAM Journal, 20(1), 8-12.

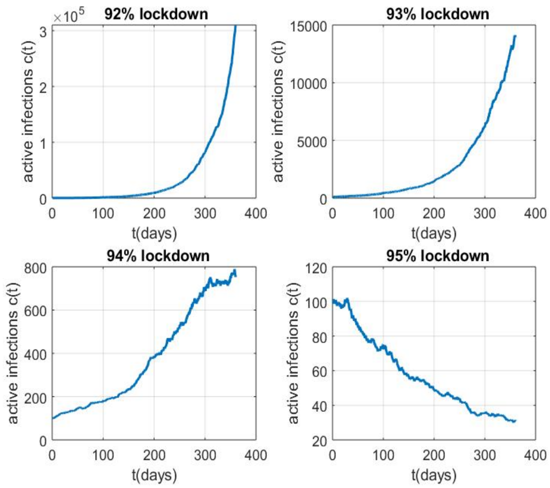

Fig. 11: 'Projections for Karnataka 365 days $92-95\%$ lockdown, using European average parameters, shows sensitivity to lockdown factor.

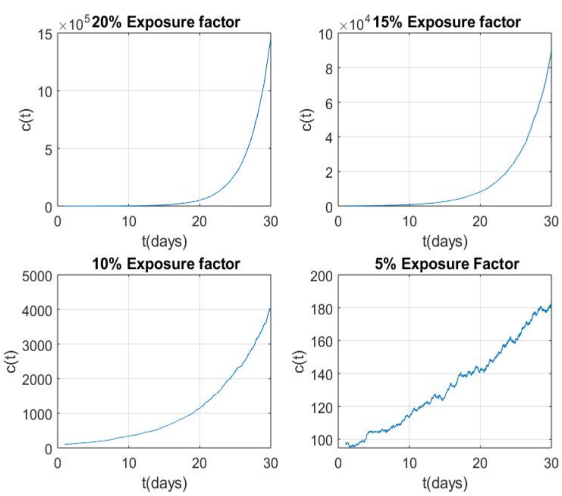

Fig. 12: Projections for Karnataka with varying rates of exposure: Italy parameters. Projections done for 30 days with 100 initial infections assumed on 25^{th} March.