Assignment - Parameter Estimation

Q1

Sol To find Max. likelihood Estimates of parameters θ_1 & θ_2 for normal pop.

Given a random sample $(x_1, x_2, \ldots x_n)$ from a normal pop. with a parameter $\mu = \theta_1$ & $\sigma^2 = \theta_2$ the likelihood of sample:

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \exp\left(-\frac{(x_i - \theta_1)^2}{2\theta_2}\right)$$

Take a log on b/s

$$l(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[-\frac{1}{2}\log(2\pi\theta_2) - \frac{(x_i - \theta_1)^2}{2\theta_2}\right]$$

To find MLE, solve for θ_1 & θ_2

For θ_1

$$\frac{dl}{d\theta_1} = \sum_{i=1}^{n} \frac{(x_i - \theta_1)}{\theta_2} = 0$$

$$\Rightarrow \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

For θ_2:

$$\frac{dl}{d\theta_2} = \sum_{i=1}^{n}\left[-\frac{1}{2\theta_2} + \frac{(x_i-\theta_1)^2}{2\theta_2^2}\right] = 0$$

$$\Rightarrow \frac{1}{\theta_2} = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

Max L.Es. are

$$\hat{\theta}_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\hat{\theta}_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{\theta}_1)^2$$

so these are sol. for mean (θ_1) & var (θ_2) of Normal pdf.

(2.)

Sol Given a random sample $X_1, X_2, \ldots X_n$ from a binomial distribution $B(m, \theta)$, then is

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

log on b/s

$$l(\theta) = \sum_{i=1}^{n} \left[\log \binom{m}{x_i} + x_i \log(\theta) + (m - x_i) \log(1-\theta) \right]$$

Take log on b/s

$$l(\theta) = \sum_{i=1}^{n} \left[\log \binom{m}{x_i} + x_i \log(\theta) + (m - x_i) \log(1-\theta) \right]$$

To find MLE for θ, we diff. w.r.t θ with resp to θ, set derivative to zero & solve for θ

$$\frac{dl}{d\theta} = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right] = 0$$

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right] = 0$$

$$\sum_{i=1}^{n} x_i - nm\theta = 0$$

$$\theta = \frac{1}{nm} \sum_{i=1}^{n} x_i$$

MLE for θ in Bd is

$$\boxed{\hat{\theta} = \frac{1}{nm} \sum_{i=1}^{n} x_i}$$