In [1]:
```
using BenchmarkTools
using Plots; pyplot()
```

Out[1]:
```
Plots.PyPlotBackend()
```

Divide-and-conquer matrix multiplication

In [2]:
```
function divide_and_conquer(A, B)

    n = size(A)[1]

    if n == 1
        return A*B
    end

    n2 = Int(n/2)

    prod = zeros(n, n)

    prod[1:n2,     1:n2]     = A[1:n2,     1:n2]*B[1:n2, 1:n2]     + A[1:n2,     n2+1:end]*B[n2+1:end, 1:n2]
    prod[1:n2,     n2+1:end] = A[1:n2,     1:n2]*B[1:n2, n2+1:end] + A[1:n2,     n2+1:end]*B[n2+1:end, n2+1:end]
    prod[n2+1:end, 1:n2]     = A[n2+1:end, 1:n2]*B[1:n2, 1:n2]     + A[n2+1:end, n2+1:end]*B[n2+1:end, 1:n2]
    prod[n2+1:end, n2+1:end] = A[n2+1:end, 1:n2]*B[1:n2, n2+1:end] + A[n2+1:end, n2+1:end]*B[n2+1:end, n2+1:end]

    return prod

end
```

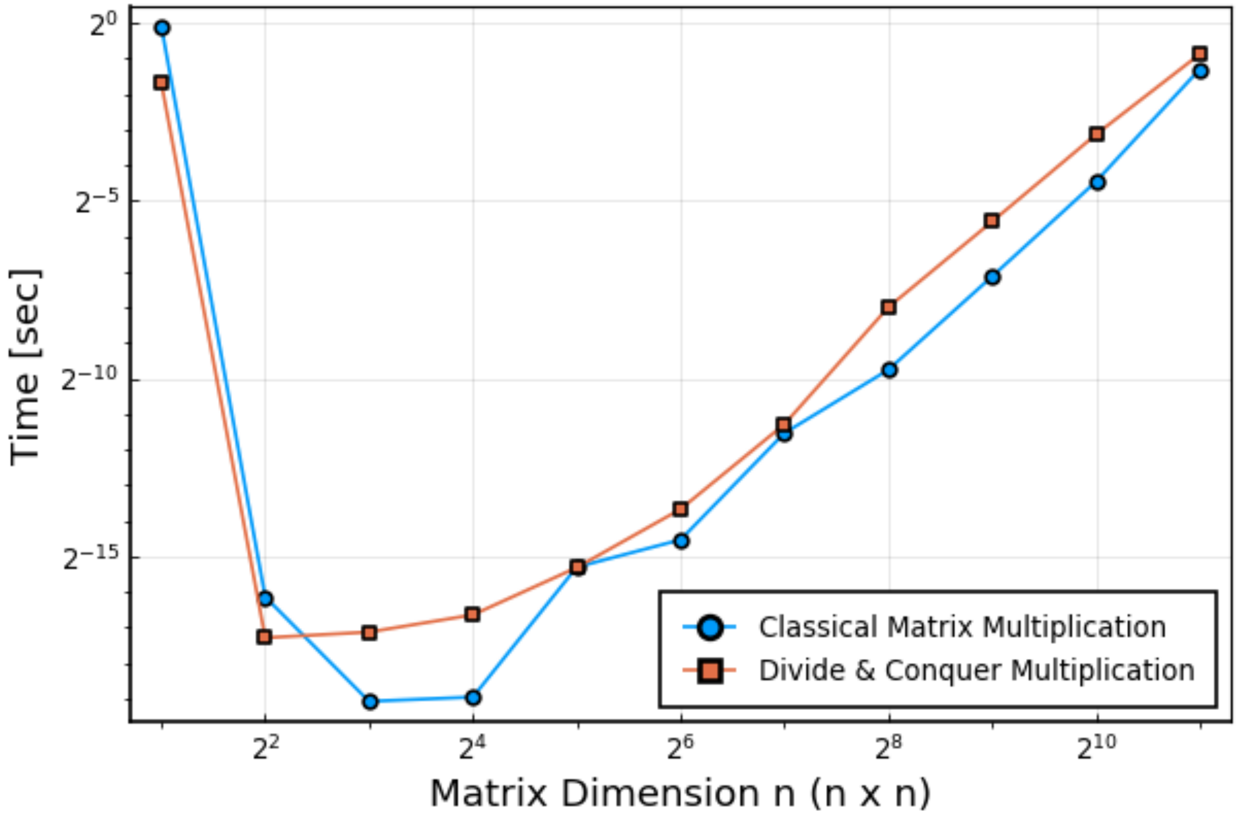
Out[2]:
```
divide_and_conquer (generic function with 1 method)
```

In [3]:
```
ns = Int.(2 .^ range(1, 11, step=1))
time_cmm = zeros(11)
time_dcm = zeros(11)

for (i, n) in enumerate(ns)

    A = rand(n, n)

    val, time_cmm[i] = @timed A*A
    val, time_dcm[i] = @timed divide_and_conquer(A, A)

end
```

In [4]:

```
plot( ns, time_cmm, xscale=:log2, yscale=:log2, label="Classical Matrix Multiplication",
        box=:on, thickness_scaling=1.2, marker=:auto)#, size=(800,600))
plot!(ns, time_dcm, label="Divide & Conquer Multiplication", marker=:auto)
xlabel!("Matrix Dimension n (n x n)")
ylabel!("Time [sec]")
#png("cmm_v_dcm.png")
```

Out[4]:



AMM on sampled data (multivariate Gaussian, multivariate t-distribution)

In [5]:

```
using LinearAlgebra
using Distributions
```

In [6]:

```
n = 500
d = 50;
```

In [7]:

```julia
μ = ones(d)
Σ = [2*0.5^abs(i-j) for i in 1:d, j in 1:d]

μt = zeros(d)

GA = MvNormal(μ, Σ)
T1 = MvTDist(1, μt, Σ)
T3 = MvTDist(3, μt, Σ)

function rand_matrix_from_row_dist(dist, n, d)

    A = zeros(n, d)

    for i in 1:n
        A[i, :] = rand(dist)
    end

    return A

end
```

Out[7]:

```
rand_matrix_from_row_dist (generic function with 1 method)
```

In [8]:

```julia
A_GA = rand_matrix_from_row_dist(GA, n, d)
A_T3 = rand_matrix_from_row_dist(T3, n, d)
A_T1 = rand_matrix_from_row_dist(T1, n, d);
```

In [9]:

```julia
function row_score_probabilities(A)

    row_scores = [norm(row, 2)^2 for row in eachrow(A)]
    probs      = normalize(row_scores, 1)

end
```

Out[9]:

```
row_score_probabilities (generic function with 1 method)
```

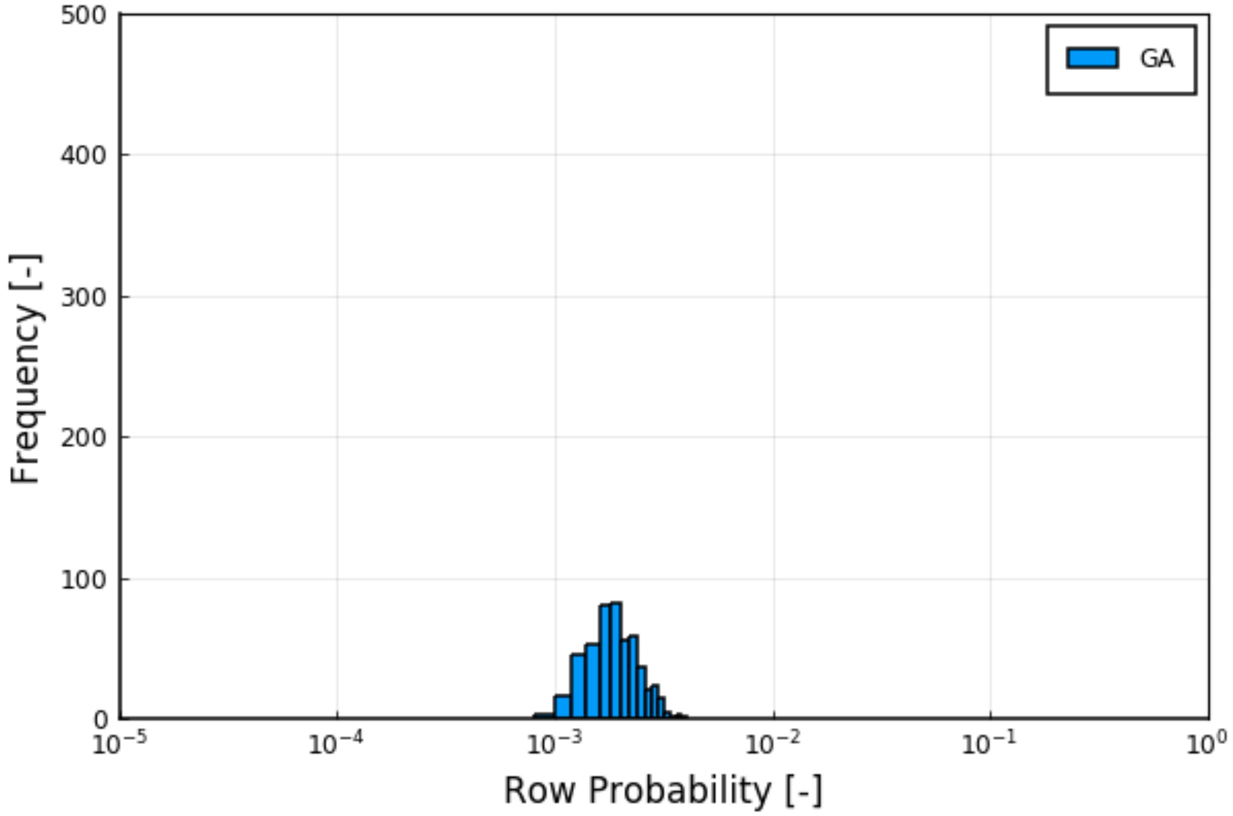
In [10]:

```julia
p_GA = row_score_probabilities(A_GA)
p_T3 = row_score_probabilities(A_T3)
p_T1 = row_score_probabilities(A_T1);
```

In [11]:

```
histogram(p_GA, label="GA",
            xscale=:log10, xlims=(1E-5, 1), ylims=(0, 500), thickness_scaling=1.1, box=:on,
            xlabel="Row Probability [-]", ylabel="Frequency [-]")
#png("row_score_dist_ga.png")
```

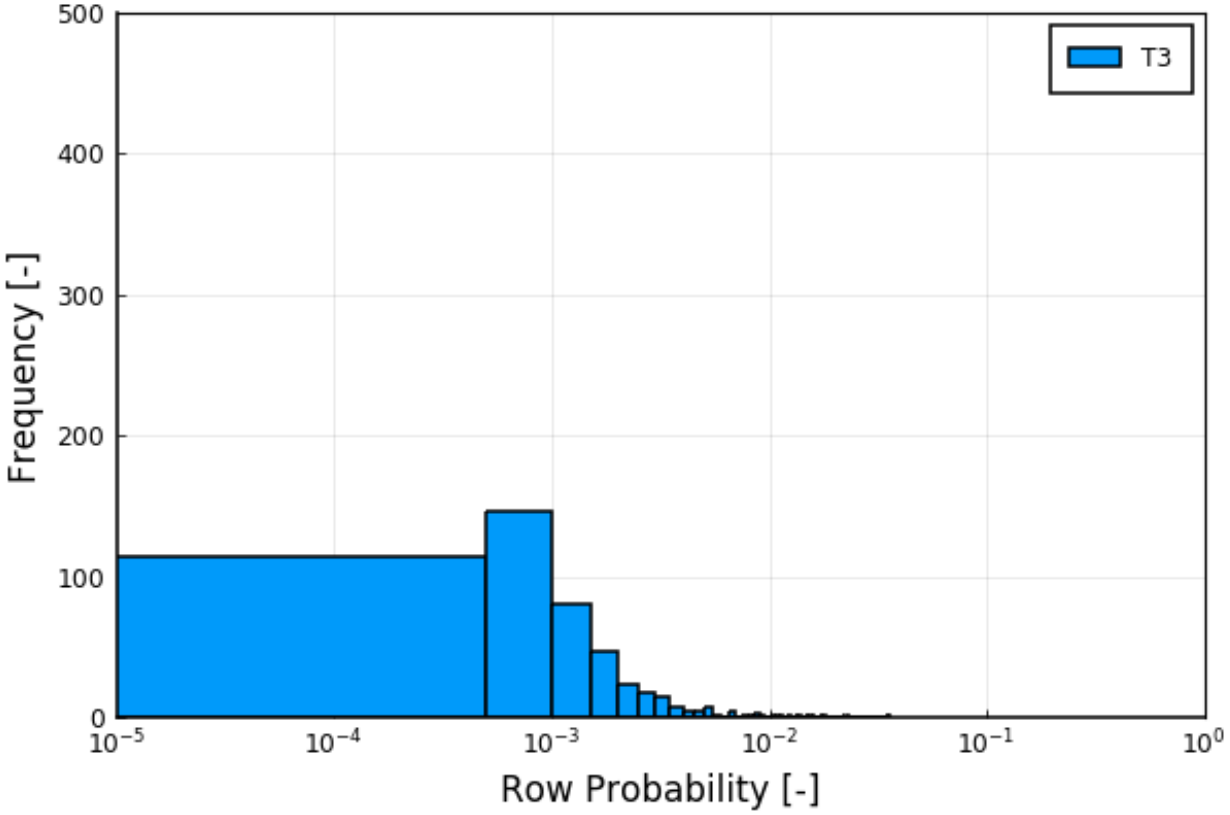
Out[11]:



In [12]:

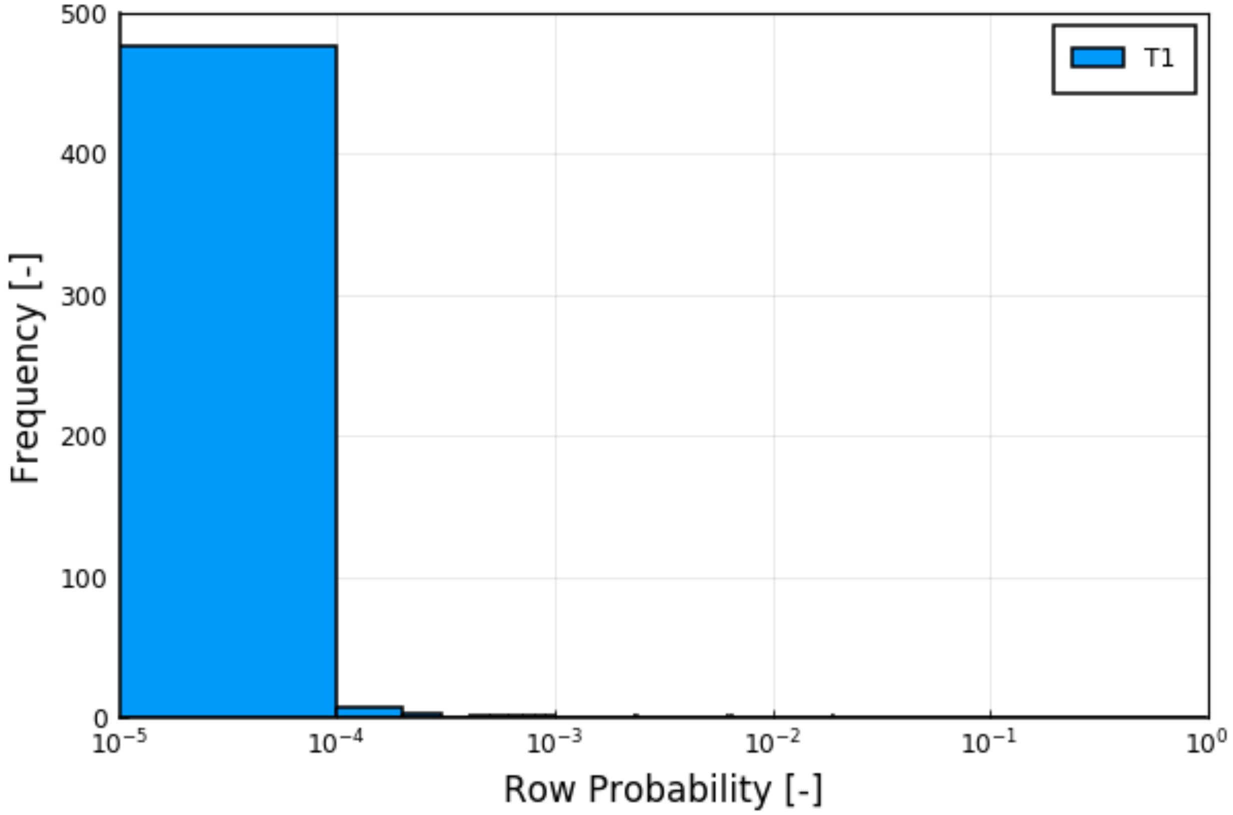
```
histogram(p_T3, label="T3",
            xscale=:log10, xlims=(1E-5, 1), ylims=(0, 500), thickness_
scaling=1.1, box=:on,
            xlabel="Row Probability [-]", ylabel="Frequency [-]")
#png("row_score_dist_t3.png")
```

Out[12]:



```
In [13]: histogram(p_T1, label="T1",
                     xscale=:log10, xlims=(1E-5, 1), ylims=(0, 500), thickness_
         scaling=1.1, box=:on,
                     xlabel="Row Probability [-]", ylabel="Frequency [-]")
         #png("row_score_dist_t1.png")
```

Out[13]:



In [14]:

```
function approximate_matrix_multiplication(A, B, dist, m)

    p = probs(dist)

    AB = zeros(size(A)[1], size(B)[2], )

    AB_exact    = A * B
    frob_exact =   norm(AB_exact)
    spec_exact = opnorm(AB_exact)

    frob_error = zeros(m)
    spec_error = zeros(m)

    for i in 1:m

        ik = rand(dist)
        pk = p[ik]

        AB *= i-1
        AB += 1/pk * A[:, ik] * B[ik, :]'
        AB /= i

        frob_error[i] = abs(  norm(AB) - frob_exact) / frob_exact
        spec_error[i] = abs(opnorm(AB) - spec_exact) / spec_exact

    end

    return prod, frob_error, spec_error

end
```

Out[14]:

```
approximate_matrix_multiplication (generic function with 1 method)
```

Row-norm sampling

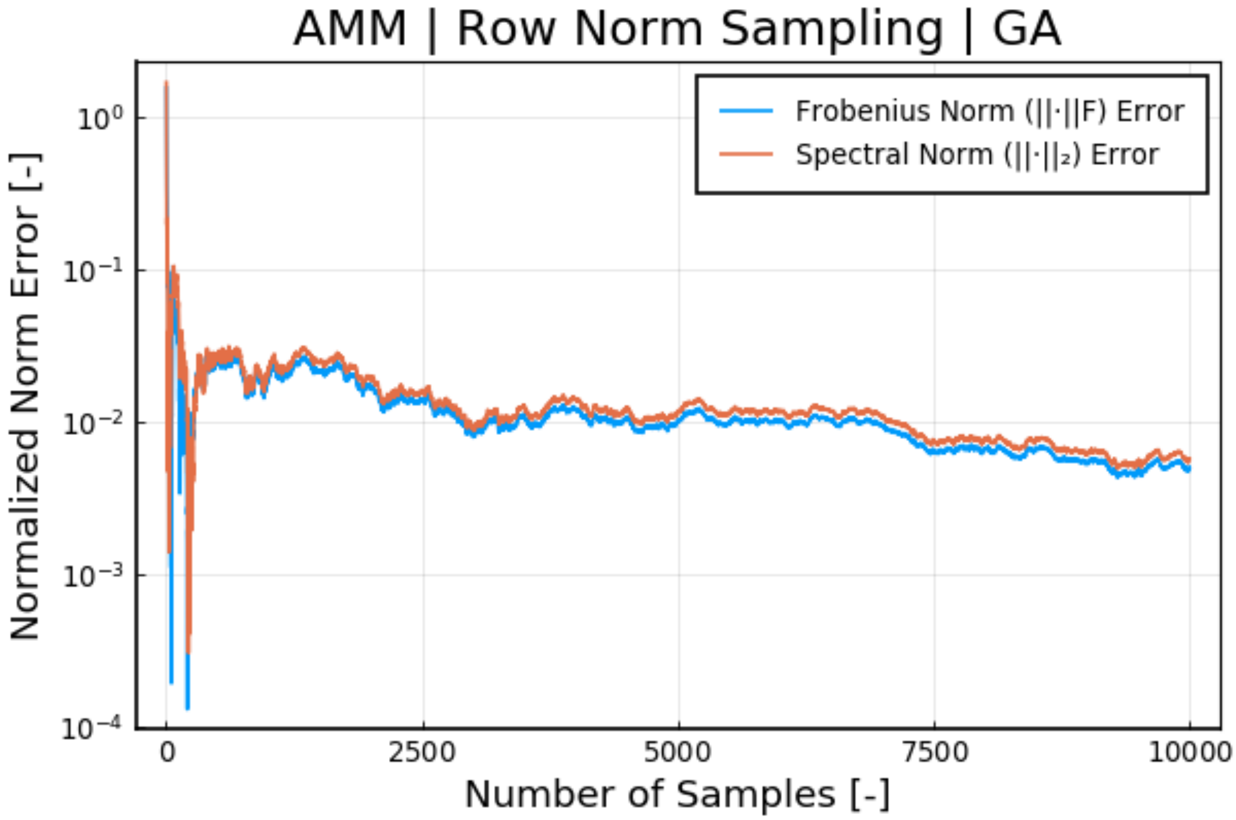
In [15]:

```
AT_GA = transpose(A_GA)
dist_GA = DiscreteNonParametric([i for i in 1:n], p_GA)
m = 10000

ATA_GA, frob_GA, spec_GA = approximate_matrix_multiplication(AT_GA, A_GA, dist_GA, m);
```

In [16]:

```
plot( 1:m, frob_GA, label="Frobenius Norm (||·||F) Error",
        box=:on, thickness_scaling=1.2, yscale=:log10)
plot!(1:m, spec_GA, label="Spectral Norm (||·||₂) Error")
title!("AMM | Row Norm Sampling | GA")
xlabel!("Number of Samples [-]")
ylabel!("Normalized Norm Error [-]")
#png("amm_row_norm_sampling_ga.png")
```

Out[16]:



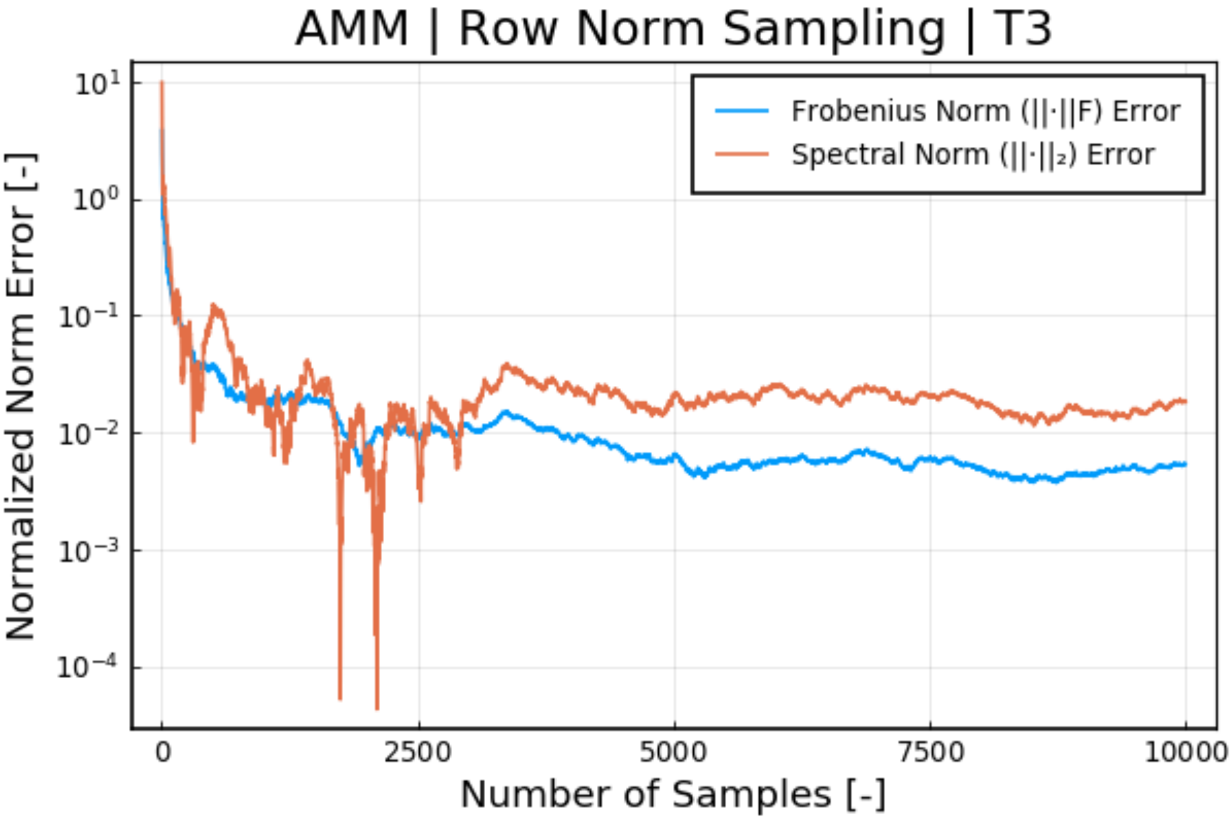
In [17]:

```
AT_T3 = transpose(A_T3)
dist_T3 = DiscreteNonParametric([i for i in 1:n], p_T3)
m = 10000

ATA_T3, frob_T3, spec_T3 = approximate_matrix_multiplication(AT_T3, A_
T3, dist_T3, m);
```

In [18]:

```
plot( 1:m, frob_T3, label="Frobenius Norm (||·||F) Error",
         box=:on, thickness_scaling=1.2, yscale=:log10)
plot!(1:m, spec_T3, label="Spectral Norm (||·||₂) Error")
title!("AMM | Row Norm Sampling | T3")
xlabel!("Number of Samples [-]")
ylabel!("Normalized Norm Error [-]")
#png("amm_row_norm_sampling_t3.png")
```

Out[18]:

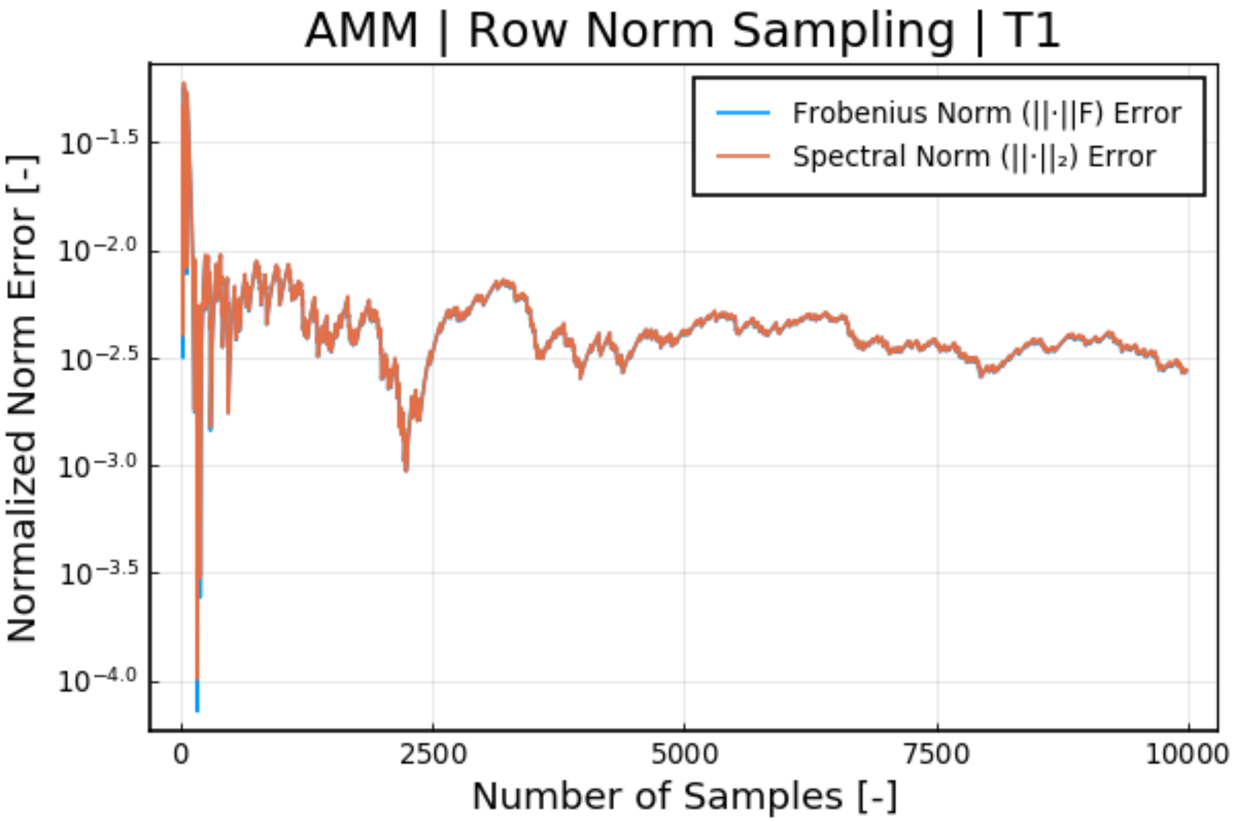


In [19]:

```
AT_T1 = transpose(A_T1)
dist_T1 = DiscreteNonParametric([i for i in 1:n], p_T1)
m = 10000

ATA_T1, frob_T1, spec_T1 = approximate_matrix_multiplication(AT_T1, A_
T1, dist_T1, m);
```

```
In [20]: plot( 1:m, frob_T1, label="Frobenius Norm (||·||F) Error",
                 box=:on, thickness_scaling=1.2, yscale=:log10)
         plot!(1:m, spec_T1, label="Spectral Norm (||·||₂) Error")
         title!("AMM | Row Norm Sampling | T1")
         xlabel!("Number of Samples [-]")
         ylabel!("Normalized Norm Error [-]")
         #png("amm_row_norm_sampling_t1.png")
```

Out[20]:



Uniform sampling

```
In [21]: uniform_dist = DiscreteNonParametric([i for i in 1:n], [1/n for i in 1
         :n])
```

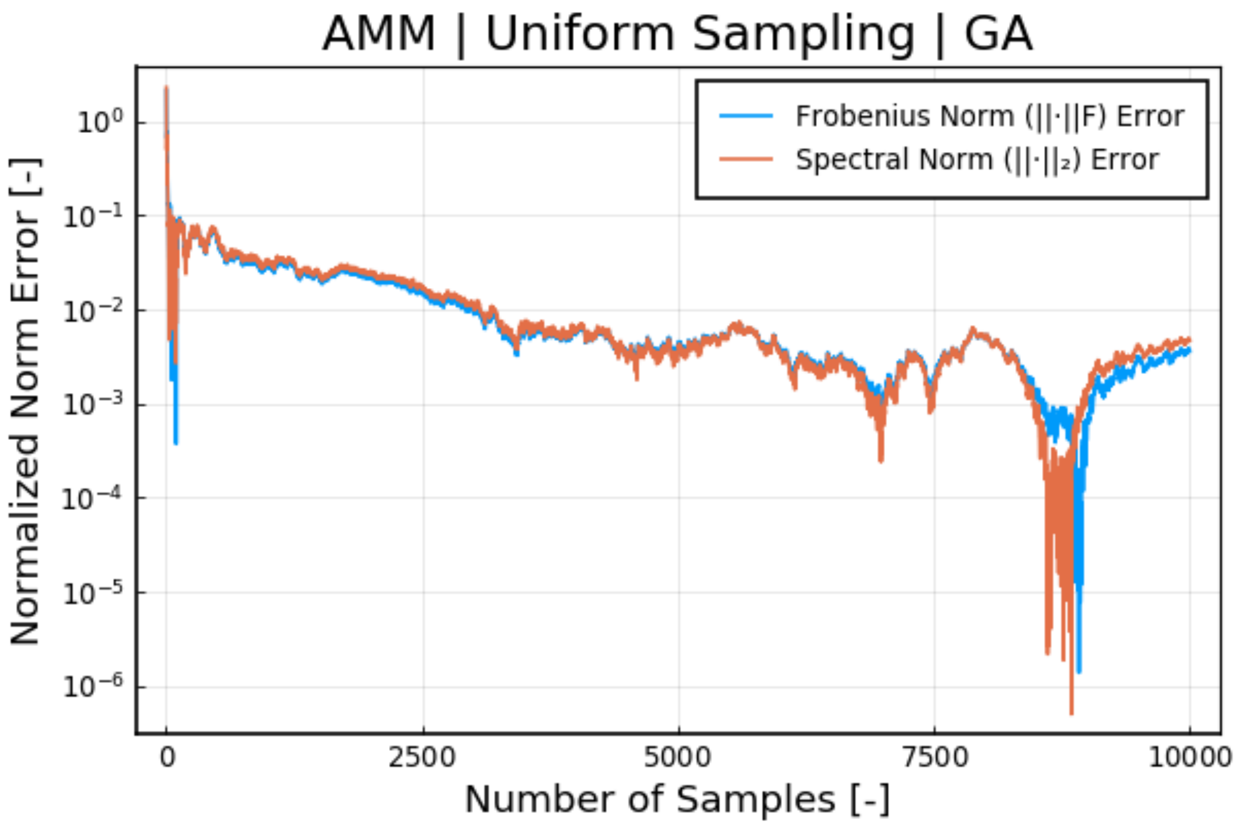
```
Out[21]: DiscreteNonParametric{Int64,Float64,Array{Int64,1},Array{Float64,1}}
         (
         support: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10  …  491, 492, 493, 494, 495,
         496, 497, 498, 499, 500]
         p: [0.002, 0.002, 0.002, 0.002, 0.002, 0.002, 0.002, 0.002, 0.002, 0
         .002  …  0.002, 0.002, 0.002, 0.002, 0.002, 0.002, 0.002, 0.002, 0.0
         02, 0.002]
         )
```

```
In [22]: ATA_GA, frob_GA, spec_GA = approximate_matrix_multiplication(AT_GA, A_
         GA, uniform_dist, m);
```

In [23]:

```
plot( 1:m, frob_GA, label="Frobenius Norm (||·||F) Error",
        box=:on, thickness_scaling=1.2, yscale=:log10)
plot!(1:m, spec_GA, label="Spectral Norm (||·||₂) Error")
title!("AMM | Uniform Sampling | GA")
xlabel!("Number of Samples [-]")
ylabel!("Normalized Norm Error [-]")
#png("amm_uniform_sampling_ga.png")
```

Out[23]:



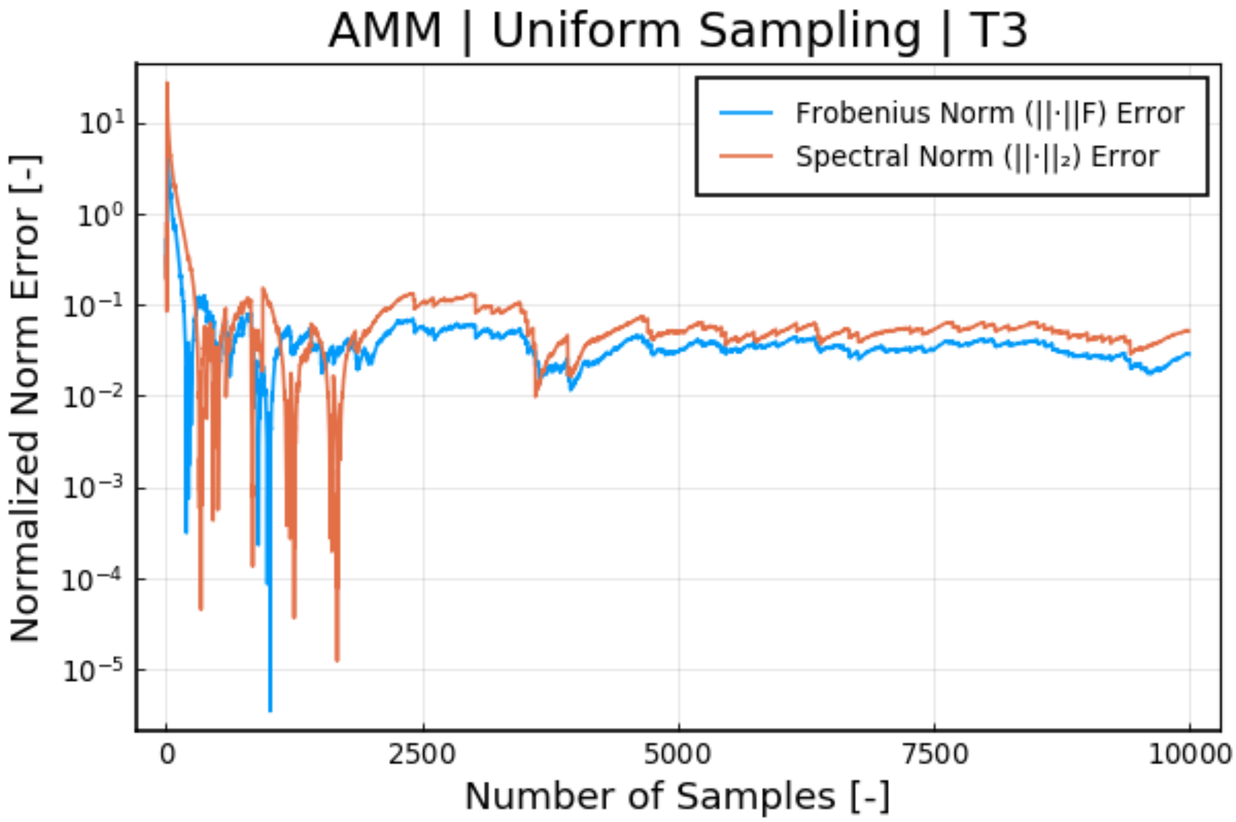
In [24]:

```
ATA_T3, frob_T3, spec_T3 = approximate_matrix_multiplication(AT_T3, A_
T3, uniform_dist, m);
```

In [25]:

```
plot( 1:m, frob_T3, label="Frobenius Norm (||·||F) Error",
          box=:on, thickness_scaling=1.2, yscale=:log10)
plot!(1:m, spec_T3, label="Spectral Norm (||·||₂) Error")
title!("AMM | Uniform Sampling | T3")
xlabel!("Number of Samples [-]")
ylabel!("Normalized Norm Error [-]")
#png("amm_uniform_sampling_t3.png")
```

Out[25]:

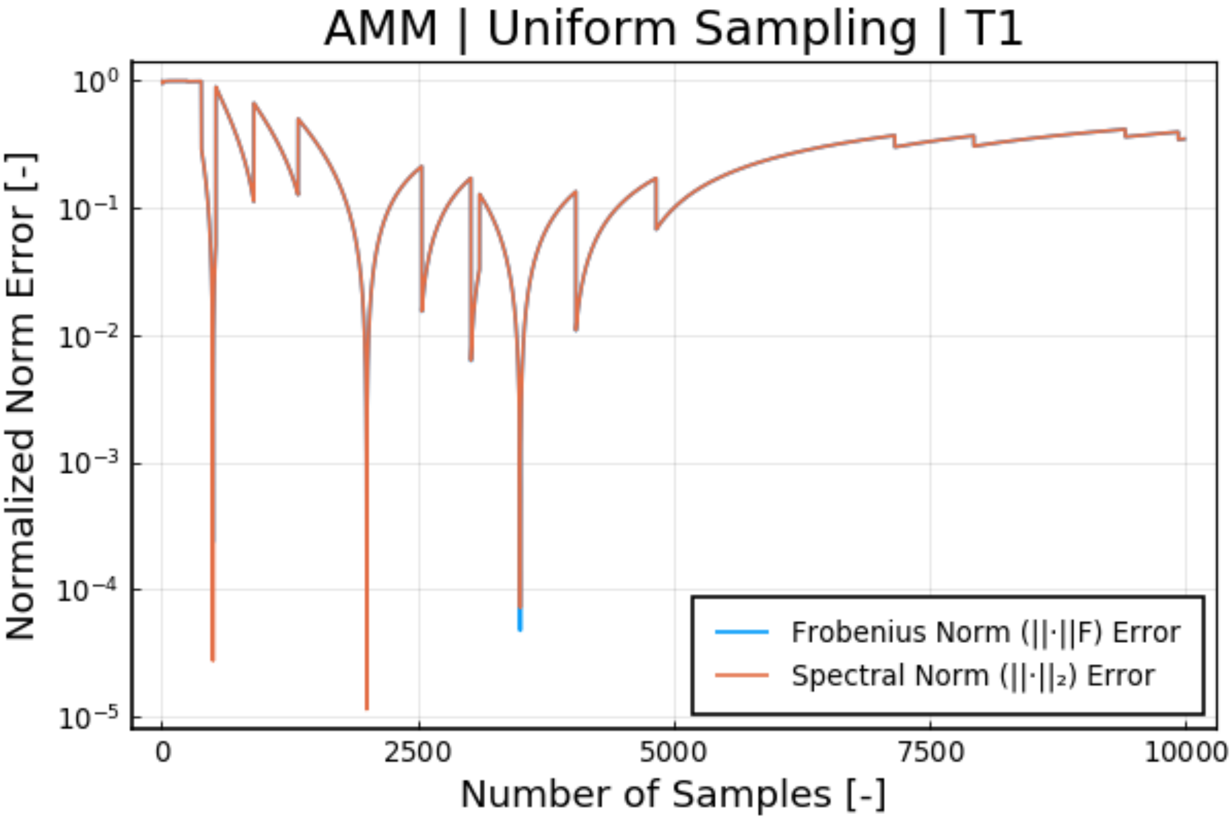


In [26]:

```
ATA_T1, frob_T1, spec_T1 = approximate_matrix_multiplication(AT_T1, A_
T1, uniform_dist, m);
```

```
In [27]: plot( 1:m, frob_T1, label="Frobenius Norm (||·||F) Error",
                 box=:on, thickness_scaling=1.2, yscale=:log10)
         plot!(1:m, spec_T1, label="Spectral Norm (||·||₂) Error")
         title!("AMM | Uniform Sampling | T1")
         xlabel!("Number of Samples [-]")
         ylabel!("Normalized Norm Error [-]")
         #png("amm_uniform_sampling_t1.png")
```

Out[27]:



AMM on MNIST

```
In [28]: using MAT
```

```
WARNING: could not import HDF5.HDF5Group into _hdf5_implementation
WARNING: could not import HDF5.HDF5Dataset into _hdf5_implementation
┌ Warning: Error requiring `HDF5` from `Plots`
│   exception = (LoadError("/Users/rossalexander/.julia/packages/Plo
ts/uCh2y/src/backends/hdf5.jl", 162, UndefVarError(:HDF5Group)), Uni
on{Ptr{Nothing}, Base.InterpreterIP}[Ptr{Nothing} @0x000000010943b7f
f, Ptr{Nothing} @0x00000001094d71d3, Ptr{Nothing} @0x00000001094d905
b, Ptr{Nothing} @0x00000001094d6c5f, Ptr{Nothing} @0x00000001094d6ed
c, Base.InterpreterIP in top-level CodeInfo for Plots._hdf5_implemen
tation at statement 4, Ptr{Nothing} @0x00000001094f0dde, Ptr{Nothing
} @0x00000001094efed0, Ptr{Nothing} @0x00000001094f05e1, Ptr{Nothing
} @0x00000001094f0ce6, Ptr{Nothing} @0x00000001094c9e77, Ptr{Nothing
} @0x00000001094f1ced, Ptr{Nothing} @0x000000010e360c73, Ptr{Nothing
} @0x0000000134a72a7c, Ptr{Nothing} @0x00000001094d907f, Ptr{Nothing
} @0x00000001094d6c5f, Ptr{Nothing} @0x00000001094d6edc, Base.Interp
reterIP in top-level CodeInfo for Plots at statement 10, Ptr{Nothing
} @0x00000001094f0dde, Ptr{Nothing} @0x00000001094f1b07, Ptr{Nothing
} @0x00000001471a3c1f, Ptr{Nothing} @0x00000001471a3c3c, Ptr{Nothing
} @0x0000000134a5cc50, Ptr{Nothing} @0x00000001471a3b7d, Ptr{Nothing
} @0x00000001471a3b9c, Ptr{Nothing} @0x000000011aeea4ab, Ptr{Nothing
} @0x00000001471a3ad3, Ptr{Nothing} @0x00000001471a3afc, Ptr{Nothing
} @0x00000001094ceac8, Ptr{Nothing} @0x00000001094ced85, Ptr{Nothing
} @0x000000011ae950d1, Ptr{Nothing} @0x00000001094ceac8, Ptr{Nothing
} @0x00000001094ced85, Ptr{Nothing} @0x0000000134a6822a, Ptr{Nothing
} @0x0000000134a68ca2, Ptr{Nothing} @0x0000000134a6e57f, Ptr{Nothing
} @0x0000000134a5d3f2, Ptr{Nothing} @0x00000001470bb3c5, Ptr{Nothing
} @0x00000001094f1899, Ptr{Nothing} @0x00000001094f074b, Ptr{Nothing
} @0x00000001094d6bc7, Ptr{Nothing} @0x00000001094d6edc, Base.Interp
reterIP in top-level CodeInfo for Main at statement 0, Ptr{Nothing}
@0x00000001094f0dde, Ptr{Nothing} @0x00000001094f1b07, Ptr{Nothing}
@0x0000000134a9af01, Ptr{Nothing} @0x00000001470ede46, Ptr{Nothing}
@0x00000001094ceac8, Ptr{Nothing} @0x00000001094ced85, Ptr{Nothing}
@0x000000011ae46bef, Ptr{Nothing} @0x000000011ae47164, Ptr{Nothing}
@0x000000011ae4717c, Ptr{Nothing} @0x00000001094dd12a])
└ @ Requires /Users/rossalexander/.julia/packages/Requires/035xH/src
/require.jl:44
```

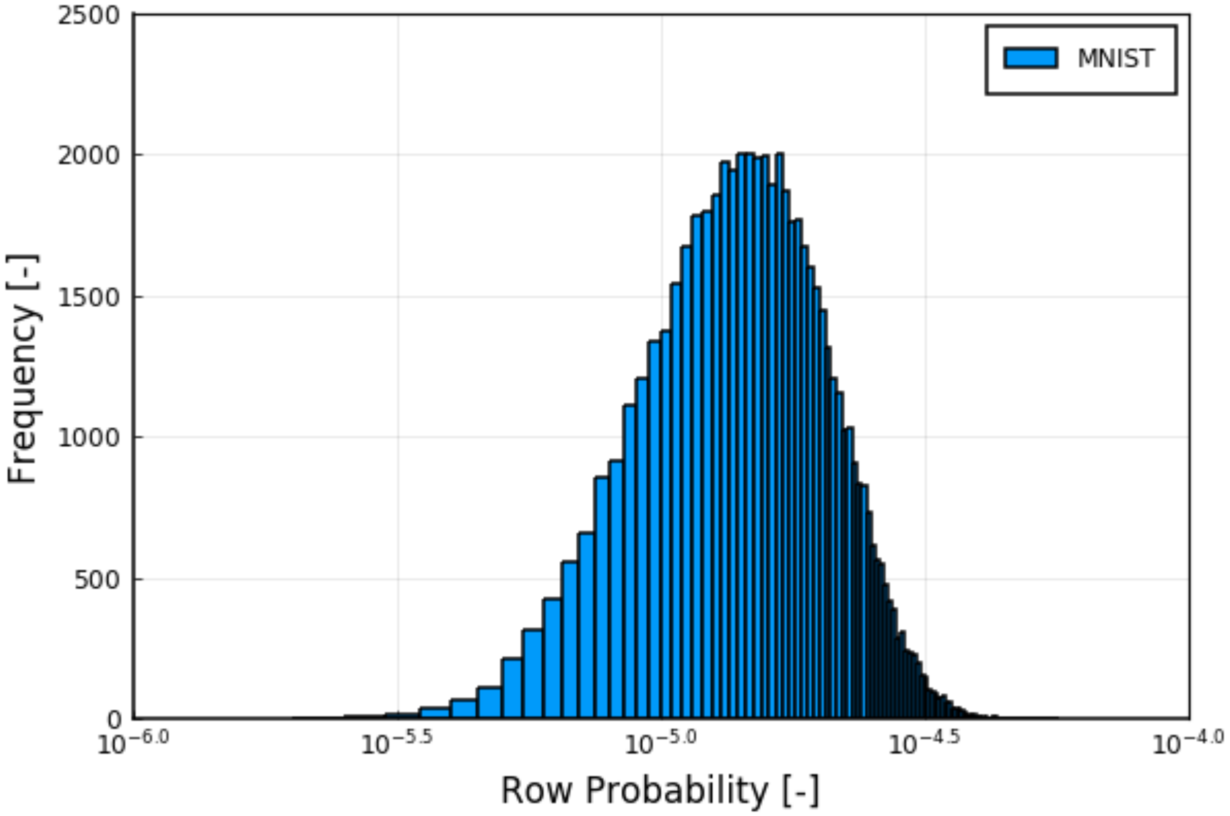
In [29]:

```
file = matopen("mnist_matrix.mat")
A_MNIST = read(file, "A")
close(file)
```

In [30]:

```
p_MNIST = row_score_probabilities(A_MNIST)

histogram(p_MNIST, label="MNIST",
            xscale=:log10, xlims=(1E-6, 1E-4), ylims=(0, 2500), thickness_scaling=1.1, box=:on,
            xlabel="Row Probability [-]", ylabel="Frequency [-]")
#png("row_score_dist_mnist.png")
```

Out[30]:



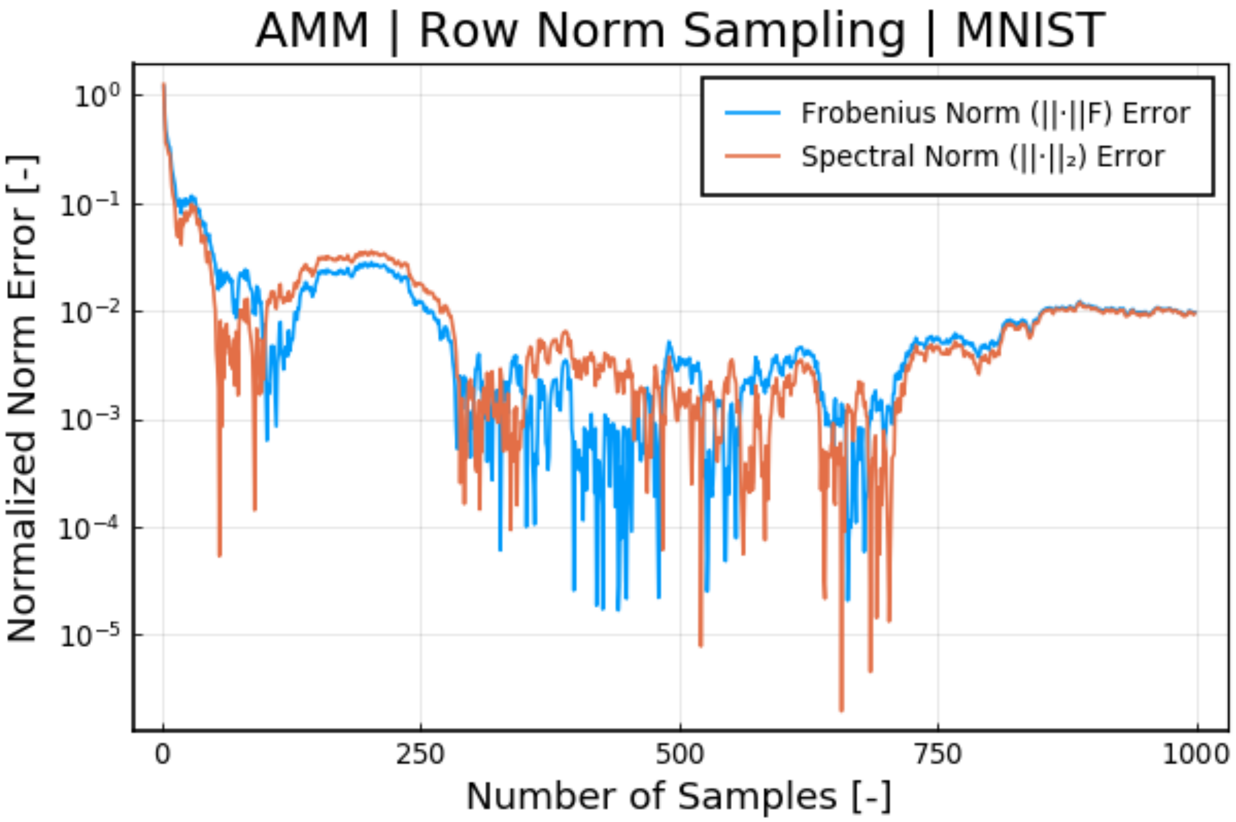
In [31]:

```
AT_MNIST = transpose(A_MNIST)
dist_MNIST = DiscreteNonParametric([i for i in 1:size(A_MNIST)[1]], p_MNIST)
m = 1000

ATA_MNIST, frob_MNIST, spec_MNIST = approximate_matrix_multiplication(AT_MNIST, A_MNIST, dist_MNIST, m);
```

In [32]:

```
plot( 1:m, frob_MNIST, label="Frobenius Norm (||·||F) Error",
        box=:on, thickness_scaling=1.2, yscale=:log10)
plot!(1:m, spec_MNIST, label="Spectral Norm (||·||₂) Error")
title!("AMM | Row Norm Sampling | MNIST")
xlabel!("Number of Samples [-]")
ylabel!("Normalized Norm Error [-]")
#png("amm_row_norm_sampling_mnist.png")
```

Out[32]:


AMM | Row Norm Sampling | MNIST
Frobenius Norm (||·||F) Error
Spectral Norm (||·||₂) Error
Normalized Norm Error [-]
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
250
500
750
1000
Number of Samples [-]