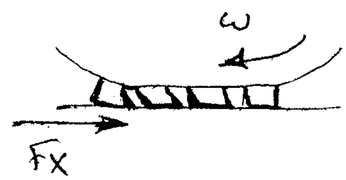
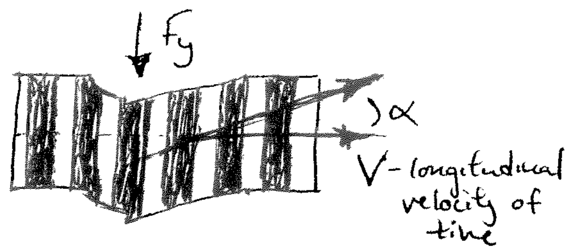
Coupled Tire forces

Since the lateral and longitudinal tire force models have the same form (when described with the appropriate variables), they can easily be extended to a combined force model



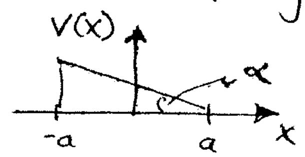


The contact patch can only support as much deformation as friction will allow. What is important is the overall magnitude of the combined lateral and longitudinal deformation.

The longitudinal force per unit length required is

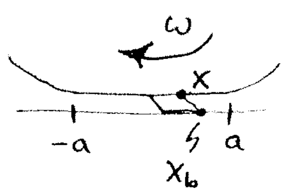
$$q_x(x) = C_{px}(a-x)\frac{\lambda}{1+\lambda} \quad \text{as before}$$

The lateral force per unit length required must be modified to account for longitudinal slip and the shifting of the contact patch.



$$q_y(x) = C_{py}(a-x_b)\tan\alpha$$

before we did not distinguish x from x_b



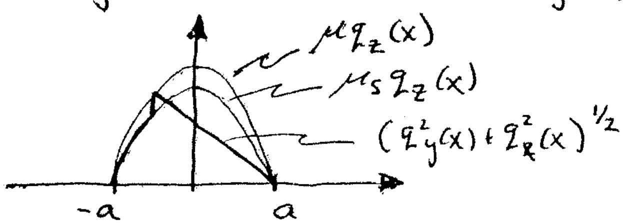
$$x_b = x + u(x) = x + (a-x)\frac{\lambda}{1+\lambda}$$

$$\Rightarrow q_y(x) = C_{py}\left[(a-x) - (a-x)\left(\frac{\lambda}{1+\lambda}\right)\right]\tan\alpha$$

$$= C_{py}(a-x)\frac{\tan\alpha}{1+\lambda}$$

This $1+$ term due to tread deformation is new

Sliding now occurs when $\left(q_y^2(x) + q_x^2(x)\right)^{1/2} = q_z(x)\cdot\mu$



Sliding thus begins when the combination of deformation due to longitudinal and lateral motion exceeds available friction $\Rightarrow$ Tire forces are coupled!

Stepping back through the formulation of the force in the contact patch, the total force magnitude is given by

$$F = \underset{\text{adhesion region}}{\int} \left(q_x^2(x) + q_y^2(x) \right) dx \;+\; \underset{\text{sliding region}}{\int} \mu_s \, q_z(x) \, dx$$

Rewriting the previous force equations slightly, we have

$$f = \sqrt{C_x^2 \left(\frac{\lambda}{1+\lambda}\right)^2 + C_\alpha^2 \left(\frac{\tan\alpha}{1+\lambda}\right)^2}$$

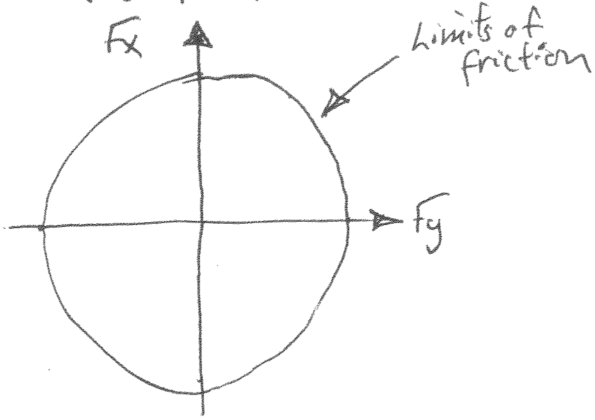
$$F = \begin{cases} f - \dfrac{1}{3\mu F_z}\left(2 - \dfrac{\mu_s}{\mu}\right) f^2 + \dfrac{1}{9\mu^2 F_z^2}\left(1 - \dfrac{2\mu_s}{3\mu}\right) f^3 & f \leq 3\mu F_z \\ \mu_s F_z & f > 3\mu F_z \end{cases}$$

Breaking into components in the lateral and longitudinal direction depending upon the ratio of force demanded gives:

$$F_x = \frac{C_x \left(\frac{\lambda}{1+\lambda}\right)}{f} F \qquad F_y = \frac{-C_\alpha \left(\frac{\tan\alpha}{1+\lambda}\right)}{f} F$$

This collapses to our previous expressions when we have only lateral or longitudinal force.

These coupled equations describe the fact that there is only so much force that can be generated in the contact patch. What is used for longitudinal force cannot be used for lateral force and vice versa. This is often called the "Friction Circle" concept or the "Friction Ellipse".

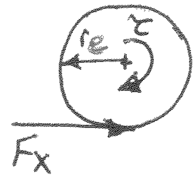


Racing is about keeping the car at the edge of the friction circle at all times.

Coupled Tire Forces - Simplified Model

A coupled tire model can be derived by considering both the slip angle, α, and the longitudinal slip, κ, in the tire contact patch.

Since $\lambda = \frac{R\omega - V}{V}$ where ω is the angular velocity of the wheel and V (in this case) is the longitudinal velocity of the tire, implementing such a model requires a new state, ω, for each wheel. A simple force balance around the wheel gives the state equation:



$$\tau - r_e F_x = J_w \dot{\omega}$$

J_w: moment of inertia for the tire and wheel

This can be added to the model but the planar model of the car with 3 states (U_x, U_y, r) now grows to seven. A simpler way of including the longitudinal forces in the model can be derived by making two assumptions:

(1) The wheel inertia is much less than that of the car
(2) We do not command more brake torque then the tire can support (otherwise we'd lock the wheel)

With these assumptions, the wheel slip dynamics are ignored since they are much faster than the dynamics of turning or accelerating the car. This is reasonable since it is much easier to spin a tire then the whole car (think of a burnout here). We then assume a static relationship between the torque we apply to the wheel and the longitudinal force:

$$\tau = r_e F_x$$

While we neglect wheel speed dynamics, the coupling of lateral and longitudinal forces is still important. With this model, the torque (drive or brake torque) applied to the wheel can be thought of as "de-rating" the tire in the lateral direction. In other words, a given value of longitudinal force applied to the tire reduces the peak lateral force available. It also changes the lateral force produced at a certain slip angle.

The maximum lateral force possible with a given level of longitudinal force is:

$$F_{y\,max} = \sqrt{(\mu F_z)^2 - F_x^2}$$

We will simplify a bit and take a single coefficient of friction for this model so

$$\mu_p = \mu_s = \mu$$

Writing in a slightly different form:

$$F_{y\,max} = \xi \mu F_z$$

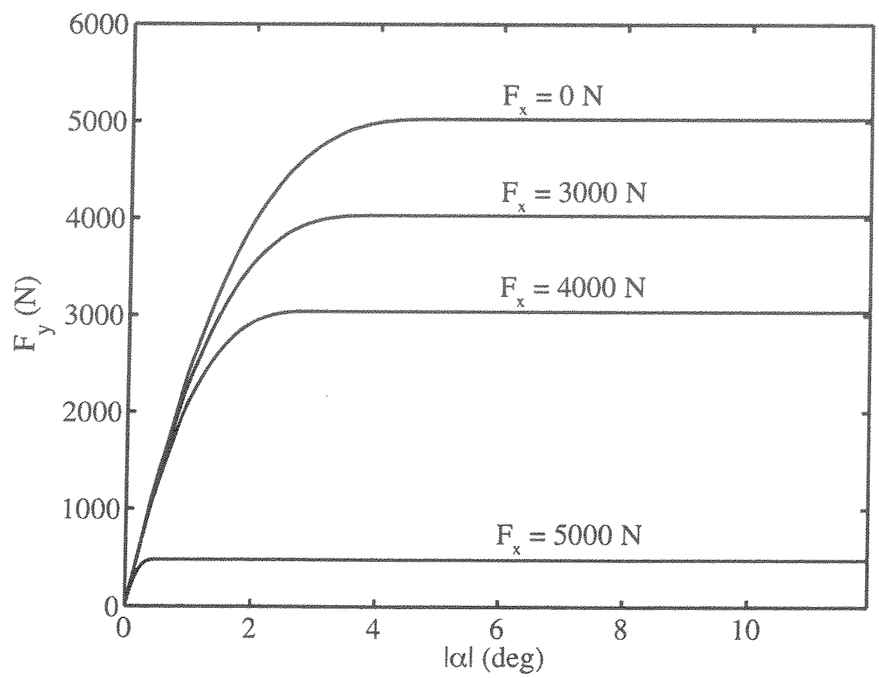
$$\text{where} \quad \xi = \frac{\sqrt{(\mu F_z)^2 - F_x}}{\mu F_z}$$

If we treat this as a new peak force and return to the Fiala model, this gives

$$F_y = \begin{cases} -C_\alpha \tan\alpha + \dfrac{C_\alpha^2}{3\xi\mu F_z}|\tan\alpha|\tan\alpha - \dfrac{C_\alpha^3}{27\xi^2\mu^2 F_z^2}\tan^3\alpha & |\alpha| \le \alpha_{sl} \\ -\xi\mu F_z \,\mathrm{sgn}(\alpha) & |\alpha| > \alpha_{sl} \end{cases}$$

$$\text{where} \quad \alpha_{sl} = \tan^{-1}\left(\frac{3\xi\mu F_z}{C_\alpha}\right)$$

So the given level of longitudinal force creates different lateral force curves. This looks like:



This model assumes that we get F_x and then figures out F_y. That works well in many cases but sometimes the competition between lateral and longitudinal slip is important to capture.