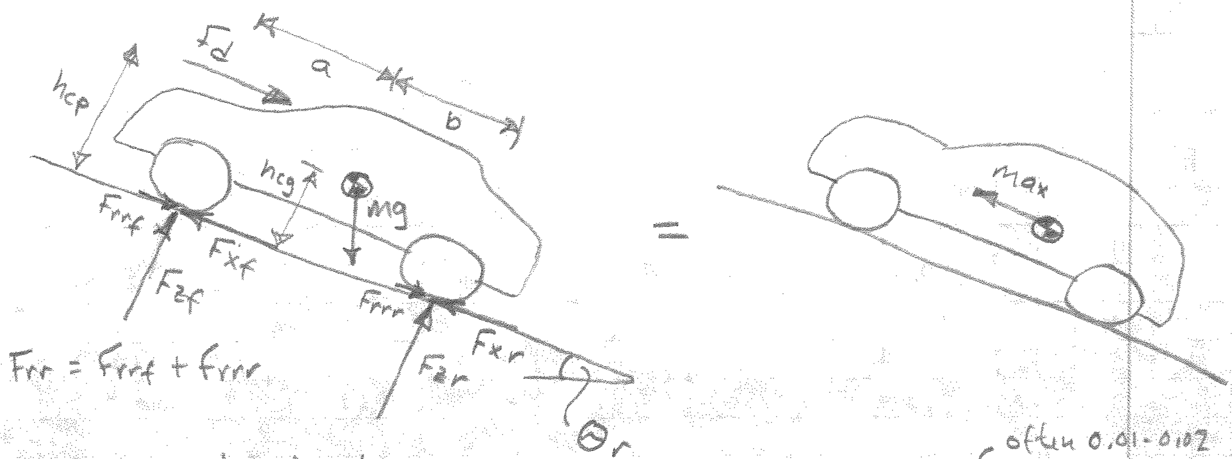
Longitudinal forces

A simple free body diagram of a car driving up a grade looks like:



F_{xf}, F_{xr} - tractive/braking forces
F_{rrf}, F_{rrr} - rolling resistances
F_{zf}, F_{zr} - normal forces
F_d - aerodynamic drag

$F_{rr} = f_{rr} mg$ (often 0.01-0.02)

$F_d = \frac{1}{2}\rho C_D A U_x^2$ (ρ: air density; C_D: coefficient of drag; A: frontal area)

When traveling straight, $a_x = \dot{U}_x$ and so

$$m\dot{U}_x = F_{xf} + F_{xr} - F_{rr} - F_d - mg\sin\theta_r$$

$F_{xf}, F_{xr} > 0$ for acceleration
$F_{xf}, F_{xr} < 0$ for braking

The grade term can grow to be quite large and can be a limiting factor for heavy vehicles on steep grades. The aerodynamic drag becomes significant at higher speeds.

You can see from this diagram that the normal load on the tires changes from acceleration and grade. We aren't quite ready to go there so, for now, consider the c.g. remaining on the ground.

If we want to get a desired level of acceleration, we can solve for the net force required by accounting for drag, rolling resistance and grade.

Let $F_x = F_{xf} + F_{xr}$

$$m\dot{U}_{x_{des}} = F_x - F_{rr} - F_d - mg\sin\theta_r$$

So $$F_x = m\dot{U}_{x_{des}} + F_{rr} + F_d + mg\sin\theta_r$$

should produce $\dot{U}_{x_{des}}$ if all forces are known. Any unknown forces act as disturbances to the system.

So it is straight forward to design a speed control for a flat road. Assume we have a desired speed profile V_{des} and its acceleration a_{des}

$$\dot{V}_{des} = a_{des}$$

Let's also suppose that we want the system to have first order dynamics with time constant λ so

$$\frac{d}{dt}(V_{des} - U_x) = -\lambda\,(V_{des} - U_x)$$

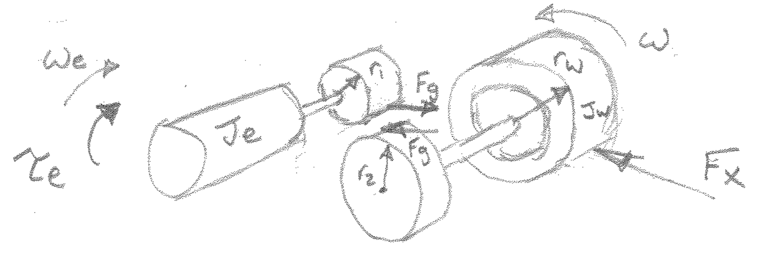
and $U_x \to V_{des}$ as $t \to \infty$.

This means $\dot{V}_{des} - \dot{U}_x = -\lambda(V_{des} - U_x)$

$$a_{des} - \frac{1}{m}\left[F_x - F_{rr} - F_d\right] = -\lambda(V_{des} - U_x)$$

$$\Rightarrow\; F_x = \underbrace{m a_{des} + F_{rr} + F_d}_{\text{Feedforward}} + \underbrace{m\lambda(V_{des} - U_x)}_{\text{Feedback}}$$

Since acceleration and braking forces are produced by engine (or electric motor) torque and a drivetrain or braking torques on the wheel, the wheel dynamics can also be important. Consider the following simplified model of an engine or motor connected to a wheel (which happens to be exact for the case of our electrified DeLorean)



$$\tau_e = J_e\dot{\omega}_e + F_g r_1 \qquad\qquad F_g r_2 = J_w\dot{\omega} + F_x r_w$$

$$F_g = \frac{\tau_e - J_e\dot{\omega}_e}{r_1} \qquad\qquad \frac{r_2}{r_1}\left[\tau_e - J_e\dot{\omega}_e\right] = J_w\dot{\omega} + F_x r_w$$

since $\frac{\omega_e}{\omega} = \frac{r_2}{r_1}$ ...

$$\frac{r_2}{r_1}\left[\tau_e - J_e\frac{r_2}{r_1}\dot{\omega}\right] = J_w\dot{\omega} + F_x r_w$$

$$\left[J_w + \left(\frac{r_2}{r_1}\right)^2 J_e\right]\dot{\omega} = \frac{r_2}{r_1}\tau_e - F_x r_w$$

$$J_{eff}\,\dot{\omega} = \frac{r_2}{r_1}\tau_e - F_x r_w$$

So the wheels have an effective inertia consisting of their actual moment of inertia and inertia of components on the other side of the gearbox (the engine or motor) magnified by the square of the gear ratio.

All of this inertia must be accelerated to accelerate the vehicle. If there is no slip at the wheels

$$U_x = r_w \cdot \omega$$

So for a rear wheel drive vehicle under acceleration

$$m\dot{U}_x = \frac{r_2}{r_1}\left(\frac{\tau_e}{r_w}\right) - \frac{J_{eff}}{r_w} \cdot \left(\frac{\dot{U}_x}{r_w}\right) - F_{rr} - F_d - mg\sin\theta_r$$

$$m_{eff}\,\dot{U}_x = \left(\frac{r_2}{r_1}\right)\left(\frac{\tau_e}{r_w}\right) - F_{rr} - F_d - mg\sin\theta_r$$

for some effective mass $m_{eff} = m + \frac{J_{eff}}{r_w^2}$

If the drivetrain inertia becomes significant, the car responds to changes in engine torque as if it had a higher mass.

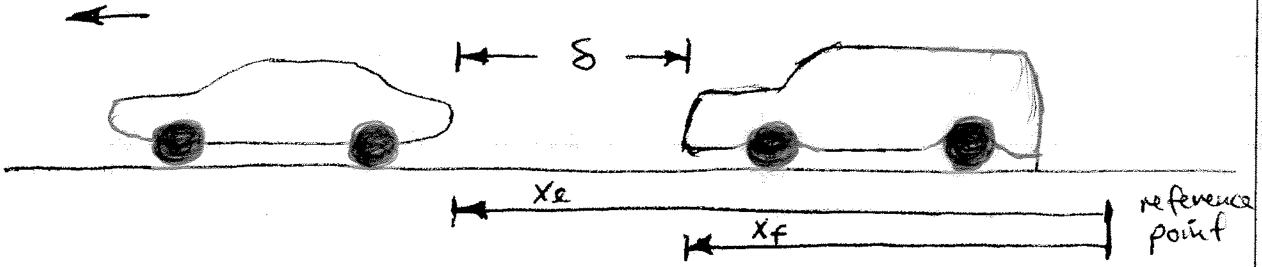
In fact, there is slip at the tires in order to produce longitudinal forces, similar to the way in which slip angles are necessary to produce lateral forces. Thus the wheels and the vehicle do not actually move according to the ratio $U_x = r_w \cdot \omega$.

This difference is often negligible but becomes significant in cases of excessive braking (where the wheel can lock up) or drive torque (which can produce wheel spin).

Vehicle Following

Just as we can use longitudinal dynamics to develop a speed controller and give us cruise control functionality, we can also design following controllers. These enable the car to follow a lead vehicle at some desired spacing.

The logical flow is very similar - we determine the desired spacing error dynamics, calculate the longitudinal acceleration we need to achieve those dynamics then calculate the necessary longitudinal force. Finally we apply brakes or use our engine/motor to develop this force.



Define some terms: x_l - position of lead vehicle
$\dot{x}_l$ - velocity " " "
$\ddot{x}_l$ - acceleration " " "

$x_f, \dot{x}_f, \ddot{x}_f$ - same quantities for follower

$\delta \triangleq$ actual spacing between vehicles: $\delta = x_l - x_f$
$\delta_{des} \triangleq$ desired spacing (more on this shortly)

$\varepsilon \triangleq \delta - \delta_{des}$ spacing error

In this formulation, our control objective is for the spacing error to go to zero (or at least be small) so that we track the desired spacing.

What should the desired spacing be?

* Constant - Proposed for some automated highway schemes (as close as 2m). Assumes no human input and that low relative velocity impacts (should they occur) would be mild.
* Brick Wall Stop - From basic physics, if the vehicle can decelerate at a rate D_x, it will travel a distance ($\dot{x}_f(0)/2D_x$) before coming to rest. For safety in the event that the lead vehicle stops immediately we could choose

$$\delta_{des} = \frac{\dot{x}_f}{2D_x} + \text{margin for delay, etc.}$$

* Constant Time Headway - This is just a fancy name for the "two second rule" taught in most Driver's Ed courses. The desired spacing should grow with the velocity of the follower.

$$\delta_{des} = \dot{x}_f \underbrace{t_h}_{\text{time headway}} + \underbrace{\delta_o}_{\text{offset}}$$

* What a Driver Would Do - Base the deceleration on range (δ) and range rate ($\dot{\delta}$). Paul Fancher from UMTRI leads the way in this area.

How would we implement something like constant time headway in a vehicle controller?

Suppose we would like the spacing error to go to zero exponentially. Then $\dot{\varepsilon} = -\lambda \varepsilon$

$$\dot{\varepsilon} = -\lambda \varepsilon$$
$$(\dot{\delta} - \dot{\delta}_{des}) = -\lambda(\delta - \delta_{des})$$
$$(\dot{\delta} - \ddot{x}_f t_h) = -\lambda(\delta - \dot{x}_f t_h - \delta_0)$$
$$\Rightarrow \quad \ddot{x}_f = \frac{1}{t_h}\left[\dot{\delta} + \lambda(\delta - \dot{x}_f t_h - \delta_0)\right]$$

If our follower vehicle's acceleration is given by the equation above, then our spacing error will decay to zero.

What do we need to know to calculate the acceleration of the following vehicle?

- $\delta, \dot{\delta}$ – Range and range rate. Requires a system like radar or laser.
- $\dot{x}_f$ – If slip is not high, can use the wheel speeds to calculate.
- λ, t_h, δ_0 – Our controller design parameters

How do we get the vehicle to have this desired acceleration?

Going back to the longitudinal equation

$$\ddot{x}_f = \underbrace{\frac{1}{m}\left[F_t - F_b\right]}_{\text{we can control these forces}} - \underbrace{\frac{1}{m}\left[F_d + F_{rr} + F_g\right]}_{\text{These forces represent various drag effects}}$$

Substituting...
$$\frac{1}{m}\left[F_t - F_b\right] = \frac{1}{m}\left[F_d + F_{rr} + F_g\right] + \frac{1}{t_h}\left[\dot{\delta} + \lambda(\delta - \dot{x}_f t_h - \delta_0)\right]$$

So we can choose the force provided by our engine or brakes to cancel the effects of drag, rolling resistance and grade and apply the necessary acceleration to track our desired spacing. This assumes:

(1) we can actuate brakes as well as engine. Some systems get by with only releasing the throttle and using the drag terms to slow the vehicle (more conservative spacing required here!).

(2) We have a good model of our system so things like mass, rolling resistance and the coefficient of drag are known. (How do we deal with changes that can occur?)

(3) The road grade is somehow known.

What happens if we don't know something exactly? for instance, what if we don't know grade and cannot compensate for it?

In this case, we choose F_t or F_b such that

$$\frac{1}{m}\left[F_t - F_b\right] = \frac{1}{m}\left[F_d + F_{rr}\right] + \frac{1}{t_h}\left[\dot{\delta} + \lambda(\delta - \dot{x}_f t_h - \delta_0)\right]$$

Then
$$\ddot{x}_f = -\frac{1}{m}F_g + \frac{1}{t_h}\left[\dot{\delta} + \lambda(\delta - \dot{x}_f t_h - \delta_0)\right]$$

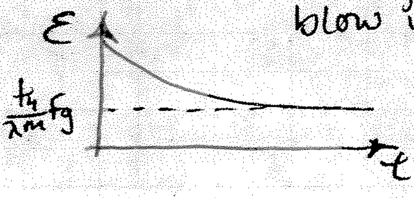
$$\dot{\varepsilon} = \dot{\delta} - \ddot{x}_f t_h$$

$$= \frac{t_h}{m}F_g - \lambda\varepsilon$$

$$\Rightarrow \quad \dot{\varepsilon} = -\lambda\varepsilon + \frac{t_h}{m}F_g$$

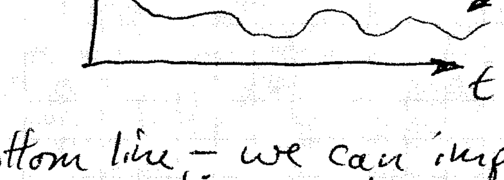
a "forcing function" for our spacing error dynamics.

If grade is constant for a stretch, $\dot{\varepsilon} = 0$ when $\varepsilon = \frac{t_h}{\lambda m}F_g$. This means there is some steady-state spacing error, but things do not blow up.



So a road angle of θ produces a spacing error of $\frac{t_h}{\lambda}g\sin\theta$.

If our grade is constantly changing (as it usually is), the spacing error will look like...



Bottom line – we can implement this without perfect information so long as we include a margin for this error.

Note – error decreases with λ, but acceleration increases with λ.
$\rightarrow$ A trade-off between comfort and tracking.

Is anything different with a constant spacing strategy on an automated highway?

$$\varepsilon = \delta - \delta_{des}$$

$$\dot{\varepsilon} = \dot{\delta} \qquad \text{since } \delta_{des} = \text{const.}$$

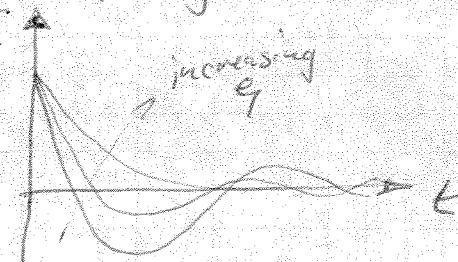
$$\ddot{\varepsilon} = \ddot{\delta} = \ddot{x}_\ell - \ddot{x}_f$$

In this case, controlling the acceleration of the follower only gives control over $\ddot{\varepsilon}$, not $\dot{\varepsilon}$.

Let's say we want $\ddot{\varepsilon} + 2\zeta\omega_n\dot{\varepsilon} + \omega_n^2\varepsilon = 0$

So we could choose spacing error response by picking ω_n and ζ.

We can alter speed of response and damping.



How do we get this?

$$\ddot{x}_\ell - \ddot{x}_f = -2\zeta\omega_n\dot{\delta} - \omega_n^2\delta$$

$$\rightarrow \ddot{x}_f = \ddot{x}_\ell + 2\zeta\omega_n\dot{\delta} + \omega_n^2\delta$$

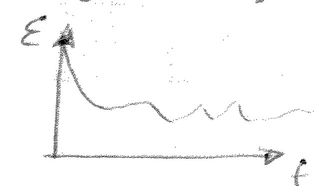
As before, choose $\frac{1}{m}[f_t - f_b] = \frac{1}{m}[f_d + f_{rr} + f_g]$

$$+ \ddot{x}_\ell + 2\zeta\omega_n\dot{\delta} + \omega_n^2\delta$$

This now requires us to know the lead vehicle acceleration which must in some way be communicated to the follower. There must also be a clean measurement of this acceleration - if the accelerometer measurement is corrupted by suspension modes, the follower will feel these in the longitudinal direction!

If the lead vehicle acceleration is not known and not included in our calculated value for f_t and f_b, then the dynamics become

$$\ddot{\varepsilon} + 2\zeta\omega_n\dot{\varepsilon} + \omega_n^2\varepsilon = \ddot{x}_\ell$$



So the dynamics are stable, but don't go to zero if the lead vehicle is accelerating or decelerating. The same holds for grade.