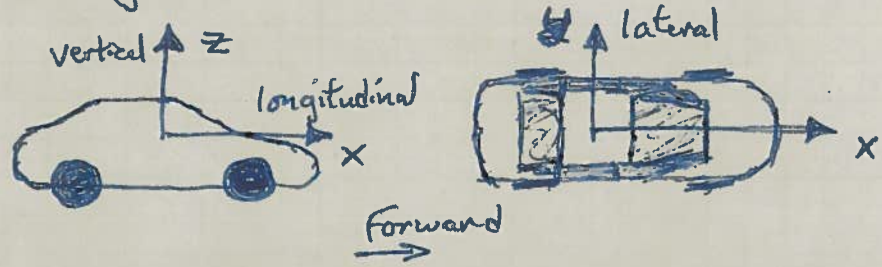
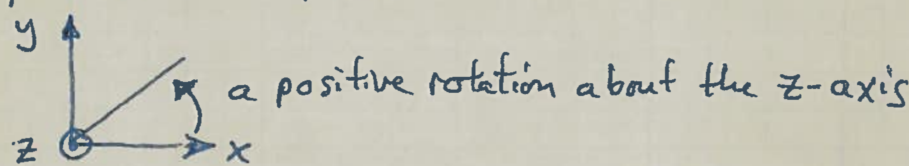
Axis System

There are different choices for axes when describing vehicle motion. In this class, we will use the ISO 8855 standard which defines the z-axis pointing up.

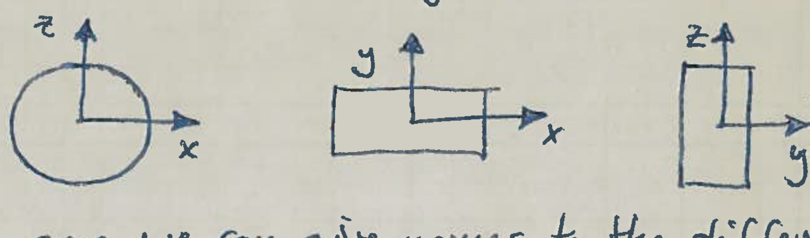


The SAE standard (derived from the aeronautics field) has the z-axis pointing down. This creates a number of potential problems when using resources such as the class text which use the SAE standard.

In this coordinate system, angles and rotations are positive in the counterclockwise direction about the positive axis of rotation.



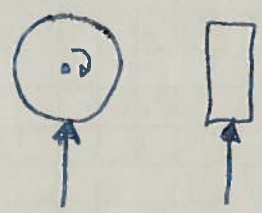
Each tire has its own axis system defined similarly



Using these axes, we can give names to the different forces and moments that act on the tires:

Normal or Vertical force

F_z

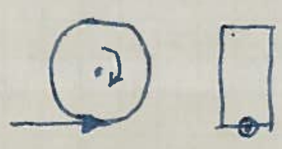


Determines ride quality and limits of adhesion for the tire

Tractive or Braking force

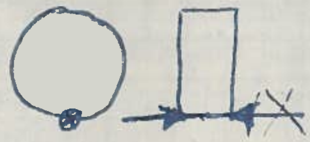
Also called longitudinal force

F_x



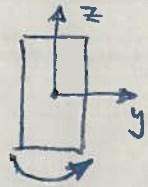
Determines acceleration and deceleration; determines tire lock-up

Cornering or Side Force
Also called lateral force
F_y



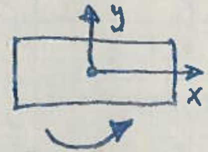
Determines lateral motion, yaw, stability and handling response.

Overturning Moment
M_x



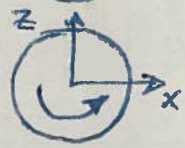
Reacted by suspension
Less important in basic vehicle dynamics

Aligning Moment
M_z



Main source of steering feel for driver feedback.

Rolling Resistance Moment
M_y

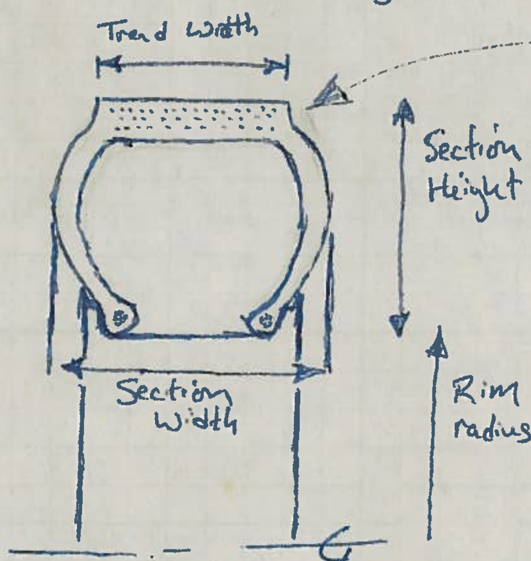


Opposes motion - important in fuel economy.

When braking or accelerating, M_y also contains torques from engine and brakes. When the wheel is freely rolling only the rolling resistance moment acts.

Tire Markings

The tire sidewall contains a lot of useful information although the parameters we need are unfortunately harder to come by.



P205/50 ZR16 87W

- P — Passenger Car
- 205 — Section width (mm)
- 50 — Aspect ratio
- R — Radial
- 16 — Rim diameter in inches
- 87 — Load rating
- W — Speed rating

$$\text{Aspect ratio} = \frac{\text{Section Height}}{\text{Section width}} \cdot 100$$

Load rating can be found in a chart (87 = 545 kg)

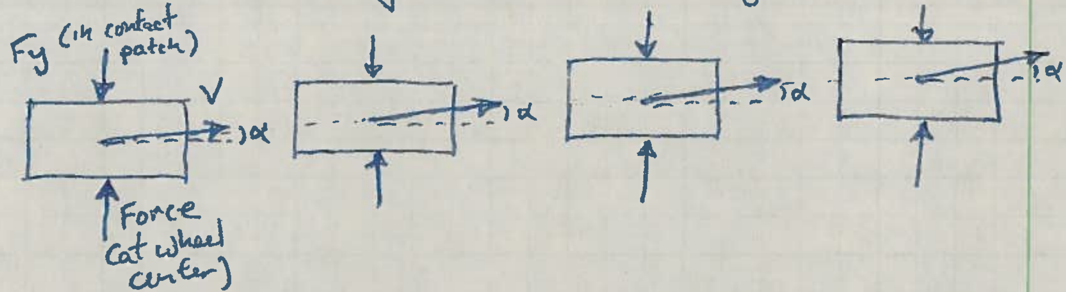
Light truck tires use 'LT' instead of 'P'

The Z speed rating was intended to be the highest rating, certifying speeds in excess of 149 mph (240 km/hr). That wasn't enough for the Autobahn so W (168 mph) and Y (186 mph) were introduced.

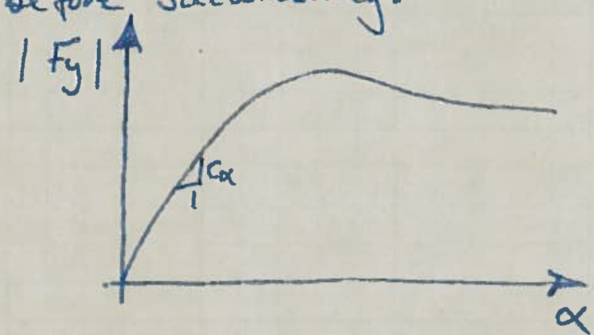
The speed rating is generally after the load rating though the 'Z' often appears in the size since it looks cool.

Lateral Tire Forces

When subjected to a small lateral force, a rolling tire tends to move laterally as well as longitudinally.



Looking at the tire motion, there is an angle called the slip angle, α, between the tire centerline and the tire's velocity vector. This angle appears for very small amounts of force and builds with the applied force before saturating.

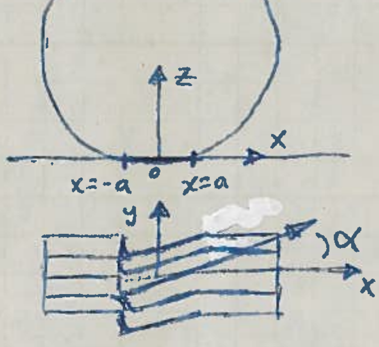


The initial part of the curve is very linear, so in this region we can write:

$$F_y = -C_\alpha \alpha$$

$C_\alpha \equiv$ cornering stiffness

But why is this the case? Shouldn't the tire remain stuck to the road until the friction between the tire and road has been overcome? Actually, it does. To see this, it is important to consider the contact patch over which the tire contacts the road.



If we assume that the tire remains stuck to the road initially then the tire contact patch must produce a deformation in the tire. The tread element enters the contact patch at x=a and deforms according to the direction of travel. At the end of the contact patch, it snaps back to its undeformed state.

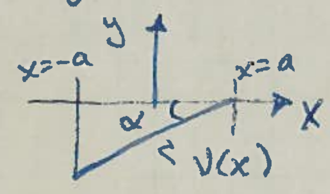
The tire lateral force is the force needed to produce this deformation in the contact patch?

How much force is necessary to deform the tire in this manner?

Assume that we can define a "foundation lateral stiffness", c_{py}, which is the amount of force needed to produce a unit of lateral deflection in a unit length of contact patch. Note that this is not simply a material property like Young's modulus of the rubber, Instead it is an overall measure of the resistance to deformation and thus depends upon such things as:

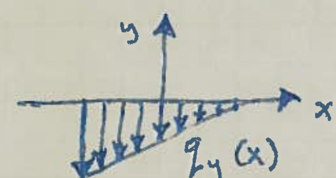
* Rubber material properties
* Material properties and arrangement of steel belts
* Height of sidewall
* Inflation pressure

If we model the deflection in the contact patch as having a simple triangular shape, then the deflection v in the y-direction is



$$v(x) = -(a-x)\tan\alpha$$

Producing this deformation requires a distributed lateral force, $q_y(x)$. This force can be calculated in terms of the deflection and the stiffness as



$$q_y(x) = c_{py}\, v(x)$$

$q_y(x)$ has units of force per unit length and has the same triangular shape as $v(x)$.

The total lateral force is: $F_y = \int_{-a}^{a} q_y\, dx$

$$= -\int_{-a}^{a} c_{py}(a-x)\tan\alpha\, dx$$

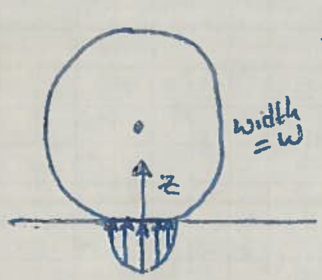
$$= -c_{py}\tan\alpha \left(ax - \frac{x^2}{2}\right)\Big|_{-a}^{a}$$

$$= \underbrace{-2c_{py}a^2}_{C_\alpha}\tan\alpha$$

So the behavior of the tire in the linear region depends upon material properties and not friction. Similarly, the cornering stiffness is independent of friction.

This assumes that we can produce this level of $q_y(x)$. But ultimately we are limited by friction so

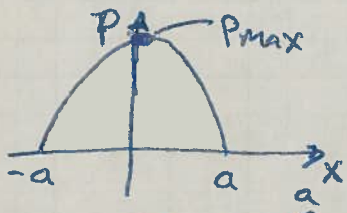
$$|q_y(x)| \le \mu\, q_z(x)$$ what does this look like?



The force distribution required to deform the circular tire into a flat contact patch is approximately parabolic.

We will assume we can model the pressure in the contact patch by

$$p = p_{max}\,\frac{(a^2 - x^2)}{a^2}$$



How does the pressure vary in terms of the normal load?

$$F_z = \int_{-a}^{a}\int_{-\frac{w}{2}}^{\frac{w}{2}} p\,dx\,dy = \int_{-a}^{a}\int_{-\frac{w}{2}}^{\frac{w}{2}} p_{max}\,\frac{(a^2-x^2)}{a^2}\,dx\,dy$$

$$= \frac{p_{max}\,w}{a^2}\int_{-a}^{a} a^2dx - x^2dx$$

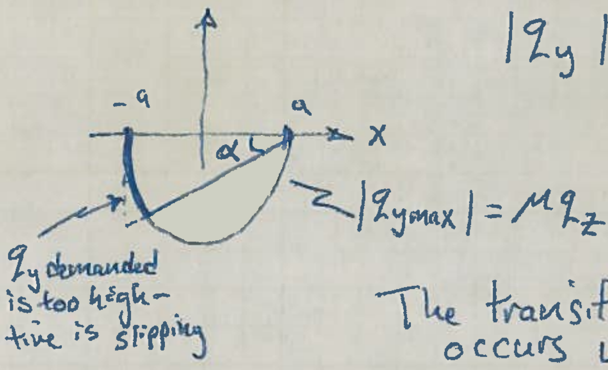
$$= \frac{p_{max}\,w}{a^2}\left(a^2x - \frac{x^3}{3}\right)\Big|_{-a}^{a} = \frac{4}{3}\,w\,a\,p_{max}$$

So $$p = \frac{3F_z}{4wa}\left(\frac{a^2-x^2}{a^2}\right)$$

Since $q_z(x) = w\,p(x)$, $q_z(x) = \frac{3F_z}{4a}\left(\frac{a^2-x^2}{a^2}\right)$

Graphically, we have a condition that looks like this:

$$|q_y| \le \mu q_z = \frac{3\mu F_z}{4a}\left(\frac{a^2-x^2}{a^2}\right)$$



The back of the contact patch is therefore sliding while in the front we still have adhesion.

The transition between adhesion and sliding occurs when $x = x_{sl}$ and $|q_y| = \mu q_z$

$$\Rightarrow \quad |(a - x_{sl})\tan\alpha| = \frac{3\mu F_z}{4a}\left(\frac{a^2 - x_{sl}^2}{a^2}\right)$$

$$\cancel{(a - x_{sl})}\,|\tan\alpha| = \frac{3\mu F_z}{4a^3}\,\cancel{(a - x_{sl})}(a + x_{sl})$$

$$x_{sl} = \frac{4c_{py}\,a^3\,|\tan\alpha|}{3\mu F_z} - a$$

As the slip angle increases this point moves further forward until the whole patch is slipping ($x_{sl} = a$)

This occurs at α_{sl} defined by $|\tan\alpha_{sl}| = \frac{3\mu F_z}{2c_{py}\,a^2}$

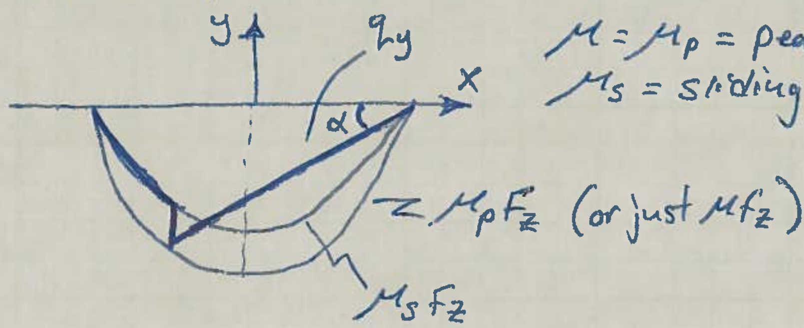
Rewriting in terms of the cornering stiffness, the point of transition between adhesion and sliding is:

$$x_{sl} = \frac{2C_\alpha a}{3\mu F_z}|\tan\alpha| - a$$

and the whole contact patch is sliding when $\alpha = \alpha_{sl}$

$$|\tan\alpha_{sl}| = \frac{3\mu F_z}{C_\alpha}$$

With the understanding that the tire has an adhesion region at the front of the contact patch determined by deformation and a friction-limited sliding region at the back, we can integrate to get the lateral force. Before doing so, it makes sense to introduce one slight addition to the model - separate coefficients of friction for kinetic (sliding) and static (peak) friction.



$\mu = \mu_p$ = peak μ

μ_s = sliding μ

The transition between adhesion and sliding is determined by the peak friction coefficient but, once sliding, the lateral force per unit length, q_{ys} is limited by the sliding friction coefficient μ_s.

Putting it all together:

$$F_y = \underset{\substack{\text{adhesion}\\\text{region}}}{\int} q_y(x)\,dx + \underset{\substack{\text{sliding}\\\text{region}}}{\int} q_y(x)\,dx$$

Need to get direction correct

$$= -\int_{x_{sl}}^{a} c_{py}(a-x)\tan\alpha\,dx - \int_{a}^{x_{sl}} \frac{3\mu F_z}{4a}\left(\frac{a^2-x^2}{a^2}\right)\operatorname{sgn}(\alpha)$$

$$= -c_{py}\tan\alpha\left[ax - \frac{x^2}{2}\right]\Bigg|_{x_{sl}}^{a} - \frac{3\mu_s F_z}{4a}\left[a^2x - \frac{x^3}{3}\right]\Bigg|_{-a}^{x_{sl}}$$

After much algebra...

$$= -C_\alpha\tan\alpha + \frac{C_\alpha^2}{3\mu F_z}\left(2 - \frac{\mu_s}{\mu}\right)\tan\alpha|\tan\alpha| - \frac{C_\alpha^3}{9\mu^2F_z^2}\tan^3\alpha\left(1 - \frac{2\mu_s}{3\mu}\right)$$

This is a variant of a tire model originally proposed by Fiala

In final form, it is

$$F_y(\alpha) = \begin{cases} -C_\alpha \tan\alpha + \dfrac{C_\alpha^2}{3\mu F_z}\left(2 - \dfrac{\mu_s}{\mu}\right)|\tan\alpha|\tan\alpha - \dfrac{C_\alpha^3}{9\mu^2 F_z^2}\tan^3\alpha\left(1 - \dfrac{2\mu_s}{3\mu}\right) & |\alpha| < \alpha \\ -\mu_s F_z \,\mathrm{sgn}(\alpha) & |\alpha| \geq \alpha_{sl} \end{cases}$$