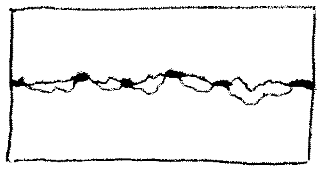
With materials such as metal, the true area of contact is much less than the apparent area of contact:

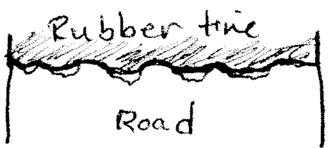


contact only at asperities

The real area of contact grows in proportion to the normal load and the friction force arises from the need to break these asperities, so

$$F_f = \mu N \quad \text{with a fairly constant } \mu$$

Rubber, however, deforms. This makes the real area of contact much greater than with metals.

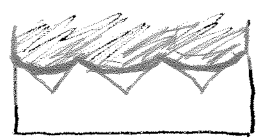


Thus "μ" is initially much higher for rubber.

As the normal load increases, the contact area also increases but not proportionally since rubber can only deform so much:

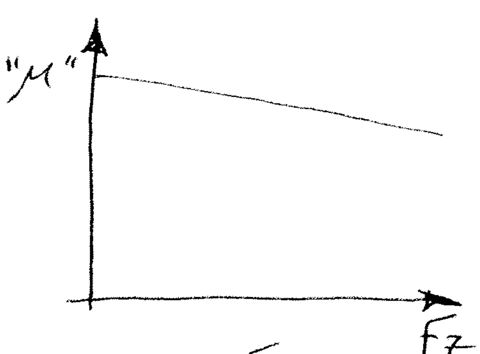
Light load

More load

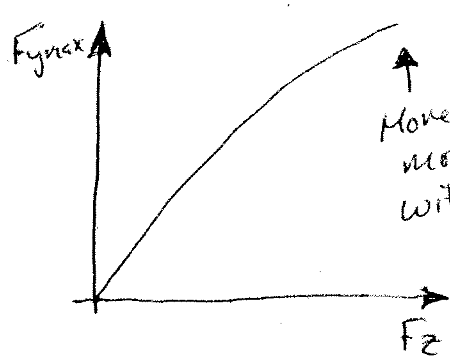


Still more load
Deflection

This leads to the following shape for the friction characteristics of a tire:



$$\mu = \frac{F_{y\max}}{F_z}$$



More F_z means more $F_{y\max}$ but with diminishing returns

This characteristic means that the more heavily loaded tire will have a lower friction capability, all other things being equal.

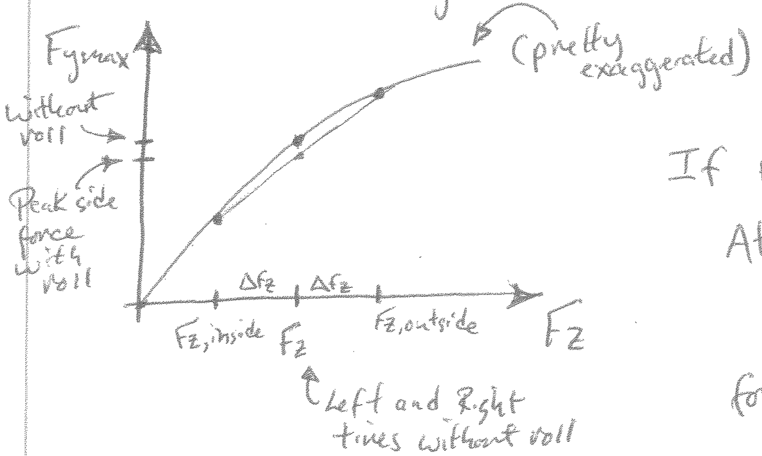
A car with four equal tires will then have a tendency towards the following handling behavior due to weight balance:

Weight on front $\Rightarrow$ Primary US and limit US

Weight on rear $\Rightarrow$ Primary OS and limit OS

The last piece of the puzzle for tuning the handling behavior is to realize the importance of the car's suspension. Among other things, the suspension impacts how much load is transferred from the inside to the outside tire as the car rolls.

What does weight transfer do?



If $F_{ymax} = c_1 F_z - c_2 F_z^2$

At straight ahead conditions,

$$F_{ymax} = 2(c_1 F_z - c_2 F_z^2)$$

for the pair of tires

With roll F_{ymax} for the pair of tires on the axle is:

$$F_{ymax} = c_1(F_z + \Delta F_z) - c_2(F_z + \Delta F_z)^2 + c_1(F_z - \Delta F_z) - c_2(F_z - \Delta F_z)^2$$

$$F_{ymax} = \underbrace{2c_1 F_z - 2c_2 F_z^2}_{\text{Straight ahead}} - \underbrace{2c_2 \Delta F_z^2}_{\text{Reduction due to load transfer}}$$

The lesson here is simple: As far as peak friction (or peak cornering or braking behavior) is concerned,

Weight Transfer is Bad!

It can, however, be very useful in tuning handling performance in the nonlinear region.

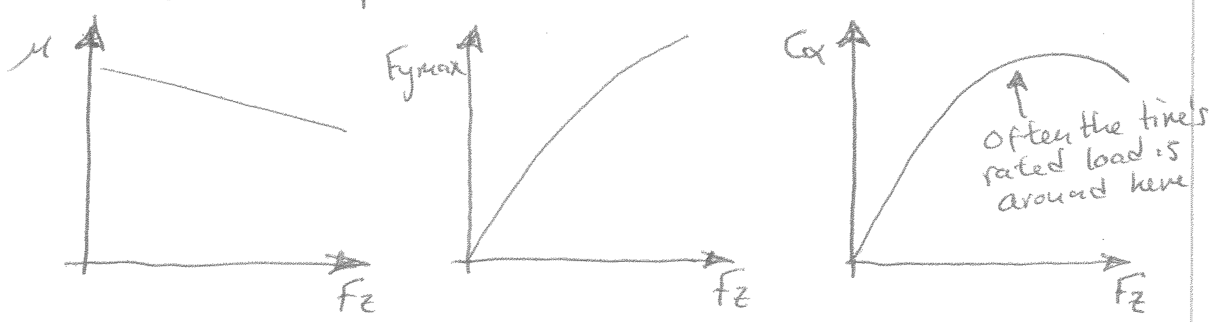
Roll Effects and the Static Roll Model

The lateral weight transfer from the inside tire to the outside tire is extremely important at higher levels of lateral acceleration. Tuning this weight transfer by choice of suspension parameters enables:

* Race car designers to design a faster car (one where the driver can get closer to peak friction)

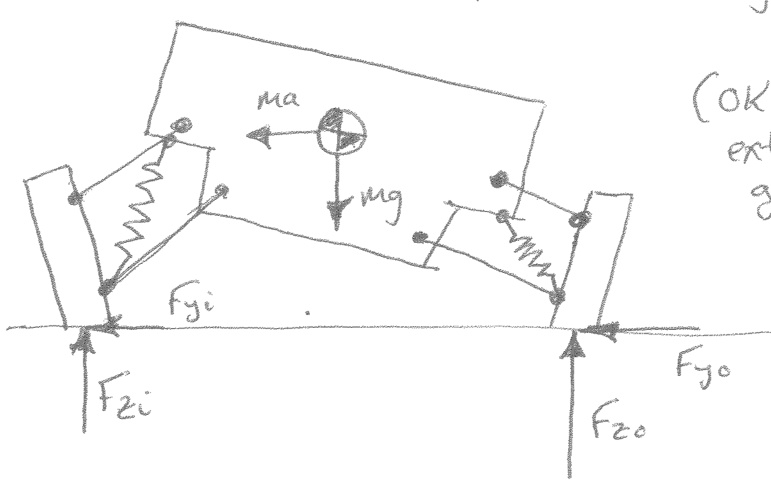
* Passenger car designers to design cars that are limit understeering in a variety of conditions.

The effect of weight transfer can be seen graphically in several plots:



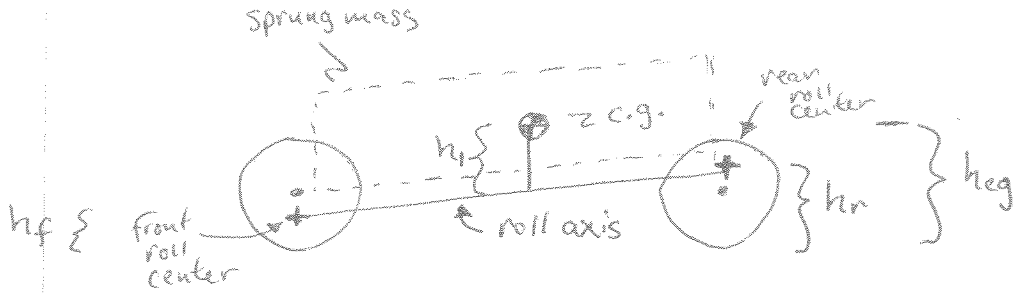
This means that weight transfer always reduces the peak cornering capability. It also has some effect on the cornering stiffness of the tire, so the effect of weight transfer will show up before the handling limits.

A car with independent suspension turning looks like:

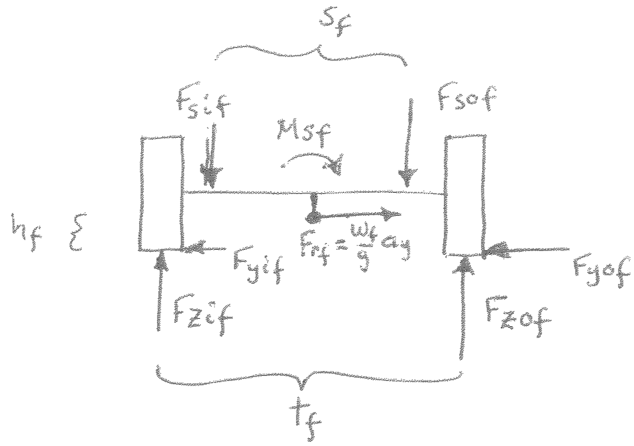


(OK, that's a bit extreme, but you get the idea)

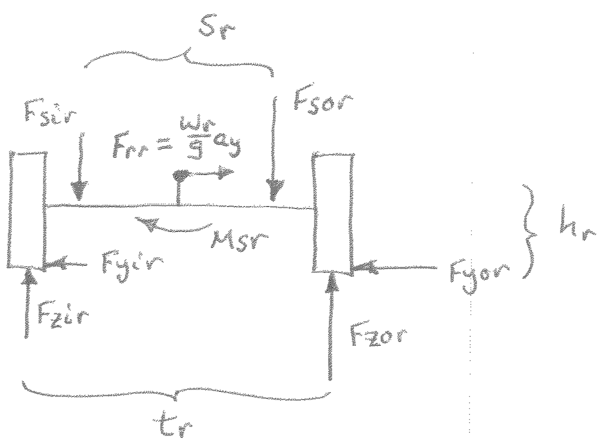
We can analyze systems like this using virtual work but risk losing intuition. Instead, the general approach is to look at a roll center model (we will justify the use of this for independent suspensions later)



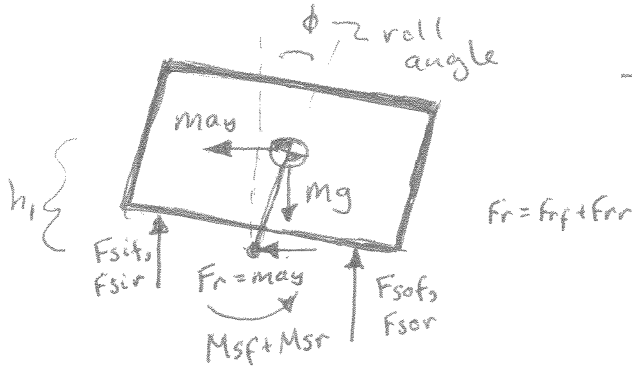
Roll axis definition



Front axle FBD



Rear axle FBD



Sprung mass FBD

These diagrams illustrate a left turn viewed from the rear of the vehicle.

$F_s \triangleq$ spring force
$F_z \triangleq$ normal (vertical) force
$F_y \triangleq$ side (lateral) force
$F_r \triangleq$ reaction force
$M_s \triangleq$ moment from anti-roll bar

This may not look like a car with independent suspension but by careful choice of the roll center it can be a good approximation. How to find the roll center and how to judge the usefulness of this model will be topics of the next lectures.

This model assumes the existence of a roll axis. It might seem reasonable to treat this as the axis around which the sprung mass rotates, but such an axis doesn't really exist. It is better to think of this as the axis around which internal suspension forces produce no roll of the sprung mass (we can locate an axis like this). We also assume that the springs carry all of the vertical force, an assumption that will be revisited.

The basic approach we will follow here is to
(1) Get the roll angle in terms of lateral acceleration
(2) Get the weight transfer on each axle
(3) Determine the effect of this transfer on handling and how springs and roll bars are used to tune handling.

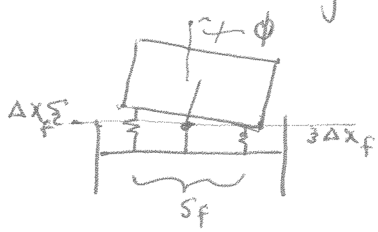
In this model, the only things that can resist the moments about the roll axis caused by the weight and acceleration at the c.g. are springs and roll bars (for now, consider the roll bar as adding a moment).

Thus summing the moments on the sprung mass...

$$mgh_1 \sin\phi - M_{sf} - M_{sr} - \tfrac{1}{2}\left[F_{sof} - F_{sif}\right] s_f \cos\phi$$
$$- \tfrac{1}{2}\left[F_{sor} - F_{sir}\right] s_r \cos\phi = -ma_y h_1 \cos\phi$$

with small angles $\quad mgh_1\phi - M_{sf} - \tfrac{1}{2}\left[F_{sof} - F_{sif}\right] s_f - M_{sr} - \tfrac{1}{2}\left[F_{sor} - F_{sir}\right] s_r$
$$= -ma_y h_1$$

The spring forces and anti-roll bar moments are related to the roll angle



static force

$$F_{sof} = F_{sfs} + K_{sf}\,\Delta x$$
$$= F_{sfs} + K_{sf}\left(\tfrac{s_f}{2}\right)\phi$$
$$F_{sif} = F_{sfs} - K_{sf}\left(\tfrac{s_f}{2}\right)\phi$$

$$\Rightarrow \quad \tfrac{1}{2}\left[F_{sof} - F_{sif}\right] s_f = \tfrac{1}{2}\left[+2K_{sf}\left(\tfrac{s_f}{2}\right)\phi\right] s_f$$
$$= +\tfrac{1}{2} K_{sf}\, s_f^2\, \phi$$

and $\quad \tfrac{1}{2}\left[F_{sor} - F_{sir}\right] s_r = +\tfrac{1}{2} K_{sr}\, s_r^2\, \phi$

We have assumed that inside and outside springs are the same stiffness (this is actually rather important).

Stabilizer (anti-roll) bars can also be modeled as giving a moment proportional to roll angle so

$$M_{sf} = K_{stabf}\,\phi \qquad M_{sr} = K_{stabr}\,\phi$$

(more on where this comes from physically later).

Rewriting the moment balance:

$$mgh_1\phi - \underbrace{\left[K_{stabf} + \tfrac{1}{2}K_{sf}\,s_f^2\right]}_{K_{\phi f}\text{ - front roll stiffness}}\phi - \underbrace{\left[K_{stabr} + \tfrac{1}{2}K_{sr}\,s_r^2\right]}_{K_{\phi r}\text{ - rear roll stiffness}}\phi = -ma_yh_1$$

$$(\underbrace{K_{\phi f} + K_{\phi r}}_{K_\phi\text{ - total roll stiffness}} - mgh_1)\phi = ma_yh_1$$

$$\Rightarrow \quad \phi = \frac{Wh_1\left(\frac{a_y}{g}\right)}{K_\phi - Wh_1} = \underbrace{\frac{Wh_1}{K_\phi - Wh_1}}_{R_\phi\text{ - roll rate}}\underbrace{\left(\frac{V^2}{Rg}\right)}_{\text{lat accel in "g"s}}$$

The roll angle depends on the roll stiffness, mass, height of the c.g. over the roll axis and acceleration

Roll rate (deg/g)	Application
1.5	race cars
3-4	sports cars
5	sport sedan
7	pretty soft...
8	late 60's, early 70's

These are general ranges, don't take too literally.

From a practical standpoint, the small angle assumptions seem perfectly valid.

Now get the weight transfer from the axle FBD...

$$\sum M_{\substack{roll\\center}} = M_{sf} + s_f F_{sof} - s_f F_{sif} + \frac{t_f}{2}F_{zif} - \frac{t_f}{2}F_{zof} + h_f F_{yif} + h_f F_{yof} = 0$$

$$\Rightarrow \quad K_{\phi f}\,\phi - t_f\underbrace{\left[\tfrac{1}{2}(F_{zof} - F_{zif})\right]}_{\Delta F_{zf}} + h_f\frac{W_f}{g}a_y = 0$$

$$K_{\phi f}\,\phi - t_f\,\Delta F_{zf} + h_f W_f\frac{V^2}{Rg} = 0$$

$$\Rightarrow \Delta F_{zf} = \frac{1}{t_f}\left[K_{\phi f}\,\phi + W_f h_f\frac{V^2}{Rg}\right]$$

Similarly $\Delta F_{zr} = \dfrac{1}{t_r}\left[K_{\phi r}\,\phi + W_r h_r\dfrac{V^2}{Rg}\right]$

Where $F_{zof} = \dfrac{W_f}{2} + \Delta F_{zf}$ $\quad F_{zif} = \dfrac{W_f}{2} - \Delta F_{zf}$

$F_{zor} = \dfrac{W_r}{2} + \Delta F_{zr}$ $\quad F_{zir} = \dfrac{W_r}{2} - \Delta F_{zr}$

Increasing the roll stiffness on the front and rear axles equally <u>decreases</u> the load transfer ΔF_z on each axle. Increasing the roll stiffness on only one axle <u>increases</u> the load transfer on that axle and <u>decreases</u> the load transfer on the other.

To see this, assume we scale the front stiffness by a ($a>1$)

Is $\Delta F_{zf}|_{new} > \Delta F_{zf}|_{old}$?

$$\frac{1}{t_f}\left[aK_{\phi f}\left(\frac{Wh_1}{aK_{\phi f}+K_{\phi r}-Wh_1}\right)+W_f h_f\right]\frac{V^2}{Rg} > \frac{1}{t_f}\left[K_{\phi f}\left(\frac{Wh_1}{K_{\phi f}+K_{\phi r}-Wh_1}\right)+W_f h_f\right]\frac{V^2}{Rg} \ ?$$

$$\frac{a}{aK_{\phi f}+K_{\phi r}-Wh_1} > \frac{1}{K_{\phi f}+K_{\phi r}-Wh_1} \ ?$$

$$aK_{\phi f}+aK_{\phi r}-aWh_1 > aK_{\phi f}+K_{\phi r}-Wh_1 \ ?$$

$$a(K_{\phi r}-Wh_1) > (K_{\phi r}-Wh_1) \ ?$$

$$a>1 \ ? \quad \underline{\text{Yes}}!$$

$\Delta F_{zr}|_{new} < \Delta F_{zr}|_{old}$ by inspection (denominator increases, numerator does not).

Now increase both front and rear by scaling by a

Is $\Delta F_{zf}|_{old} > \Delta F_{zf}|_{new}$?

$$\frac{a}{aK_{\phi f}+aK_{\phi r}-Wh_1} < \frac{1}{K_{\phi f}+K_{\phi r}-Wh_1} \ ?$$

$$aK_{\phi f}+aK_{\phi r}-aWh_1 < aK_{\phi f}+aK_{\phi r}-Wh_1$$

$$-aWh_1 < -Wh_1$$

$$a>1 \ ? \quad \underline{\text{Yes}}!$$

So for race cars the idea is to make both suspensions stiff in roll to avoid weight transfer (and loss of peak side force), then make adjustments by front/rear tuning. For passenger cars, comfort concerns prevent achieving the same total roll stiffness as race cars, but the basic idea is the same.