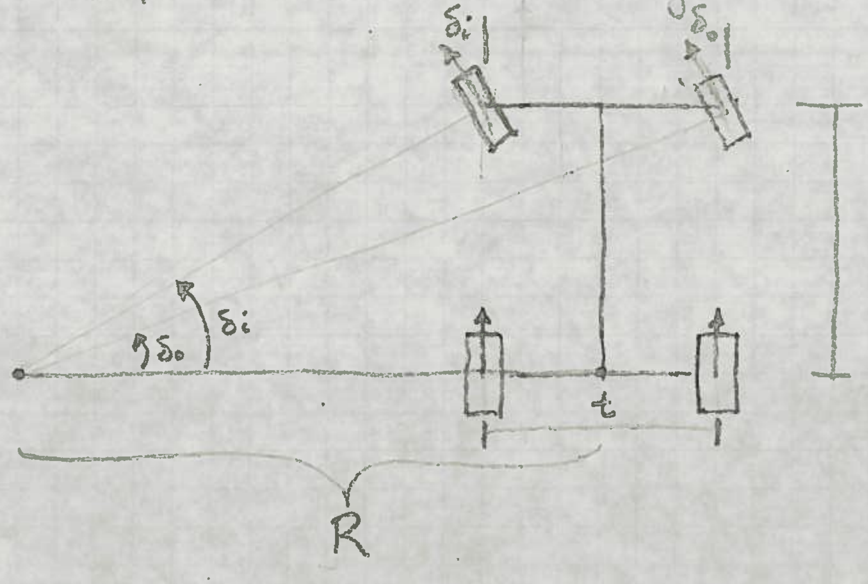
Kinematic Vehicle Model

A common assumption made when modeling the vehicle at low speed is that the velocity vector at each wheel is parallel to the wheel's longitudinal axis:

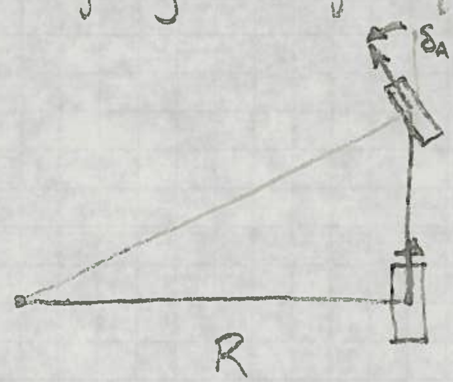


For this to hold true for a four wheeled vehicle when turning, the front wheels need to turn at different angles with the inside wheel having a larger angle:

$$\tan \delta_i = \frac{L}{R - t/2}$$

$$\tan \delta_o = \frac{L}{R + t/2}$$

The geometry that accomplishes this in a steering system is known as Ackerman geometry. Instead of keeping track of all four wheels, the car is often modeled as a single track or "bicycle" model by imagining a single equivalent wheel on the centerline:



To arrive at the same turning behavior as the four wheeled model, the bicycle model must turn at an angle:

$$\tan \delta_A = \frac{L}{R}$$

for a turn of radius R

This angle δ_A is known as the Ackerman angle for that radius. Sometimes small angles are assumed when calculating the Ackerman angle giving:

$$\delta_A \approx \frac{L}{R}$$

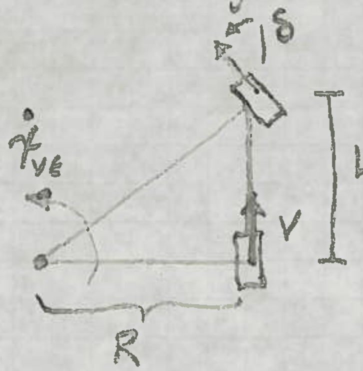
To get a feel for this, consider the angles associated with four different turning radii for a vehicle with a wheelbase $L = 2.5m$

Radius	δ_A	δ_A (small angle)	δ_i	δ_o
5m	26.6°	28.6°	30.5°	23.5°
10m	14.0°	14.3°	15.1°	13.1°
20m	7.13°	7.16°	7.40°	6.87°
40m	3.57°	3.58°	3.64°	3.51°

The difference between these angles is pronounced at very tight turning radii, particularly those close to the car's physical limits (which is generally a steer angle of 30-35°). For cars that spend a lot of time in tight turns like delivery vehicles, Ackerman geometry is important to avoid tire wear.

At the higher radii associated with higher speed driving, the difference among all of these angles is fairly negligible. Purpose-built race cars may be designed without Ackerman steering since they only see small radii in the pits.

It is straightforward to develop equations of motion for the Kinematic bicycle model:



$$\tan\delta = \frac{L}{R}$$

$$V = R\,\dot{\psi}_{VE}$$

$$\frac{1}{R} = \frac{\dot{\psi}_{VE}}{V}$$

$$\Rightarrow \tan\delta = \frac{L}{V}\dot{\psi}_{VE}$$

$$\dot{\psi}_{VE} = \frac{V}{L}\tan\delta$$

If we choose the rear axle as our vehicle reference point:

$$\underset{\sim}{V}_{ov,v} = \begin{bmatrix} U_x \\ U_y \\ 0 \end{bmatrix} = \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix}$$ (Remember $\kappa_v = \frac{1}{R}$)

$$\dot{s} = \left(\frac{1}{1-eK_1}\right)(U_x\cos\Delta\psi - U_y\sin\Delta\psi)$$

$$\boxed{\dot{s} = \left(\frac{1}{1-eK_1}\right) V\cos\Delta\psi}$$

$$\dot{e} = U_x\sin\Delta\psi + U_y\cos\Delta\psi$$

$$\boxed{\dot{e} = V\sin\Delta\psi}$$

$$\Delta\dot{\psi} = \dot{\psi}_{VE} - \dot{\psi}_{PE}$$

$$\boxed{\Delta\dot{\psi} = \frac{V}{L}\tan\delta - K_1\dot{s}}$$

This is a complete set of equations for determining the motion of a kinematic model with inputs of δ and V relative to a path in space.

If we assume that $K_1 e \ll 1$ (the path has a large radius and the vehicle is close to the path) and that the heading error is small...

$$\dot{s} = V$$

$$\dot{e} = V\Delta\psi$$

$$\Delta\dot{\psi} = \frac{V}{L}\delta - K_1 V$$

which is a linearized kinematic model in path coordinates.

This is a very simple model but it still tells us some interesting things. For instance, we reduce the lateral error by changing the heading error with our steering and enabling that to integrate.