Bode Plots

It is very easy to get a feel for the frequency response of a system from a sketch based on a few simple rules. These plots are easiest if made on a log-log scale for magnitude and log-linear for phase. Plots of the frequency response in this form are known as Bode plots.

For the root locus, we put the transfer function in the form:

$$KG(s) = K\frac{(s-z_1)(s-z_2)\dots(s-z_m)}{(s-p_1)(s-p_2)\dots(s-p_n)}$$

for Bode plots, we rearrange this slightly to get:

$$KG(j\omega) = K_o\frac{(j\omega\tau_1+1)(j\omega\tau_2+1)\dots(j\omega\tau_m+1)}{(j\omega\tau_a+1)(j\omega\tau_b+1)\dots(j\omega\tau_{\_}+1)}$$

If we look at taking the logarithm (base 10) of the magnitude:

$$\log|KG(j\omega)| = \log|K_o| + \log|j\omega\tau_1+1| + \log|j\omega\tau_2+1| + \dots$$
$$-\log|j\omega\tau_a+1| - \log|j\omega\tau_b+1| - \dots$$

everything simply adds!

For the phase angle:

$$\angle KG(j\omega) = \angle K_o + \angle(j\omega\tau_1+1) + \angle(j\omega\tau_2+1) + \dots$$
$$-\angle(j\omega\tau_a+1) - \angle(j\omega\tau_b+1) - \dots$$

This is also additive.

We can therefore sketch the frequency response of the system as a sum of individual elements

(1) $K_o(j\omega)^n$

(2) $(j\omega\tau+1)^{\pm 1}$

(3) $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1\right]^{\pm 1}$

When plotting the magnitude, it is customary to use dB

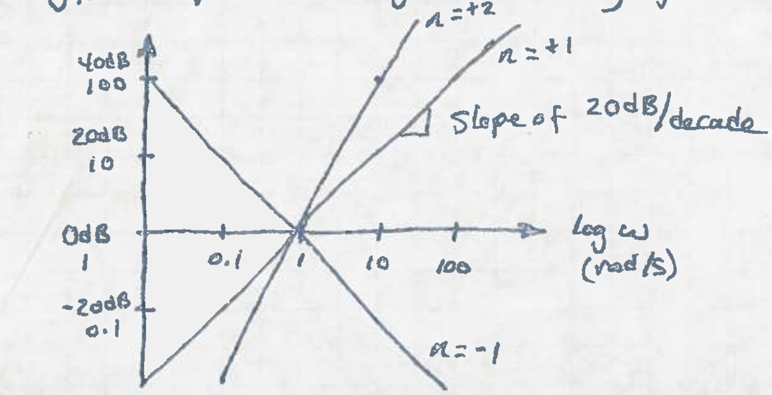
Magnitude in dB $= 20\log_{10}(G(j\omega))$

Magnitude of $1 = 0$ dB

Order of magnitude is 20 dB (from 0.1 to 1 for instance)

(1) $K_o (j\omega)^n$

$$\log_{10} K_o (j\omega)^n = \log K_o + n \log |j\omega|$$



plot of $n \log|j\omega|$

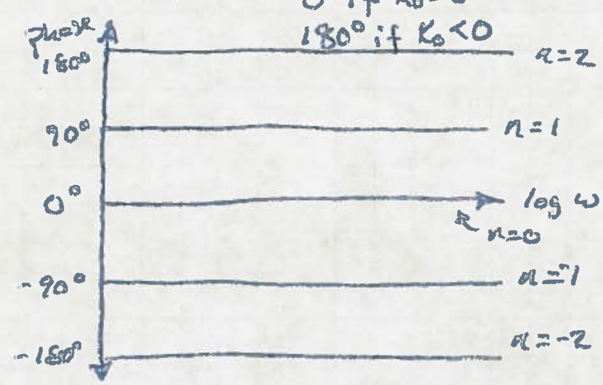
The magnitude is just a straight line on the log-log plot. The slope is $^{+}$20dB/decade/zero or $^{-}$20dB/decade/pole

The value of K_o just shifts the plot up or down (the magnitude at 1 rad/s is K_o or $\log K_o$ in dB).

$$\angle K_o (j\omega)^n = \angle K_o + \angle (j\omega)^n$$

$\angle K_o$: 0° if $K_o > 0$, 180° if $K_o < 0$

$\angle (j\omega)^n$: $n \times 90°$

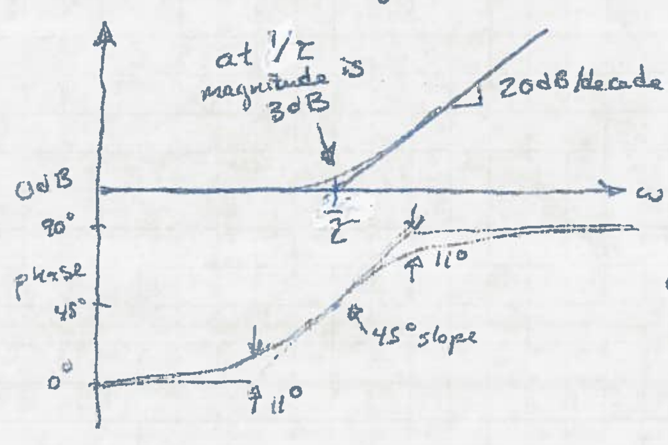


plot of $\angle (j\omega)^n$

(2) $j\omega\tau + 1$

when $\omega\tau \ll 1$ $\quad |j\omega\tau + 1| \approx 1 \quad \angle(j\omega\tau+1) \approx 0°$

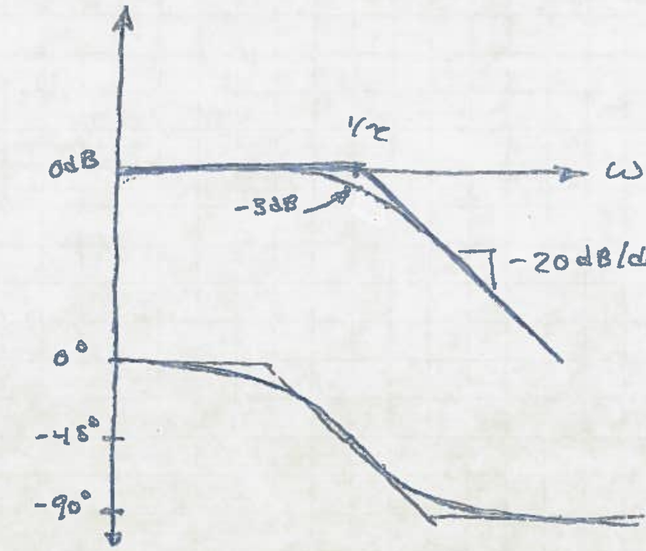
$\omega\tau \gg 1$ $\quad |j\omega\tau+1| \approx \omega\tau \quad \angle(j\omega\tau+1) \approx 90°$



@ $\omega\tau = 1$
$|j\omega\tau + 1| = \sqrt{2}$
or +3dB

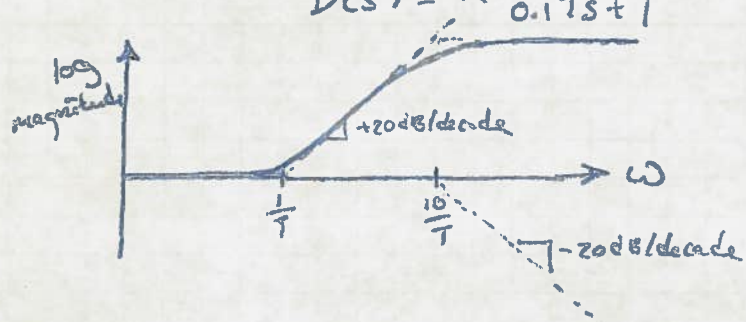
Can draw phase with asymptotes as well.

(2) $\frac{1}{j\omega\tau + 1}$ This is easy - just flip sign on phase and magnitude (on logarithmic scale).



With these rules we could sketch our lead compensator very easily

$$D(s) = K\,\frac{Ts+1}{0.1Ts+1} \qquad \alpha = 0.1 \text{ in this example}$$



(3) $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1\right]^{\pm 1}$

The attached page shows $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1\right]^{-1}$

To sketch, the plot is centered around ω_n with a resonance peak and phase angle slope that are determined by the damping ratio,

Away from ω_n (by a factor of 10 or more in frequency), this looks similar to the contribution of two real poles.

$\omega \ll \omega_n$ 0dB and 0° phase
$\omega \gg \omega_n$ -40dB/dec slope and -180° phase

To find the response for $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1\right]$ just invert the plot.

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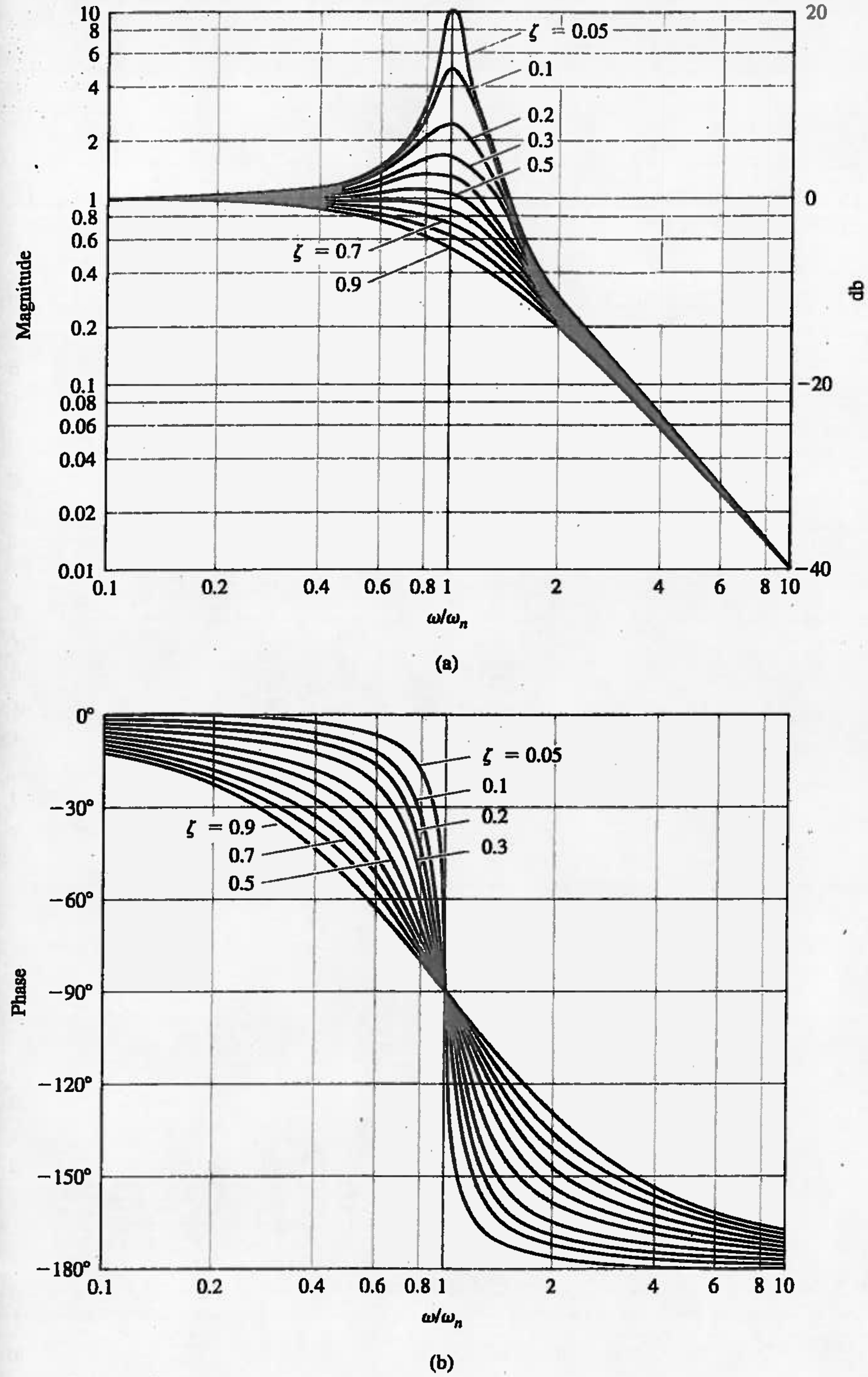


Figure 6.3
(a) Magnitude; (b) phase of Eq. (6.9)

Right Half Plane Poles and Zeros

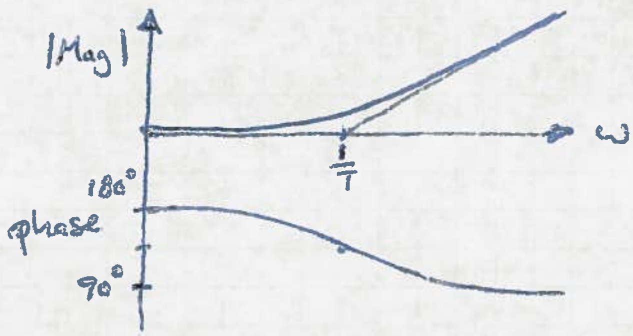
When poles or zeros lie in the right half plane, they have the same magnitude characteristics as their left half plane counterparts but different phase response.

Zero

$(j\omega T - 1)$

magnitude $= \sqrt{(\omega^2 T^2) + 1}$

phase $= \tan^{-1}\left(\frac{\omega T}{-1}\right)$ $\Rightarrow$ $180°$ at low ω
$90°$ at high ω



Magnitude is same as LHP zero

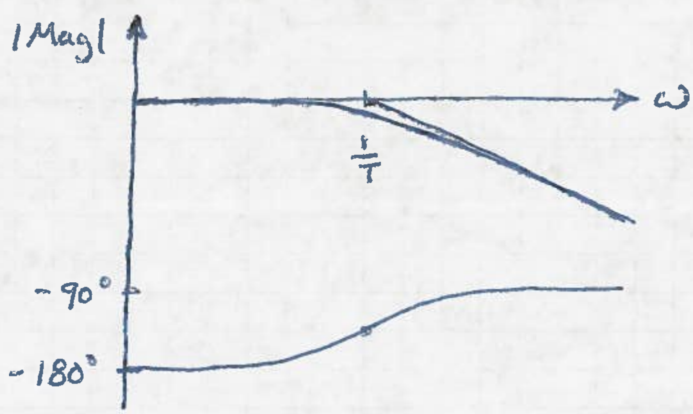
Phase starts at 180° then decreases to 90°.

Pole

$$\frac{1}{j\omega T - 1} = \frac{1}{j\omega T - 1} \cdot \frac{-j\omega T - 1}{-j\omega T - 1} = \frac{-j\omega T - 1}{\omega^2 T^2 + 1}$$

$$\text{magnitude} = \sqrt{\frac{\omega^2 T^2}{(\omega^2 T^2 + 1)^2} + \frac{1}{(\omega^2 T^2 + 1)^2}} = \sqrt{\frac{1}{\omega^2 T^2 + 1}}$$

phase $= \tan^{-1}\left(\frac{-\omega T}{-1}\right)$ $\Rightarrow$ $180°$ or $-180°$ at low ω
$-90°$ at high ω



Magnitude is same as LHP pole.

Phase starts at -180° increases to -90°