Project Report

Effectively Tracking a Curved Path Using a Lookahead Controller and a PID Controller

Team 9 (Abrupt Grade Change)

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Executive Summary

In this report, we detail the design process of creating two longitudinal and lateral controllers for an autonomous vehicle following a race track. The first controller is a lookahead controller that is designed from the first principles of vehicle dynamics. The second controller is a PID (proportional-integral-derivative) controller, which is a more general controller that we apply to our specific vehicle control problem. For each of these controllers, we use fundamental techniques, including root locus and pole plots, to estimate the initial values of the controller gains. We implement the two controllers in MATLAB, observe their performance on low-fidelity simulations, and tune the gains to meet the desired performance specifications. Using these roughly-tuned controllers, we simulate vehicle performance on simulators of increasing fidelity and slightly tune the gains to account for additional effects not present in the low-fidelity simulator (noise, saturation, rate limits, etc.). Ultimately, both controllers are able to meet the desired performance specifications and track the path successfully.

Controller 1: Lookahead Controller

The lookahead controller is a combination of a simple longitudinal feedforward and feedback controller and a simple lateral feedforward and feedback controller. The performance of these controllers is governed by three gains: the lookahead distance x_{la}, the lookahead gain K_{la}, and the longitudinal control gain K_{long}.

Longitudinal Controller

The longitudinal control law is a feedforward term with an additional feedback term that is proportional to the difference between the actual longitudinal velocity and the desired longitudinal velocity. The initial gain value for the longitudinal control gain came from fundamental calculations using a desired performance specification of 0.25 g/(m/s error). Cancelling out the drag and rolling resistance terms in the $U_{x,des,dot}$ term yields that:

$$K_{long,\, initial} = \frac{F_x}{m(U_{x,des} - U_x)} = \frac{ma_{x,des}}{m(U_{x,des} - U_x)} = \frac{a_{x,des}}{U_{x,des} - U_x} = \frac{0.25g}{1} = 2.45 \approx 2.5\, s^{-1}$$

Lateral Controller

Based on previous homeworks, we note that a reasonable lookahead distance is typically in the range of 5 - 20 m. As a result, we selected multiple lookahead distances for the purpose of evaluating multiple lookahead gains through pole plots. Since longitudinal velocity is a free variable in the gain formulation, we reviewed the speed profile and chose to fix the longitudinal velocity at values of 5 and 10 m/s as this is a rough approximation of the average velocity of the profile. In Figure 1, we show pole plots for a longitudinal velocity of 5 m/s. In each subplot, we fix the lookahead distance and then produce pole locations for varying lookahead gains. In Figure 2, we replicate the analysis for a longitudinal velocity of 10 m/s.

At low speeds, a gain value (K_{la}) in the range of 3,000-4,000 resulted in poles that were farthest away from the imaginary axis and gave us the highest margins for stability. At the highest longitudinal velocity of 10 m/s, we observed that a gain value of nearly 3,500 produced a pair of well-damped poles, i.e., we were able to limit the oscillatory nature of the responses while still meeting the required specifications for the controller. Hence, we finalized K_{la} at 3,500 N/m.

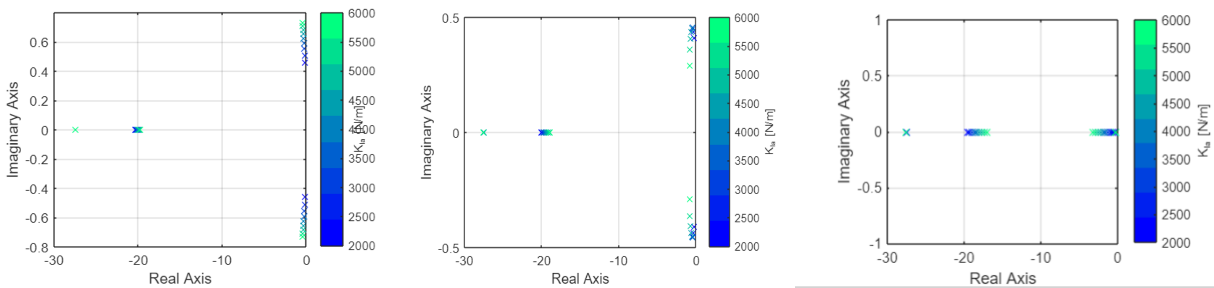

Figure 1: U_x = 5 m/s, x_{la} = 5, 10, 20 m, K_{la} = 2,600:6,000

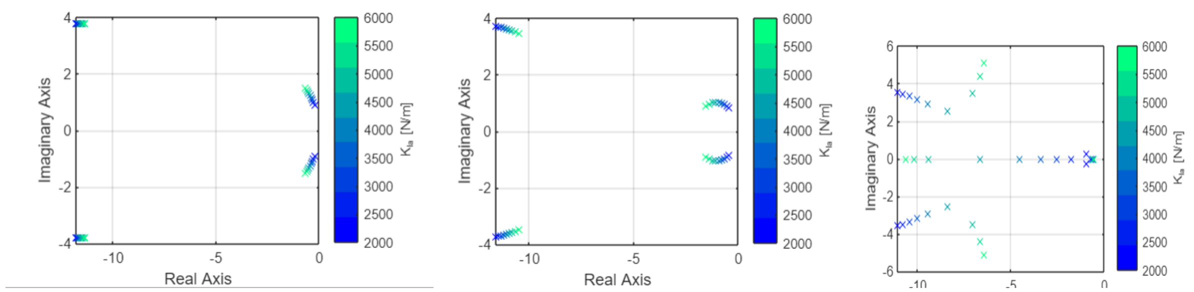

Figure 2: U_x = 10 m/s, x_{la} = 5, 10, 20 m, K_{la} = 2,600:6,000

After fixing K_{la} at 3,500 N/m, we produced pole plots, shown in Figure 3, spanning longitudinal velocities of 5 - 10 m/s for three different lookahead distances. The pole plots verified that the choices for the lookahead distance (22 m) and K_{la} resulted in dominant poles that had adequate damping to produce a smoother response.

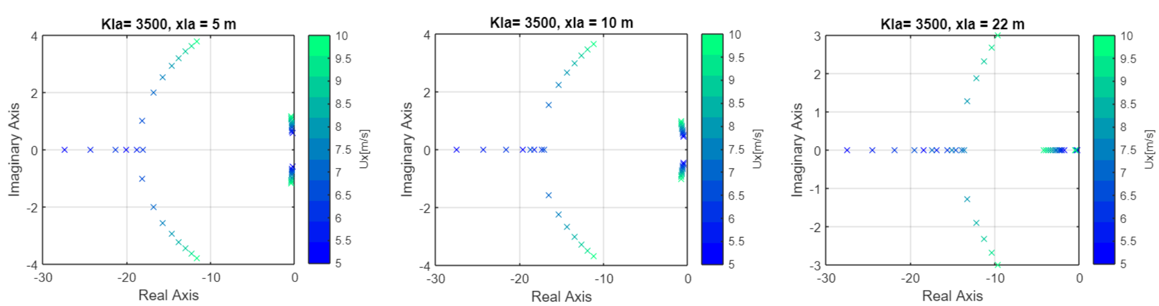

Figure 3: U_x = 5:10 m/s, x_{la} = 5, 10, 22 m, K_{la} = 3,500

With gains selected for the lookahead and longitudinal controllers, we now wanted to tune the lookahead distance. Evaluating this was a simple search where we adjusted the lookahead distance and evaluated the maximum lateral error performance specification over a few set intervals, after which we chose the best interval and conducted a more targeted search in that interval. The results are shown in Tables 1 and 2 below, which led to a lookahead distance choice of 22 meters.

Table 1: Lookahead Distance Search, First Iteration

Lookahead Distance x_{la} (m)	10	15	20	25	30
Maximum Lateral Error e_{max} (m)	0.2653	0.2186	0.2006	0.2055	0.2199

Table 2: Lookahead Distance Search, Second Iteration

Lookahead Distance x_{la} (m)	19	20	21	**22**	23
Maximum Lateral Error e_{max} (m)	0.2031	0.2006	0.1990	**0.1988**	0.2031

These procedures led to our initial gains of K_{long} = 2.5 s^{-1}, K_{la} = 3500 N/m, and x_{la} = 22 m.

High-Fidelity Simulation & Tuning

After settling on controller properties for the original simulation, we tested our controller on the more complex simulations and found that the gain selections did not meet the performance specifications. In particular, the maximum lateral error increased from 0.1988 m on sim mode 0 to 0.2897 m on sim mode 1, 0.3030 m on sim mode 2, and 0.3049 m on sim mode 3, all over the allowed specification of 0.2 m. As such, we conducted another coordinate search evaluating multiple different design points varying each control variable, and, post-design review, arrived at our final controller gain design as shown in Table 3.

Table 3: Coordinate Search for Complex Simulation Scenarios

Lookahead Distance x_{la} (m)	**Lookahead Gain K_{la} (N/m)**	**Longitudinal Gain K_{long} (s^{-1})**	**Maximum Lateral Error e_{max} (m)**	**U_x Tracking (qualitatively meets spec?)**
22	3500	2.5	.3013	-
22	3500	.5	.2840	-
22	6500	.5	.2664	-
20	6500	.5	.2549	-
15	6500	.5	.2289	-
10	6500	.5	.1896	Bad! (fix K_{long})
8	6500	5	.1948	Good!
8	**3000**	**1.8**	**.1870**	**Good!**

Controller 2: PID Controller

PID control defines the controlling steer angle as a function of lateral error, its derivative, and its integral from the initial conditions to the current time. This can be incorporated into the previous set of differential equations by adding an integral term to the vehicle state. The PID control gains can be incorporated into the open-loop A matrix in the same manner as the lookahead controller, where the eigenvalues of the closed-loop (autonomous) A matrix give the closed-loop poles.

$$\frac{d}{dt}\begin{bmatrix} \int_0^t e(\tau)d\tau \\ e \\ \dot{e} \\ \Delta\psi \\ \Delta\dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-(C_{af}+C_{ar})}{mU_x} & \frac{(C_{af}+C_{ar})}{m} & \frac{(bC_{ar}-aC_{af})}{mU_x} \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{(bC_{ar}-aC_{af})}{I_zU_x} & \frac{(aC_{af}-bC_{ar})}{I_z} & \frac{-(a^2C_{af}+b^2C_{ar})}{I_zU_x} \end{bmatrix} \begin{bmatrix} \int_0^t e(\tau)d\tau \\ e \\ \dot{e} \\ \Delta\psi \\ \Delta\dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{C_{af}}{m} \\ 0 \\ \frac{aC_{af}}{I_z} \end{bmatrix}\delta$$

$$\delta = -\left(K_P e + K_I\int_0^t e(\tau)d\tau + K_D\dot{e}\right)$$

$$\frac{d}{dt}\begin{bmatrix} \int_0^t e(\tau)d\tau \\ e \\ \dot{e} \\ \Delta\psi \\ \Delta\dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -\frac{C_{af}}{m}K_I & -\frac{C_{af}}{m}K_P & \frac{-(C_{af}+C_{ar})}{mU_x}-\frac{C_{af}}{m}K_D & \frac{(C_{af}+C_{ar})}{m} & \frac{(bC_{ar}-aC_{af})}{mU_x} \\ 0 & 0 & 0 & 0 & 1 \\ -\frac{C_{af}}{m}K_I & -\frac{C_{af}}{m}K_P & \frac{(bC_{ar}-aC_{af})}{I_zU_x}-\frac{C_{af}}{m}K_D & \frac{(aC_{af}-bC_{ar})}{I_z} & \frac{-(a^2C_{af}+b^2C_{ar})}{I_zU_x} \end{bmatrix} \begin{bmatrix} \int_0^t e(\tau)d\tau \\ e \\ \dot{e} \\ \Delta\psi \\ \Delta\dot{\psi} \end{bmatrix}$$

Solving the eigenvalue problem for the closed-loop A matrix produces the characteristic equation of the PID transfer function. Rough guidelines for system stability can be established through the Routh-Hurwitz stability criterion. The criterion can form complicated expressions that even MATLAB symbolic solver struggles to solve, so only the simplest expressions are displayed below, dictating that the derivative and integral gains must be positive.

$$\det(\lambda I - A) = a_5\lambda^5 + a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$$

$$a_5 = 1 > 0$$

$$a_4 = \frac{C_{af}I_zU_x + C_{ar}I_zU_x + C_{af}U_xa^2m + C_{ar}U_xb^2m + C_{af}I_zU_x^2K_D}{I_zU_x^2m} > 0$$

$$a_0 = \frac{C_{af}C_{ar}U_x^2K_I(a+b)}{I_zU_x^2m} > 0$$

The PID controller gains were determined one at a time in the s-plane. Proportional gain was considered first, holding the derivative and integral gains at zero. Longitudinal velocity is held at

10 m/s throughout this analysis as it is a rough approximation of the longitudinal velocity throughout the given speed profile. The locations of the poles as a function of K_P is given in Figure 4.

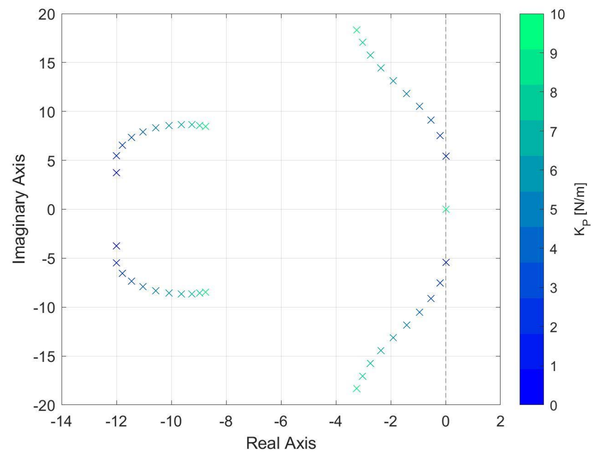

Figure 4: U_x = 10 m/s, K_P = 0:10, K_I = 0, K_D = 0

The proportional gain was selected to be 1. This gain moves all but 4 of the poles off of the imaginary axis, while minimizing the imaginary component of all poles. A low proportional gain is also desirable to prevent exaggerated responses to instantaneous errors. Next, derivative gain was considered, holding proportional gain at 1 and integral gain at 0. Figure 5 shows the location of the closed-loop poles as a function of K_D.

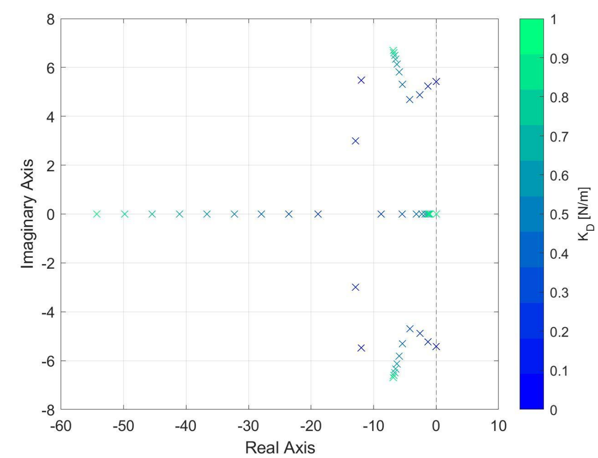

Figure 5: U_x = 10 m/s, K_P = 1, K_I = 0, K_D = 0:1

Derivative gain was chosen to be 0.3. This value maximizes the damping of the dominant poles of the system while placing all other poles on the real axis. Finally, integral control was considered, holding proportional gain at 1 and derivative gain at 0.3 (Figure 6).

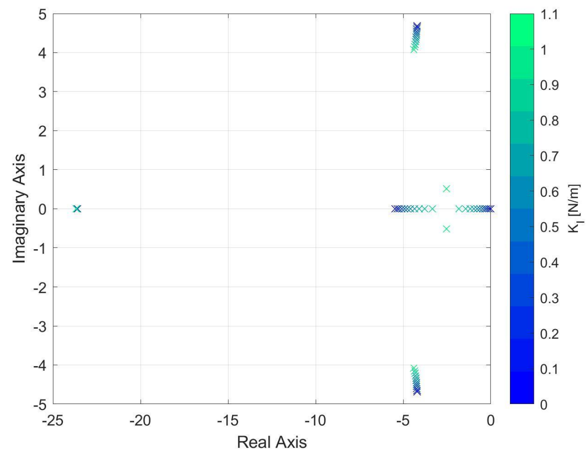

Figure 6: U_x = 10 m/s, K_P = 1, K_I = 0:1.1, K_D = 0.3

Integral gain was chosen to be 1.1. This value moves the dominant poles as far into the right half plane as possible, making the system stable and simultaneously increasing damping.

To prevent the integral term from dominating the response, logic was added to the PID controller to limit the product of the integral term and integral gain to a magnitude of 5 degrees. This makes the controller robust against integral build-up during a delayed start.

Discussion

Lessons learned when moving to the hard simulator

What did you see when adding noise and actuator dynamics that you did not see in the simpler simulation?

For the lookahead controller, the magnitude of the error increased significantly after adding the actuator dynamics. Velocity tracking became worse at lower speeds (5 m/s) than at higher speed which the controller was designed for. The lateral velocity and the yaw rate became noisier in the higher simulation modes. Additionally, in these modes, the longitudinal acceleration reached the lower limit of -4 m/s^2.

For the PID controller, the velocity tracking was still worse around 5 m/s but the lateral velocity, yaw rate were not significantly noisier (unlike controller 1). Both lateral and longitudinal acceleration values were slightly more oscillatory with both values hitting the limits of -4 m/s^2. Oscillations were also observed for the front lateral forces around 0 N.

Which of the factors (noise, actuator dynamics, delayed start…) made an impact on your controller design?

Due to steering delay, the trends in the lookahead distance were reversed. For the easier simulations, increasing the look ahead distance led to lowering of the lateral error but for the harder simulation, reducing the look ahead distance resulted in better tracking.

The limit on the PID integral contribution to the steering input was prompted by the delayed start in Sim 3. Otherwise, integrator build-up caused a large corrective action at the beginning of the trajectory, pushing the controller out of specification for acceleration.

Noise did not have a large effect on the performance of either controller. The response of the system showed some leftover noise in the output, but this did not cause either control to violate one of the specifications. Noise would have had a greater effect magnitude if the controllers had used a higher proportional gain, and both controllers were designed to have the lowest gain necessary to achieve project specifications.

Lessons learned in your control design

You solved this problem with two different controllers. How do they compare in terms of tracking performance, smoothness of the steering response and robustness to disturbances and noise in the hard simulator? What difference is there in the pole locations between your two controllers and how might that help to explain the different results?

The lookahead controller places its poles at -11.4849 ± 3.73118j and -0.51559 ± 1.02839j, while the PID controller places its poles at -23.6957 + 0.0000i, -4.3771 ± 4.0804i, and -2.5322 ± 0.5092i. Converting the dominant poles, we get ω_n = 1.150 rad/s and ζ = 0.448 for the lookahead controller and ω_n = 2.583 rad/s and ζ = 0.980. Based on this primary analysis, the dominant poles of the lookahead controller match the increased stability we see on the response paths below.

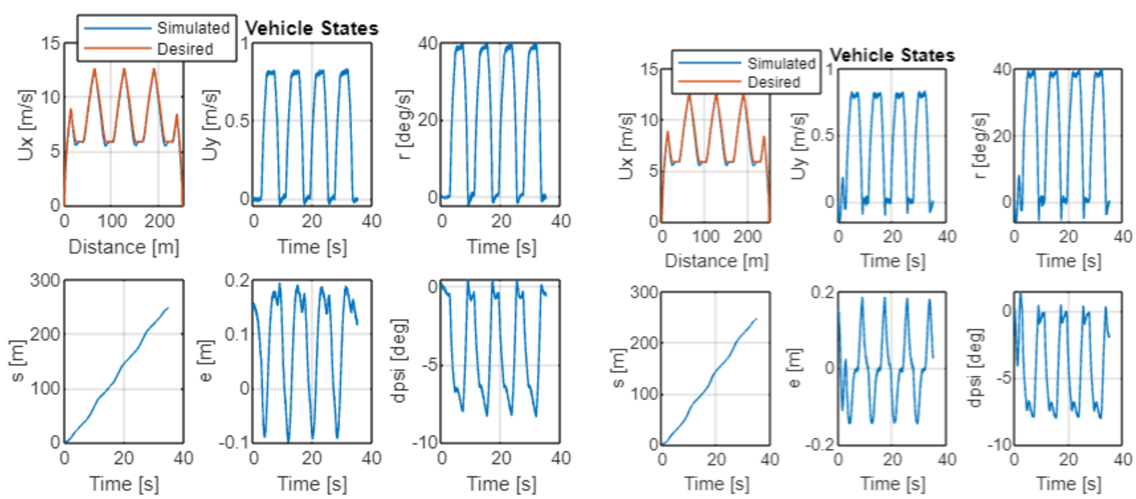

Figure 7: Lookahead (left) and PID Controller (right) Performance on Sim Mode 2

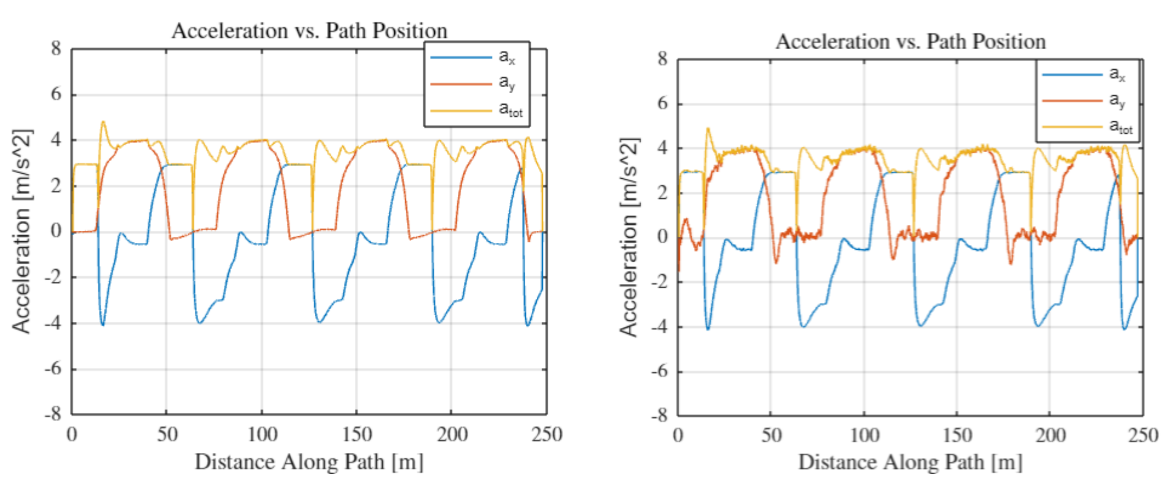

Figure 8: Lookahead (left) and PID Controller (right) Performance on Sim Mode 2

On sim mode 3, with PID, we observe increased oscillations at 0 degrees of steer angle. Due to the time delay, the integrator accumulates the error for the first 5 seconds and then the controller action is executed. This explains the dip that we observe in the plot for the steer angle which is absent in the first plot (from the lookahead controller). We also observe increased oscillations for lateral acceleration around 0 m/s^2 and a small peak in the beginning due to the time delay.

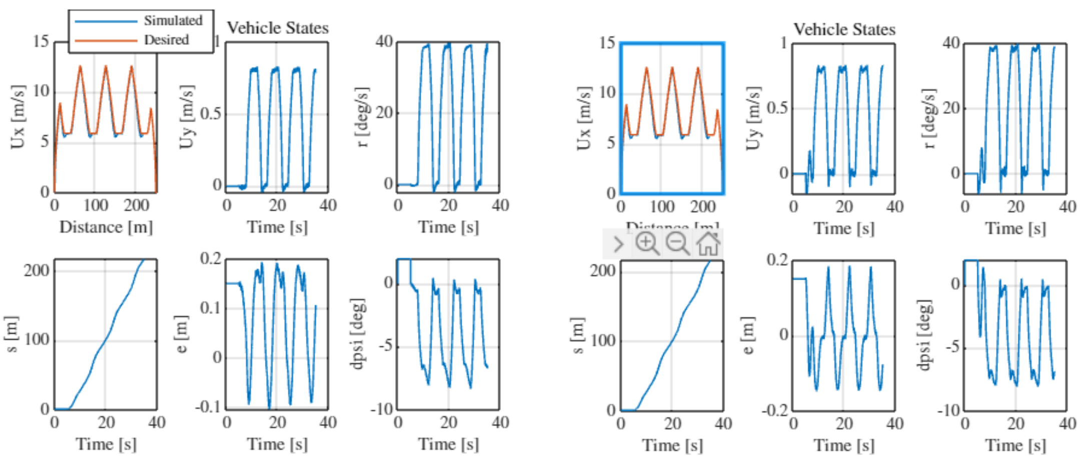

Figure 9: Lookahead (left) and PID Controller (right) Performance on Sim Mode 3

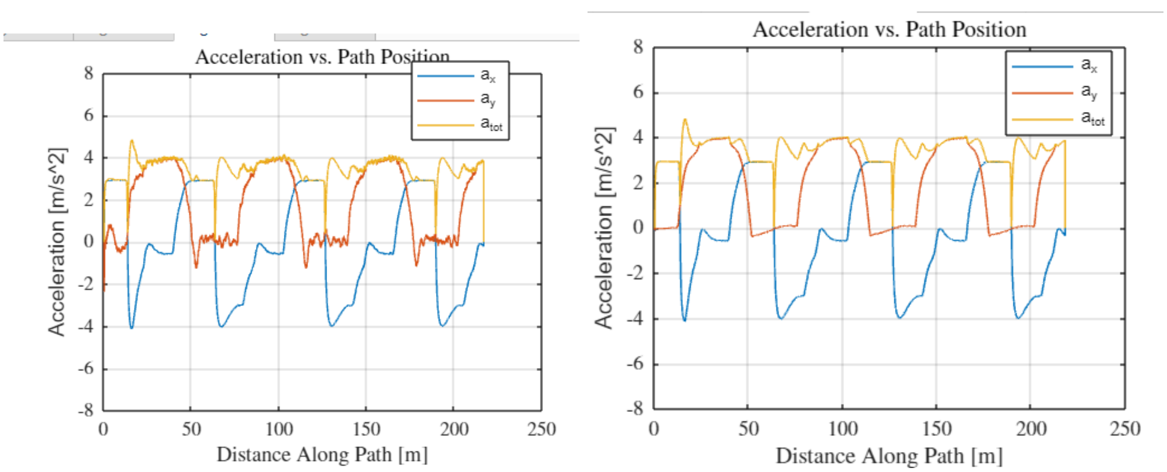

Figure 10: Lookahead (left) and PID Controller (right) Performance on Sim Mode 3

Appendix (submitted to Gradescope): Results Using Sim Mode 2

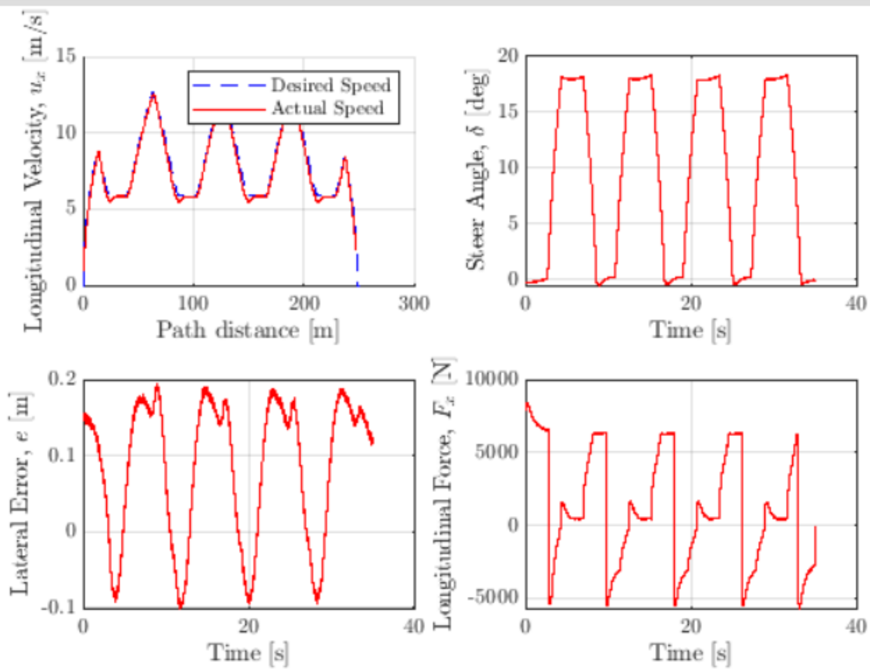

Figure 11: Lookahead Controller Performance on Sim Mode 2

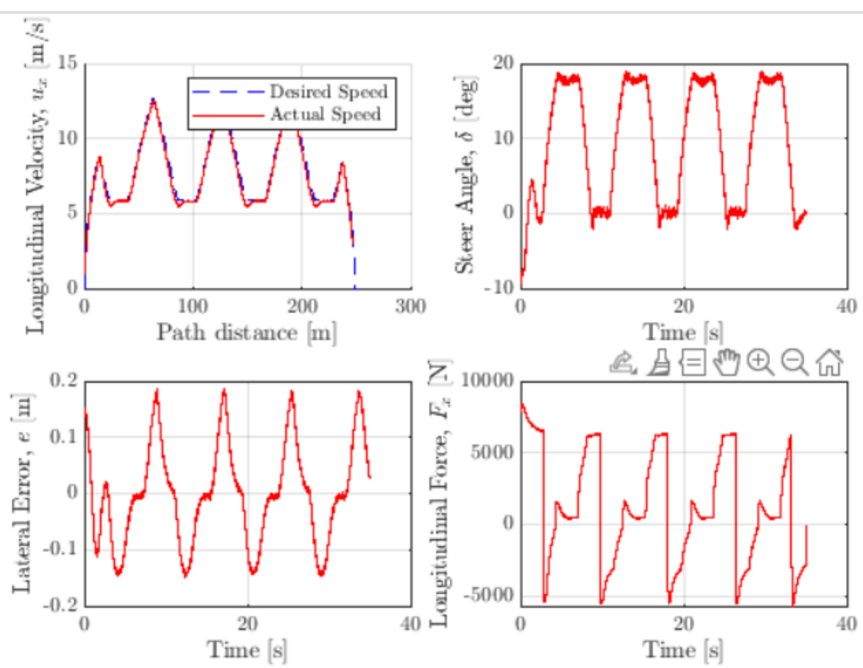

Figure 12: PID Controller Performance on Sim Mode 2