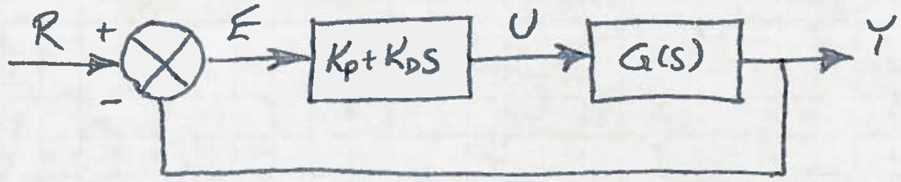
# Derivative Control

When the steady-state error response does not require integral control, a PD controller may be used instead.



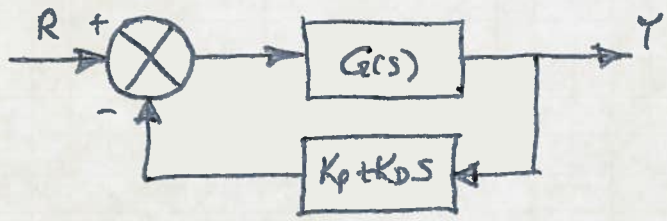

$$\frac{Y(s)}{R(s)} = \frac{(K_p + K_D s)\,G(s)}{1 + (K_p + K_D s)\,G(s)}$$

For the transfer function $G(s) = \dfrac{1}{ms^2 + bs}$

$$\Rightarrow \quad \frac{Y(s)}{R(s)} = \frac{K_p + K_D s}{ms^2 + (b + K_D)s + K_p}$$

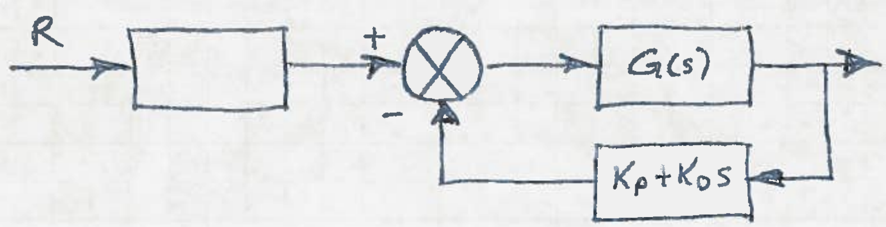
The denominator looks like a familiar form (now the system is second order since there is no integral control) but the system has a zero.

What happens if we put the controller in the feedback loop instead of the forward path?




$$\frac{Y(s)}{R(s)} =$$

To deal with the steady-state error, we can scale the reference. This has no effect on the stability of the control system since it occurs outside the feedback loop:




$$\frac{Y(s)}{R(s)} =$$

If we equate the terms to those in the standard form:

$$\frac{K_p}{ms^2+(b+K_D)s+K_p} = \frac{K_p/m}{s^2+(b+K_D)/m\, s+K_p/m} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$K_p$ can be used to set the natural frequency
$K_D$ can be used to set the damping ratio

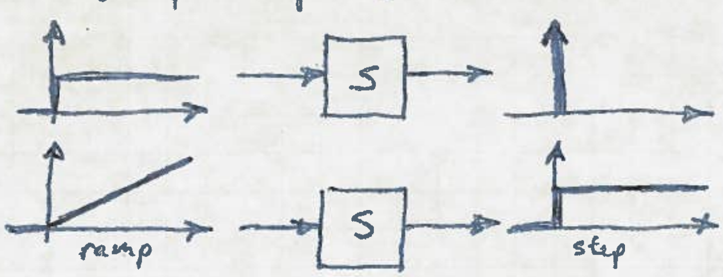
The behavior of the closed-loop poles as we change gains can be viewed graphically:


We can use these gains to achieve specifications like rise time and peak overshoot simultaneously.

Derivative control can be a powerful tool in achieving the desired performance of a system. However, there is a challenge that pure derivative control is not realizable.

$D(s) = K_p + K_D s$ $\Rightarrow$ numerator has higher order than the denominator.

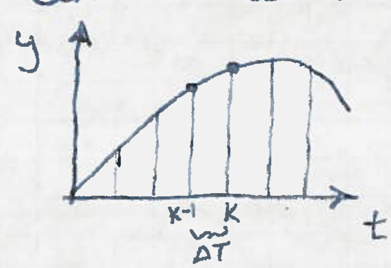
Systems with a higher numerator order than denominator cannot be built with passive elements like resistors, capacitors and inductors. They also have issues with causality and power. Consider a couple of example inputs:




Finite input gives an infinite output.

Output changes before the input does.

In digital systems, we might think of approximating a derivative with a difference.




$$\dot{y}(k) \approx \frac{y(k)-y(k-1)}{\Delta T}$$

This is inherently looking backwards and not the true derivative.

This approach often results in a very noisy signal since the differencing process amplifies noise.

Suppose $y_m(k) = y(k) + n(k)$

$y_m$ - measured output
$y$ - actual output
$n$ - noise

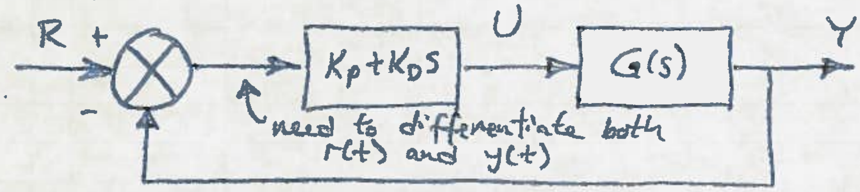
Then $$\dot{y}(k) = \frac{y(k) - y(k-1)}{\Delta T} + \frac{n(k) - n(k-1)}{\Delta T}$$

Amplify noise difference by $\Delta T$

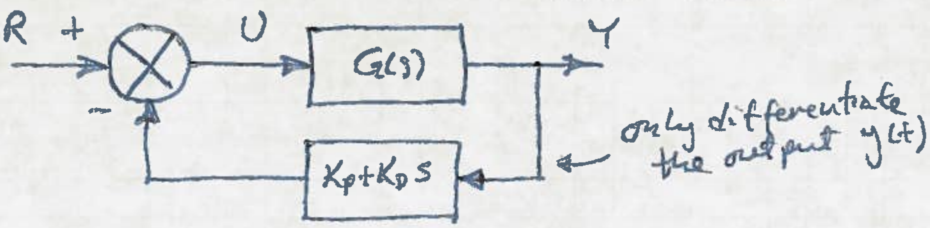
As $\Delta T \to 0$, the approximation of the derivative gets better but the noise amplification gets worse!

With very clean signals (like a high resolution optical encoder) this may work to approximate the derivative. In other cases, it can result in big problems.

The challenge of physically implementing a derivative also is a deciding factor in considering whether to put the controller in the feedback or the forward path:




$$u(t) = K_p\left(r(t) - y(t)\right) + K_D\left(\dot{r}(t) - \dot{y}(t)\right)$$




$$u(t) = r(t) - K_p y(t) - K_D \dot{y}(t)$$

If the reference signal is noisy, it may be preferable to have the controller in the feedback path to avoid differentiating it. The feedback path is also better if the reference input makes step changes (what will the input try to do in such cases?).

The issues with implementing pure derivative control can be resolved by using a lead compensator. This is a circuit that can be built (remember the first homework assignment) and filters out the high frequency noise (that will be clear in a couple of weeks).

The lead compensator is just the PD controller with a fast pole:

$$D(s) = \frac{K_P + K_D s}{\frac{1}{a}s + 1} \qquad \text{pole at } s = -a$$

As the value of a increases, the lead compensator resembles the PD controller more closely.

Another difference between choosing to put the PD controller in the feedback path or the forward path is the zero.

$$\frac{Y(s)}{R(s)} = \frac{K_P + K_D s}{ms^2 + (b+K_D)s + K_P} \qquad \frac{Y(s)}{R(s)} = \frac{K_P}{ms^2 + (b+K_D)s + K_P}$$

forward path          feedback path

What is the effect of this zero? There are several ways to look at zeros and this will be the focus of the next lecture. For now, consider breaking up the two parts of the numerator

$$\frac{Y(s)}{R(s)} = \underbrace{\frac{K_P}{ms^2 + (b+K_D)s + K_P}}_{\text{our standard 2nd order system form}} + \underbrace{s\,\frac{K_D}{ms^2 + (b+K_D)s + K_P}}_{\text{the derivative of the standard 2nd order form}}$$

If $K_D << K_P$, the system response looks a lot like the standard second order system. In this case, the zero location is far from the imaginary axis since

$$z = -K_P/K_D \quad \text{is the zero location.}$$

Therefore zeros that are far from the imaginary axis have little impact on the system behavior when they are in the left half plane.

As $K_D$ becomes larger, however, the system response includes more of this derivative term. This can lead to, for instance, greater overshoot to a step response.