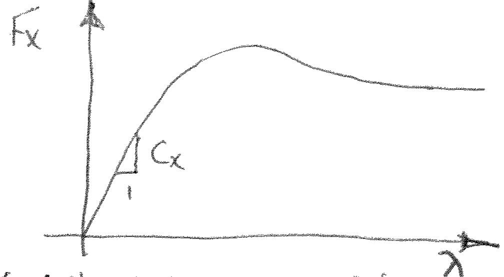
Longitudinal Tire Forces

Forces that accelerate or brake the car must also go through the tire contact patches. Just as the lateral tire forces are coupled to the slip angle, the longitudinal forces have a related longitudinal slip

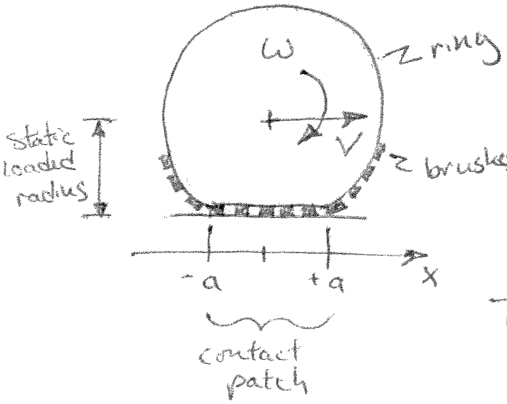


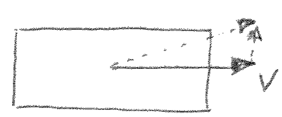
(If this looks like something you've seen before, you are correct)

$$\lambda = \frac{R_e\omega - V}{V}$$

V is the component of the wheel center velocity over ground along the tire's longitudinal axis.

For longitudinal slip, λ, to exist, the wheel must rotate at a different speed than a solid disc model (with $V = R\omega$) would imply. This may sound strange but then so did the idea of a slip angle at first. This is just the longitudinal equivalent and can be explained in terms of the brush and ring model.



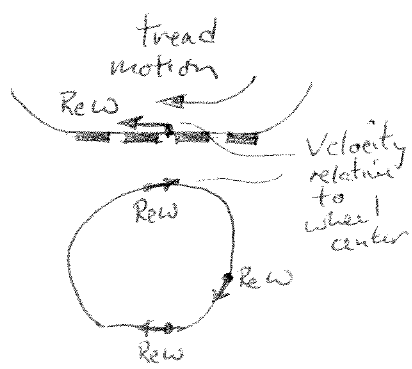


Tire may also have lateral velocity. V is the longitudinal component.

For now, consider the case where the tire has zero slip angle.

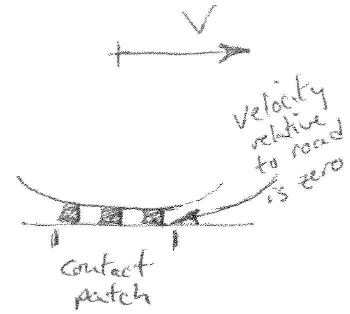
The tire has two main parts:
(1) An inextensible ring (like the steel belts)
(2) Flexible brushes (the tire tread)

First look at the tire in a free rolling condition with no applied wheel torques or longitudinal forces.



The velocity of a point on the ring must be $R_e\omega$ for some effective radius since the ring is not able to change its circumference. The effective radius is not the tire outside radius or the static loaded radius but more closely the radius of the steel belts. We define it in terms of pure rolling:

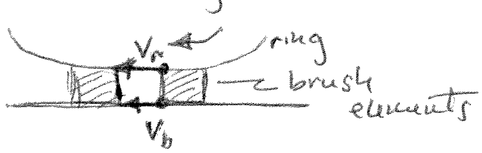
$$R_e = \frac{\int_0^t V\,dt}{\int_0^t \omega\,dt}$$



If the tire is originally stuck to the road, the velocity of the tread relative to the road is zero,
The velocity of the wheel center relative to the road is V.
The velocity of the brush tips relative to the wheel center is therefore V

The slip is $\lambda = \dfrac{R_e\omega - V}{V} = 0$ since $V = R_e\omega$

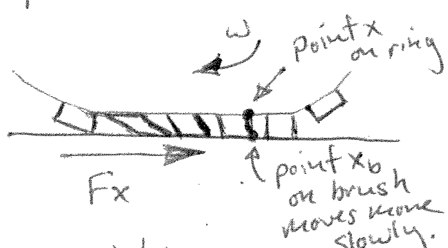
In other words, we define this free rolling condition as having zero slip and define the radius accordingly.



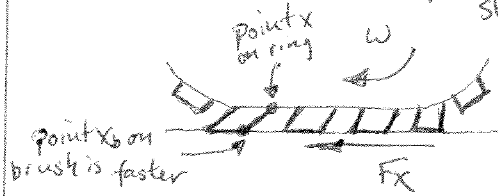
Another way to look at this is that the brush elements do not deform since $V_r = V_b$

$$V_r = R_e\omega = V = V_b$$

When braking or accelerating, the picture changes. for positive longitudinal forces, the brushes are deflected towards the front of the tire. The velocity of a point on the ring is higher than that of the corresponding point on a brush.



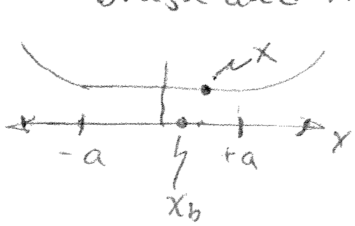
The deformation in the brushes starts at zero and increases as the brush moves through the contact patch. The deflection drops to zero when the brush leaves the contact patch.



Braking is just the opposite direction for deformation. The velocity of a point on the brush is higher than the ring.

The deflection in the brushes is the difference between the x-axis position of the corresponding points on the brush and ring.

$$u(x) = x_b - x$$



Assuming that no sliding occurs, after a time Δt in the contact patch

$$x = a - R_e\omega\Delta t \qquad \Delta t = \frac{(a-x)}{R_e\omega}$$
$$x_b = a - V\Delta t$$

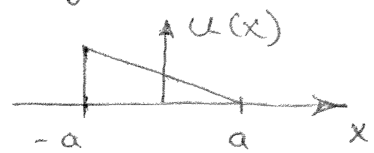
So $u(x) = a - V\Delta t - a + Re\omega\Delta t$

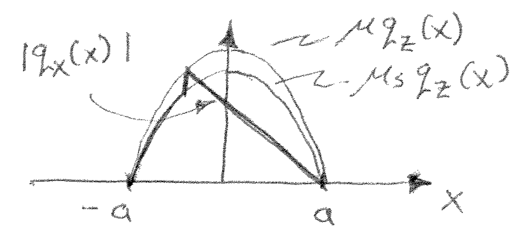
$$= [Re\omega - V]\,\Delta t$$

$$= \left[\frac{Re\omega - V}{Re\omega}\right](a-x)$$ This looks sort of like $\lambda = \frac{Re\omega - V}{V}$

$$= (a-x)\left(\frac{\lambda}{1+\lambda}\right)$$

We get a picture very much like the lateral direction:





$u(x) > 0$ for $\lambda > 0$, $F_x > 0$
$u(x) < 0$ for $\lambda < 0$, $F_x < 0$

Defining a stiffness for the brushes, c_{px}:

$$q_x(x) = c_{px}\,u(x) = (a-x)\,c_{px}\left(\frac{\lambda}{1+\lambda}\right)$$

We start sliding when we reach x_{se} at which

$$|q_x(x_{se})| = \mu q_z(x_{se})$$

$$(a-x)\,c_{px}\left|\frac{\lambda}{1+\lambda}\right| = \frac{3\mu F_z}{4a}\left(\frac{a^2 - x_{se}^2}{a^2}\right)$$

or $$x_{se} = \frac{4c_{px}a^3\left|\frac{\lambda}{1+\lambda}\right|}{3\mu F_z} - a$$

So our total longitudinal force is

$$F_x = \int_{\substack{\text{adhesion}\\\text{region}}} q_x(x)\,dx + \int_{\substack{\text{sliding}\\\text{region}}} q_x(x)\,dx$$

$$= \int_{x_{se}}^{a} c_{px}(a-x)\left(\frac{\lambda}{1+\lambda}\right)dx + \int_{a}^{x_{se}} \frac{3\mu_s F_z}{4a}\left(\frac{a^2-x^2}{a^2}\right)\operatorname{sgn}\left(\frac{\lambda}{1+\lambda}\right)$$

For the lateral direction, we had

$$F_y = -\int_{x_{se}}^{a} c_{py}(a-x)\tan\alpha\,dx - \int_{-a}^{x_{se}} \frac{3\mu_s F_z}{4a}\left(\frac{a^2-x^2}{a^2}\right)\operatorname{sgn}(\alpha)$$

$$x_{se} = \frac{4c_{py}a^3|\tan\alpha|}{3\mu F_z} - a$$

$\Rightarrow$ Same form of equation, just change sign and replace

(1) c_{py} with c_{px} (2) $\tan\alpha$ with $\frac{\lambda}{1+\lambda}$

So defining $C_x = 2c_{px} a^2$

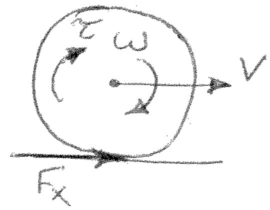
$$F_x = C_x \left(\frac{\lambda}{1+\lambda}\right) - \frac{C_x^2}{3\mu F_z}\left(2 - \frac{\mu_s}{\mu}\right)\left(\frac{\lambda}{1+\lambda}\right)\left|\frac{\lambda}{1+\lambda}\right| + \frac{C_x^3}{9\mu^2 F_z^2}\left(\frac{\lambda}{1+\lambda}\right)^3\left(1 - \frac{2\mu_s}{3\mu}\right)$$

This expression is valid up to total sliding which occurs at

$$\left|\frac{\lambda_{sl}}{1+\lambda_{sl}}\right| = \frac{3\mu F_z}{C_x} \quad \text{then} \quad F_x = \mu_s F_z \, \mathrm{sgn}(\lambda)$$

Although this expression is in terms of $\frac{\lambda_{sl}}{1+\lambda_{sl}}$, keep in mind that λ_{sl} is small before we reach total sliding, on the order of a few percent. Thus qualitatively, you can think of longitudinal force being a function of slip (aka λ) alone.

To use these expressions for the longitudinal forces, we need to know the slip, λ. This means we need both the wheel speed and the longitudinal velocity of the wheel center over ground. The latter we can easily get from the planar model; the former requires adding new states for wheel speeds. The differential equations are:



$$\tau - R_e F_x = J_w \dot{\omega}$$

where J_w is the polar moment of inertia of the wheel and τ is the applied torque. An engine torque would be positive, a brake torque negative.

Dynamics of Braking

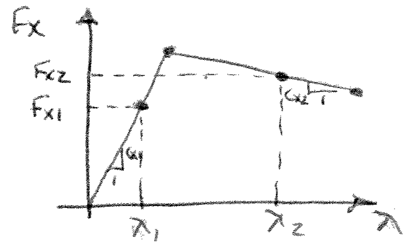
The longitudinal forces are governed by the wheel slip:

$$\lambda = \frac{R_e\omega - V}{V}$$

which in turn depends upon the wheel speed dynamics:

$$J_w \dot{\omega} = \tau - R_e F_x(\lambda)$$

These dynamics can be particularly challenging during braking. To see why, consider a very simple bilinear model of the tire curve:



In braking, $\tau < 0$, $F_x < 0$, $\lambda < 0$

Consider two points where point 1 is before the peak of the tire curve and point 2 is after the peak.

At equilibrium 1, $\tau_1 = R_e F_{x1}$, $\lambda = \lambda_1$ and $\omega = \omega_1$

Generally, $\Delta\omega_1 = \omega - \omega_1 \qquad \Delta\dot{\omega}_1 = \dot{\omega} - \dot{\omega}_1 = \dot{\omega}$

$$\tau = \tau_1 + \Delta\tau_1, \quad \lambda = \lambda_1 + \Delta\lambda_1$$

$$\Delta\lambda_1 = \frac{R_e\omega - V}{V} - \frac{R_e\omega_1 - V}{V} = \frac{R_e}{V}\Delta\omega_1$$

$$F_x = F_{x1} + C_{x1}\Delta\lambda_1$$

Substituting into the equation of motion

$$J_w \dot{\omega} = \tau - R_e F_x$$

$$J_w \Delta\dot{\omega}_1 = \tau_1 + \Delta\tau_1 - R_e[F_{x1} + C_{x1}\Delta\lambda_1]$$

$$= \tau_1 - R_e F_{x1} + \Delta\tau_1 - R_e C_{x1}\left(\frac{R_e}{V}\Delta\omega_1\right)$$

$$J_w \Delta\dot{\omega}_1 = \Delta\tau_1 - \frac{R_e^2 C_{x1}}{V}\Delta\omega_1$$

$$\frac{\Delta\omega_1(s)}{\Delta\tau_1(s)} = \frac{1}{J_w s + \frac{R_e^2 C_{x1}}{V}}$$

Stable first order dynamics

$\Rightarrow$ Small perturbations in the torque produce stable responses in the rotational speed of the wheel.

We can do the same thing at equilibrium 2, everything is the same except the slope of the force-slip curve

$$\text{So} \quad F_x = F_{x2} - C_{x2}\,\Delta\lambda_2$$

$$\Rightarrow \quad J_w \Delta\dot{\omega}_2 = \Delta\tau_2 + R_e C_{x2}\left(\frac{R_e}{V}\Delta\omega_2\right)$$

$$= \Delta\tau_2 + \frac{R_e^2 C_{x2}}{V}\Delta\omega_2$$

$$\frac{\Delta\omega_2(s)}{\Delta\tau_2(s)} = \frac{1}{J_w s - \frac{R_e^2 C_{x2}}{V}}$$

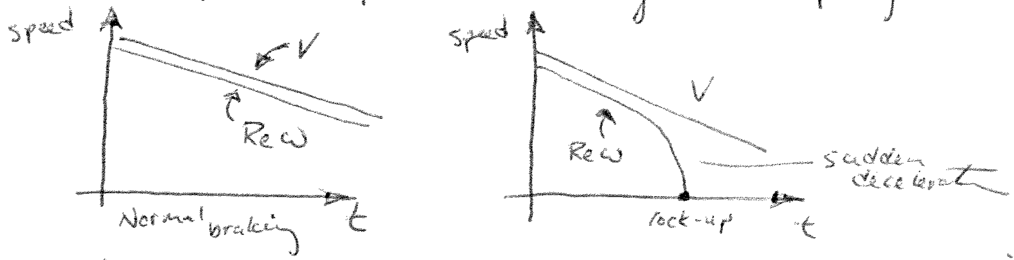
unstable first order dynamics!

So after the peak of the tire curve, the wheel speed response is <u>unstable</u>. Any slight perturbation in torque will result in a dramatic change in wheel speed.

Physically, this is the challenge of wheel lock-up. At points past the peak of the tire curve, any brake torque of greater magnitude than equilibrium will cause the slip to go to -1. This corresponds to a locked wheel.

The locked wheel is just like a sliding block of rubber. Not only does this take away the peak force capability but the high longitudinal sliding velocity takes away lateral force capability as well. This is particularly problematic when the rear wheel locks up - with insufficient rear force laterally, the car will spin.

Anti-lock brake systems modulate brake pressure to prevent this from happening. Because speed over ground (V) is not an available measurement, cars can only do a rough estimate of slip. Instead, ABS systems look primarily at wheel deceleration since with the low inertia of the wheels, wheel speed can change more rapidly than vehicle speed.



Wheel spin can be an issue with excessive positive longitudinal force (burnouts). However this requires a car with sufficient drive torque and horsepower. All cars can brake at the limits, few can drive at the limits on dry pavement