Problem 5 (Chapter 4, Exercise 10)

```
import pandas as pd
weekly = pd.read_csv("Weekly.csv")
```

```
weekly.info()
```

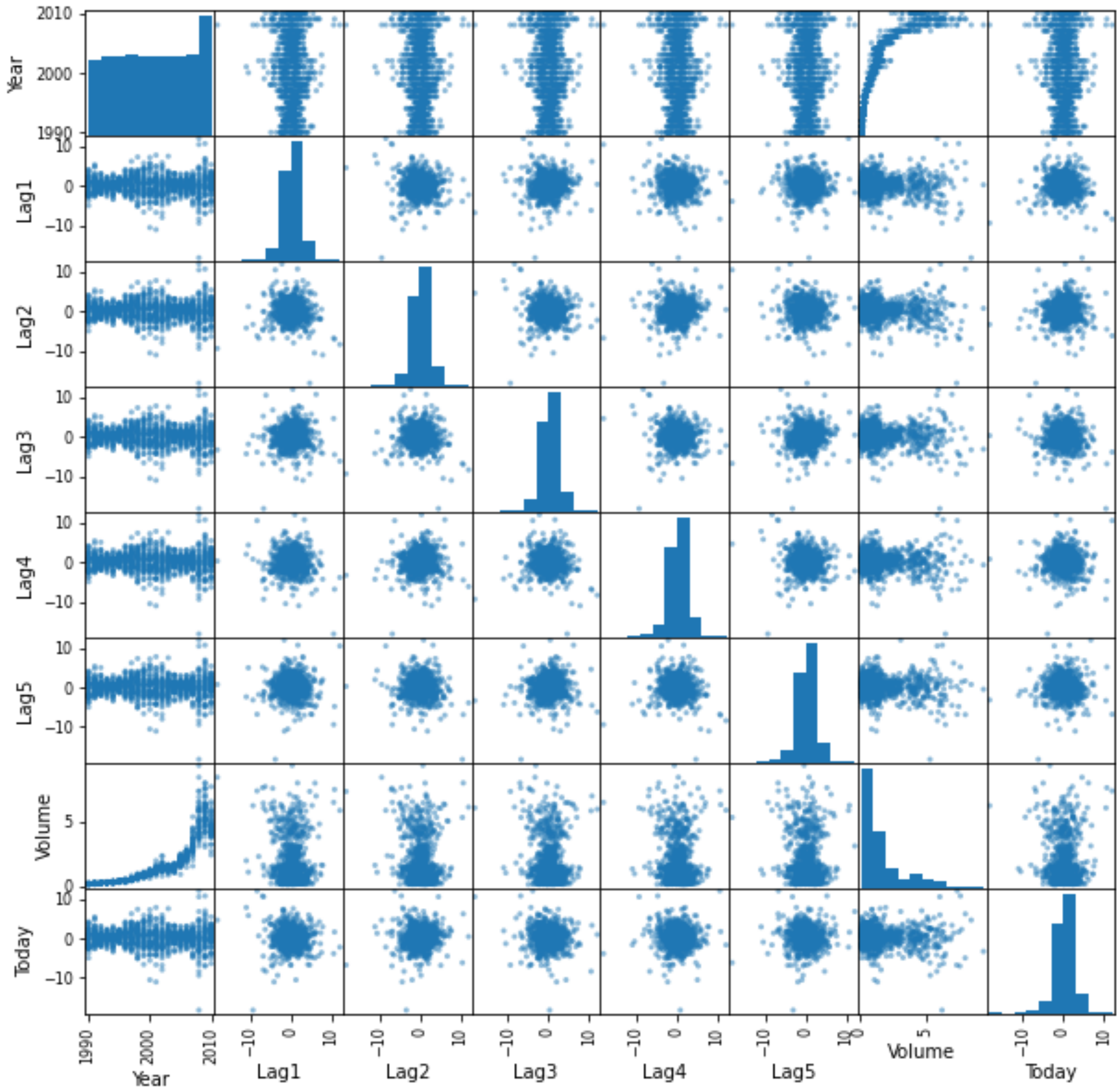
```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1089 entries, 0 to 1088
Data columns (total 9 columns):
 #   Column     Non-Null Count  Dtype  
---  ------     --------------  -----  
 0   Year       1089 non-null   int64  
 1   Lag1       1089 non-null   float64
 2   Lag2       1089 non-null   float64
 3   Lag3       1089 non-null   float64
 4   Lag4       1089 non-null   float64
 5   Lag5       1089 non-null   float64
 6   Volume     1089 non-null   float64
 7   Today      1089 non-null   float64
 8   Direction  1089 non-null   object 
dtypes: float64(7), int64(1), object(1)
memory usage: 76.7+ KB
```

Problem 5(a)

```
weekly.describe(include='all')
```

	Year	Lag1	Lag2	Lag3	Lag4	Lag5	
count	1089.000000	1089.000000	1089.000000	1089.000000	1089.000000	1089.000000	1(
unique	NaN	NaN	NaN	NaN	NaN	NaN	
top	NaN	NaN	NaN	NaN	NaN	NaN	
freq	NaN	NaN	NaN	NaN	NaN	NaN	
mean	2000.048669	0.150585	0.151079	0.147205	0.145818	0.139893	
std	6.033182	2.357013	2.357254	2.360502	2.360279	2.361285	
min	1990.000000	-18.195000	-18.195000	-18.195000	-18.195000	-18.195000	
25%	1995.000000	-1.154000	-1.154000	-1.158000	-1.158000	-1.166000	
50%	2000.000000	0.241000	0.241000	0.241000	0.238000	0.234000	
75%	2005.000000	1.405000	1.409000	1.409000	1.409000	1.405000	
max	2010.000000	12.026000	12.026000	12.026000	12.026000	12.026000	

```
pd.plotting.scatter_matrix(weekly, figsize=(10,10));
```



```
weekly.corr()
```

	Year	Lag1	Lag2	Lag3	Lag4	Lag5	Volume	Tod
Year	1.000000	-0.032289	-0.033390	-0.030006	-0.031128	-0.030519	0.841942	-0.032
Lag1	-0.032289	1.000000	-0.074853	0.058636	-0.071274	-0.008183	-0.064951	-0.075
Lag2	-0.033390	-0.074853	1.000000	-0.075721	0.058382	-0.072499	-0.085513	0.059
Lag3	-0.030006	0.058636	-0.075721	1.000000	-0.075396	0.060657	-0.069288	-0.071
Lag4	-0.031128	-0.071274	0.058382	-0.075396	1.000000	-0.075675	-0.061075	-0.007
Lag5	-0.030519	-0.008183	-0.072499	0.060657	-0.075675	1.000000	-0.058517	0.011
Volume	0.841942	-0.064951	-0.085513	-0.069288	-0.061075	-0.058517	1.000000	-0.033
Today	-0.032460	-0.075032	0.059167	-0.071244	-0.007826	0.011013	-0.033078	1.000

Problem 5(b)

```
import statsmodels.api as sm

X = weekly[['Lag1', 'Lag2', 'Lag3', 'Lag4', 'Lag5', 'Volume']]
X = sm.add_constant(X)

y = weekly['Direction'].astype('category').cat.codes

log_reg = sm.Logit(y, X).fit()
```

```
/usr/local/lib/python3.7/dist-packages/statsmodels/tools/_testing.py:19: Futu
  import pandas.util.testing as tm
Optimization terminated successfully.
         Current function value: 0.682441
         Iterations 4
```

```
print(log_reg.summary())
```

```
                           Logit Regression Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 108
Model:                          Logit   Df Residuals:                     108
Method:                           MLE   Df Model:                          
Date:                Fri, 16 Jul 2021   Pseudo R-squ.:               0.00658
Time:                        19:36:57   Log-Likelihood:              -743.1
converged:                       True   LL-Null:                     -748.1
Covariance Type:            nonrobust   LLR p-value:                  0.131
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975
------------------------------------------------------------------------------
const          0.2669      0.086      3.106      0.002       0.098       0.43
Lag1          -0.0413      0.026     -1.563      0.118      -0.093       0.01
Lag2           0.0584      0.027      2.175      0.030       0.006       0.11
Lag3          -0.0161      0.027     -0.602      0.547      -0.068       0.03
Lag4          -0.0278      0.026     -1.050      0.294      -0.080       0.02
Lag5          -0.0145      0.026     -0.549      0.583      -0.066       0.03
Volume        -0.0227      0.037     -0.616      0.538      -0.095       0.05
==============================================================================
```

Problem 5(c)

```
print(log_reg.pred_table())
```

```
[[ 54. 430.]
 [ 48. 557.]]
```

```
acc = (54 + 557) / (54 + 430 + 48 + 557)
print(acc)
```

```
0.5610651974288338
```

```
print(54/(54+430))
print(557/(557+48))
```

```
0.1115702479338843
0.9206611570247933
```

Problem 5(d)

```
weekly1990to2008 = weekly.copy()
weekly1990to2008 = weekly1990to2008[weekly1990to2008['Year'] >= 1990]
weekly1990to2008 = weekly1990to2008[weekly1990to2008['Year'] <= 2008]
X_train = weekly1990to2008['Lag2']
X_train = sm.add_constant(X_train)

y_train = weekly1990to2008['Direction'].astype('category').cat.codes

log_reg = sm.Logit(y_train, X_train).fit()
```

```
Optimization terminated successfully.
         Current function value: 0.685555
         Iterations 4
```

```
weekly2009to2010 = weekly.copy()
weekly2009to2010 = weekly2009to2010[weekly2009to2010['Year'] >= 2009]
weekly2009to2010 = weekly2009to2010[weekly2009to2010['Year'] <= 2010]
X_test = weekly2009to2010['Lag2']
X_test = sm.add_constant(X_test)

y_test = weekly2009to2010['Direction'].astype('category').cat.codes
y_pred = (log_reg.predict(X_test) >= 0.5).astype(int)

import numpy as np
cm = np.zeros((2,2))
for i in range(2):
  for j in range(2):
    cm[i, j] = sum(yt==i and yp==j for yt, yp in zip(y_test, y_pred))

cm
```

```
array([[ 9., 34.],
       [ 5., 56.]])
```

```
acc = (9 + 56) / (9 + 56 + 5 + 34)
print(acc)
```

```
0.625
```

```
print(9/(9+34))
print(56/(5+56))
```

```
0.20930232558139536
0.9180327868852459
```

Problem 5(e)

```
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis as LDA

lda = LDA()
lda.fit(X_train, y_train)
```

```
LinearDiscriminantAnalysis(n_components=None, priors=None, shrinkage=None,
                           solver='svd', store_covariance=False, tol=0.0001)
```

```
from sklearn.metrics import confusion_matrix

y_pred = lda.predict(X_test)
cm = confusion_matrix(y_test, y_pred)
print(cm)
```

```
[[ 9 34]
 [ 5 56]]
```

```
acc = (9 + 56) / (9 + 56 + 5 + 34)
print(acc)
```

```
0.625
```

```
print(9/(9+34))
print(56/(5+56))
```

```
0.20930232558139536
0.9180327868852459
```

Problem 5(f)

```
from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis as QDA

qda = QDA()
qda.fit(X_train, y_train)
```

```
/usr/local/lib/python3.7/dist-packages/sklearn/discriminant_analysis.py:691: U
  warnings.warn("Variables are collinear")
QuadraticDiscriminantAnalysis(priors=None, reg_param=0.0,
                              store_covariance=False, tol=0.0001)
```

```
y_pred = qda.predict(X_test)
cm = confusion_matrix(y_test, y_pred)
print(cm)
```

```
[[43  0]
 [61  0]]
/usr/local/lib/python3.7/dist-packages/sklearn/discriminant_analysis.py:715: R
  X2 = np.dot(Xm, R * (S ** (-0.5)))
/usr/local/lib/python3.7/dist-packages/sklearn/discriminant_analysis.py:715: R
  X2 = np.dot(Xm, R * (S ** (-0.5)))
/usr/local/lib/python3.7/dist-packages/sklearn/discriminant_analysis.py:718: R
  u = np.asarray([np.sum(np.log(s)) for s in self.scalings_])
```

```
acc = (43) / (43+61)
print(acc)
```

```
0.41346153846153844
```

```
print(43/(43+0))
print(0/(61+0))
```

```
1.0
0.0
```

Problem 5(g)

```
from sklearn.neighbors import KNeighborsClassifier as KNN

knn = KNN(1)
knn.fit(X_train, y_train)
```

```
KNeighborsClassifier(algorithm='auto', leaf_size=30, metric='minkowski',
                     metric_params=None, n_jobs=None, n_neighbors=1, p=2,
                     weights='uniform')
```

```
y_pred = knn.predict(X_test)
cm = confusion_matrix(y_test, y_pred)
print(cm)
```

```
[[21 22]
 [31 30]]
```

```
acc = (21+30) / (21+30+22+31)
print(acc)
```

```
0.49038461538461536
```

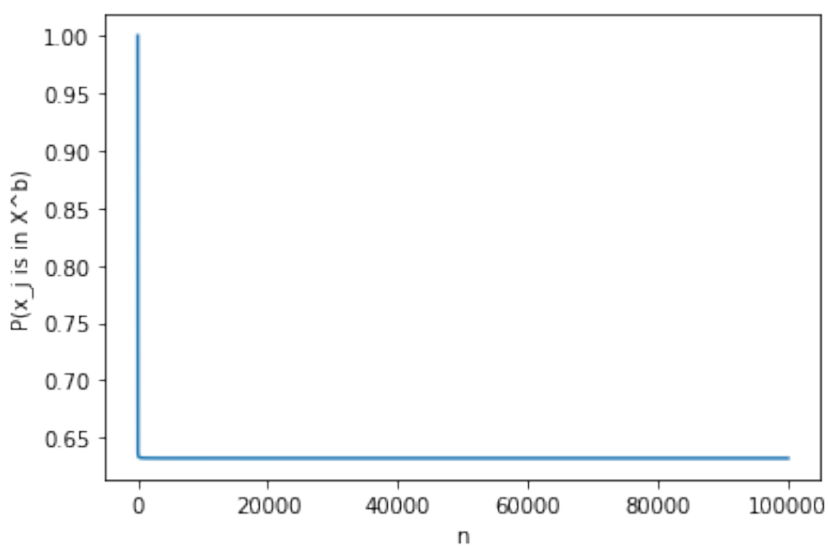
```
print(21/(21+22))
print(30/(31+30))
```

```
0.4883720930232558
0.4918032786885246
```

Problem 6 (Chapter 5, Exercise 2)

Problem 6(g)

```
import matplotlib.pyplot as plt
plt.plot(np.arange(1, 100000), [1-(1-1/n)**n for n in np.arange(1, 100000)])
plt.xlabel("n")
plt.ylabel("P(x_j is in X^b)");
```



Problem 6(h)

```
np.random.seed(1)

ctr = 0
for i in range(10000):
  a = np.random.choice(range(100), size=100, replace=True)
  if 4 in a:
    ctr += 1

print(ctr)
```

```
6302
```

Problem 7 (Chapter 5, Exercise 5)

```
default = pd.read_csv("Default.csv")
```

```
np.random.seed(1)
```

Problem 7(a)

```
X = default[['income', 'balance']]
X = sm.add_constant(X)

y = default['default'].astype('category').cat.codes

log_reg = sm.Logit(y, X).fit()
```

```
Optimization terminated successfully.
         Current function value: 0.078948
         Iterations 10
```

Problem 7(b)

```
from sklearn.model_selection import train_test_split

X_train, X_test, y_train, y_test = train_test_split(X, y)

log_reg = sm.Logit(y_train, X_train).fit()

y_pred = (log_reg.predict(X_test) >= 0.5).astype(int)

error = np.mean([yt != yp for yt, yp in zip(y_test, y_pred)])
print(error)
```

```
Optimization terminated successfully.
         Current function value: 0.078638
         Iterations 10
0.0252
```

Problem 7(c)

```
# trial 1
X_train, X_test, y_train, y_test = train_test_split(X, y)

log_reg = sm.Logit(y_train, X_train).fit()

y_pred = (log_reg.predict(X_test) >= 0.5).astype(int)

error = np.mean([yt != yp for yt, yp in zip(y_test, y_pred)])
print(error)

# trial 2
X_train, X_test, y_train, y_test = train_test_split(X, y)

log_reg = sm.Logit(y_train, X_train).fit()

y_pred = (log_reg.predict(X_test) >= 0.5).astype(int)

error = np.mean([yt != yp for yt, yp in zip(y_test, y_pred)])
print(error)

# trial 3
X_train, X_test, y_train, y_test = train_test_split(X, y)

log_reg = sm.Logit(y_train, X_train).fit()

y_pred = (log_reg.predict(X_test) >= 0.5).astype(int)

error = np.mean([yt != yp for yt, yp in zip(y_test, y_pred)])
print(error)
```

```
Optimization terminated successfully.
         Current function value: 0.079030
         Iterations 10
0.0236
Optimization terminated successfully.
         Current function value: 0.076866
         Iterations 10
0.0276
Optimization terminated successfully.
         Current function value: 0.079056
         Iterations 10
0.0272
```

Problem 7(d)

```
default['student'] = default['student'].astype('category').cat.codes

X = default[['income', 'balance', 'student']]
X = sm.add_constant(X)

X_train, X_test, y_train, y_test = train_test_split(X, y)

log_reg = sm.Logit(y_train, X_train).fit()

y_pred = (log_reg.predict(X_test) >= 0.5).astype(int)

error = np.mean([yt != yp for yt, yp in zip(y_test, y_pred)])
print(error)
```

```
Optimization terminated successfully.
         Current function value: 0.076317
         Iterations 10
0.028
```

Problem 8 (Chapter 5, Exercise 6)

```
default = pd.read_csv("Default.csv")
```

```
np.random.seed(1)
```

Problem 8(a)

```
X = default[['income', 'balance']]
X = sm.add_constant(X)

y = default['default'].astype('category').cat.codes

log_reg = sm.Logit(y, X).fit()
```

```
Optimization terminated successfully.
         Current function value: 0.078948
         Iterations 10
```

```
print(log_reg.summary())
```

```
                              Logit Regression Results
==============================================================================
Dep. Variable:                      y   No. Observations:                10000
Model:                          Logit   Df Residuals:                     9997
Method:                           MLE   Df Model:                            2
Date:                Fri, 16 Jul 2021   Pseudo R-squ.:                  0.4594
Time:                        19:36:59   Log-Likelihood:                -789.48
converged:                       True   LL-Null:                       -1460.3
Covariance Type:            nonrobust   LLR p-value:                 4.541e-292
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const        -11.5405      0.435    -26.544      0.000     -12.393     -10.688
income      2.081e-05   4.99e-06      4.174      0.000     1.1e-05    3.06e-05
balance        0.0056      0.000     24.835      0.000       0.005       0.006
==============================================================================

Possibly complete quasi-separation: A fraction 0.14 of observations can be
perfectly predicted. This might indicate that there is complete
quasi-separation. In this case some parameters will not be identified.
```

Problem 8(b)

```
def boot_fn(default, indices):

  X = default[['income', 'balance']]
  X = X.iloc[indices, :]
  X = sm.add_constant(X)

  y = default['default'].astype('category').cat.codes
  y = y.iloc[indices]

  log_reg = sm.Logit(y, X).fit()

  return log_reg.params[1:]
```

Problem 8(c)

```
beta_is, beta_bs = [], []
B = 100

for i in range(B):
```

```
beta_i, beta_b = boot_fn(default, np.random.choice(range(10000), size=10000));
beta_is.append(beta_i)
beta_bs.append(beta_b)
 Optimization terminated successfully.
          Current function value: 0.070837
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.075469
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.074056
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.080399
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.079418
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.075494
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.076585
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.078997
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.077621
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.077644
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.078250
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.076597
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.082268
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.079406
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.080869
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.075792
          Iterations 10
 Optimization terminated successfully.
          Current function value: 0.079341
          Iterations 10
 Optimization terminated successfully.
```

```
Optimization terminated successfully.
         Current function value: 0.076661
         Iterations 10
Optimization terminated successfully.
         Current function value: 0.086022
         Iterations 10
Optimization terminated successfully.
         Current function value: 0.083204
         Iterations 10
```

```
beta_i_mean = np.mean(beta_is)
beta_i_se   = np.sqrt(1/(B-1)*np.sum( [(beta_i - beta_i_mean)**2 for beta_i in be

beta_b_mean = np.mean(beta_bs)
beta_b_se   = np.sqrt(1/(B-1)*np.sum( [(beta_b - beta_b_mean)**2 for beta_b in be

print("beta_income  (mean & SE)", beta_i_mean, beta_i_se)
print("beta_balance (mean & SE)", beta_b_mean, beta_b_se)
```

```
beta_income  (mean & SE) 2.016973494321125e-05 5.4383662396369285e-06
beta_balance (mean & SE) 0.005677686072213888 0.00022069873134705593
```

Problem 9 (Chapter 5, Exercise 8)

Problem 9(a)

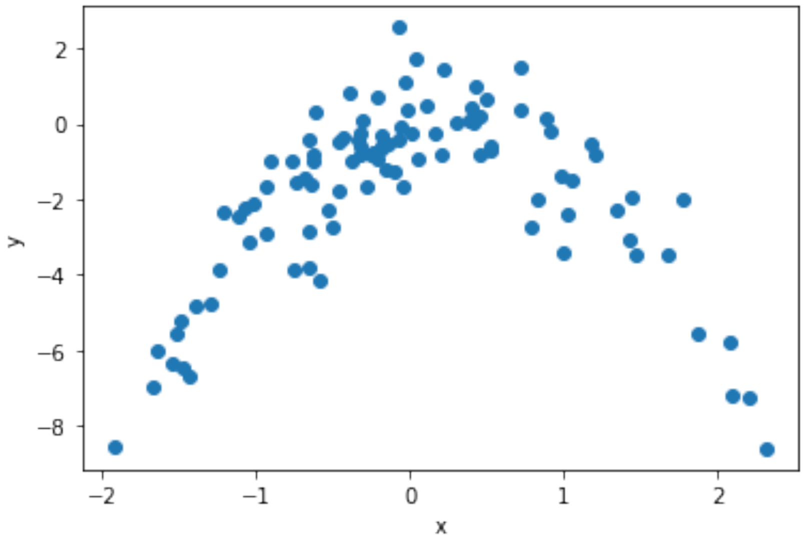
```
data = pd.read_csv("ch5_ex8.csv")
```

```
data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 100 entries, 0 to 99
Data columns (total 2 columns):
 #   Column  Non-Null Count  Dtype  
---  ------  --------------  -----  
 0   x       100 non-null    float64
 1   y       100 non-null    float64
dtypes: float64(2)
memory usage: 1.7 KB
```

Problem 9(b)

```
plt.scatter(data.x, data.y)
plt.xlabel("x")
plt.ylabel("y");
```



Problem 9(c)

```
np.random.seed(1)
```

```
from sklearn.preprocessing import PolynomialFeatures

X = np.array(data['x']).reshape(-1, 1)
y = np.array(data['y']).reshape(-1, 1)

for k in [1, 2, 3, 4]:

  loocv_errors = []

  for i in range(data.shape[0]):

    X_loo  = np.delete(X, i).reshape(-1, 1)
    y_loo  = np.delete(y, i).reshape(-1, 1)

    pf     = PolynomialFeatures(degree=k)
    Xp_loo = pf.fit_transform(X_loo)

    lin_reg = sm.OLS(y_loo, Xp_loo).fit()
    y_pred = lin_reg.predict(pf.transform(X[i].reshape(1,1)))
    loocv_error = (y_pred - y[i])**2

    loocv_errors.append(loocv_error)

  loocv_errors = np.array(loocv_errors)
  print("degree %d LOOCV error:" %k, "%+3.2e" %loocv_errors.mean())
```

```
degree 1 LOOCV error: +5.89e+00
degree 2 LOOCV error: +1.09e+00
degree 3 LOOCV error: +1.10e+00
degree 4 LOOCV error: +1.11e+00
```

Problem 9(d)

```
np.random.seed(2)
```

```
for k in [1, 2, 3, 4]:

  loocv_errors = []

  for i in range(data.shape[0]):

    X_loo  = np.delete(X, i).reshape(-1, 1)
    y_loo  = np.delete(y, i).reshape(-1, 1)

    pf     = PolynomialFeatures(degree=k)
    Xp_loo = pf.fit_transform(X_loo)

    lin_reg = sm.OLS(y_loo, Xp_loo).fit()
    y_pred = lin_reg.predict(pf.transform(X[i].reshape(1,1)))
    loocv_error = (y_pred - y[i])**2

    loocv_errors.append(loocv_error)

  loocv_errors = np.array(loocv_errors)
  print("degree %d LOOCV error:" %k, "%+3.2e" %loocv_errors.mean())
```

```
degree 1 LOOCV error: +5.89e+00
degree 2 LOOCV error: +1.09e+00
degree 3 LOOCV error: +1.10e+00
degree 4 LOOCV error: +1.11e+00
```

Problem 9(f)

```
X = np.array(data['x']).reshape(-1, 1)
y = np.array(data['y']).reshape(-1, 1)

for k in [1, 2, 3, 4]:

  pf = PolynomialFeatures(degree=k)
  Xp = pf.fit_transform(X)

  lin_reg = sm.OLS(y, Xp).fit()
  print(lin_reg.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.01
Model:                            OLS   Adj. R-squared:                 -0.00
Method:                 Least Squares   F-statistic:                    0.961
Date:                Fri, 16 Jul 2021   Prob (F-statistic):              0.32
Time:                        19:37:03   Log-Likelihood:                -226.8
No. Observations:                 100   AIC:                            457.
```

```
No. Observations:                 100   AIC:                             457.
Df Residuals:                      98   BIC:                             462.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975
------------------------------------------------------------------------------
const         -1.8185      0.236     -7.692      0.000      -2.288      -1.34
x1             0.2430      0.248      0.981      0.329      -0.249       0.73
==============================================================================
Omnibus:                       17.572   Durbin-Watson:                   2.19
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               20.49
Skew:                          -1.051   Prob(JB):                     3.55e-0
Kurtosis:                       3.704   Cond. No.                         1.0
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correc
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.81
Model:                            OLS   Adj. R-squared:                  0.80
Method:                 Least Squares   F-statistic:                     210.
Date:                Fri, 16 Jul 2021   Prob (F-statistic):           5.10e-3
Time:                        19:37:03   Log-Likelihood:                -143.5
No. Observations:                 100   AIC:                             293.
Df Residuals:                      97   BIC:                             300.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975
------------------------------------------------------------------------------
const         -0.0954      0.133     -0.715      0.476      -0.360       0.16
x1             0.8996      0.113      7.961      0.000       0.675       1.12
x2            -1.8666      0.092    -20.399      0.000      -2.048      -1.68
==============================================================================
Omnibus:                        1.794   Durbin-Watson:                   2.23
Prob(Omnibus):                  0.408   Jarque-Bera (JB):                1.22
Skew:                          -0.183   Prob(JB):                        0.54
Kurtosis:                       3.399   Cond. No.                         2.4
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correc
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.81
Model:                            OLS   Adj. R-squared:                  0.80
Method:                 Least Squares   F-statistic:                     139.
Date:                Fri, 16 Jul 2021   Prob (F-statistic):           8.04e-3
Time:                        19:37:03   Log-Likelihood:                -143.5
No. Observations:                 100   AIC:                             295.
Df Residuals:                      96   BIC:                             305.
```

Problem 10 (Chapter 5, Exercise 9)

```
boston = pd.read_csv("Boston.csv")
```

Problem 10(a)

```
mu_hat = boston['medv'].mean()
print(mu_hat)
```

```
22.532806324110698
```

Problem 10(b)

```
SE_mu_hat = boston['medv'].std()/np.sqrt(boston.shape[0])
print(SE_mu_hat)
```

```
0.4088611474975351
```

Problem 10(c)

Problem 10(d)

Problem 10(e)

Problem 10(f)

Problem 10(g)

Problem 10(h)