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A modified observer/Kalman filter identification (OKID) algorithm employing output residuals

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Abstract

The observer/Kalman filter identification (OKID) is an algorithm widely used for the identification of state space models. The standard OKID algorithm involves the estimation of the Kalman filter and system Markov parameters, followed by the realization of a state space model of the system using the eigensystem realization algorithm (ERA). In this paper, a modified and conceptually simple version of the OKID algorithm, termed the residual-based observer/Kalman filter identification (ROKID), is proposed. The ROKID algorithm uses ordinary least square method twice to solve two linear regression problems yielding the Kalman filter residuals and the system Markov parameters, respectively. Finally, the ERA algorithm is used to obtain a state space model of the system. The efficacy of the proposed algorithm is examined and compared with the standard OKID algorithm and the recently proposed OKID with deterministic projection (OKID/DP) algorithm via a simulation example. The results show that the proposed algorithm outperforms the standard OKID algorithm. Although its performance is less than that of the OKID/DP algorithm, due to its simplicity, the proposed algorithm represents a useful tool for linear state space model identification.

Keywords State-space models · Kalman filtering · Markov parameters · Linear systems · Closed-loop identification

1 Introduction

In the fields of estimation, filtering, and control system design, state space representation plays an important role beside the input–output-based transfer function modeling. This is a main motivation for the research interest in the identification of state space models from input output data. For this purpose, subspace methods [1,2] have been proven to be very effective. They are relying on robust linear algebra algorithms, noniterative, and suitable for multi-input multi-output (MIMO) systems without the need to special pre-parametrization. In addition to subspace methods, the observer/ Kalman identification (OKID) algorithm [3] is an algorithm commonly used for the identification of linear state space models. The OKID algorithm has been applied in many areas, e.g., iterative learning control [4], identification of aircraft [5], autonomous underwater vehicle [6], suspen-dome structures [7], dynamical behavior of polymers [8], and even non-engineering areas such as animal behavior identification [9]. One advantage of the OKID algorithm, compared to the standard subspace methods in the early literature, is its direct applicability to closed-loop data [10]. Closed-loop identification has its advantage, from economical and safety perspectives, without requiring open-loop identification tests.

In the first step of the standard OKID algorithm, the Markov parameters of the optimal observer (i.e., the Kalman filter) are estimated using an ordinary least squares problem. Second, the process Markov parameters are computed from the predictor Markov parameters. Finally, a state space model of the system is obtained using the eigensystem realization algorithm (ERA) [3]. Recently, Vicario et al. [11] proposed an algorithm termed OKID with deterministic projection (OKID/DP) in which the first step of the standard

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OKID algorithm is modified such that it provides, using an ordinary least square method, an estimate of the residuals instead of the predictor Markov parameters. In the second step, the input and estimated residuals, along with the Kalman filter predicted output, are fed to an existing deterministic subspace algorithm [1] to obtain the state space model matrices. A similar approach of using Kalman filter residuals for system identification of Box–Jenkins models is also, recently, proposed by Doraiswami and Cheded [12].

In this paper, the approach proposed by Vicario et al. [11] is used to formulate a modified version of the OKID algorithm. The first step of the OKID/DP algorithm is adopted in the proposed approach. In the second step, however, the input and estimated residuals, along with the Kalman filter predicted output data, are used to obtain system Markov parameters through another least square problem. Finally, a state space model is realized using the estimated system Markov parameters using ERA algorithm [13]. By reviewing two OKID-based algorithms—the standard OKID and the OKID/DP algorithms, this paper proposes a conceptually simple ROKID algorithm consisting of two linear regressions and one realization step.

The rest of the paper is organized as follows. In Sect. 2, innovation and predictor state space representations are given. The standard OKID and OKID/DP algorithms are reviewed in Sect. 3. In Sect. 4, the proposed ROKID algorithm is presented. Simulation results are provided to illustrate the performance of the proposed method in Sect. 5. Finally some conclusions are drawn in Sect. 6.

2 Innovation and predictor state space representations

A state space model with process and measurement noise can be represented in the following so-called innovation form [2]:

$$x(k+1) = Ax(k) + Bu(k) + Ke(k), \tag{1}$$
$$y(k) = Cx(k) + Du(k) + e(k), \tag{2}$$

where $x(k) \in \mathcal{R}^n$ is the state, $u(k) \in \mathcal{R}^m$ is the input, $y(k) \in \mathcal{R}^l$ is the output, and $e(k) \in \mathcal{R}^l$ is a zero-mean Gaussian white noise called the innovation. The matrices A, B, C, D are the state space model matrices of appropriate dimensions and $K \in \mathcal{R}^{n\times l}$ is the Kalman gain.

The process Markov parameters corresponding to the model (1) and (2) are given as:

$$g_i = \begin{cases} D, & i = 0, \\ CA^{i-1}B, & i \geq 1. \end{cases} \tag{3}$$

$$h_i = \begin{cases} I_{l\times l}, & i = 0, \\ CA^{i-1}K, & i \geq 1. \end{cases} \tag{4}$$

where $I_{l\times l}$ is the identity matrix of size $l \times l$.

For the innovation state space model, the system identification problem can be stated as follows: *Given a set of N input–output data, $u(k)$, $y(k)$, $k = 0, 1, 2, \ldots, N-1$, find the state space matrices A, B, C, D up to a similarity transformation, in addition to the Kalman gain, K.*

In the literature, the innovation model (1) and (2) is usually transformed into the so-called predictor form, which is more useful, specially for lightly-damped systems where the poles of the system are close to the unit circle [3]. For this purpose, the innovation term $e(k)$ from (2) is substituted into (1) to give:

$$\hat{x}(k+1) = \bar{A}\hat{x}(k) + \bar{B}u(k) + Ky(k), \tag{5}$$
$$\hat{y}(k) = C\hat{x}(k) + Du(k), \tag{6}$$

where $\bar{A} = A - KC$ and $\bar{B} = B - KD$. The objective of the predictor is to provide a one-step ahead predicted estimate of the system state $\hat{x}(k)$ and output $\hat{y}(k)$ of the original system (1) and (2). Similarly, the predictor Markov parameters of the model (5) and (6), from predictor inputs, $u(k)$ and $y(k)$, to predictor output, $\hat{y}(k)$, are given as:

$$\bar{g}_i = \begin{cases} D, & i = 0, \\ C\bar{A}^{i-1}\bar{B}, & i \geq 1. \end{cases} \tag{7}$$

$$\bar{h}_i = \begin{cases} 0_{l\times m}, & i = 0, \\ C\bar{A}^{i-1}K, & i \geq 1, \end{cases} \tag{8}$$

where $0_{l\times m}$ is a zero matrix of size $l \times m$. The process and predictor Markov parameters are related as follows [14]:

$$g_i = \begin{cases} \bar{g}_0, & i = 0, \\ \bar{g}_i + \sum_{j=1}^{i} \bar{h}_j g_{i-j} & i \geq 1. \end{cases} \tag{9}$$

$$h_i = \begin{cases} \bar{h}_1, & i = 1, \\ \bar{h}_i + \sum_{j=1}^{i-1} \bar{h}_j h_{i-j} & i \geq 2. \end{cases} \tag{10}$$

The above relationships hold under the assumption that the underlying Kalman filter has reached the steady-state so that the Kalman gain becomes constant [14].

3 The OKID algorithms

In this section, the standard OKID algorithm as well as the idea of converting the stochastic identification problem into deterministic leading to the OKID/DP algorithm are briefly reviewed.

3.1 Standard OKID algorithm

The problem of identifying state space models is so challenging because while input and output data are known, the state sequence is unknown. In order to address this problem, the standard OKID algorithm employs the predictor model (5). Using this model, the state $\hat{x}(k)$ can be expressed as a function of input and output sequences of length p into the past in addition to the initial state $\hat{x}(k-p)$ as follows:

$$\hat{x}(k) = \bar{A}^p\hat{x}(k-p) + \left[\,\bar{B}\ \bar{A}\bar{B}\ \dots\ \bar{A}^{p-1}\bar{B}\,\right]\begin{bmatrix} u(k-1)\\ u(k-2)\\ \vdots\\ u(k-p)\end{bmatrix} + \left[\,K\ \bar{A}K\ \dots\ \bar{A}^{p-1}K\,\right]\begin{bmatrix} y(k-1)\\ y(k-2)\\ \vdots\\ y(k-p)\end{bmatrix}. \tag{11}$$

The OKID parameter p, the length of the past horizon, plays an important role in the algorithm. It is usually chosen to be much larger than the system order, e.g., 20 times larger. Due to stability of optimal observers, it can be assumed that $\bar{A}^p \approx 0$, which means that the effect of initial state $\hat{x}(k-p)$ can be neglected. It also ensures that the underlying Kalman filter reaches steady-state so that (9) and (10) hold true. Substituting (11) into (6), neglecting the initial state term $\bar{A}^p\hat{x}(k-p)$, and noting that

$$y(k) = \hat{y}(k) + e(k), \tag{12}$$

give

$$y(k) = \left[\,D\ C\bar{B}\ C\bar{A}\bar{B}\ \dots\ C\bar{A}^{p-1}\bar{B}\,\right]\begin{bmatrix} u(k)\\ u(k-1)\\ u(k-2)\\ \vdots\\ u(k-p)\end{bmatrix} + \left[\,CK\ C\bar{A}K\ \dots\ C\bar{A}^{p-1}K\,\right]\begin{bmatrix} y(k-1)\\ y(k-2)\\ \vdots\\ y(k-p)\end{bmatrix} + e(k). \tag{13}$$

From (13), the OKID core equation can be written as [11]:

$$Y_1 = \Phi V + E, \tag{14}$$

where

$$Y_1 = \left[\,y(p)\ y(p+1)\ \dots\ y(N-1)\,\right], \tag{15}$$

$$V = \left[\begin{array}{cccc} u(p) & u(p+1) & \dots & u(N-1)\\ u(p-1) & u(p) & \dots & u(N-2)\\ \vdots & \vdots & \vdots & \vdots\\ u(0) & u(1) & \dots & u(N-p-1)\\ \hline y(p-1) & y(p) & \dots & y(N-2)\\ y(p-2) & y(p-1) & \dots & y(N-3)\\ \vdots & \vdots & \vdots & \vdots\\ y(0) & y(1) & \dots & y(N-p-1) \end{array}\right], \tag{16}$$

$$E = \left[\,e(p)\ e(p+1)\ \dots\ e(N-1)\,\right], \tag{17}$$

$$\Phi = \left[\,D\ C\bar{B}\ C\bar{A}\bar{B}\ \dots\ C\bar{A}^{p-1}\bar{B}\ \mid\ CK\ C\bar{A}K\ \dots\ C\bar{A}^{p-1}K\,\right]. \tag{18}$$

It was proven that the least square solution of (14) corresponds to the optimal observer, i.e., the Kalman filter which minimizes the expected value of the Euclidean norm squared of the state estimation error, $E[(x(k)-\hat{x}(k))^T(x(k)-\hat{x}(k))]$, at each time step [11].

The least square solution of (14) gives an estimate of the observer or predictor Markov parameters $\hat{\Phi}$, cf. (7) and (8). The standard OKID algorithm continues by using (9) and (10) to calculate process Markov parameters from the observer Markov parameters. The ERA realization algorithm in Sect. 4.1 can then be used to obtain state space model matrices of the innovation form (1) and (2).

3.2 Conversion to a deterministic problem

In a recent contribution, Vicario et al. [11] proposed to estimate the residual (innovation) terms $\hat{e}(p)$, $\hat{e}(p+1), \dots, \hat{e}(N-1)$ as defined in (17) and then feed them to the innovation model (1) and (2), leading to the following state space model:

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K\hat{e}(k), \tag{19}$$

$$\hat{y}(k) = C\hat{x}(k) + Du(k). \tag{20}$$

This model has two inputs, $u(k)$ and $\hat{e}(k)$, and one output, $\hat{y}(k)$. As the inputs are known, the problem becomes fully deterministic and can be solved with any existing deterministic subspace algorithm such as the deterministic projection (DP) algorithm [1]. This is the main idea behind the algorithm OKID/DP developed by Vicario et al. [11].

The same approach of using the estimated residual sequence as defined in (17) will be employed in this paper to develop a modified version of the standard OKID algorithm as illustrated in the next section.

4 The proposed residual-based OKID (ROKID) algorithm

In this paper, it is proposed to write the following OKID-like core equation based on the deterministic state space model (19) and (20):

$$Y_2 = \Psi W + \epsilon, \tag{21}$$

where

$$Y_2 = \left[\, \hat{y}(p+N_s)\ \hat{y}(p+N_s+1) \ldots \hat{y}(N-1) \,\right], \tag{22}$$

$$W = \left[\begin{array}{cccc} u(p+N_s) & u(p+N_s+1) & \ldots & u(N-1) \\ u(p+N_s-1) & u(p+N_s) & \ldots & u(N-2) \\ \vdots & \vdots & \vdots & \vdots \\ u(p) & u(p+1) & \ldots & u(N-N_s-1) \\ \hline \hat{e}(p+N_s-1) & \hat{e}(p+N_s) & \ldots & \hat{e}(N-2) \\ \hat{e}(p+N_s-2) & \hat{e}(p+N_s-1) & \ldots & \hat{e}(N-3) \\ \vdots & \vdots & \vdots & \vdots \\ \hat{e}(p) & \hat{e}(p+1) & \ldots & \hat{e}(N-N_s-1) \end{array}\right], \tag{23}$$

$$\epsilon = \left[\, \epsilon(p+N_s)\ \epsilon(p+N_s+1) \ldots \epsilon(N-1) \,\right], \tag{24}$$

$$\Psi = [\underbrace{D\ CB\ CAB\ \ldots\ CA^{N_s-1}B}_{\Psi_1 \in \mathcal{R}^{l\times m(N_s+1)}} \mid \underbrace{CK\ CAK\ \ldots\ CA^{N_s-1}K}_{\Psi_2 \in \mathcal{R}^{l\times N_s l}}]. \tag{25}$$

In the above equations, Ψ_1 is the system Markov parameters, and ϵ represents the approximation error of the linear regression (21). Equation (21) can be solved using ordinary least square method to yield Ψ_1, which consists of the first N_s+1 process Markov parameters, g_i, cf. (3). In order to obtain better results from the realization algorithm at the final step of the OKID algorithm, it is desirable that N_s be larger than p. Also note from (17) that the available estimate of the innovation sequence starts from $\hat{e}(p)$. Therefore, in (22), the output sequence, Y_2, has to start from $\hat{y}(p+N_s)$.

4.1 The eigensystem realization algorithm

After the estimation of the Markov parameters, g_i, a realization step is required to find state space model matrices. This can be done using e.g. the eigensystem realization algorithm [13] or Kung's algorithm [15]. The standard OKID algorithm [3] employs the ERA realization in which the system Markov parameters are used to form the following Hankel matrix, of size $il \times im$:

$$H_{i,i} = \begin{bmatrix} g_1 & g_2 & \ldots & g_i \\ g_2 & g_3 & \ldots & g_{i+1} \\ \vdots & \vdots & \ddots & \vdots \\ g_i & g_{i+1} & \ldots & g_{2i-1} \end{bmatrix} = \begin{bmatrix} CB & CAB & \ldots & CA^{i-1}B \\ CAB & CA^2B & \ldots & CA^iB \\ \vdots & \vdots & \vdots & \\ CA^{i-1}B & CA^iB & \ldots & CA^{2i-2}B \end{bmatrix}, \tag{26}$$

where i must be greater than or equal to the system order, n. In this work, if N_s is even, $i = N_s/2$, otherwise, $i = (N_s+1)/2$.

Now, define the matrices H_0 as $H_{i,i}$ excluding the last m columns and H_1 as $H_{i,i}$ excluding the first m columns. Using Matlab notation, $H_0 = H_{i,i}(:, 1 : end - m)$ and $H_1 = H_{i,i}(:, m+1 : end)$. It possible to show that the matrix H_0 can be factored as:

$$H_0 = \Gamma\Delta = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{i-1} \end{bmatrix} \left[\, B\ AB \ldots A^{i-2}B \,\right], \tag{27}$$

where $\Gamma \in \mathcal{R}^{il\times n}$ is the extended observability matrix and $\Delta \in \mathcal{R}^{n\times m(i-1)}$ is the extended controllability matrix. The decomposition in (27) is performed using the following singular value decomposition (SVD):

$$H_0 = \left[\, U_1\ U_2 \,\right] \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \approx \underbrace{U_1 S_1^{1/2}}_{\Gamma}\, \underbrace{S_1^{1/2} V_1^T}_{\Delta} \tag{28}$$

where U_1 and U_2 are orthogonal matrices whose columns are the left singular vectors, V_1 and V_2 are orthogonal matrices whose columns are the right singular vectors, $S_1 \in \mathcal{R}^{n\times n}$ is a diagonal matrix whose n diagonal elements denote the significant singular values, and S_2 is ideally zero matrix. With an estimate of Γ and Δ, the matrix C can be read directly as the first l rows in Γ and B as the first m columns in Δ. Furthermore, by realizing that $H_1 = \Gamma A \Delta$, the matrix A can be estimated as:

$$A = S_1^{-1/2} U_1^T H_1 V_1 S_1^{-1/2}. \tag{29}$$

Noting that the feed-through matrix $D = g_0$, all state space model matrices are thus obtained.

If an estimate of the Kalman gain K is required, it can be obtained in the same way as the matrix B. In this case, however, the realization algorithm is applied to a Hankel matrix formed from the Markov parameters h_i (corresponding to Ψ_2 in (25)) instead of g_i contained in Ψ_1.

4.2 Proposed ROKID algorithm summary

The proposed algorithm can be summarized as follows:

1. Collect N input–output data.

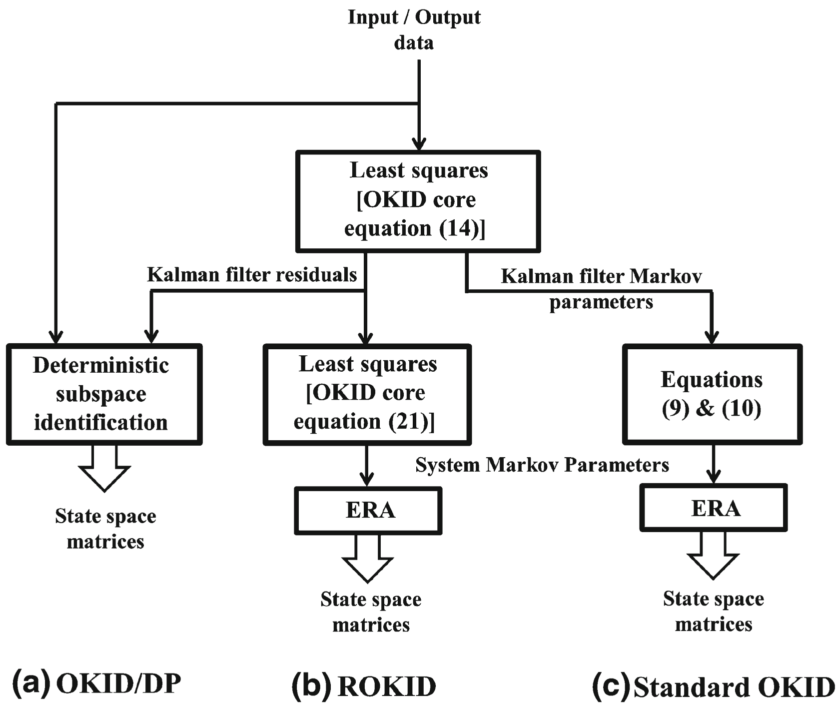

Fig. 1 A flowchart of the three OKID-based algorithms: **a** OKID/DP, **b** proposed ROKID, and **c** standard OKID

2. Form the matrices Y_1, V as defined in (15) and (16).
3. Solve the ordinary least square problem (14) for the residual sequence E defined in (17).
4. Form the matrices Y_2, W as defined in (22) and (23), respectively.
5. Solve the ordinary least square problem (21) for $\Psi_1 \in \mathcal{R}^{l\times m(N_s+1)}$, defined in (25), which corresponds to the first $N_s + 1$ process Markov parameters.
6. Apply the eigensystem realization algorithm (ERA) in Sect. 4.1 to Ψ_1 to obtain the state space model matrices, A, B, C, D, and to Ψ_2 to obtain the Kalman gain matrix K.

4.3 Comparison of OKID, ROKID, and OKID/DP algorithms

Comparisons of the OKID, ROKID, and OKID/DP algorithms are illustrated using the flowchart shown in Fig. 1. All algorithms start from the OKID core equation (14). The equation is solved using ordinary least squares and yields an estimate for the Kalman filter Markov parameters in addition to filter residuals. The standard OKID algorithm uses the estimated Markov parameters to extract the system Markov parameters. These are then realized into a state space model using the ERA algorithm. The ROKID and OKID/DP algorithms, on the other hand, use the residual sequence. The ROKID solves another OKID core equation (21) using the ordinary least square method to obtain the system Markov parameters which is realized into a state space model with the ERA algorithm. The OKID/DP [11] differs from the above algorithms in that it employs one of the deterministic subspace identification available in the literature [1] to obtain the state space model matrices.

The simplicity of the proposed ROKID algorithm lies in using the ordinary least square method in two steps followed by an ERA realization algorithm. From an algorithmic point of view, this is simpler than the standard OKID algorithm which depends on the relationship between Kalman filter and system Markov parameters (Eqs. 9, 10). It is much simpler in comparison with OKID/DP which uses deterministic subspace identification employing the following operations: matrix projections, singular value decomposition (SVD), and ordinary least square method. Therefore, the proposed ROKID provides a conceptually simple approach for the identification of linear state space models using only linear regressions and state space realization.

5 Simulation results

In order to evaluate the performance of the standard OKID [3], the proposed ROKID, and the OKID/DP [11] algorithms, the following simulation example is considered. The process to be identified is a second-order system having the following transfer function:

$$P(z) = \frac{z}{z^2 - 1.6z + 0.89}.$$

The process operates under closed-loop control as shown in Fig. 2 [16,17], where the controller and disturbance transfer functions are given, respectively, as:

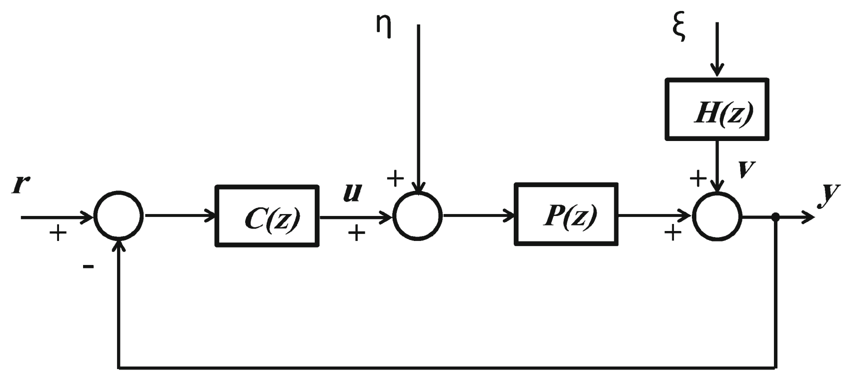

Fig. 2 Closed-loop system for the simulation example

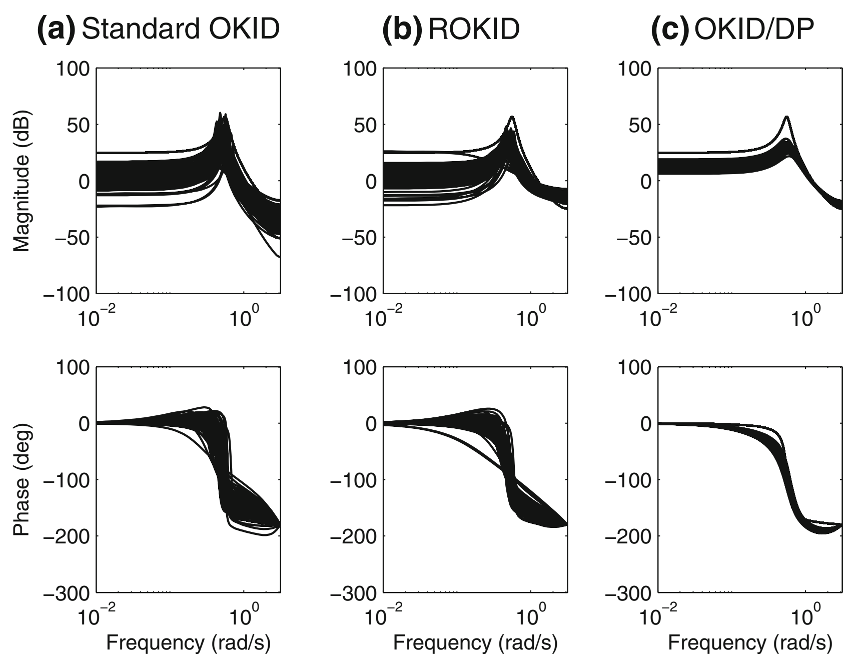

Fig. 3 Bode plot for one hundred Monte Carlo simulations using **a** standard OKID, **b** proposed ROKID, and **c** OKID/DP algorithms

$$C(z) = \frac{z - 0.8}{z}, \quad H(z) = \frac{z^3 - 1.56z^2 + 1.045z - 0.3338}{z^3 - 2.35z^2 + 2.09z - 0.6675}.$$

The signals η and ξ are both zero-mean Gaussian white noises with variances of 0.2 and 1/9, respectively. The reference signal is a sum of randomly generated sinusoids $r(t) = \rho \sum_{j=1}^{30} A_j \sin(\omega_j t + \phi_j), t = 0, 1, \ldots, N-1$, where A_j are Gaussian random numbers with $N(0, 1)$, and ω_j and ϕ_j are uniformly distributed over $(0, \pi)$. ρ is a factor to normalize the variance of reference to unity. To account for measurement noise, a zero-mean Gaussian white noise, e, of variance 0.1 is added to the output y. This corresponds to a signal to noise ratio (SNR) $= 10\times \log(var(y)/var(e)) \approx$ 12dB.

In the simulation tests, one hundred Monte Carlo Simulations are performed. In each realization, the number of data points used are 1000, and different realization is used for the reference, r, and noises, η and ξ. In all algorithms, the number of predictor Markov parameters, $p = 100$, while the number of system Markov parameters, $N_s = 200$.

The bode plots and eigenvalues of the 100 identified models are shown in Figs. 3 and 4, respectively. It can be seen that the OKID/DP is superior in terms of the spread of both frequency response and eigenvalues. On the other hand, the proposed ROKID slightly outperforms the standard OKID algorithm.

The average top ten singular values of the Hankel matrix of Markov parameters estimated over the 100 simulations are shown in Fig. 5 with the top four singular values given in Table 1. Recalling that the true system is second order, it is clear that the gap between the second and third singular values is the largest using OKID/DP algorithm among the three algorithms . Again, the proposed ROKID algorithm is slightly better than the standard OKID algorithm. This implies that the determination of system order is clearer using ROKID compared with OKID algorithm.

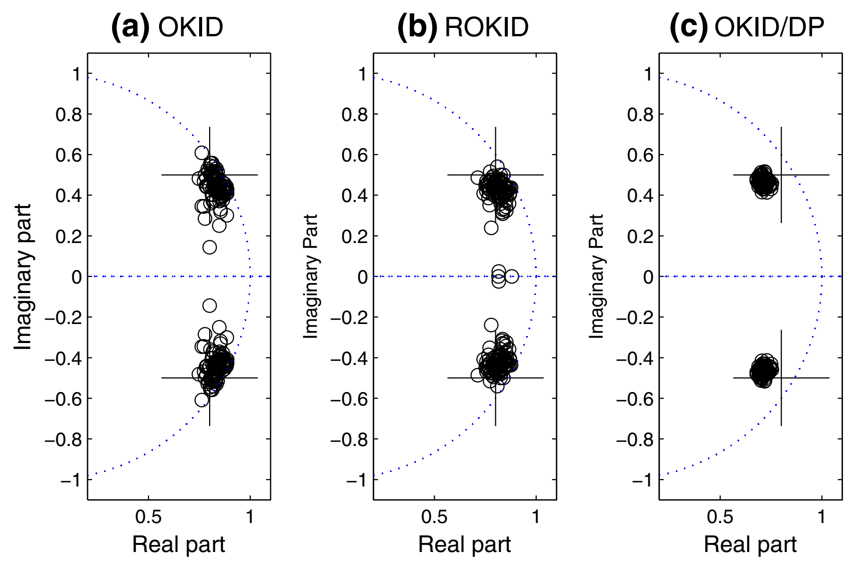

Fig. 4 Eigenvalues of identified models in one hundred Monte Carlo simulations using **a** standard OKID, **b** proposed ROKID, and **c** OKID/DP algorithms. The true eigenvalues are marked with (+)

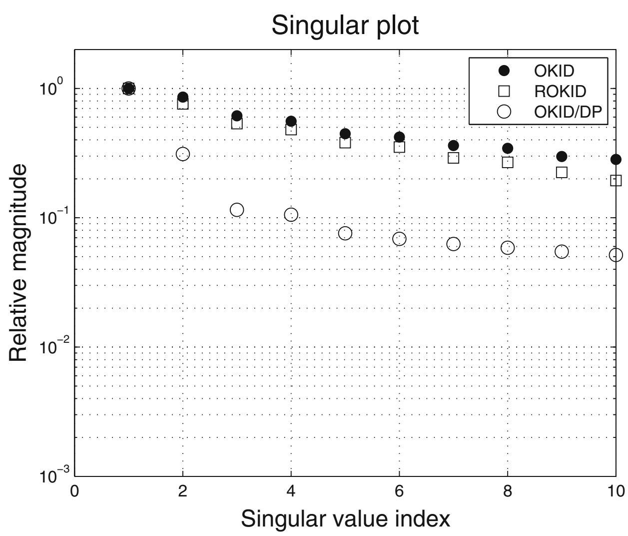

Fig. 5 The first ten singular values averaged over one hundred Monte Carlo Simulation using standard OKID, proposed ROKID, and OKID/DP algorithms

Table 1 The first four singular values averaged over 100 Monte Carlo simulations for the three algorithms: OKID, ROKID, and OKID/DP, as well as the ratio between the second and third singular values

Singular value	OKID	ROKID	OKID/DP
σ_1	1.0000	1.0000	1.0000
σ_2	0.8540	0.7686	0.3438
σ_3	0.6347	0.5461	0.1147
σ_4	0.5812	0.4851	0.1026
σ_2/σ_3	1.3455	1.4074	2.9087

It is also interesting to test the whiteness of the residual sequences obtained by the three algorithms. For this purpose, the normalized autocorrelation function (ACF) of the residual sequences is plotted in Fig. 6. According to the figure, all methods have non-zero ACF values for non-zero lags indicating that they are not white. This is expected because the dataset used for identification is of finite size. To deal with this problem, it is worth mentioning here that Phan et al. [18] have recently proposed to use a second Kalman filter to whiten the residuals from the first Kalman filter.

6 Conclusions

In this paper, an algorithm termed ROKID, is proposed for the identification of state space models. The algorithm consists of solving two ordinary least square problems. In the first one, the output residuals from the system Kalman filter are obtained. In the second, an estimate of system Markov parameters is found. Finally, the ERA realization algorithm is employed to realize a state space model of the system. Although the algorithm efficacy is less than the recently proposed algotihm OKID/DP [11] which combines both OKID and deterministic subspace identification, the ROKID algorithm is more simpler and moreover, shows slightly higher accuracy compared to the standard OKID algorithm [3].

It should be emphasised that the proposed ROKID algorithm is applicable only for the identification of linear systems with both deterministic and stochastic inputs. As a future work, the proposed algorithm can be extended to the identification of purely stochastic systems [19], bilinear systems [20], or unstable systems along the lines of [21].

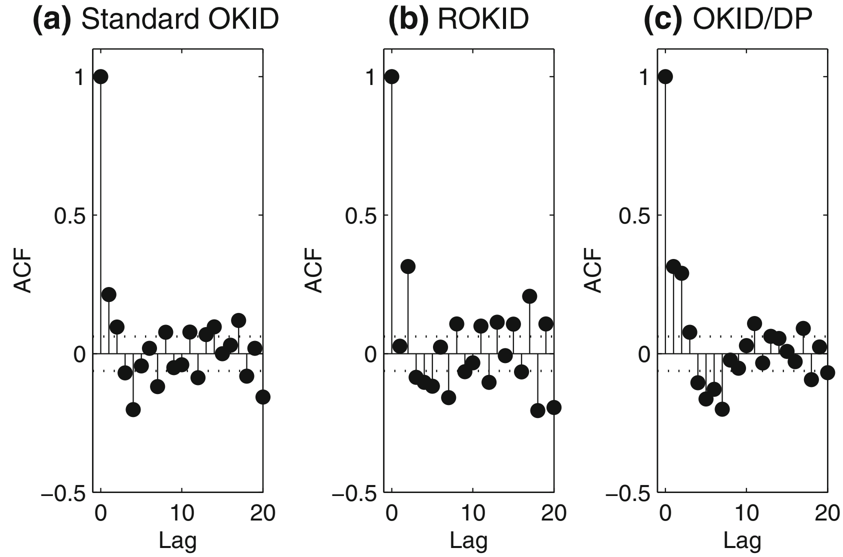

Fig. 6 Normalized autocorrelation function (ACF), averaged over 100 Monte Carlo simulations, of the Kalman filter residuals obtained using **a** standard OKID, **b** proposed ROKID, and **c** OKID/DP algorithms. The dotted lines correspond to the 95% confidence interval

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