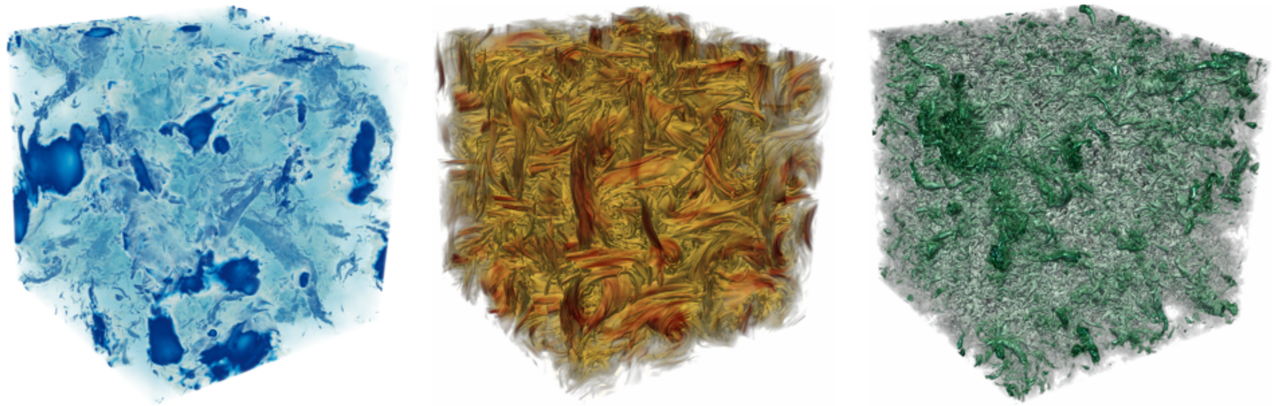
# Tensor Decompositions for Integral Histogram Compression and Look-up

Rafael Ballester-Ripoll & Renato Pajarola

IEEE VIS '18 (TVCG paper)

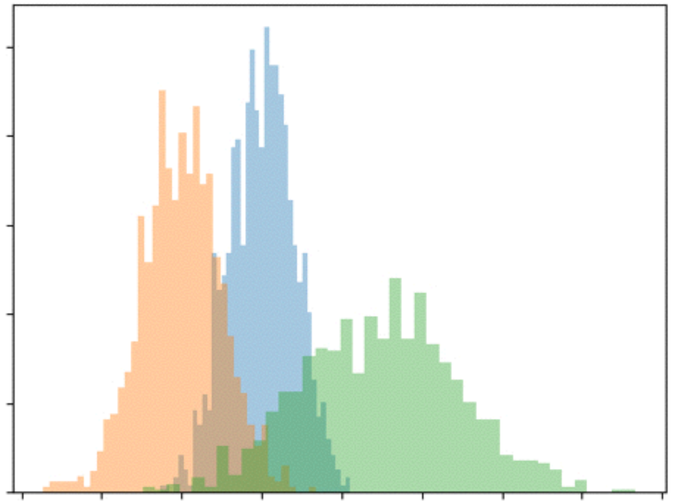
25th October 2018



University of Zurich[UZH]

VISUALIZATION AND MULTIMEDIA LAB

# Histograms



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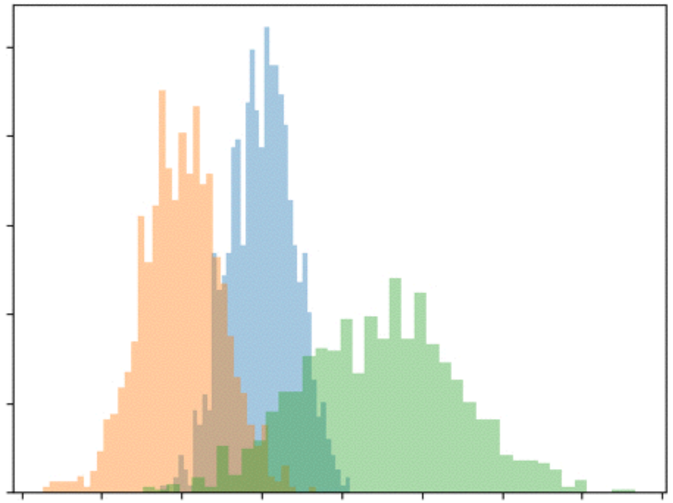

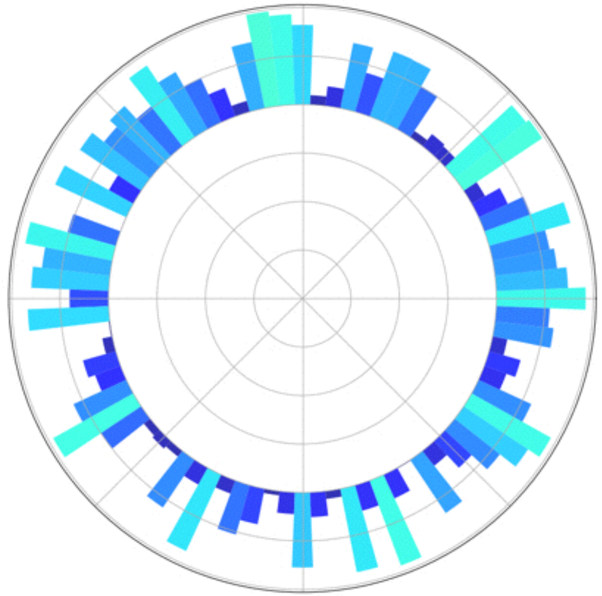

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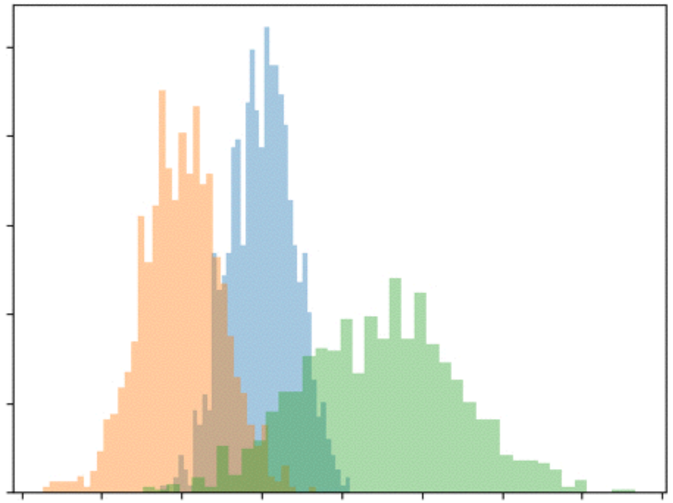
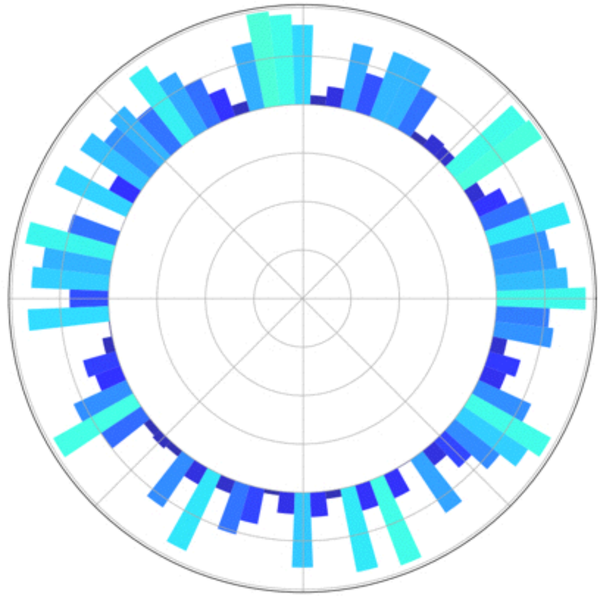
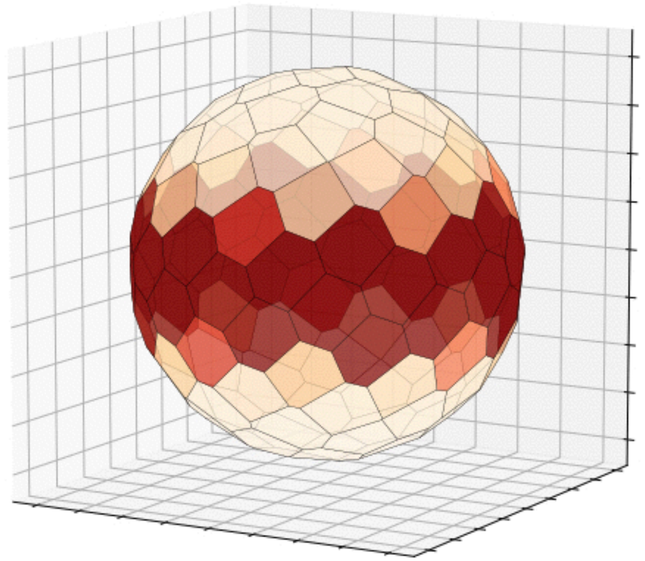
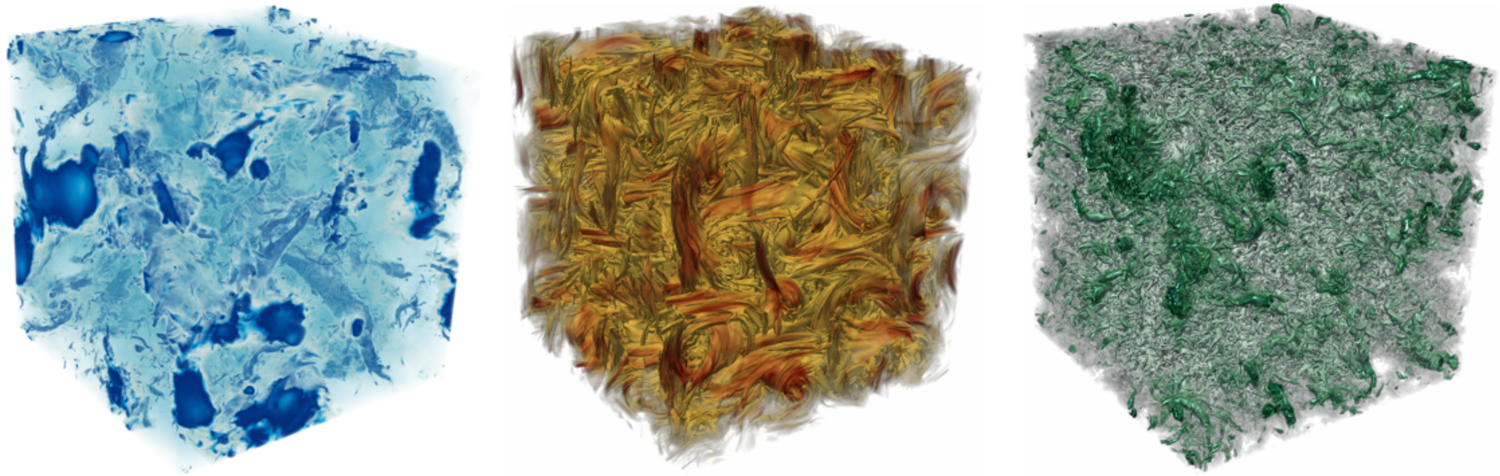






- Image editing
- Signal processing
- Feature extraction
- Computer vision

# Large-scale Data Sets



- Growing **resolution:**
  - More computing power
  - Better acquisition technology
- Multiple **dimensions**
- We focus on **regular grid data**

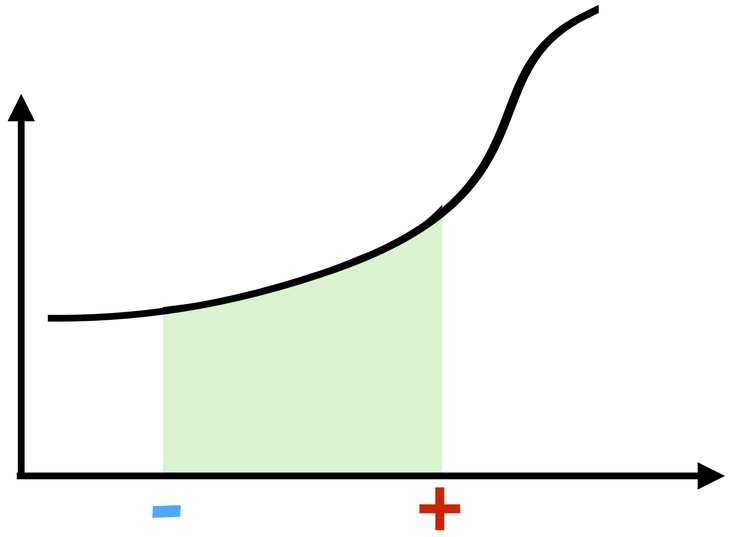
# Brute-force Histograms

- CT scan: $1024^3$ = 1 billion voxels
- Computing a **histogram over some region**: up to a billion memory accesses
- Data set fits in RAM/GPU…
  - … but many accesses are **non-sequential**
  - In this example: up to 1 million
- Typical times: 1-50ms
- What if we need **many histograms**?
  - E.g. over 1K regions ⟶ **no longer interactive**

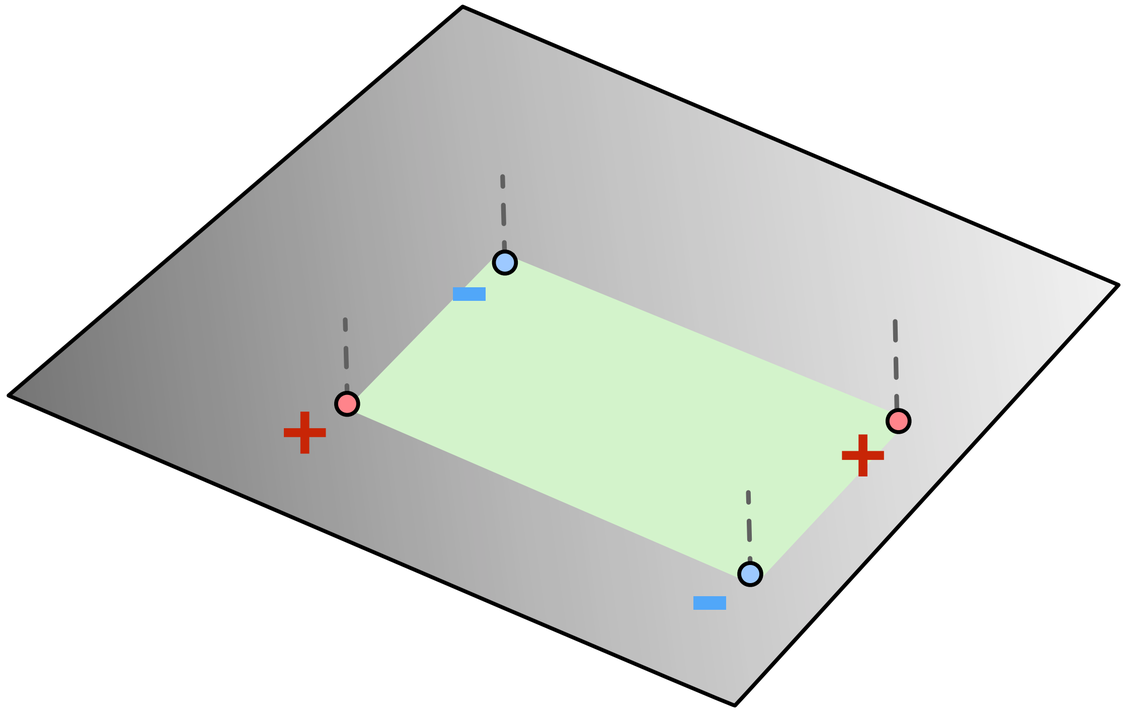
# Integral Histogram

- Data structure to **speed-up** histogram computation [Porikli '05]
  - E.g. million memory accesses ⟶ few hundreds
- Based on summed area tables (SAT)
  - One per bin $b = 1, \ldots, B$

# Summed Area Tables

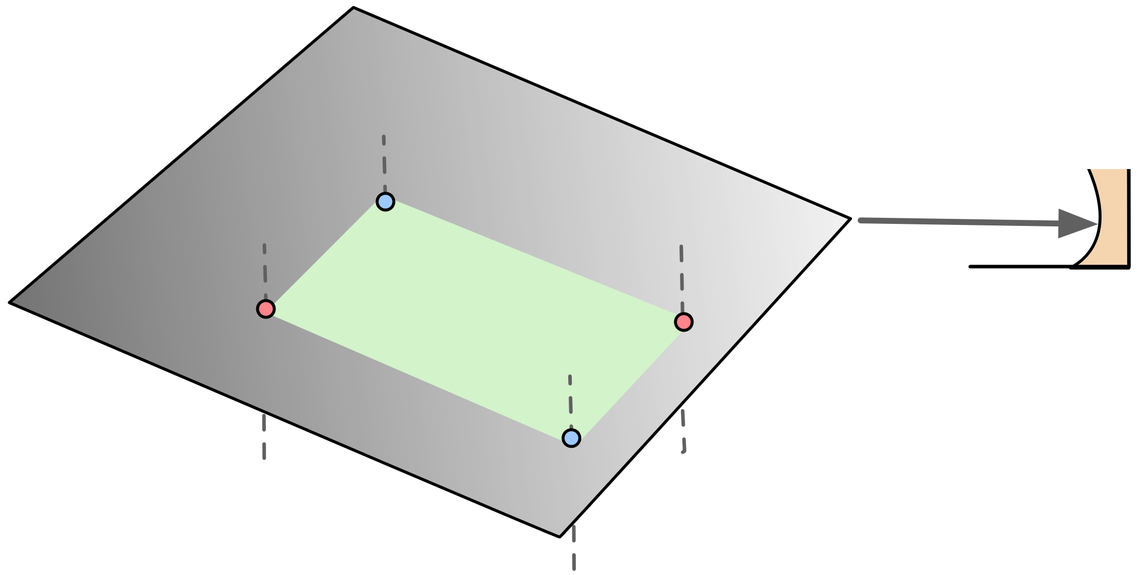



1D



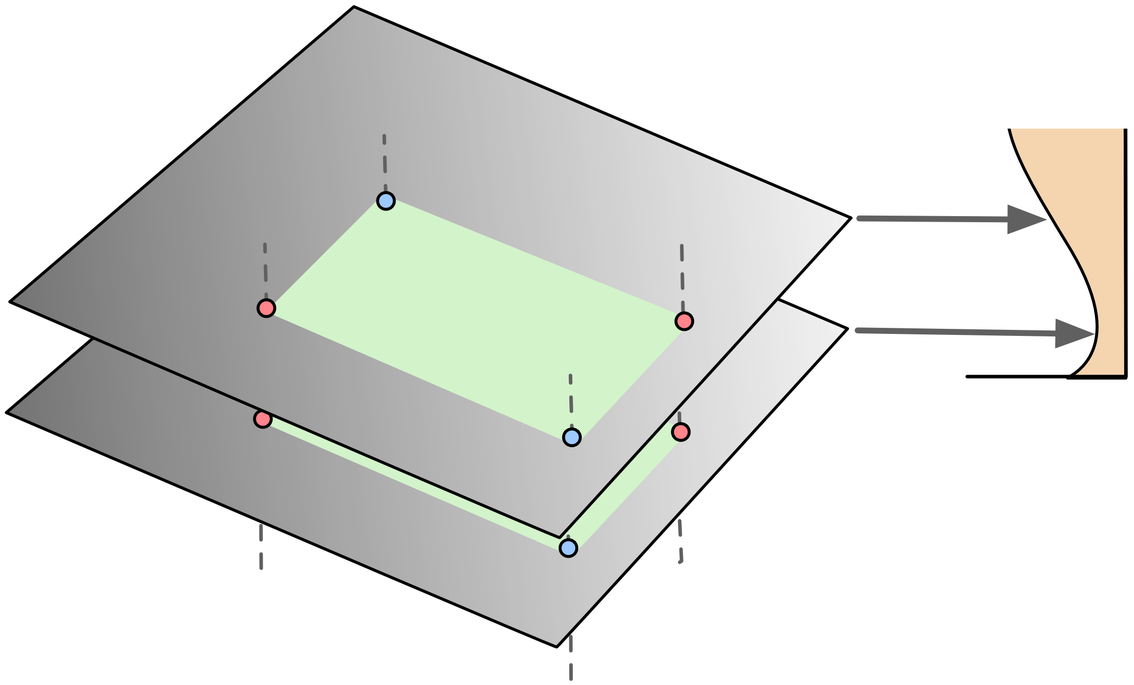

2D

# Integral Histogram



[Porikli ’05]

# Integral Histogram



[Porikli ’05]

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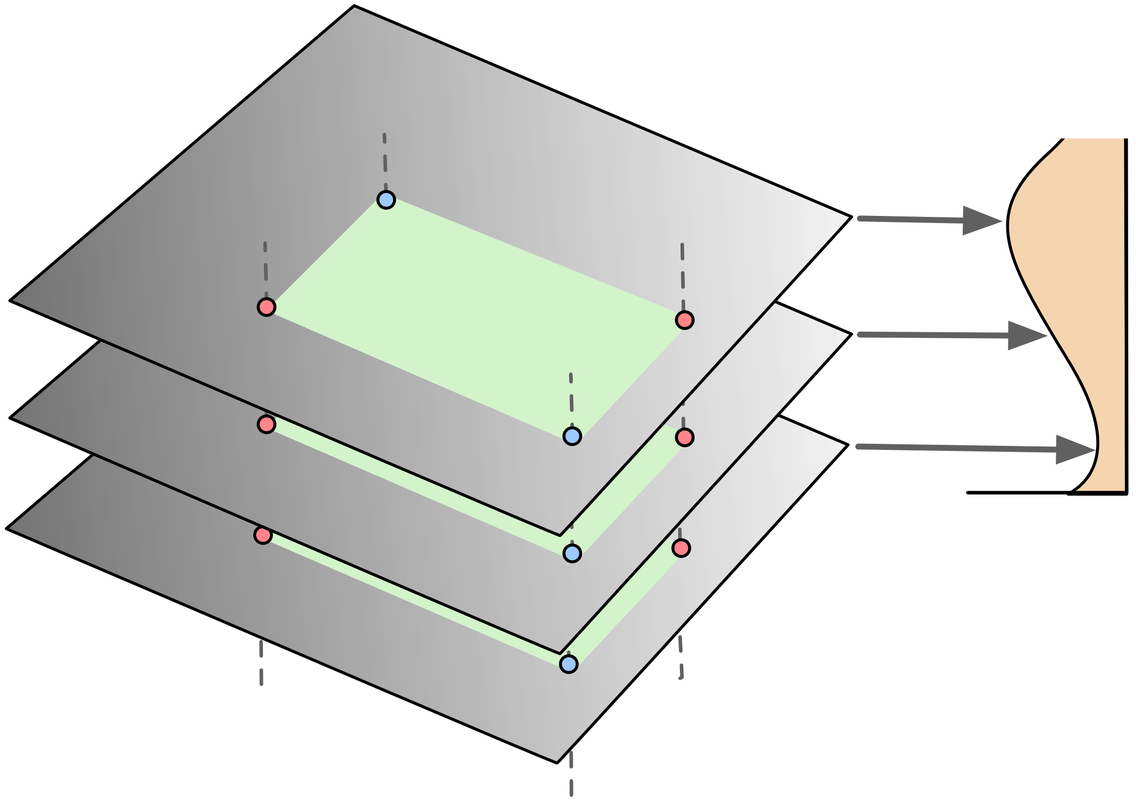

[Porikli ’05]

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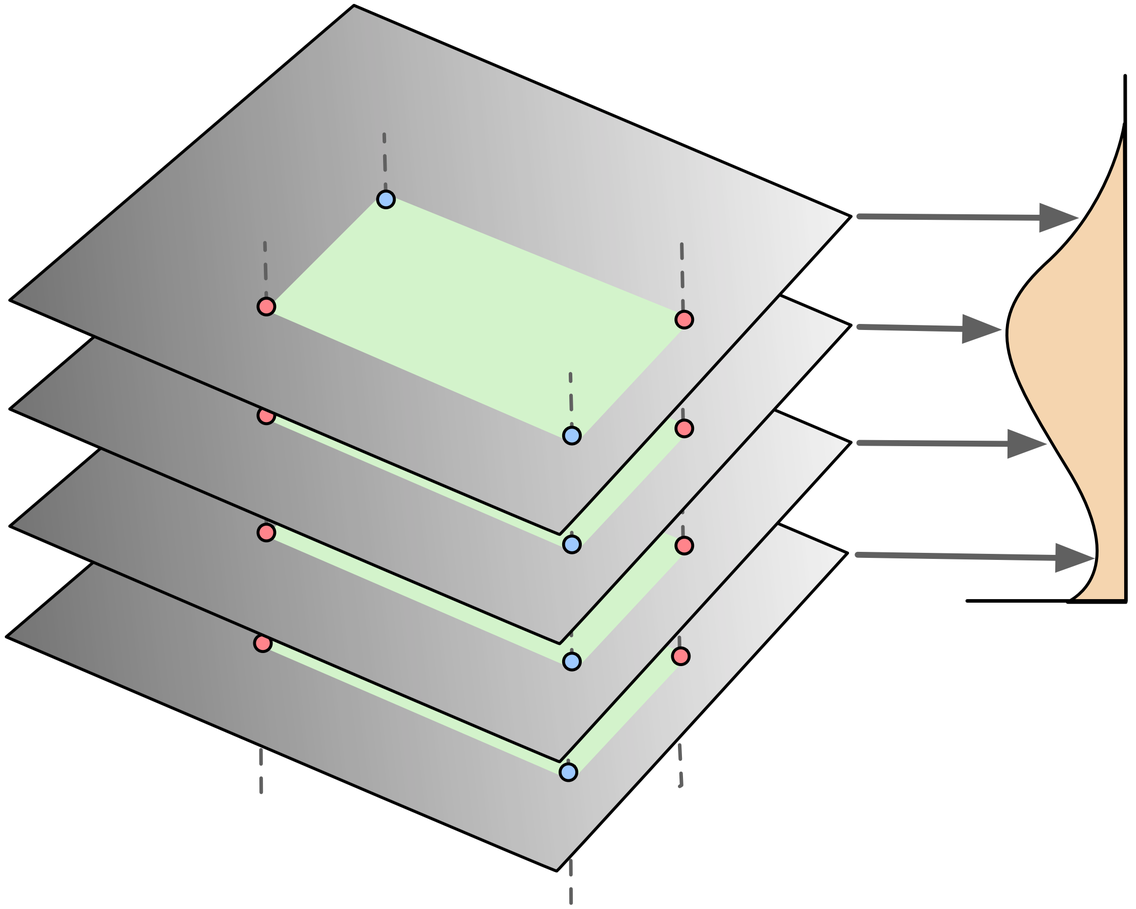

[Porikli ’05]

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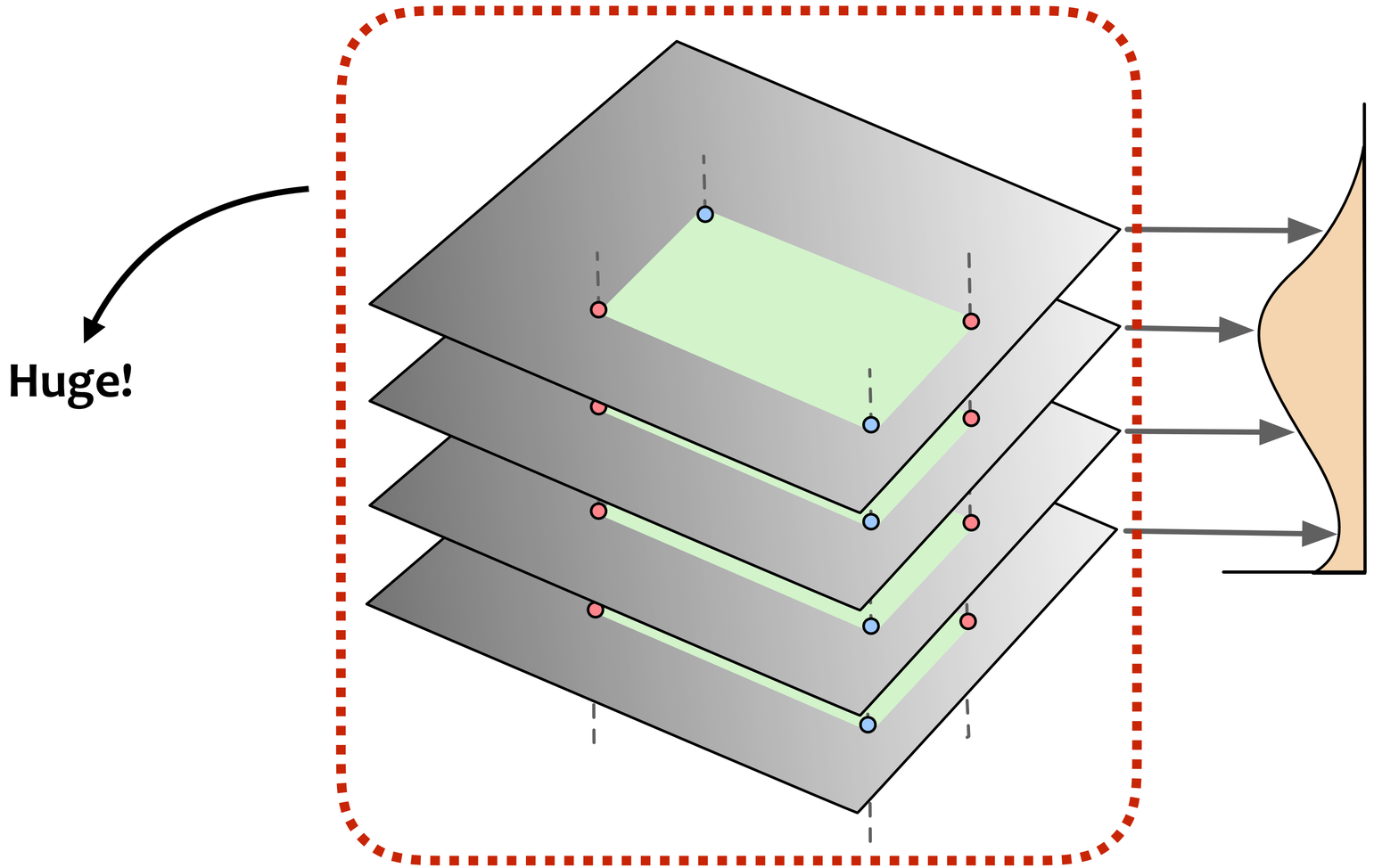
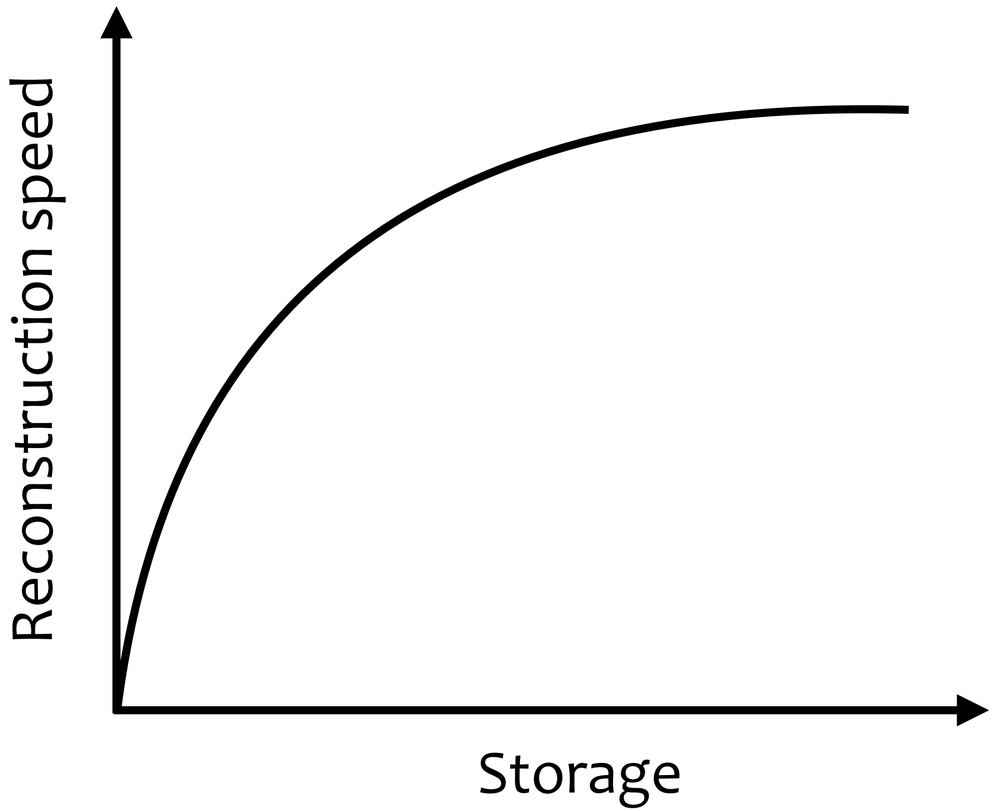



[Porikli '05]

# Integral Histogram: Drawbacks

- Trade-off: higher speed, but **gigantic storage cost**
  - E.g. 1GB and 128 bins ⟶ 512GB
  - Won't fit in GPU or even main memory
  - Must use disk, still **prohibitive**
- Massively redundant!
- Only good for rectangular regions

# Integral Histogram: Compression



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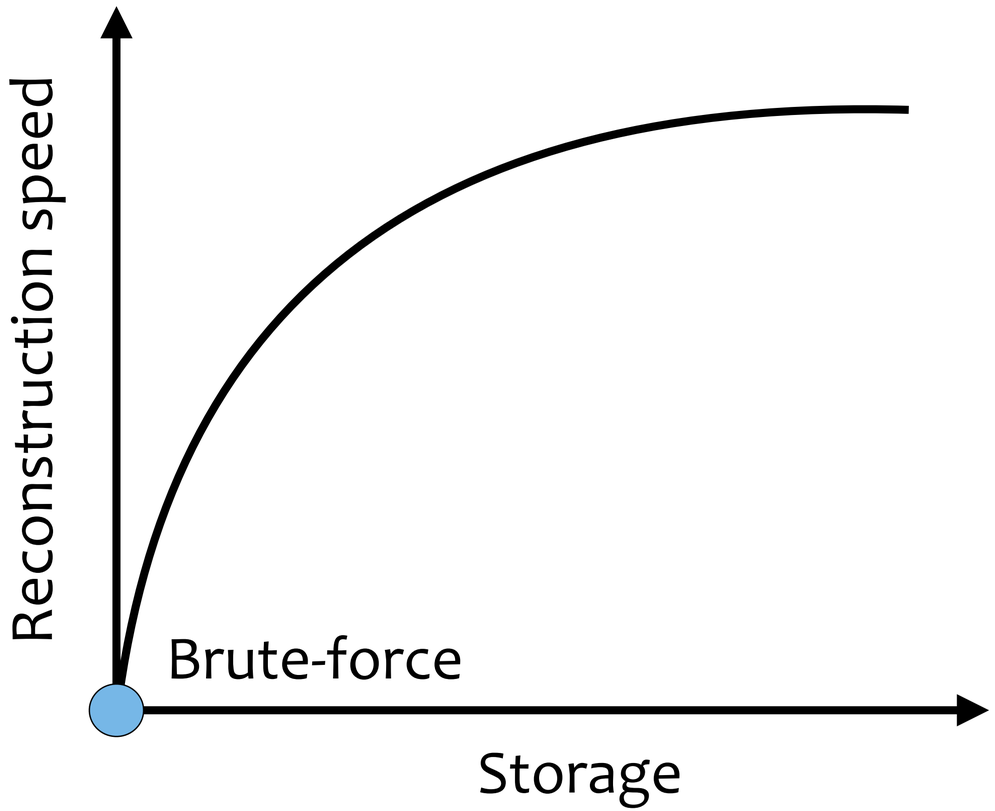

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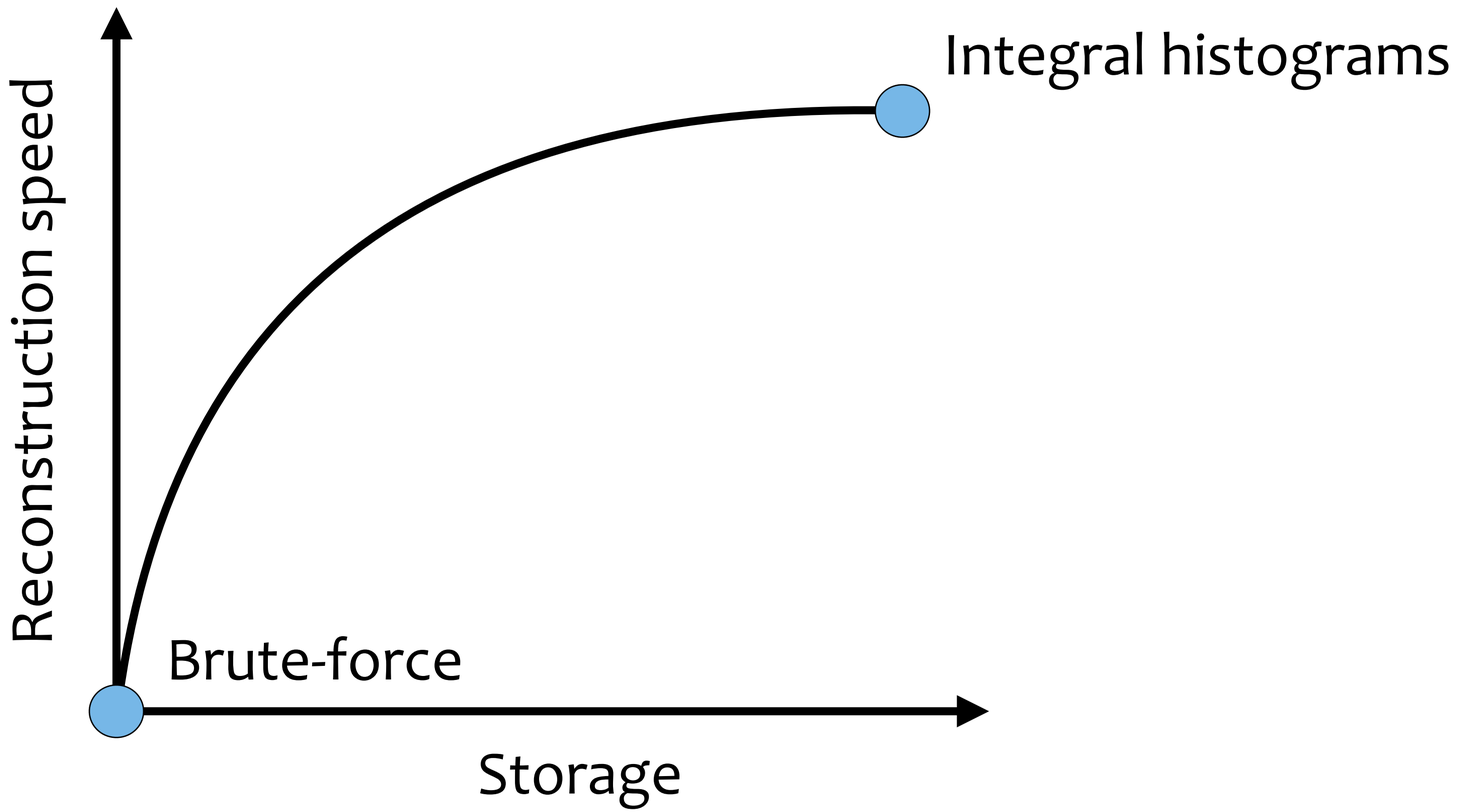
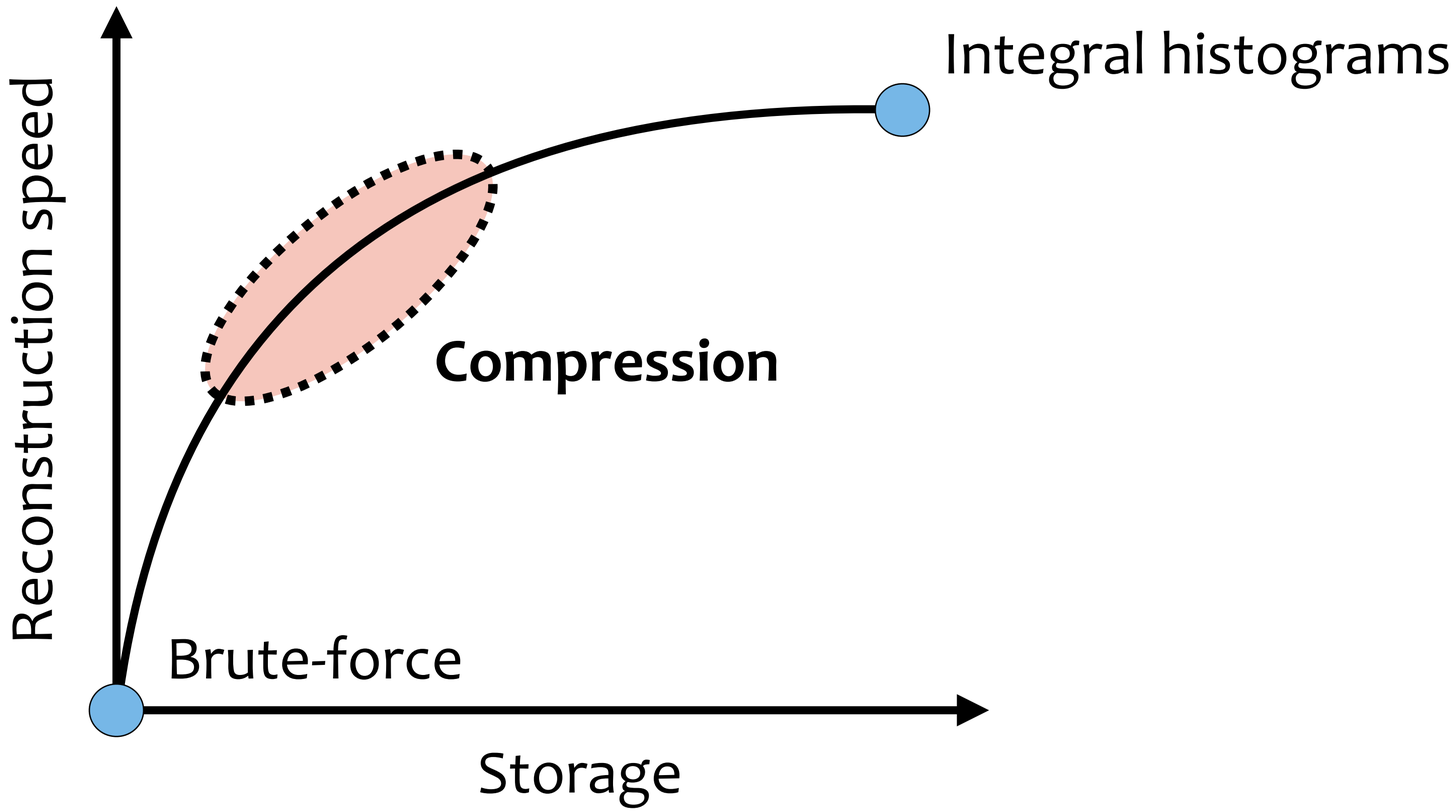


# Integral Histogram: Compression

**Let's compress the integral histogram**

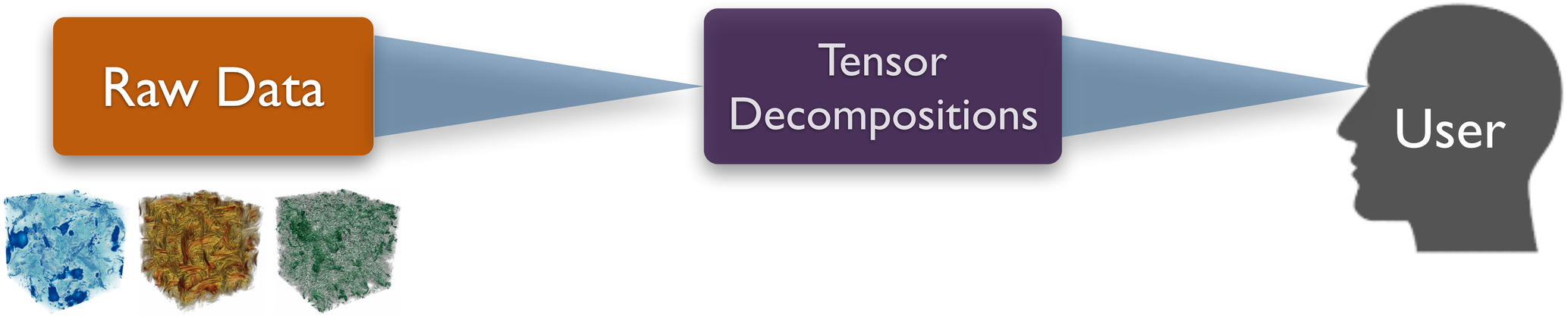


# Integral Histogram: Compression

**Let's compress the integral histogram**

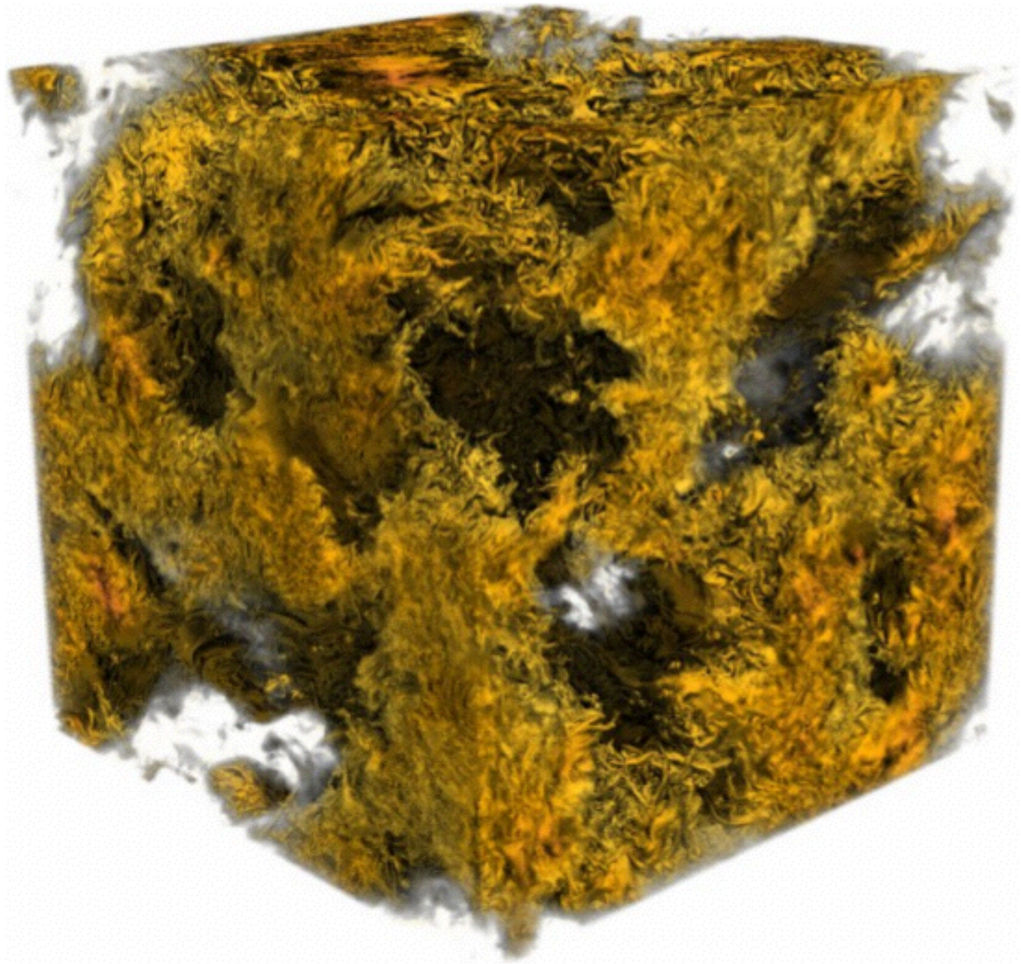
- Challenges:
  - Compressed IH **should fit in memory**
  - Decompression should be **faster** than brute-force histogram computation
- Example: Wavelet-SAT [Lee & Shen '13]
  - Fast IH decompression using wavelets
  - Lossless ⟶ moderate compression ratios
  - Only rectangular regions

# Tensor Decompositions

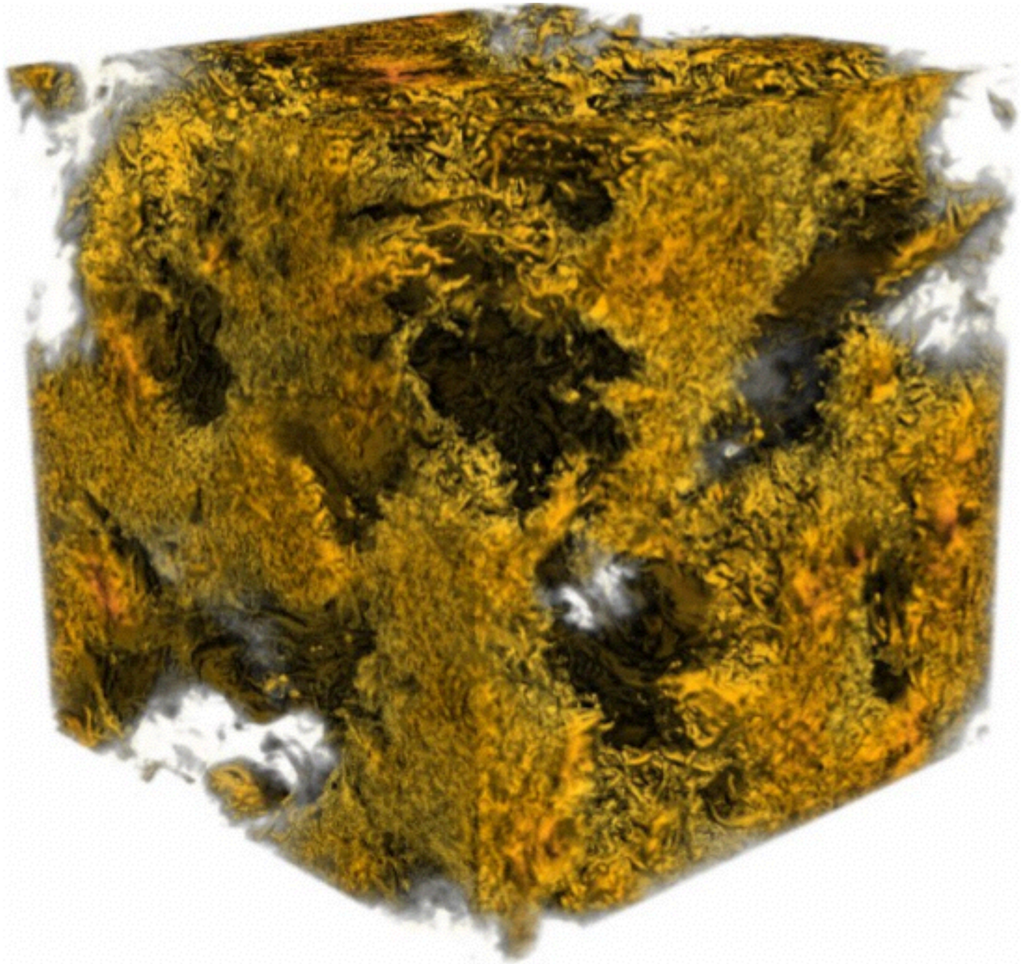



- Support for **non-rectangular** regions
- Reconstruction: matrix/tensor products
- Lossy ⟶ very **high compression rates**

# Tensor Compression



Scalar field (512MB)



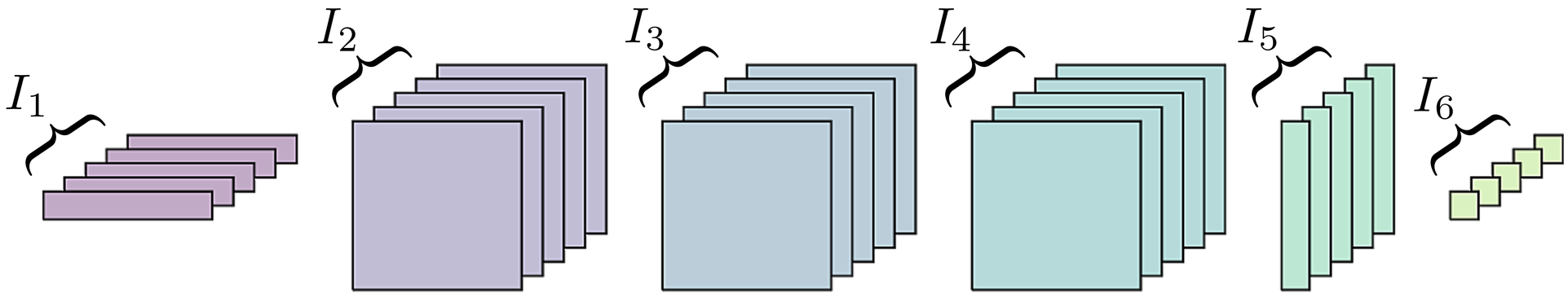
Tensor-compressed (1.4MB)



github.com/rballester/tthresh

# Tensor Train Decomposition

- Sequences of matrix products [Oseledets' 11]




$$\mathcal{T}[i_1, \ldots, i_N] \approx \mathcal{T}^{(1)}[:, i_1, :] \cdots \mathcal{T}^{(N)}[:, i_N, :]$$

- Compact, flexible, scales well with #dimensions, easy to operate with

# The Tensor Toolkit

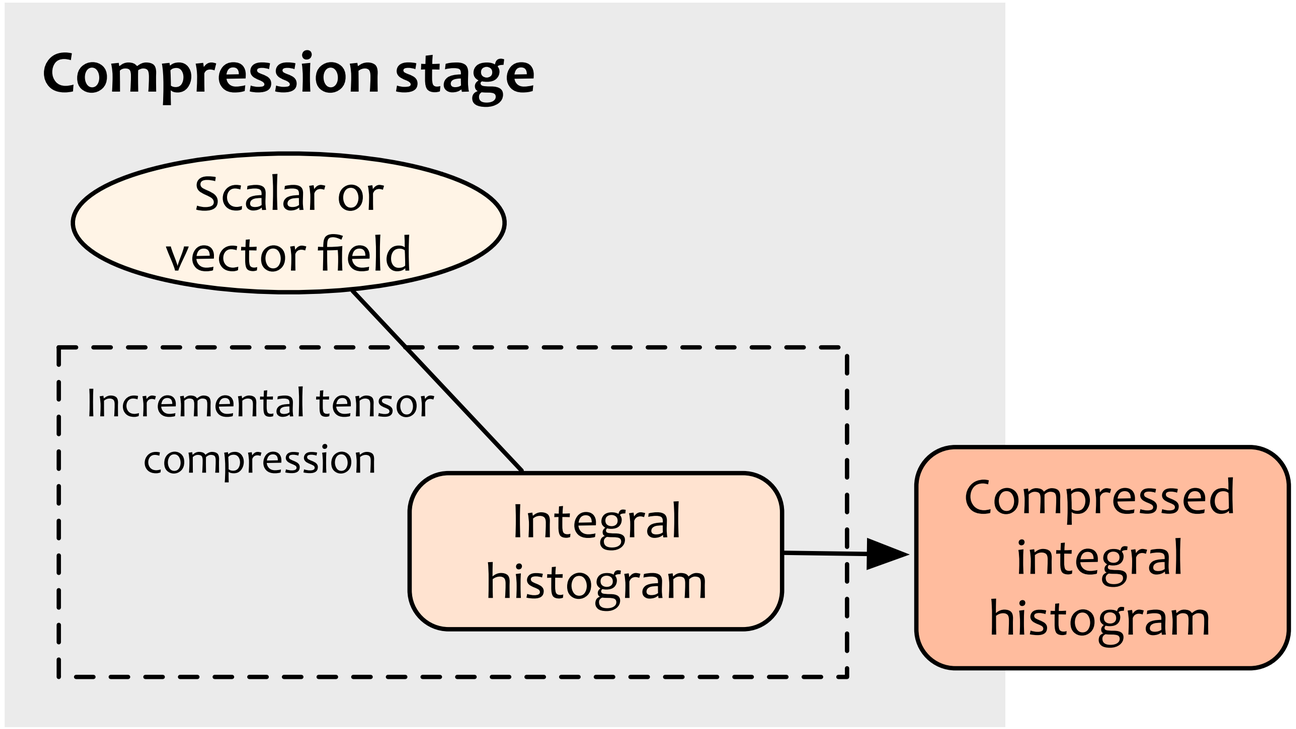
**“Easier” (fewer operations)**

- Indexing/slicing
- Decompression
- Addition/subtraction
- Editing tensor entries
- Stacking tensors
- DCT, DFT, linear transforms
- Separable convolution
- Differentiation/integration

**“Harder” (more operations)**

- Compression
- Recompression
- Product
- Elementwise functions
- General convolution
- Dot product
- Finding extrema of a tensor
- Interpolation

# Proposed Solution



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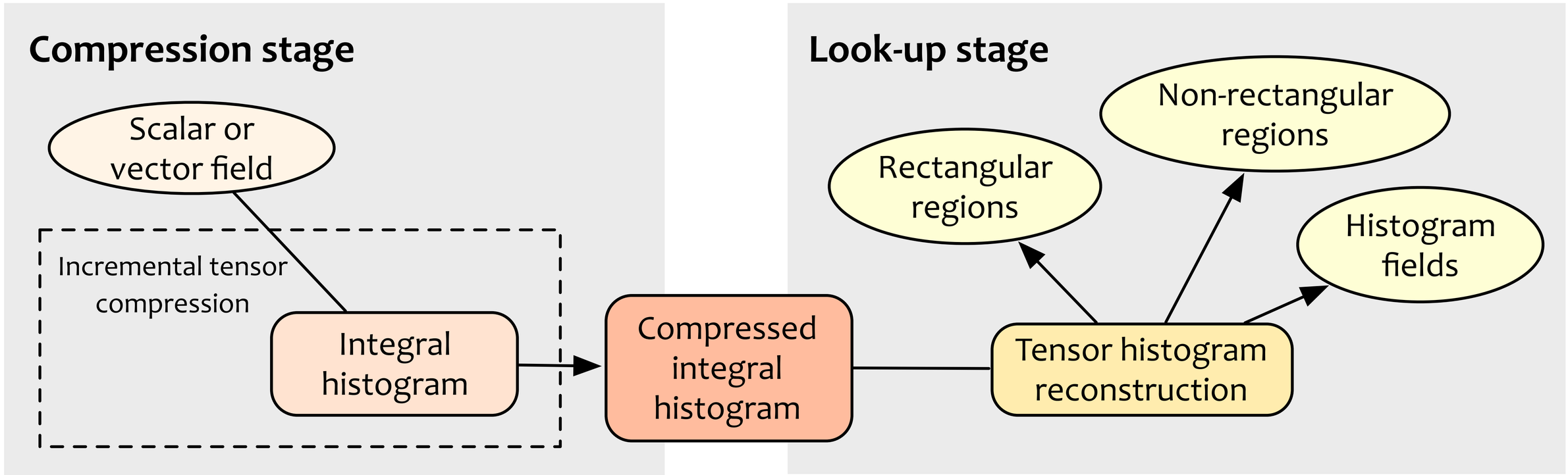
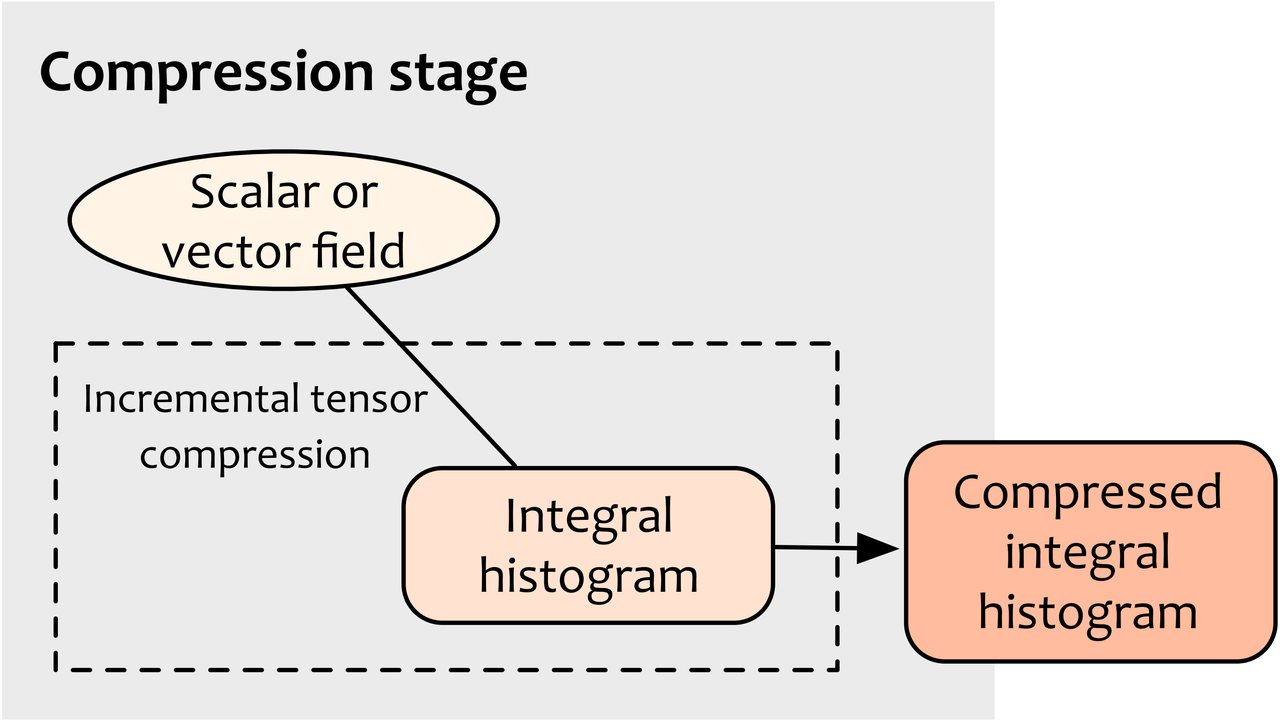


# Compression Stage



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**Compression stage**

# Tree-based Compression

Bin 1 Bin 2

# Tree-based Compression

Compress

Bin 1 Bin 2

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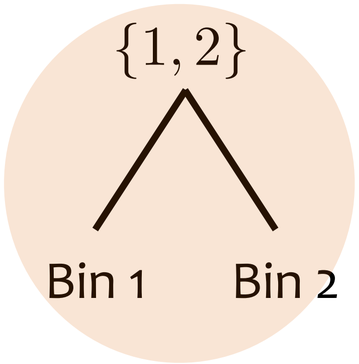
Compress

Bin 1 Bin 2

# Tree-based Compression

Stack



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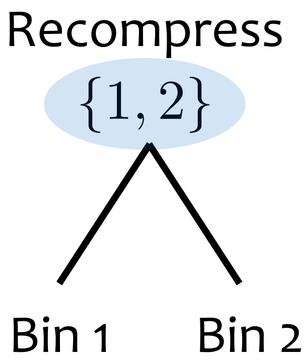

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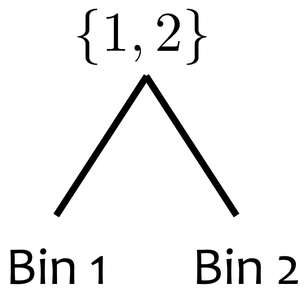

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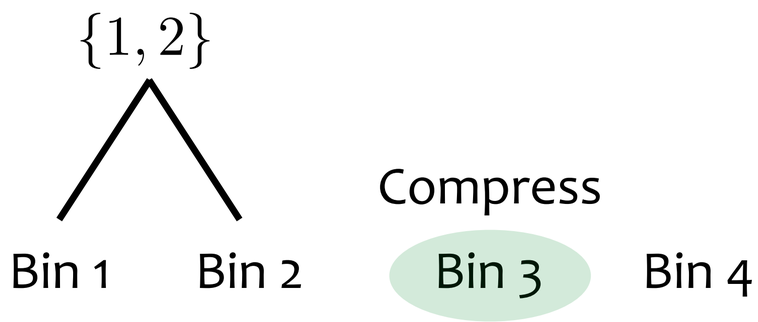

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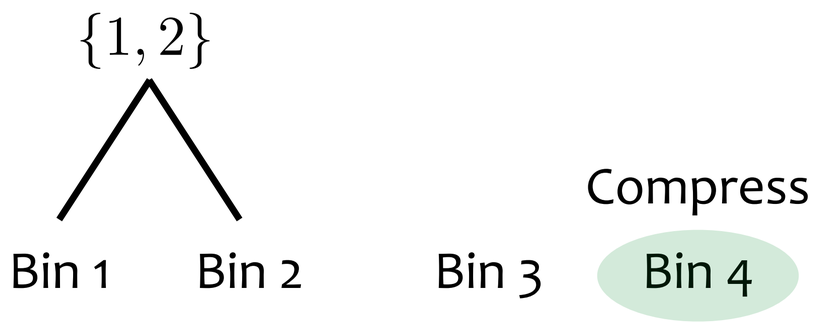

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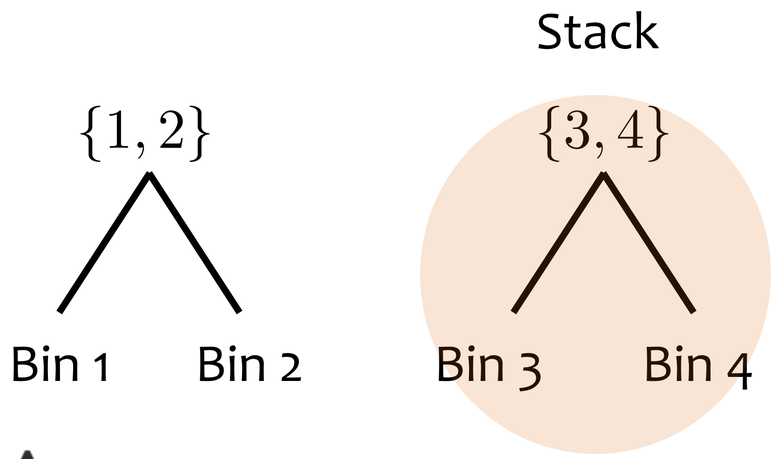

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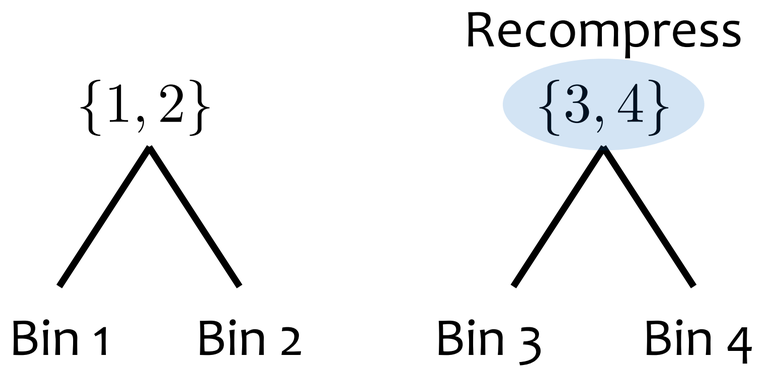

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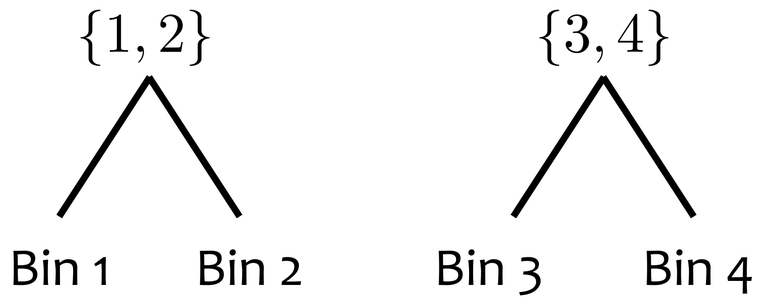

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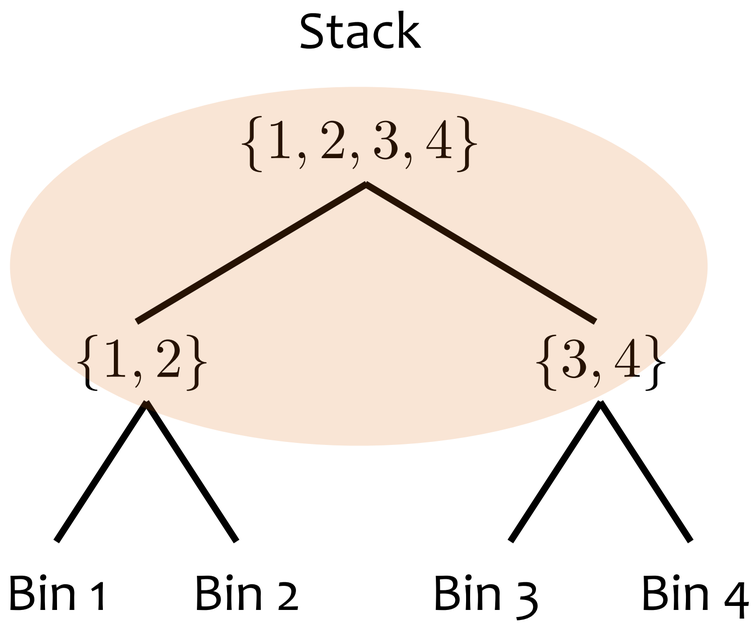

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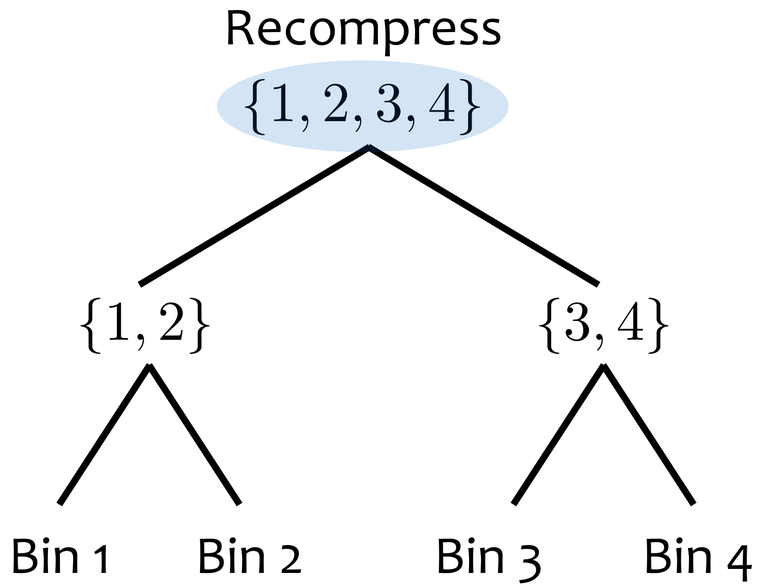

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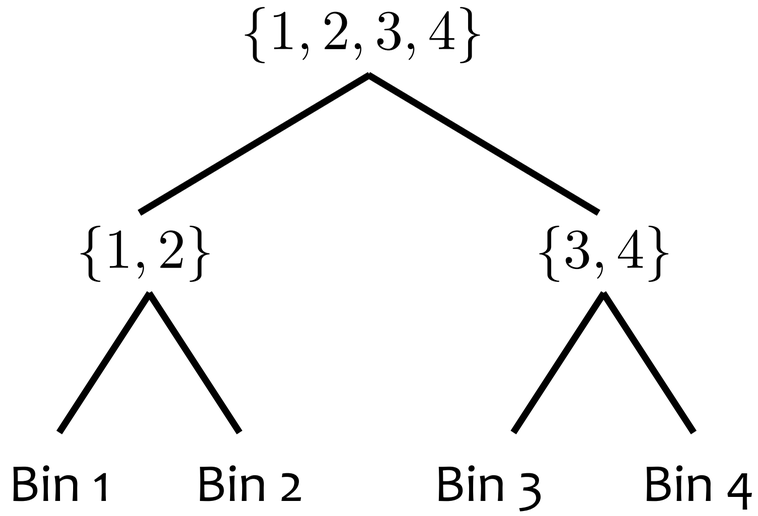

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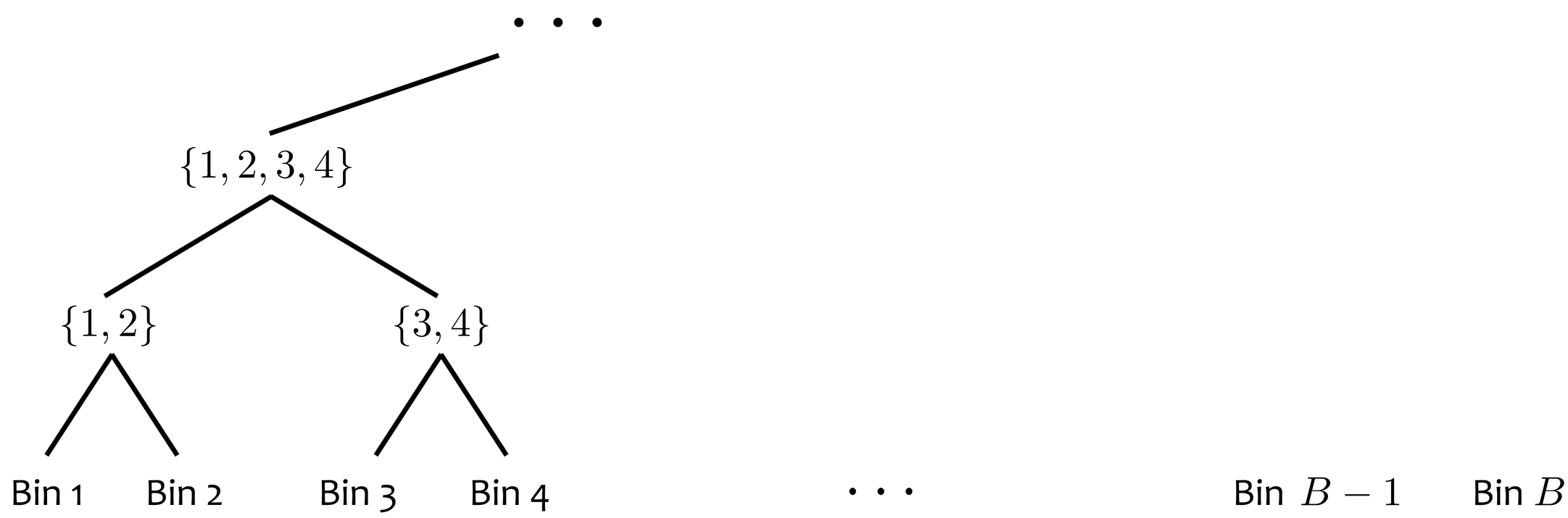

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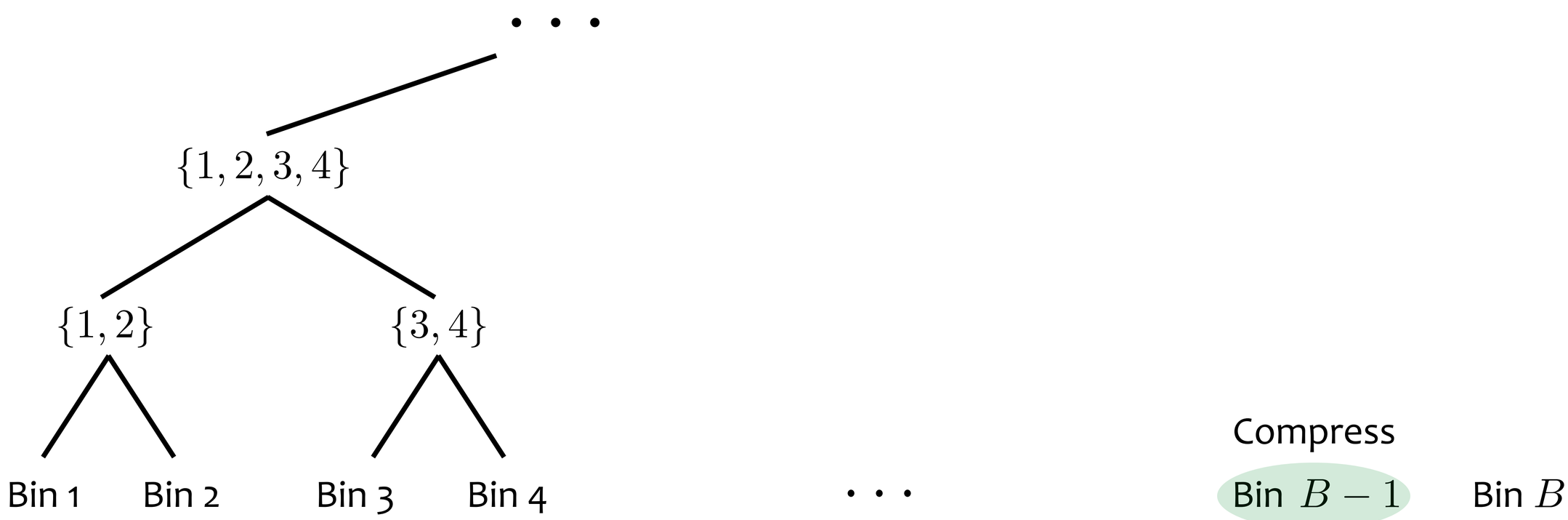

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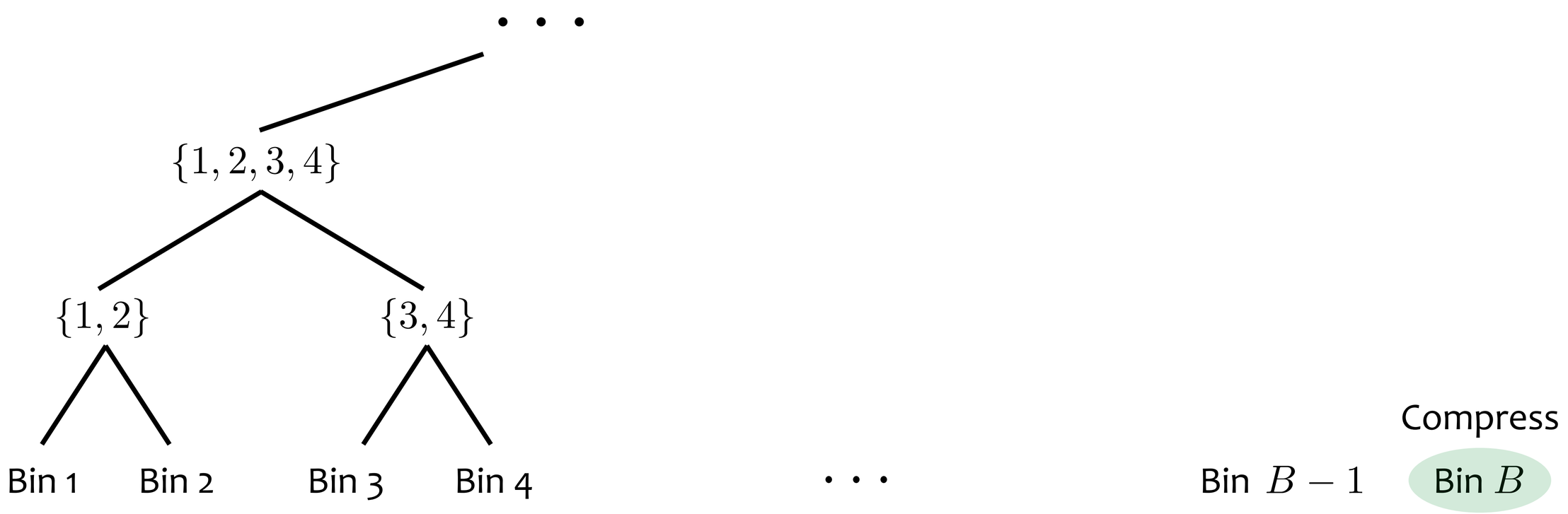

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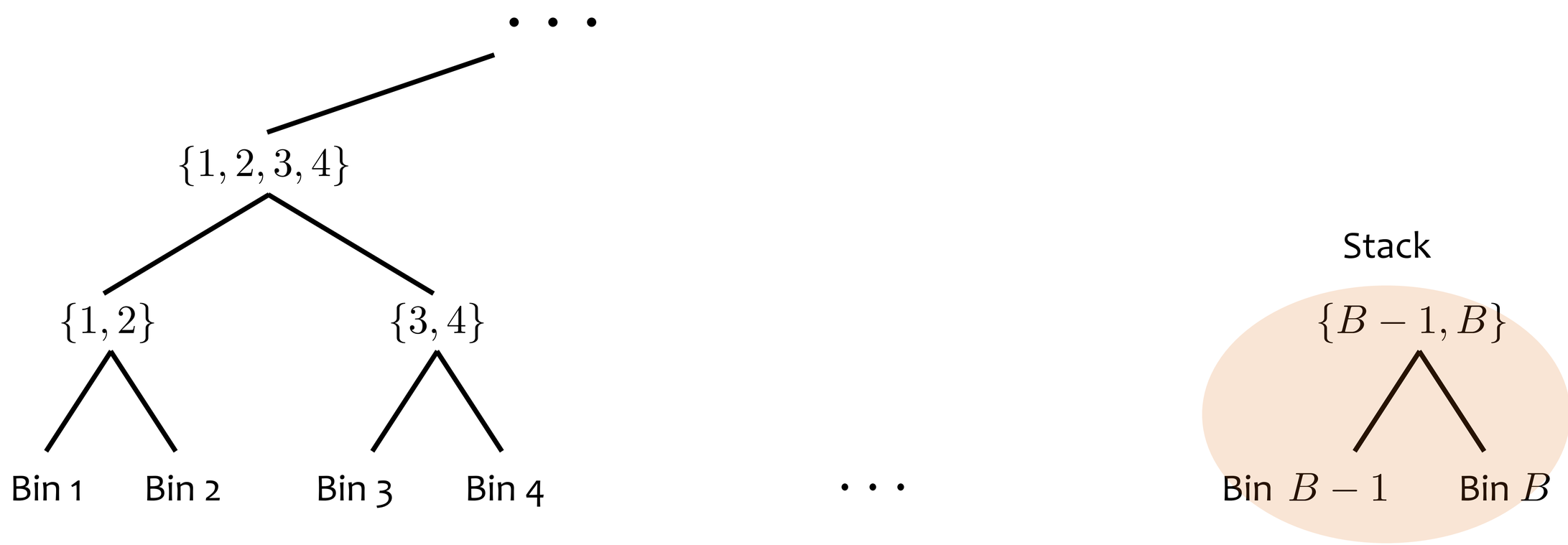

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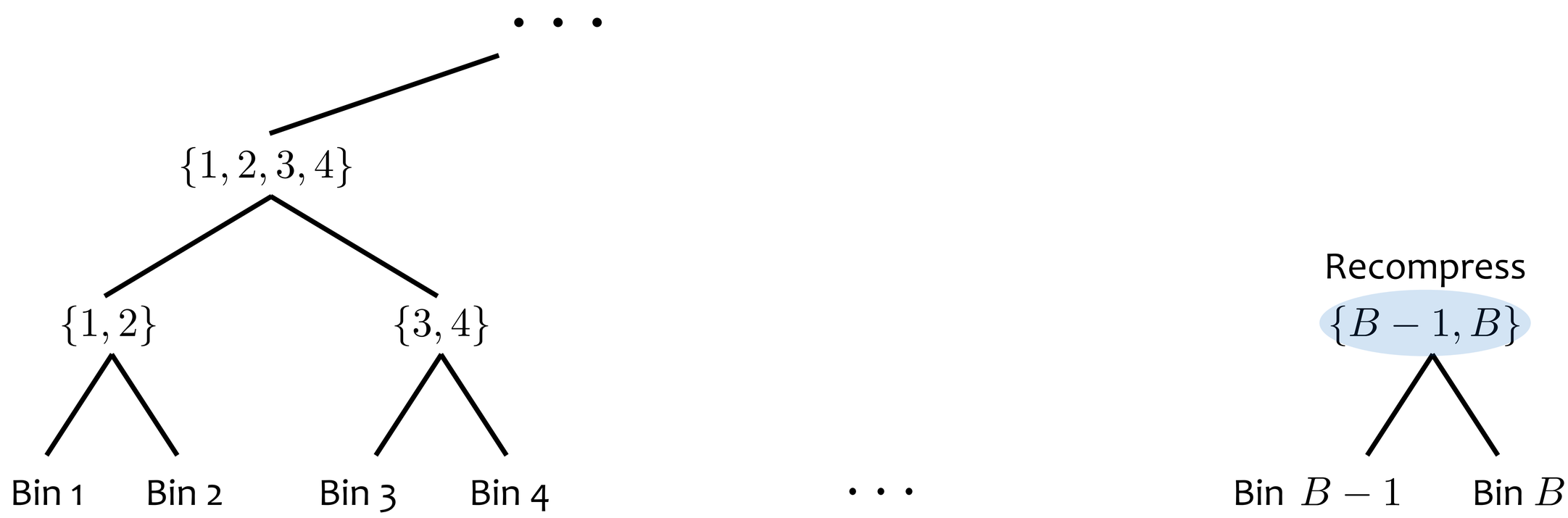

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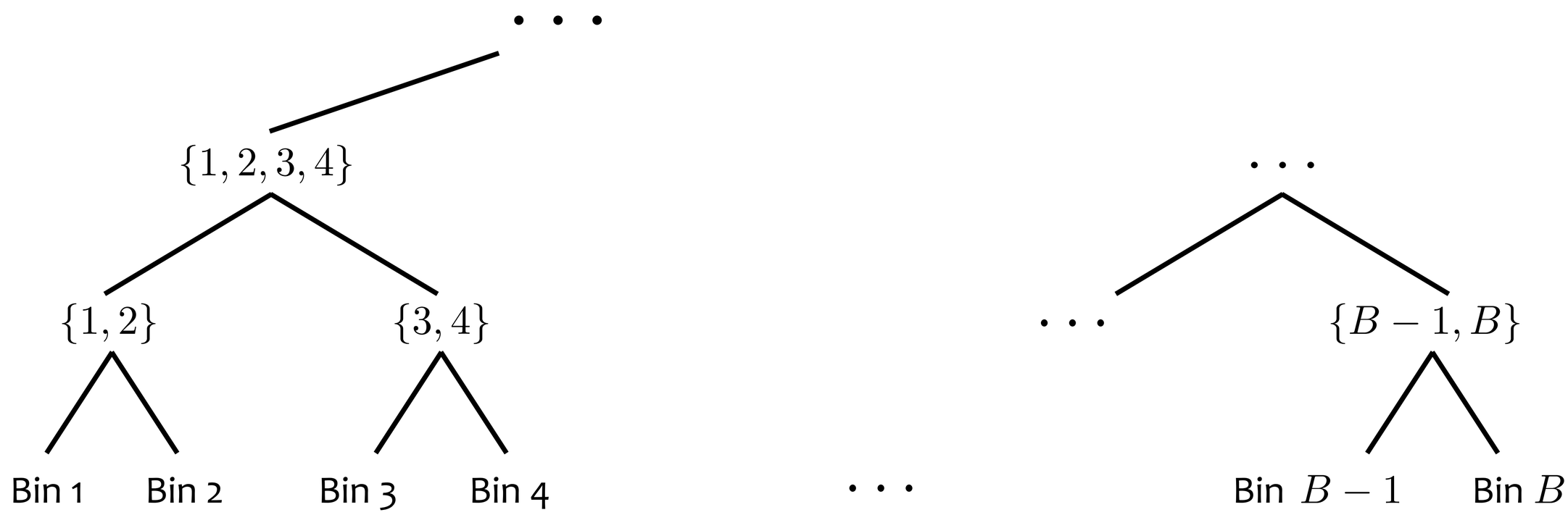

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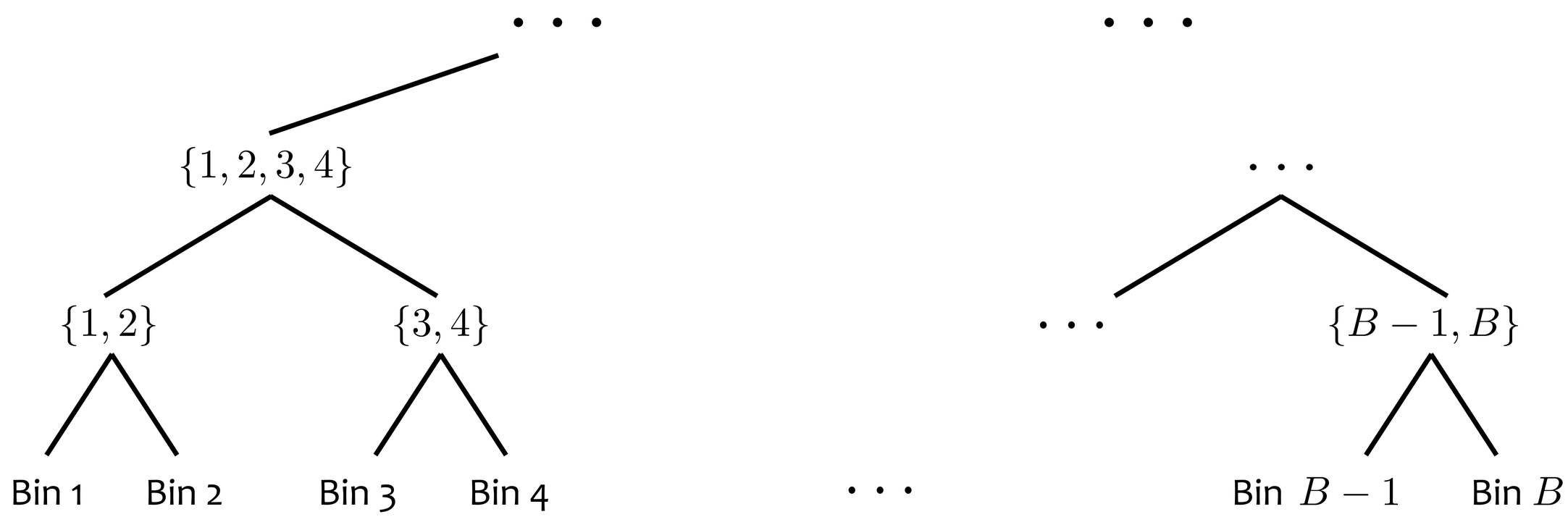

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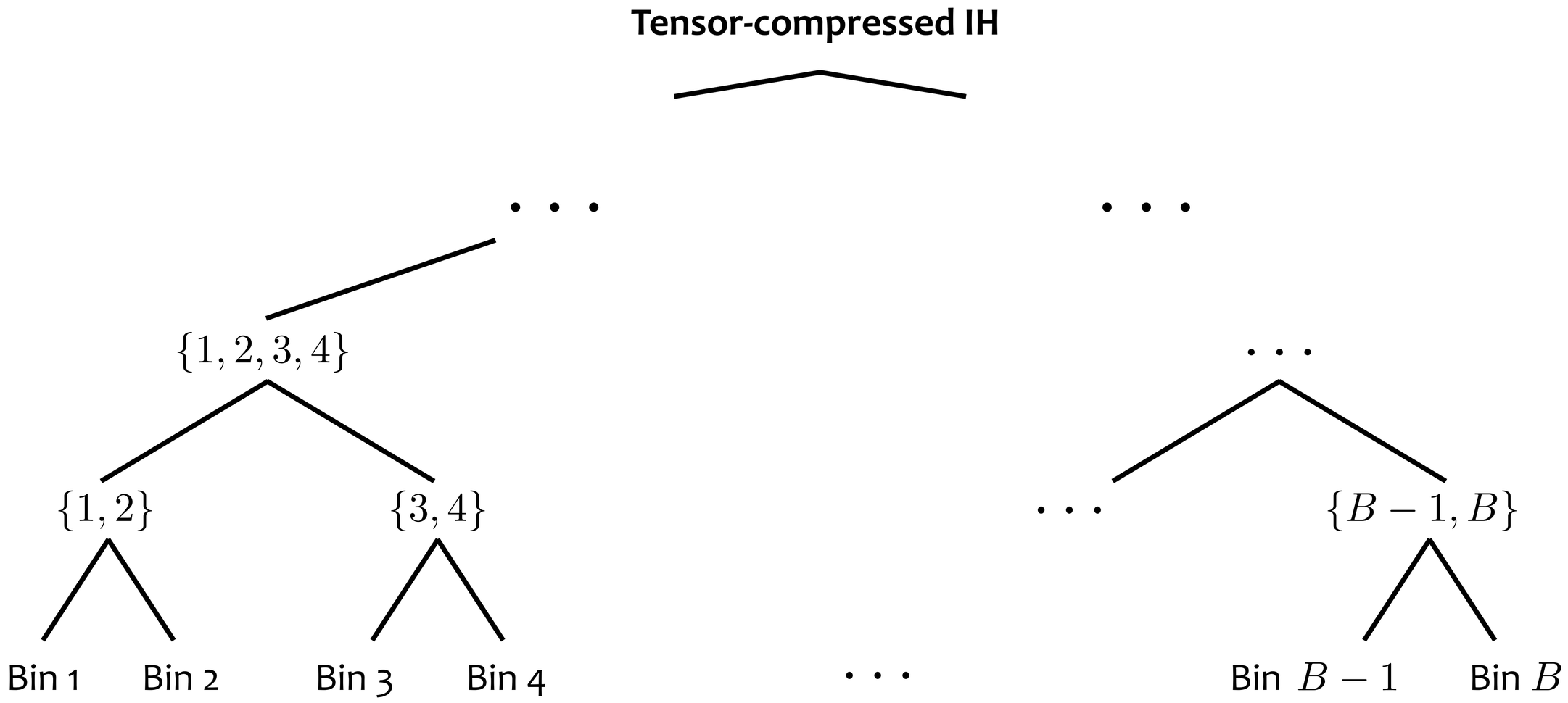
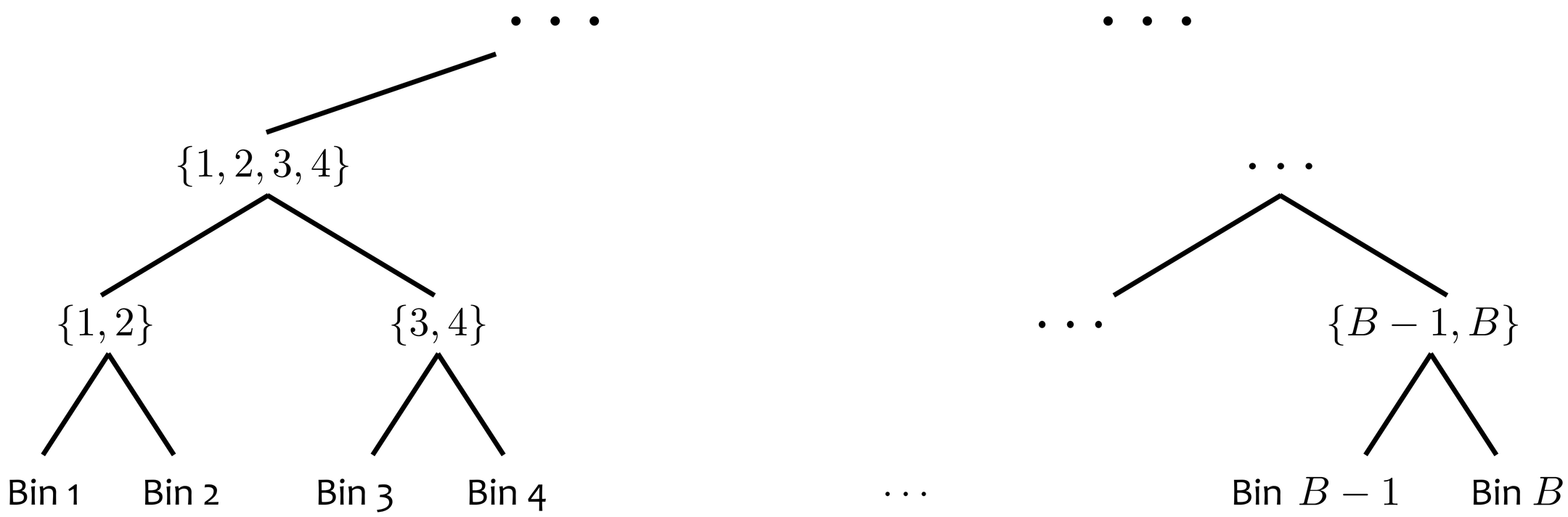
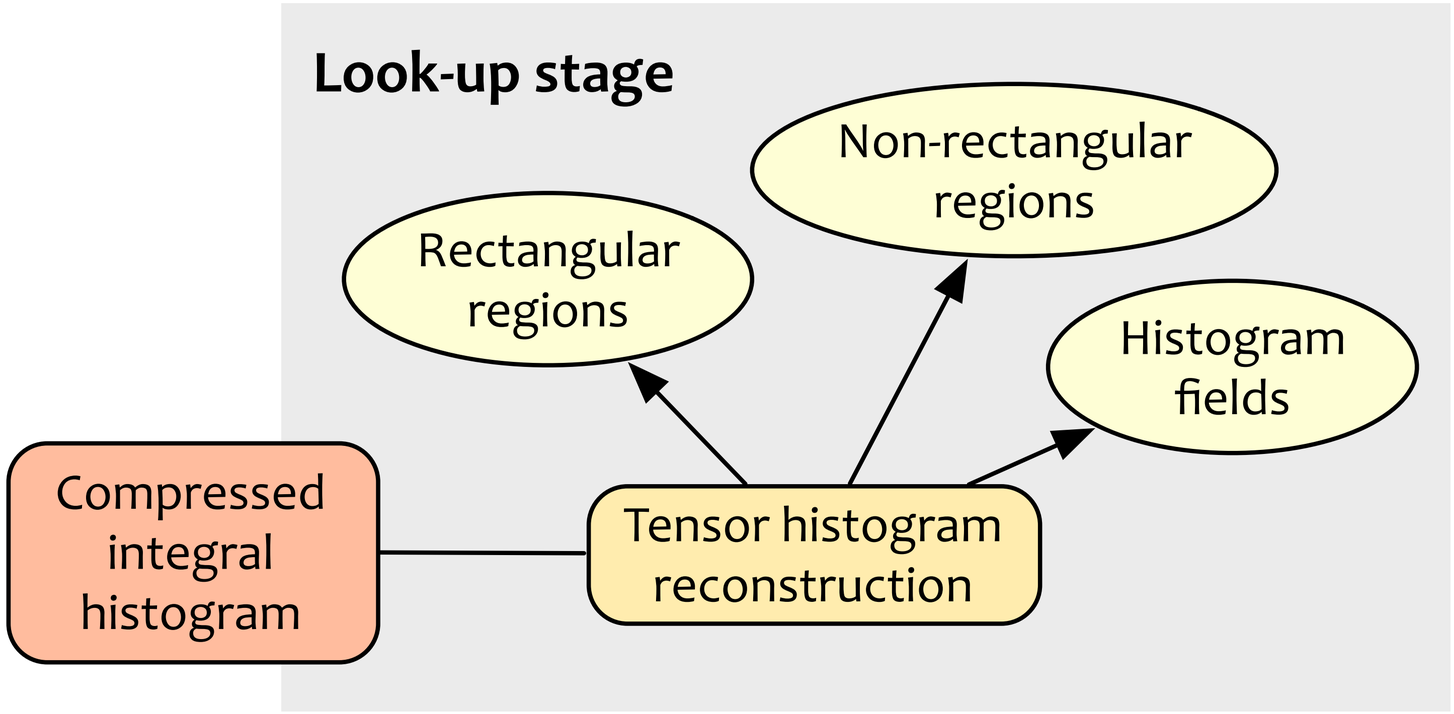


# Tree-based Compression

**Tensor-compressed IH ⟶ x100 - x1000 compression**



# Look-up Stage



# The Tensor Toolkit

**“Easier” (fewer operations)**

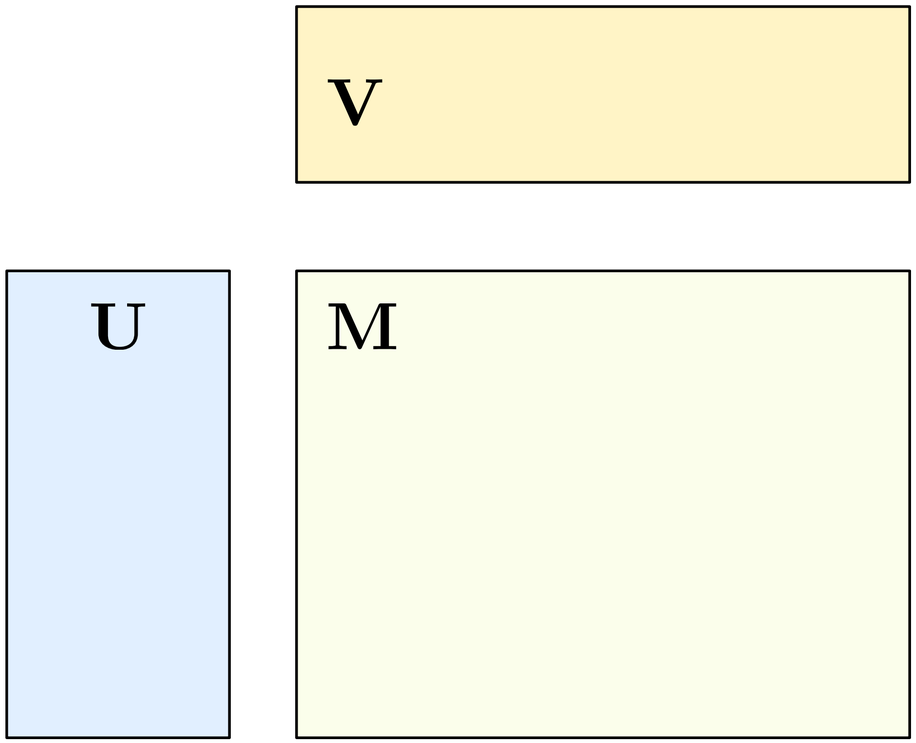
- Indexing/slicing
- Decompression
- Addition/subtraction
- Editing tensor entries
- Stacking tensors
- DCT, DFT, linear transforms
- Separable convolution
- Differentiation/integration

**“Harder” (more operations)**

- Compression
- Recompression
- Product
- Elementwise functions
- General convolution
- Dot product
- Finding extrema of a tensor
- Interpolation

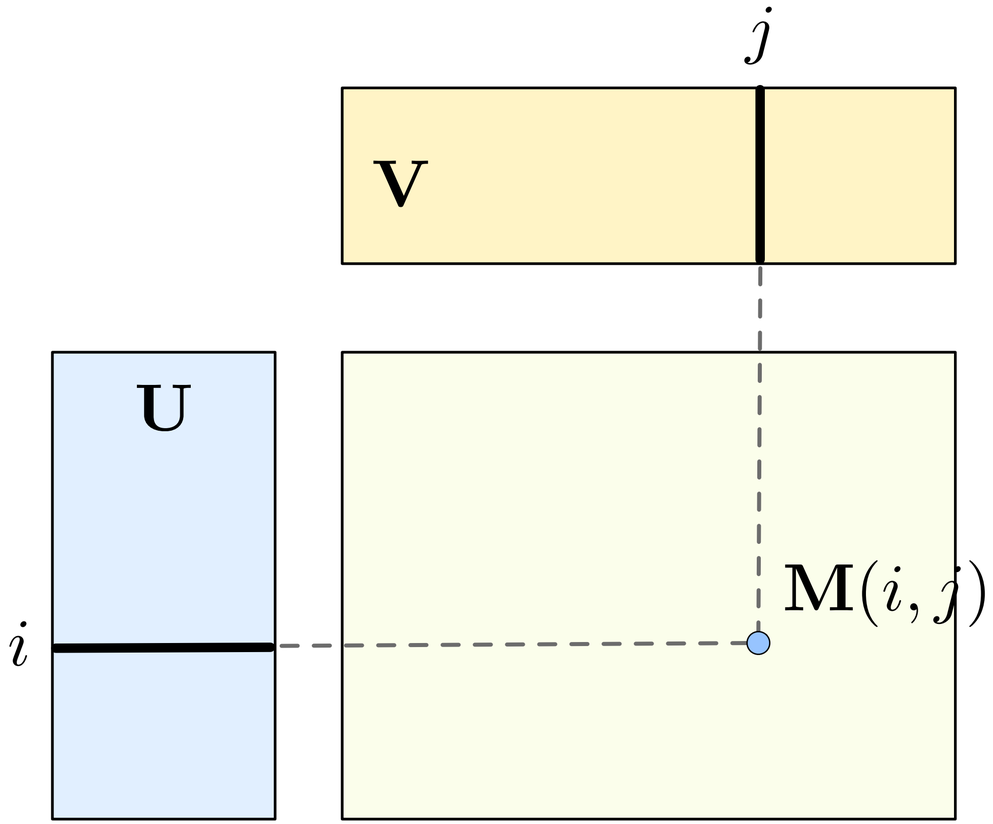
**Look-up stage**

# Tensor SAT Reconstruction



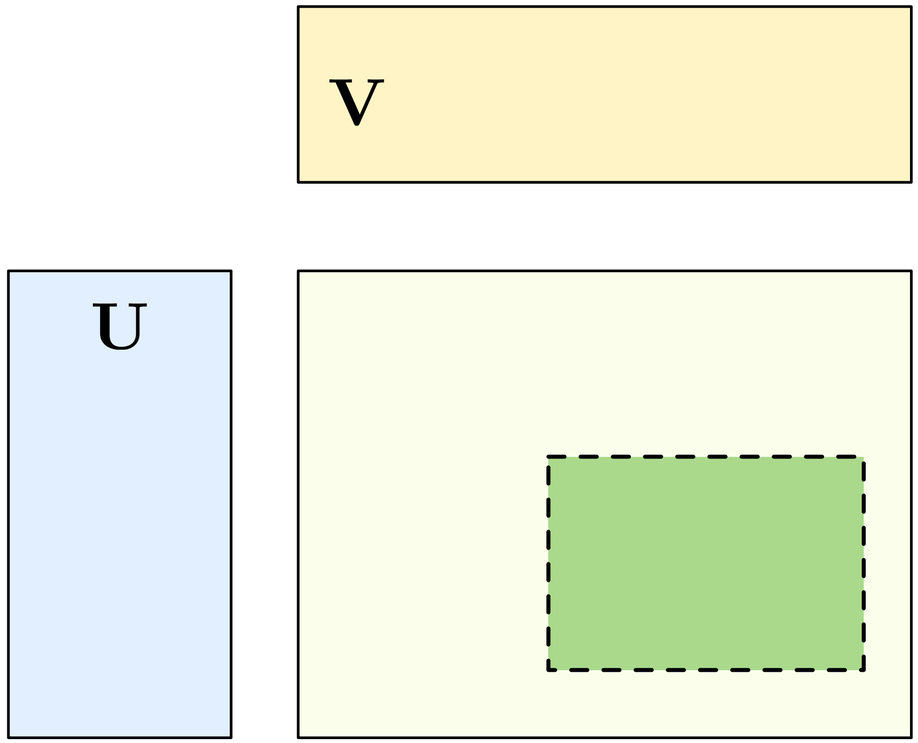

$$\mathbf{M} \approx \mathbf{U} \cdot \mathbf{V}$$

# Tensor SAT Reconstruction

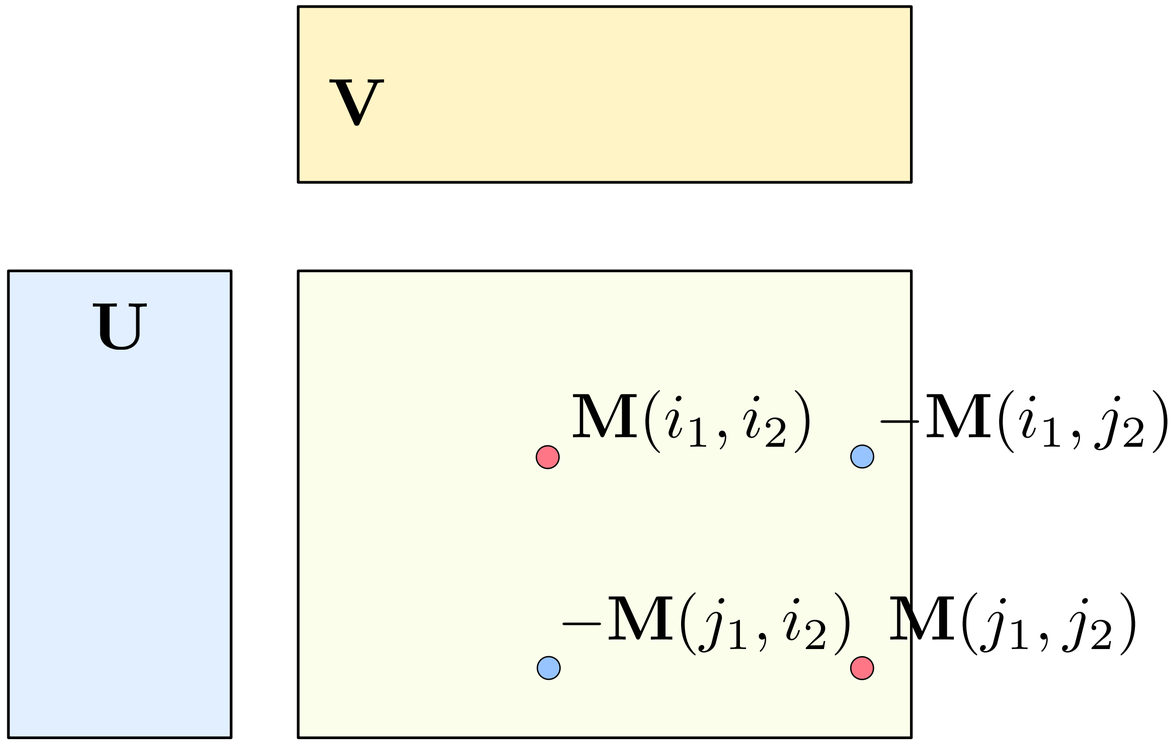



$$\mathbf{M}(i,j) \approx \boldsymbol{u}_i \cdot \boldsymbol{v}_j$$

# Tensor SAT Reconstruction

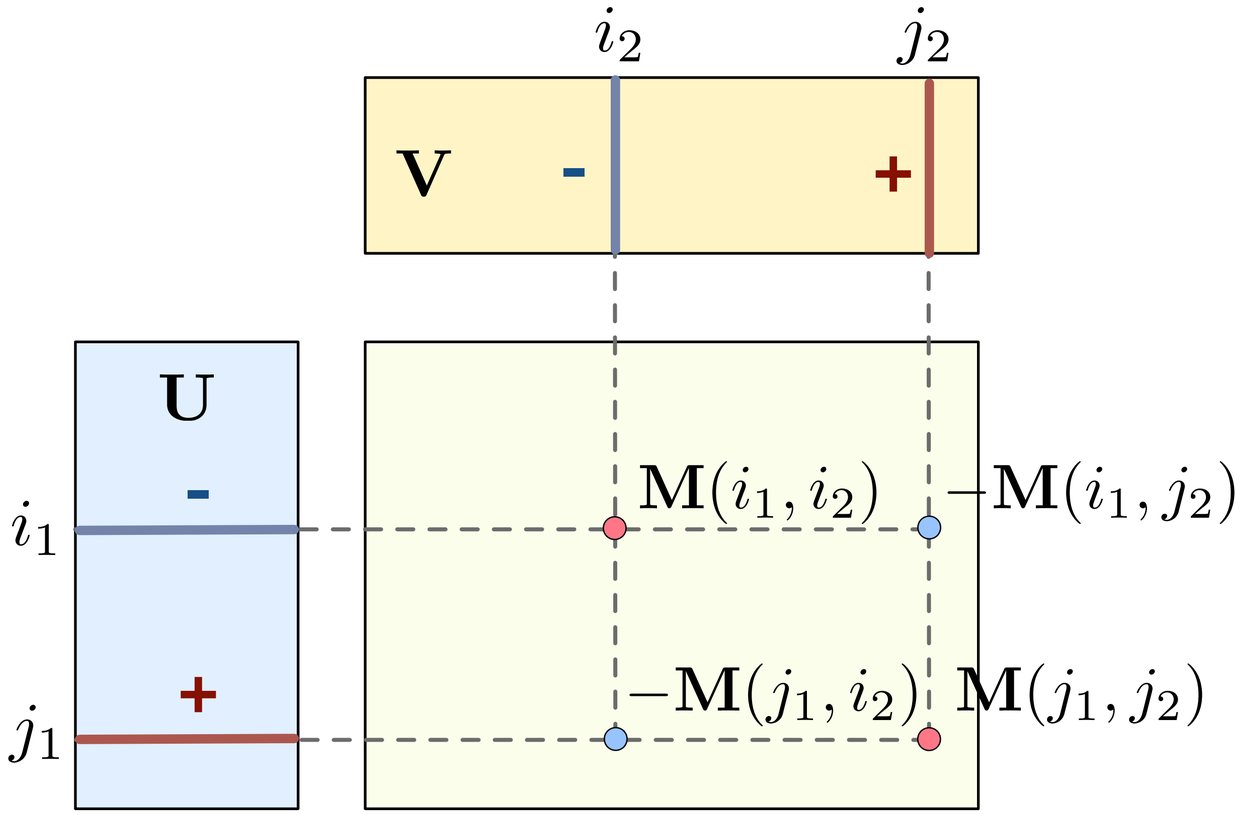


# Tensor SAT Reconstruction



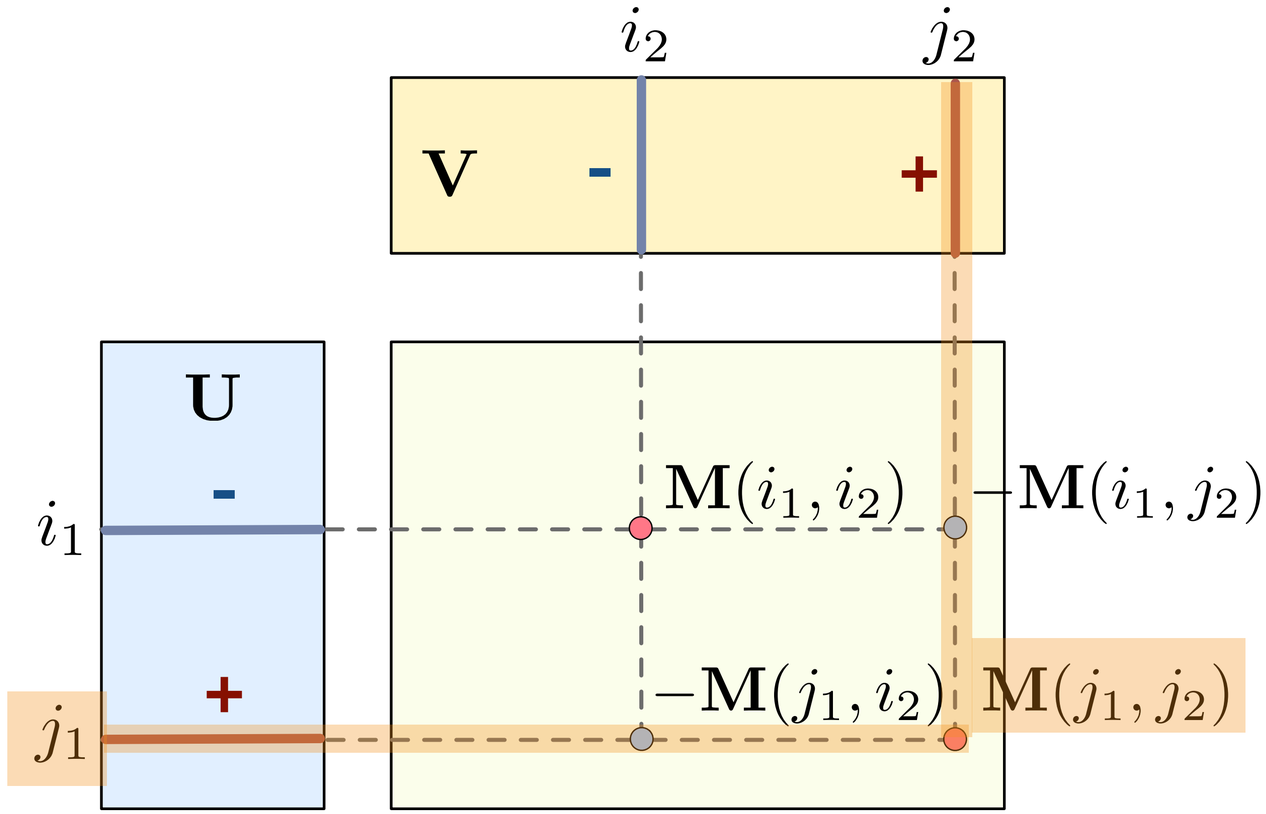

$$\mathbf{M}(j_1, j_2) - \mathbf{M}(j_1, i_2) - \mathbf{M}(i_1, j_2) + \mathbf{M}(i_1, i_2)$$

# Tensor SAT Reconstruction



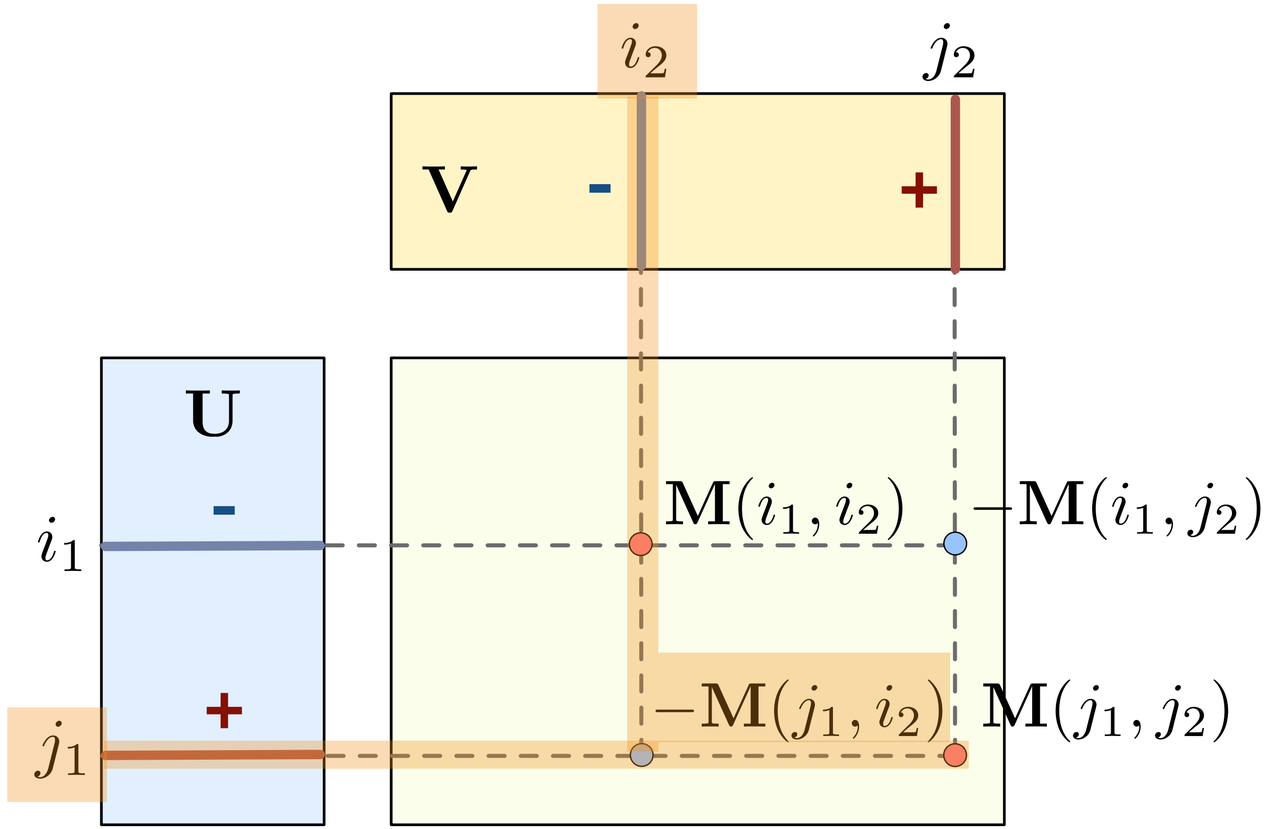

$$(\boldsymbol{u}_{j_1} - \boldsymbol{u}_{i_1}) \cdot (\boldsymbol{v}_{j_2} - \boldsymbol{v}_{i_2}) \approx \mathbf{M}(j_1, j_2) - \mathbf{M}(j_1, i_2) - \mathbf{M}(i_1, j_2) + \mathbf{M}(i_1, i_2)$$

# Tensor SAT Reconstruction



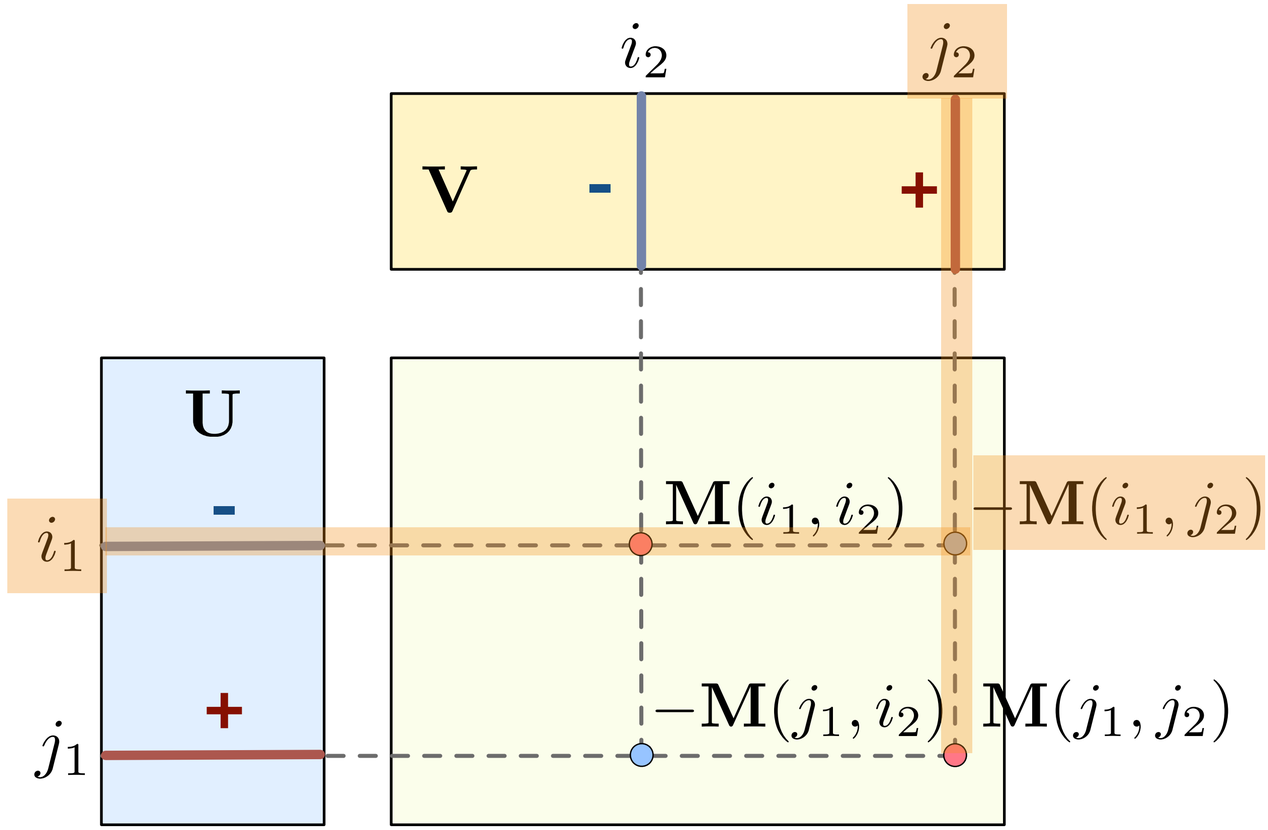

$$(\boldsymbol{u}_{j_1} - \boldsymbol{u}_{i_1}) \cdot (\boldsymbol{v}_{j_2} - \boldsymbol{v}_{i_2}) \approx \mathbf{M}(j_1, j_2) - \mathbf{M}(j_1, i_2) - \mathbf{M}(i_1, j_2) + \mathbf{M}(i_1, i_2)$$

# Tensor SAT Reconstruction



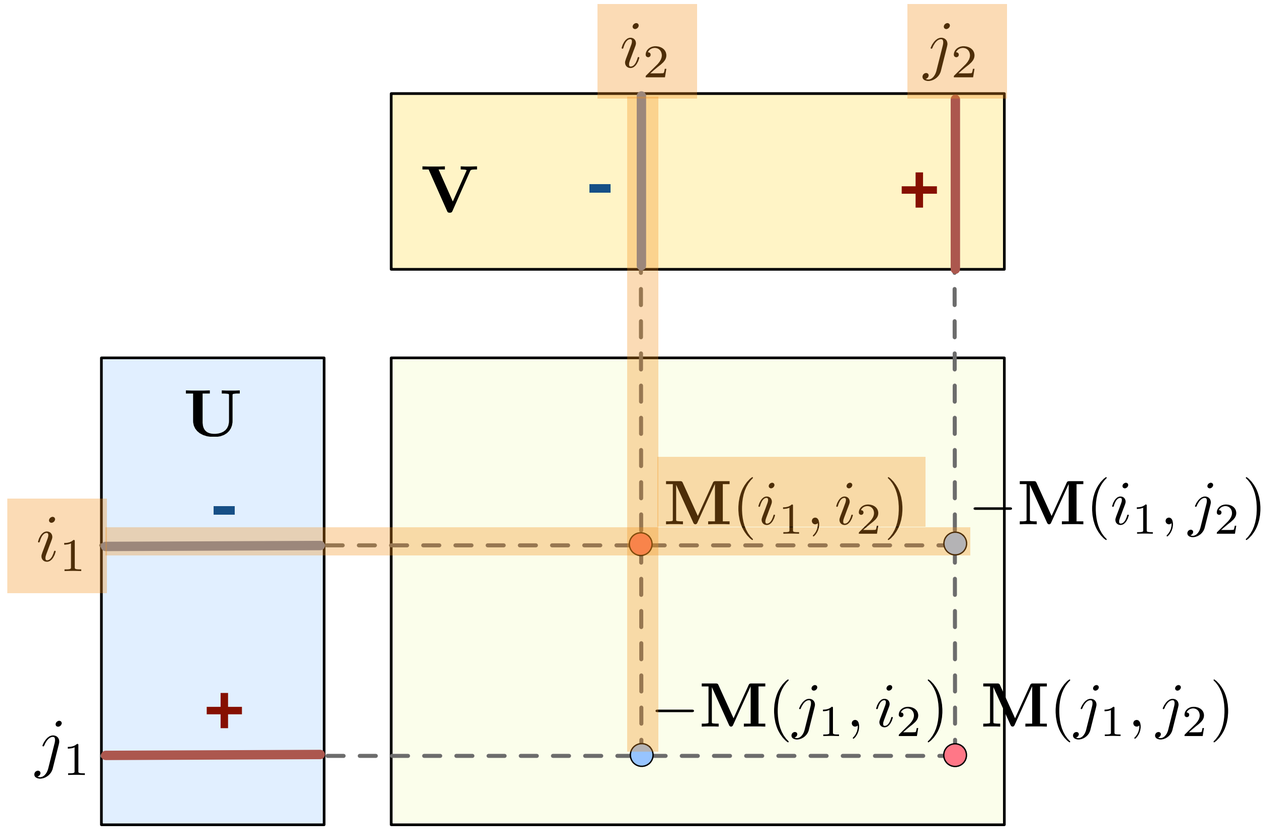

$$(\boldsymbol{u}_{j_1} - \boldsymbol{u}_{i_1}) \cdot (\boldsymbol{v}_{j_2} - \boldsymbol{v}_{i_2}) \approx \mathbf{M}(j_1, j_2) - \mathbf{M}(j_1, i_2) - \mathbf{M}(i_1, j_2) + \mathbf{M}(i_1, i_2)$$

# Tensor SAT Reconstruction




$$(\boldsymbol{u}_{j_1} - \boldsymbol{u}_{i_1}) \cdot (\boldsymbol{v}_{j_2} - \boldsymbol{v}_{i_2}) \approx \mathbf{M}(j_1, j_2) - \mathbf{M}(j_1, i_2) - \mathbf{M}(i_1, j_2) + \mathbf{M}(i_1, i_2)$$

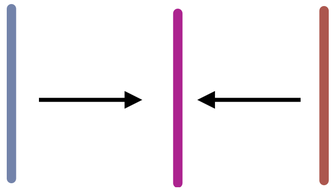
# Tensor SAT Reconstruction



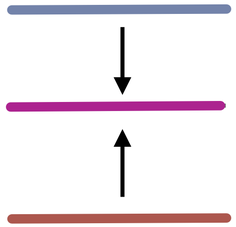

$$(\boldsymbol{u}_{j_1} - \boldsymbol{u}_{i_1}) \cdot (\boldsymbol{v}_{j_2} - \boldsymbol{v}_{i_2}) \approx \mathbf{M}(j_1, j_2) - \mathbf{M}(j_1, i_2) - \mathbf{M}(i_1, j_2) + \mathbf{M}(i_1, i_2)$$

# Tensor SAT Reconstruction

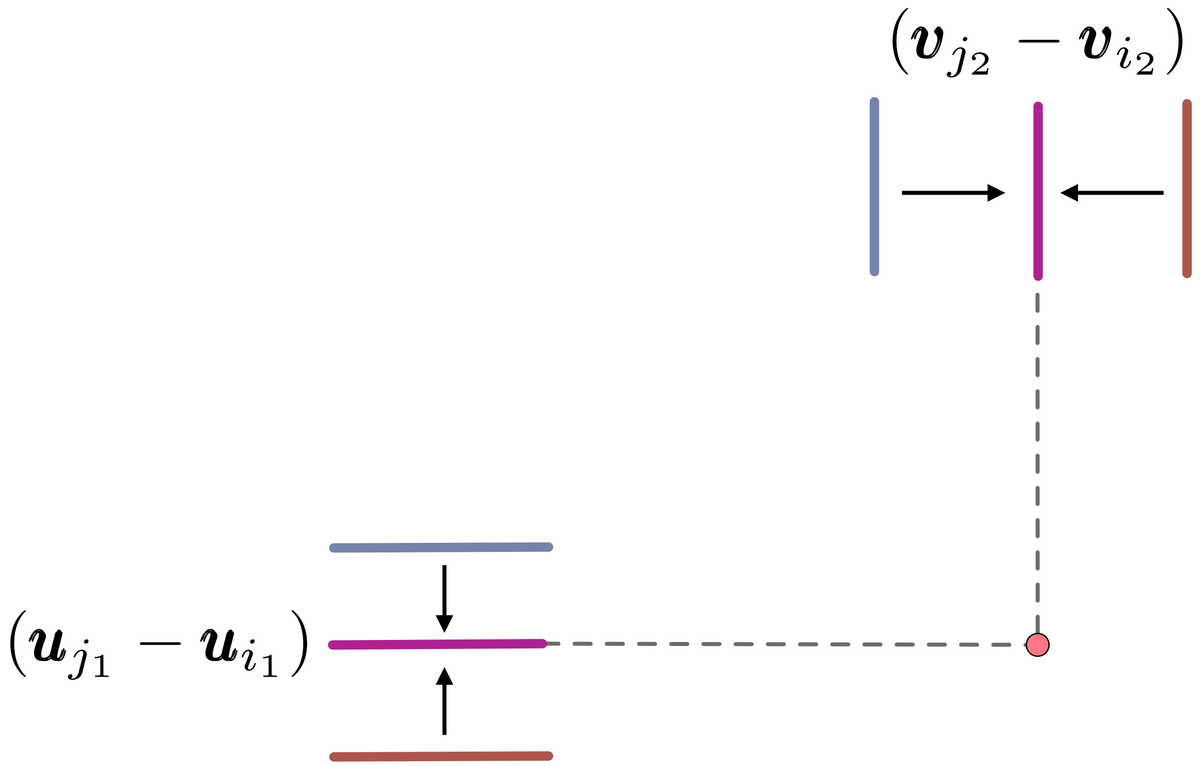
$(\boldsymbol{v}_{j_2} - \boldsymbol{v}_{i_2})$



$(\boldsymbol{u}_{j_1} - \boldsymbol{u}_{i_1})$



# Tensor SAT Reconstruction



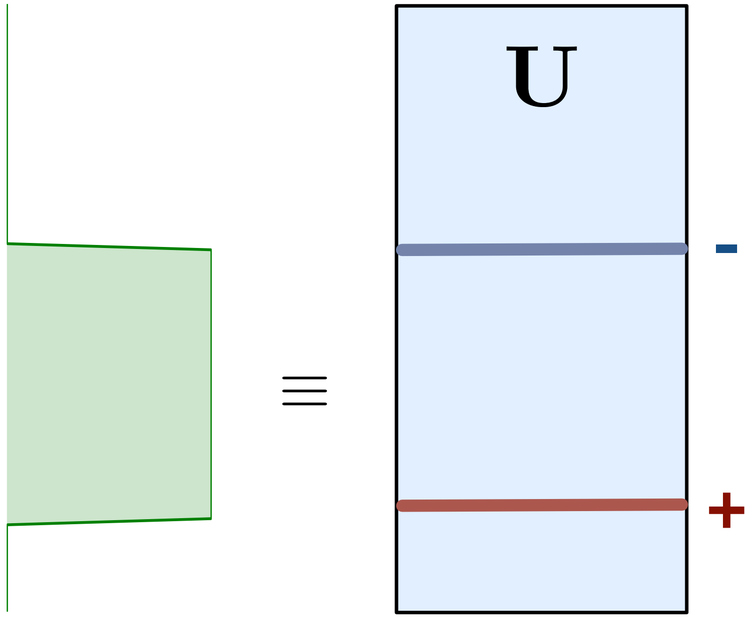

- Higher dimensions: matrix-vector (instead of vector-vector) products

# Non-rectangular ROIs

- Convolve rows together, prior to subtraction

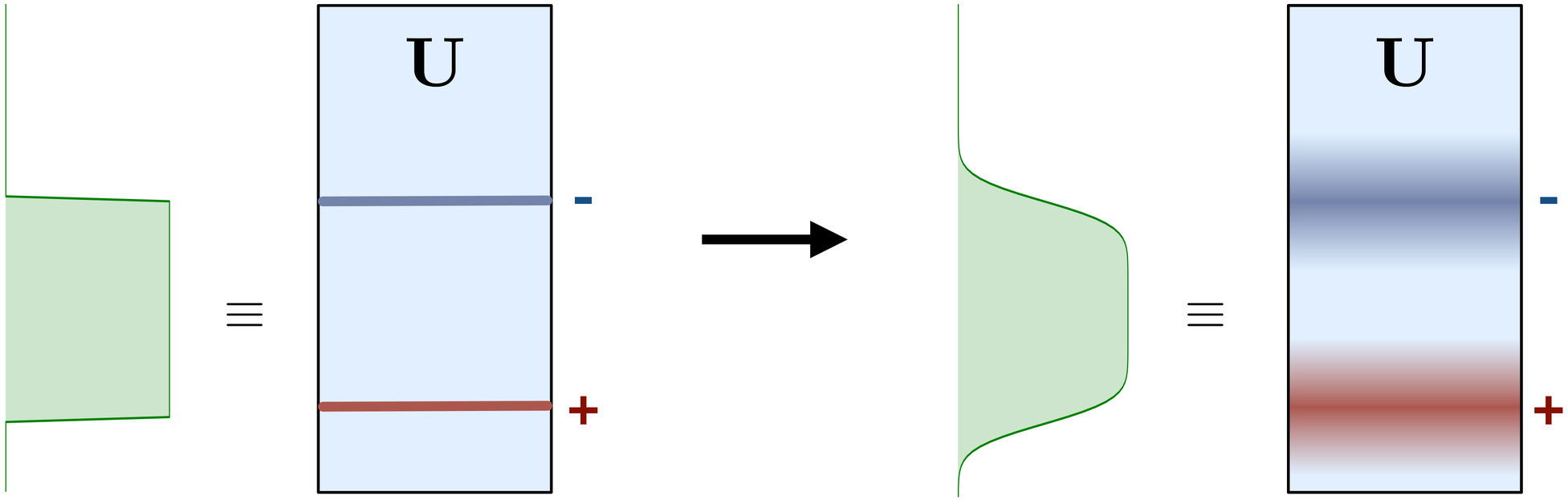
# Non-rectangular ROIs

- Convolve rows together, prior to subtraction



# Non-rectangular ROIs

- Convolve rows together, prior to subtraction



# Histogram Fields

- We were doing: matrix-vector products
- What if we want to reconstruct **many histograms** at once?
- Tensor-tensor products (**tensor contractions**)
  - Generalize matrix-matrix products
  - Just as parallelizable

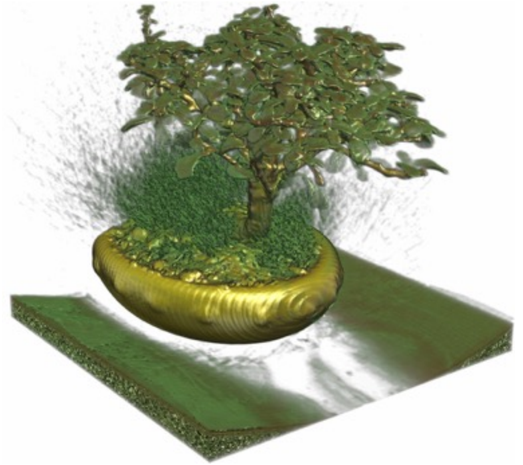
`tensorflow.einsum()` or `torch.einsum()` is all about tensor contraction!

# Results

# Scalar Fields

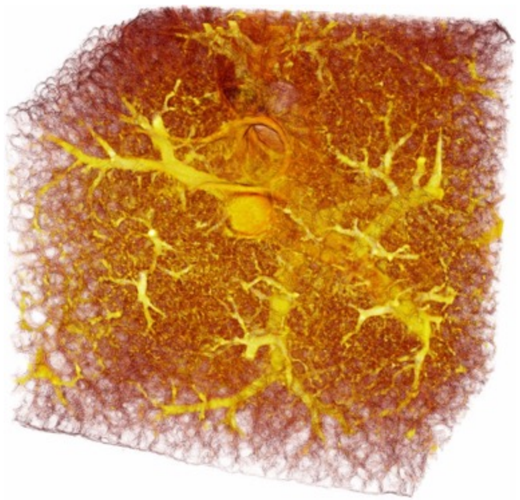
Waterfall, $4096^2$, 16MB

IH: 4GB, compressed 22:1



Bonsai, $256^3$, 256MB

IH: 8GB, compressed 214:1



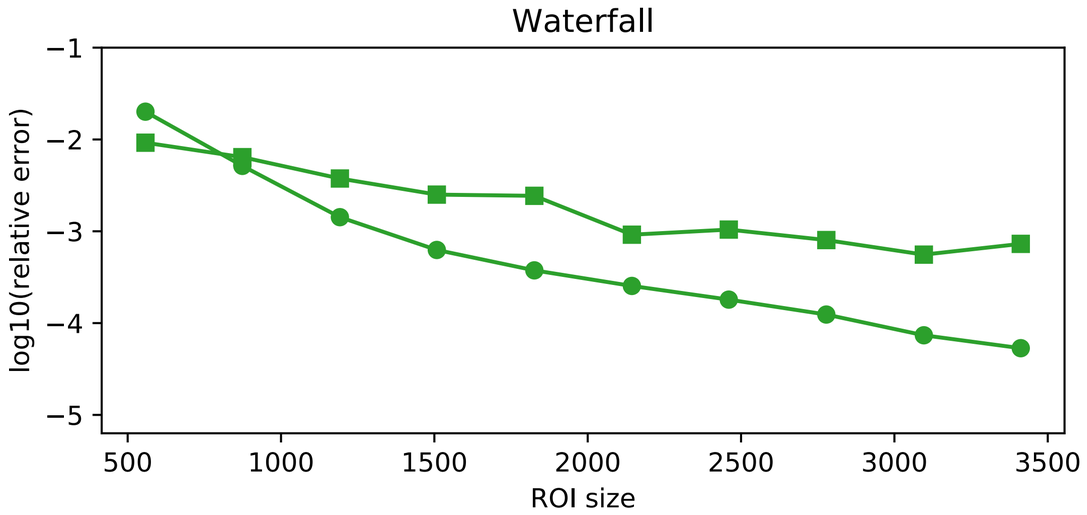
Lung, $512^3$, 128MB

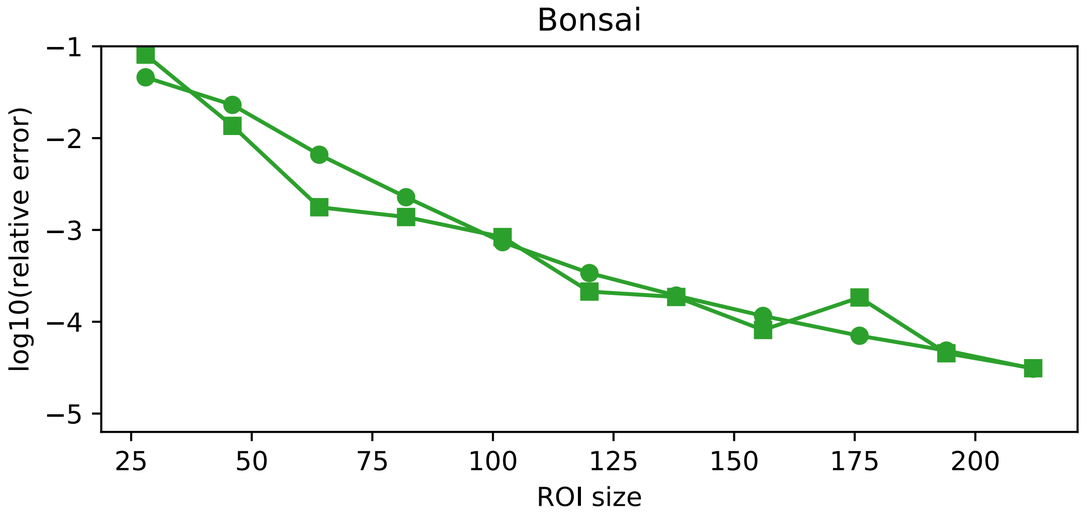
IH: 64GB, compressed 319:1

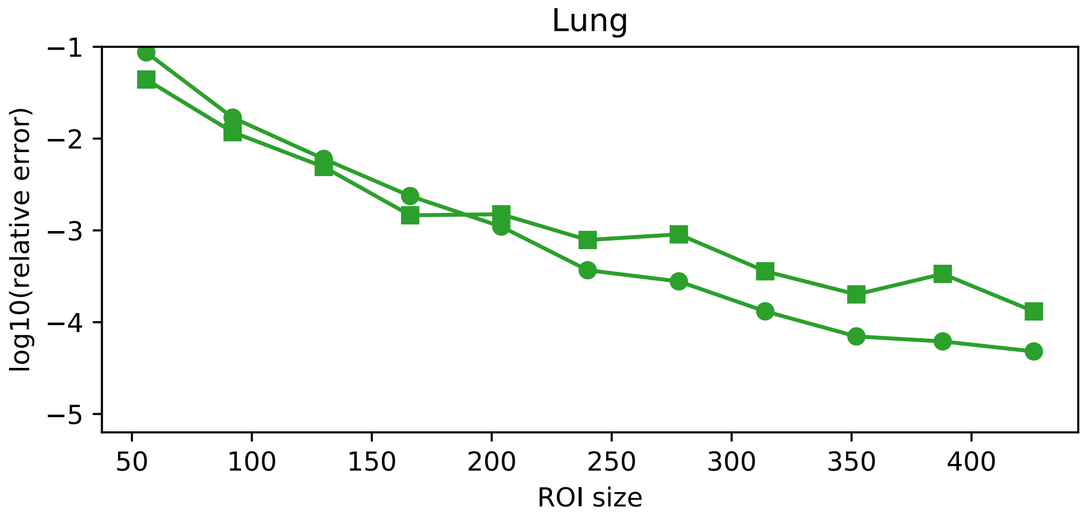
Flower, $1024^3$, 1GB

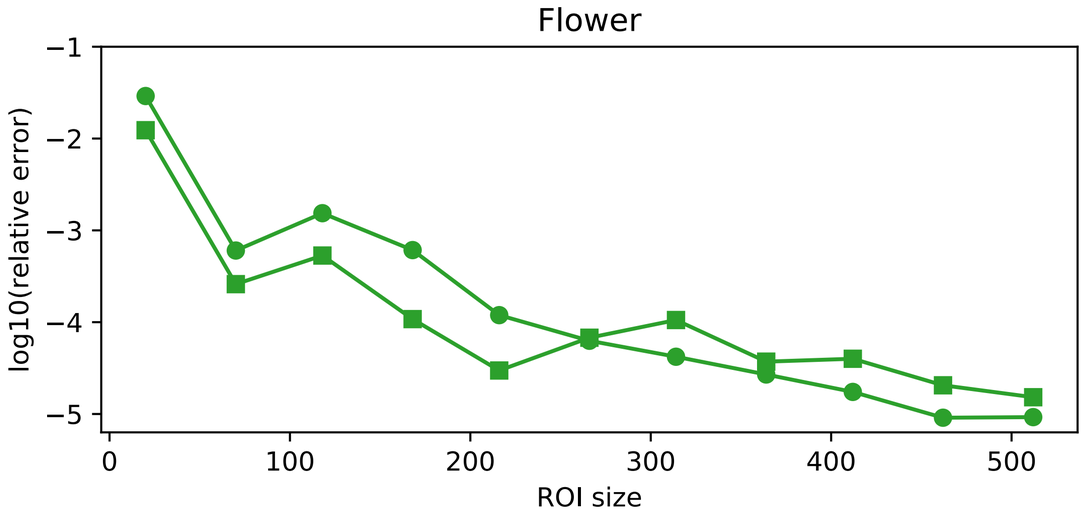
IH: 512GB, compressed 766:1

# Scalar Fields: Errors



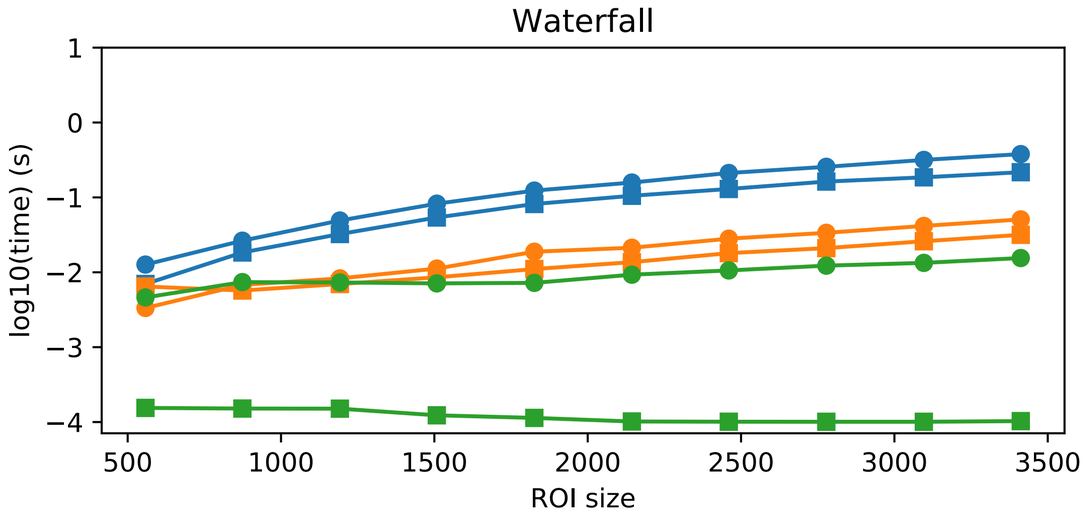



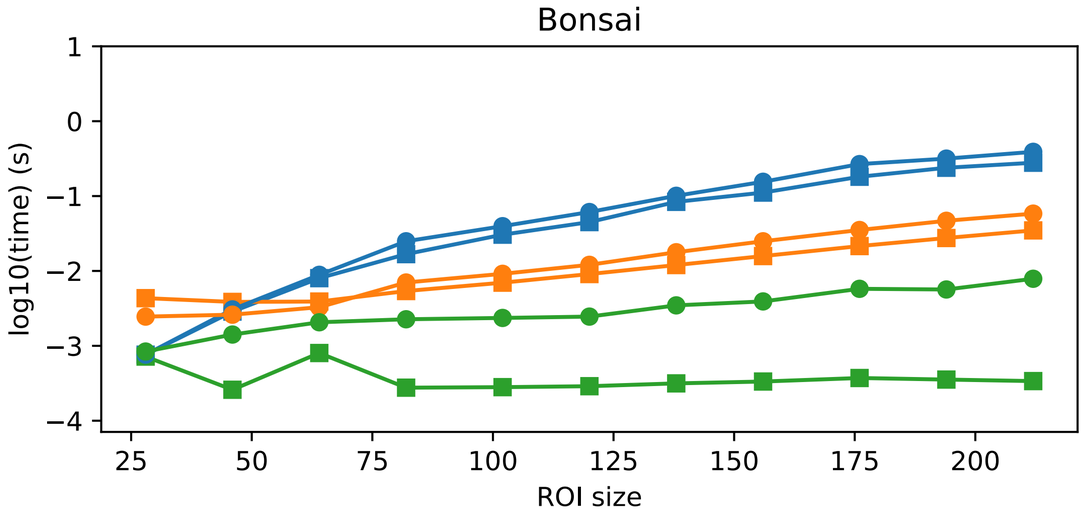

VISUALIZATIONANDMULTIMEDIALAB

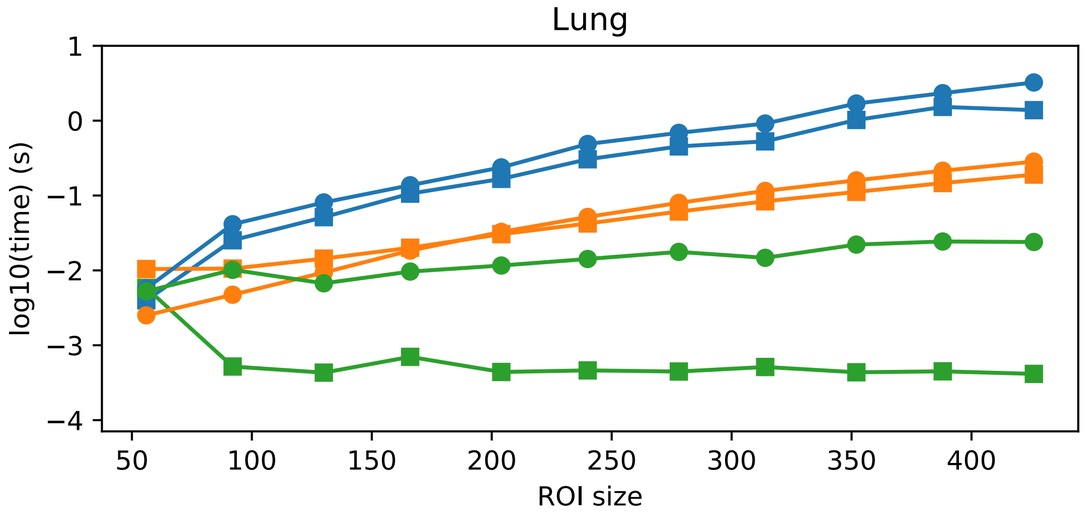
Box ROI

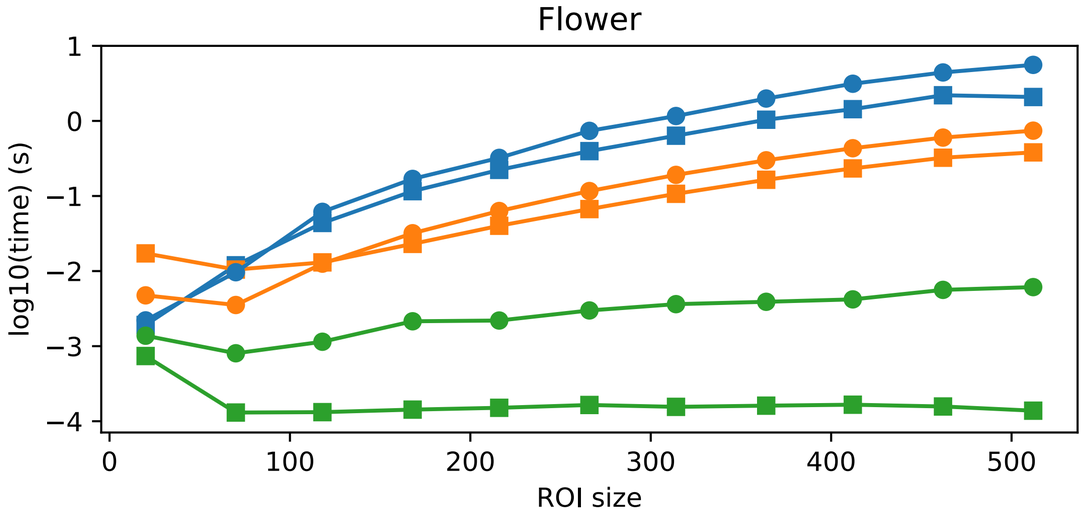
Gaussian ROI

# Scalar Fields: Timings



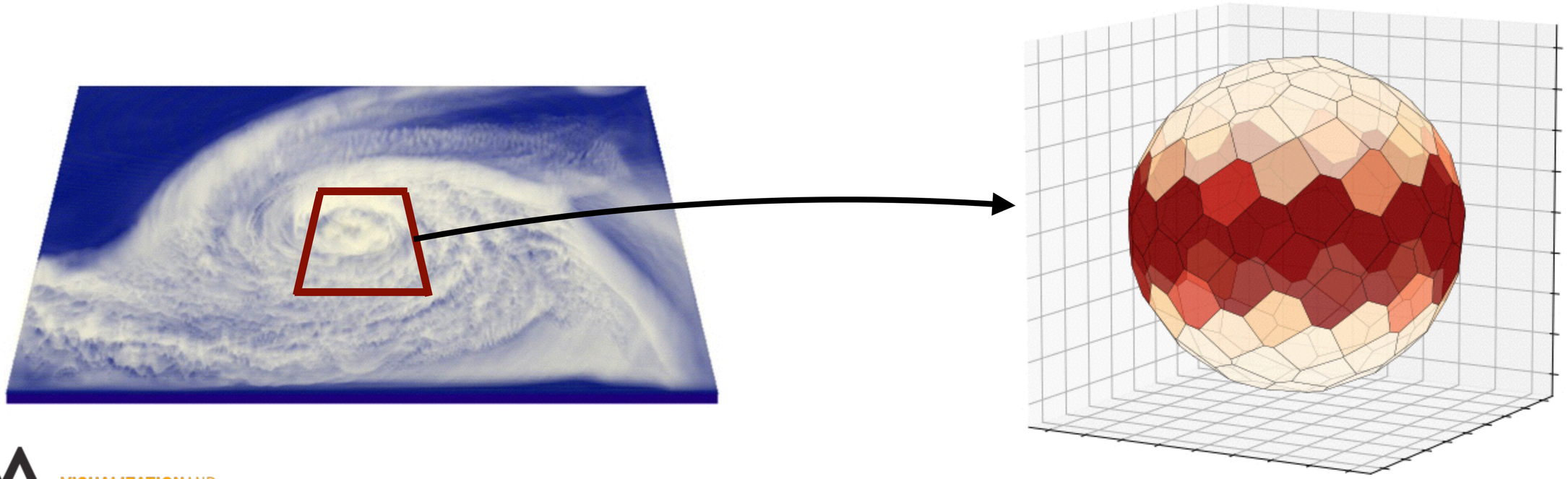




VISUALIZATION AND MULTIMEDIA LAB

Box (BF) Box (BF, CuPy) Box (ours) Gaussian (BF) Gaussian (BF, CuPy) Gaussian (ours)

# Vector Field Histograms

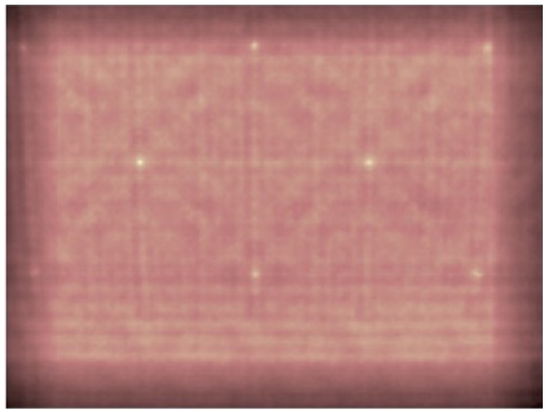
- Use case: **wind direction** in hurricane Isabel
- Dense and complex vector field — how to visualize?
- We quantize wind direction into 128 bins ⟶ **directional histograms**
- Do that for each window ⟶ feature vectors capturing **local wind structure**



# Normalized Cross-Correlation (NCC)

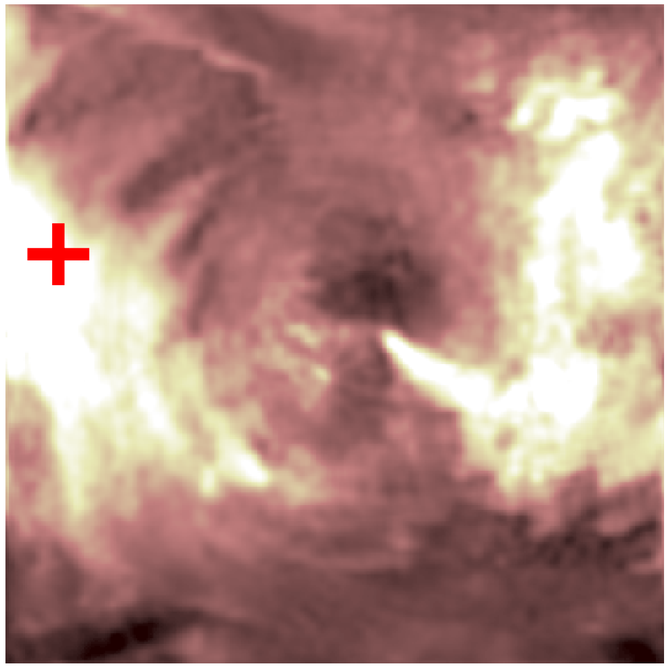
- Defined as cosine similarity between a template $\boldsymbol{t}$ and all feature vectors $\boldsymbol{v}$ :

$$\mathrm{NCC}_{\boldsymbol{t}}(\boldsymbol{v}) = \frac{\boldsymbol{t} \cdot \boldsymbol{v}}{\|\boldsymbol{t}\| \cdot \|\boldsymbol{v}\|}$$

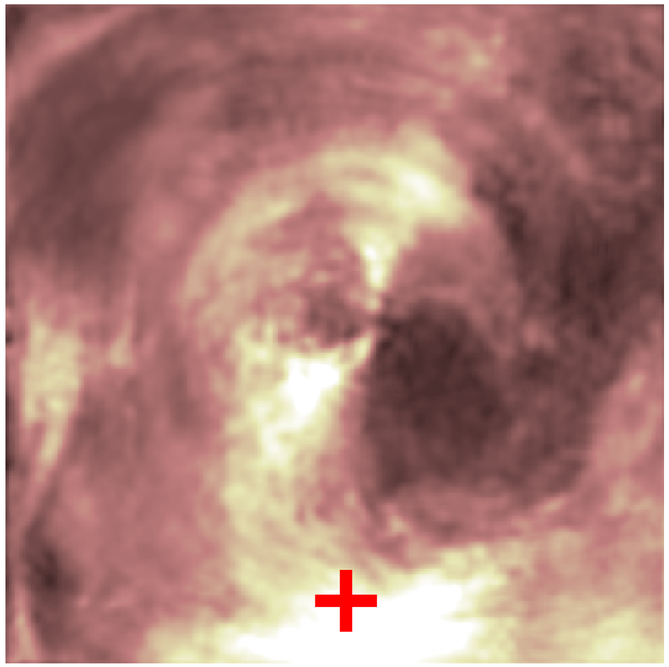
$\mathrm{NCC}$  $\left($  $\right) =$ 

- For us: the feature vectors are directional histograms
- Template: user-defined by clicking on a neighborhood

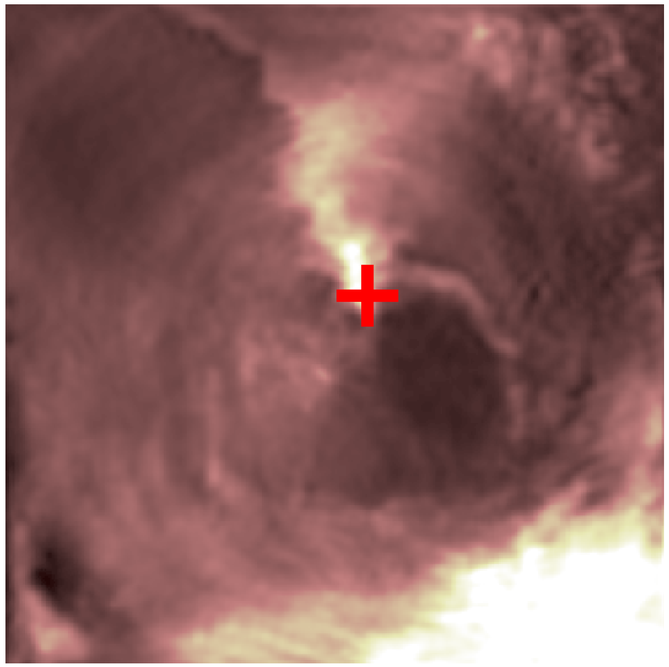
# Normalized Cross-Correlation (NCC)



# Normalized Cross-Correlation (NCC)

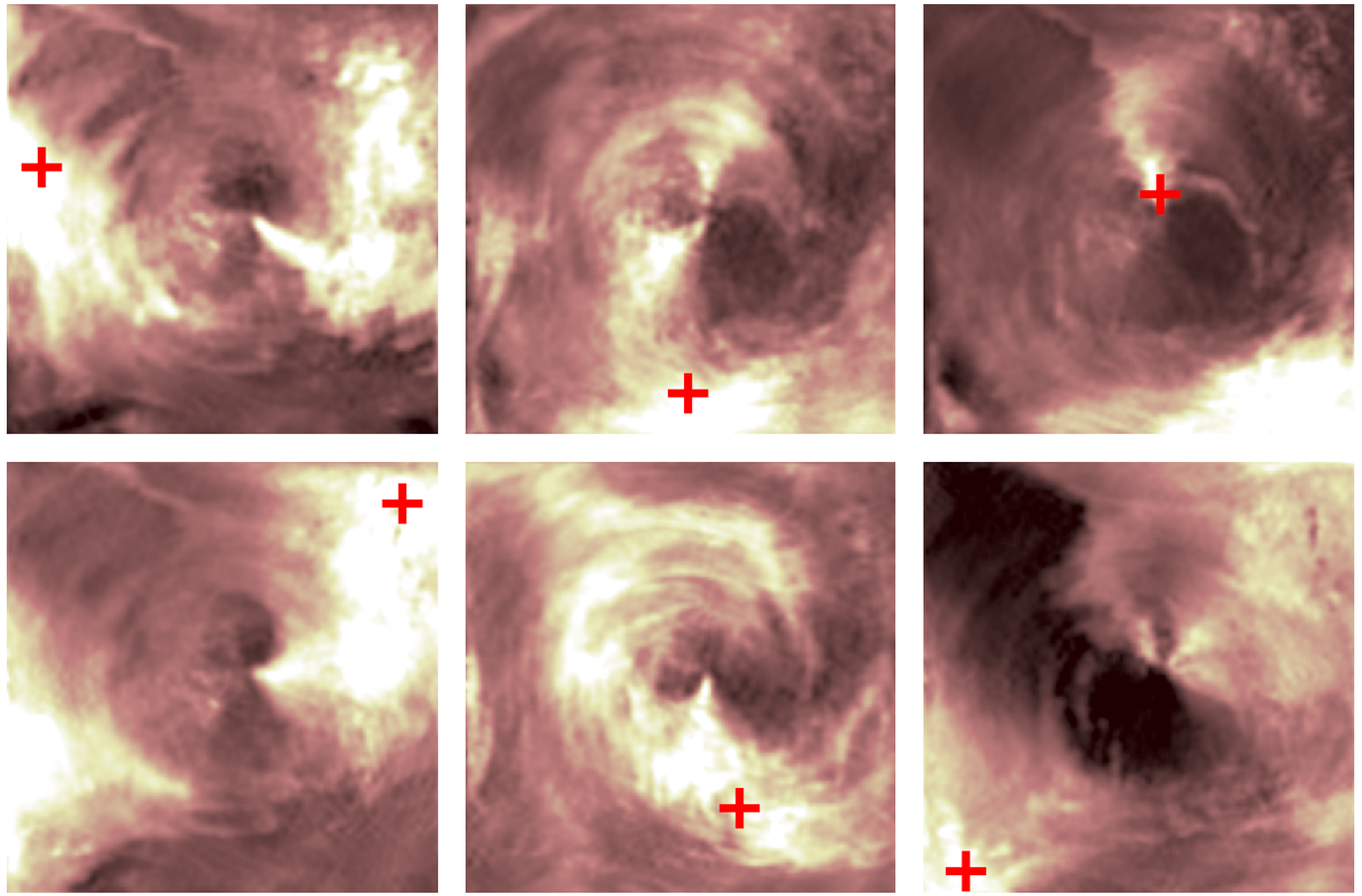


# Normalized Cross-Correlation (NCC)



# Normalized Cross-Correlation (NCC)

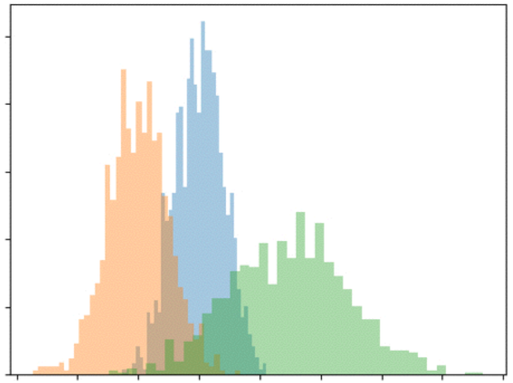
- Responsive interaction (~1s)

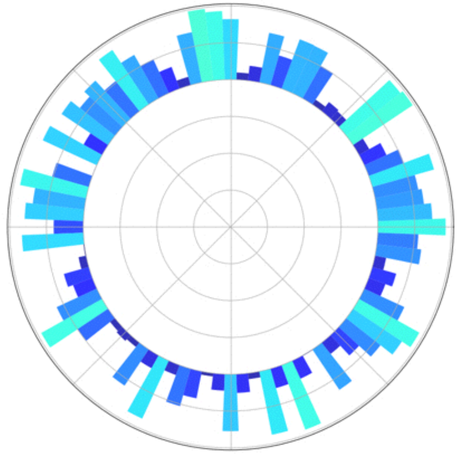


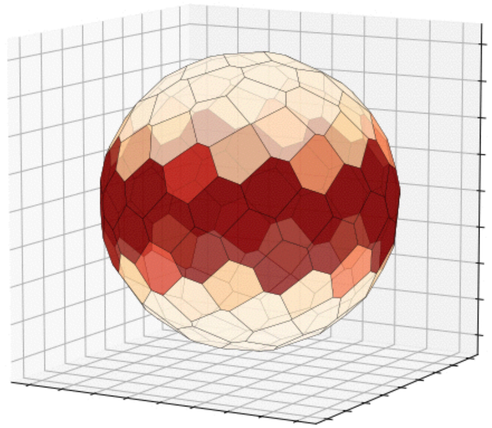
# Conclusions

- Compression/decompression for **integral histograms**
- Using **tensors** (first of such kind)
- **Lossy** compression ⟶ **huge reduction ratios**
- Ideal for **medium to large** ROIs
- Can handle **non-rectangular** ROIs
- Efficient **histogram field** reconstruction

# Thank You!







**Paper:** *"Tensor Decompositions for Integral Histogram Compression and Look-up"*

**Code:** *github.com/rballester/tthistograms*

**Slides:** *github.com/rballester/slides*

***Acknowledgment: University of Zurich CanDoc grant FK-16-012***

# Tensor Manipulation in PyTorch

*github.com/rballester/tntorch*

- Basic and advanced indexing and manipulation
- CP, Tucker, TT and others
- Regression, classification
- Sensitivity analysis

# Why Compress SATs, not the Original Table