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VISUAL-INERTIAL REQUEST FOR ATTITUDE DETERMINATION OF MULTIROTOR AERIAL VEHICLES

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Abstract:*The present paper proposes a visual-inertial attitude estimator for multirotor aerial vehicles (MAV). The vehicle is assumed to be equipped with a 3-axis rate-gyro and a downward-pointing camera, both rigidly fixed on its structure. The method is based on the REQUEST algorithm, which is a well-known solution to the problem of attitude determination form vector measurements. Here the camera provides the vector measurements defined as unit vectors pointing from the camera optical center to known landmarks. The method is evaluated via Monte Carlo simulations which shows its perfomance for different number of visible landmarks and different values of a fading factor parameter of the method.*

Keywords: *Visual-inertial estimation, attitude determination, REQUEST, multirotor aerial vehicle.*

1 INTRODUCTION

In the last fifteen years we saw a fast development of the research on automatic control of micro aerial vehicles (MAV). For example, see the paper Mahony *et al.* (2012) and the video D'Andrea (2013). Nevertheless, in order to realize acrobatic flights like those described in the afore-cited works, it was crucial to adopt an expensive tracking system for pose estimation using stationary infrared cameras (Vicon Motion Systems, 2016). In other words, there is still room for research and technological development in navigation (estimation of position and velocity) and attitude determination of MAVs, aiming at improving their performance and reliability, specially for operations in urban areas and indoors.

Motivated by applications in the aerospace area, the research on attitude determination (AD) has been developing since the 1950 decade and remain active in the current days (Yang, 2012). More recently, the literature on MAV has become concerned with AD, but we claim that it can be still enriched with the formalism, notations, and methods inherited from the aerospace literature.

The well-known Wahba Problem (Wahba, 1965) first formalized the attitude estimation from pairs of vector measurements, as a constrained least-squares problem. Until the 2000 decade, many methods, considered as the seminal ones, appeared for solving this problem; they can be classified into: batch methods (Shuster and Oh, 1981), and recursive methods (Shuster, 1993; Lefferts *et al.*, 1982; Bar-Itzhack and Reiner, 1984; Bar-Itzhack and Oshman, 1985; Bar-Itzhack and Idan, 1987; Bar-Itzhack, 1996; Markley and Mortari, 1999; Markley, 1989, 2003). The reference (Krogh, 2002) re-visits seven different methods of solution to the Wahba Problem.

Regarding the attitude representation, the attitude quaternion is the prefered one. This is because the quaternion has the minimal number of parameters (four) for a global parameterization of the 3D attitude without singularity (Stuelpnagel, 1964). Moreover, it presents a linear kinematics Eq. (Wertz, 1978), and therefore allows a good computational efficiency in attitude simulation.

In particular, a batch quaternion estimator named QUEST (Shuster and Oh, 1981) became very popular in the aerospace community (see Markley (2014), p. 189). As a batch method, the QUEST does not use vector measurements taken in the past to estimate the attitude at the current time. Therefore, to completely estimate the 3D attitude, at least two vector measurements are required at each time instant. For attitude determination of MAVs, the two conventional vectors are the direction of the gravity acceleration (measured by a 3-axis accelerometer) and the direction of the local magnetic field (measured by a 3-axis magnetometer) (Magnussen *et al.*, 2013; Martin and Salaün, 2010).

Despite of its popularity, after the publication of EKF-like methods (Lefferts *et al.*, 1982; Bar-Itzhack and Reiner, 1984; Bar-Itzhack and Oshman, 1985; Bar-Itzhack and Idan, 1987) and improvements in onboard computer technology, the QUEST became obsolete for not considering all the measurement history to estimate the quaternion at the current time. Bar-Itzhack (Bar-Itzhack, 1996) proposed a recursive version of the QUEST method, which was named REQUEST. It is worth mentioning that different from the batch methods, the recursive ones can estimate the attitude using only one vector measurement at a time, since this vector varies sufficiently throughout the vehicle's motion. However, the use of redundant vectors improve the estimator both in performance and reliability. The REQUEST method has a fading factor ρ to deal with the rate-gyro noise in a suboptimal manner. Almost ten years later, Choukroun (Choukroun *et al.*, 2001) proposed the Optimal-REQUEST algorithm, which improved the REQUEST by formulating an optimal factor ρ related to the covariances of the measurement noises. However, we argue that it is quite easy to tune the parameter ρ in the REQUEST method by trial and error, since it is scalar and it is the unique parameter to adjust.

The MAV literature has been intensively reporting the use of camera systems onboard MAVs for navigation and SLAM. Although some of these works include attitude in their estimation scheme, most of them are mainly concerned with position and velocity estimation (Zhang *et al.*, 2009). The paper Shabayek *et al.* (2012) presents a clear and extensive review about moviment estimation from camera measurements, by several distinct approaches, starting from horizon-based methods and passing through vanishing points, optical flow, and methods based on stereo camera systems.

The present paper investigates the problem of attitude determination of MAVs, using a strapdown downward-pointing camera and a 3-axis rate-gyro. A landmark map of the flight terrain is assumed to be available. The attitude estimation method adopted here is the REQUEST (Bar-Itzhack, 1996). The main contribution of this work is the formulation of vector measurements (required not only in the REQUEST, but in any Wahba-based method) as unit vectors given the direction of landmarks with respect to the optical center of the camera. The method is evaluated by Monte Carlo simulations, showing that it is a good alternative to the attitude estimation based on magnetometers and acceleromenters and revealing its performance against the number of visible landmarks. The remaining text is organized in the following manner. Section 2 formally defines the AD problem, while Section 3 presents a solution to it. Section 4 presents the Monte Carlo simulation results. Finally, Section 5 presents the concluding remarks.

2 PROBLEM STATEMENT

Consider the MAV and the two Cartesian Coordinate Systems (CCS) illustrated in Fig. 1. The body CCS $S_{\mathrm{B}} \triangleq \{\hat{x}_{\mathrm{B}}, \hat{y}_{\mathrm{B}}, \hat{z}_{\mathrm{B}}\}$ is attached to the vehicle structure, at its center of mass (denoted by P) and has its z-axis, $\hat{z}_{\mathrm{B}}$, perpendicular to the rotor plane. The reference CCS $S_{\mathrm{R}} \triangleq \{\hat{x}_{\mathrm{R}}, \hat{y}_{\mathrm{R}}, \hat{z}_{\mathrm{R}}\}$ is fixed on the ground at point O and has its z-axis, $\hat{z}_{\mathrm{R}}$, aligned with the local vertical. Fig. 1 also illustrates a set of q landmarks positioned at the points $P_1, P_2, ..., P_q$.

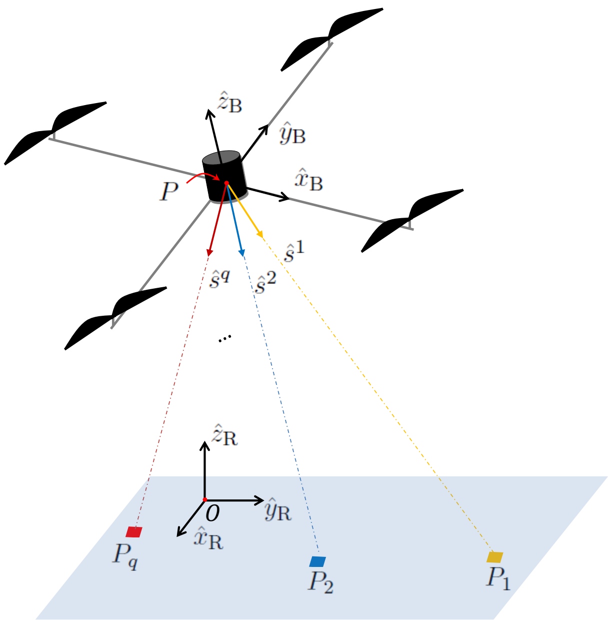

Figure 1: The Cartesian coordinate systems, the multirotor vehicle, and the flight environment.

Assume that the camera is set with its optical center at P and there is a 3-axis rate-gyro installed parallel to S_{B}. Define the unit vectors $\hat{\mathbf{s}}_i$ which describe the direction from P to P_i, for $i = 1, ..., q$. Denote the algebraic representation of $\hat{s}$ in S_{B} and S_{R} by $\mathbf{b}_i \in \mathbb{R}^3$ and $\mathbf{r}_i \in \mathbb{R}^3$, respectively. The representations $\mathbf{b}_i$ and $\mathbf{r}_i$ are related to each other by $\mathbf{b}_i = \mathbf{D}\mathbf{r}_i$, where $\mathbf{D} \in \mathrm{SO}(3)$ is the attitude matrix of S_{B} w.r.t. S_{R}.

Assume that the MAV position P and the landmark positions $P_1, P_2, ..., P_q$ w.r.t. S_{R} are known. Assume also that, at an arbitrary sampling instant k, only a number $n < q$ of landmarks is visible. Note that as the vehicle moves, the set of visible landmarks is changing. Denote the indices of such visible landmarks by $i_1, i_2, ..., i_n$. Based on the above assumptions, define the time sequence of pairs of vector measurements as:

$$\mathcal{V}(k) \triangleq \left\{ \left(\check{\mathbf{b}}_{i_1}(k), \check{\mathbf{r}}_{i_1}(k) \right), \left(\check{\mathbf{b}}_{i_2}(k), \check{\mathbf{r}}_{i_2}(k) \right), ..., \left(\check{\mathbf{b}}_{i_n}(k), \check{\mathbf{r}}_{i_n}(k) \right) \right\}, \tag{1}$$

where $\check{\mathbf{b}}_i$ and $\check{\mathbf{r}}_i$, $\forall i = i_1, ..., i_n$, are measures at instant k of $\mathbf{b}_i$ and $\mathbf{r}_i$, respectively. Note that the number of vector measurement pairs in $\mathcal{V}(k)$ varies over time k.

Now, in order to define a measurement model (relating the vector measurements with the desired attitude), consider just one arbitrary vector measurement pair $\left(\check{\mathbf{b}}_i(k), \check{\mathbf{r}}_i(k)\right) \in \mathcal{V}(k)$, for some $i \in \{i_1, ..., i_n\}$. The measurement model used here is given by

$$\check{\mathbf{b}}_i(k) = \mathbf{D}(\mathbf{q}(k))\check{\mathbf{r}}_i(k) + \delta\mathbf{b}_i(k), \tag{2}$$

where $\{\delta\mathbf{b}_i(k)\}$ is a zero-mean white Gaussian sequence with covariance $\mathbf{R}_i(k)$; and $\mathbf{q}(k) \in \mathbb{R}^4$ is the attitude quaternion which parameterizes $\mathbf{D}$ at instant k. The expression $\mathbf{D}(\mathbf{q})$ denotes the attitude matrix of S_{B} w.r.t. S_{R} corresponding to $\mathbf{q}$. It is given by (Markley, 2014)

$$\mathbf{D}(\mathbf{q}(k)) = (q^2 - \mathbf{e}^{\mathrm{T}}\mathbf{e})\mathbf{I}_3 + 2\mathbf{e}\mathbf{e}^{\mathrm{T}} - 2q[\mathbf{e}\times], \tag{3}$$

where $[\mathbf{e}\times]$ denotes the cross-product matrix of $\mathbf{e} \triangleq \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}^{\mathrm{T}}$,

$$[\mathbf{e}\times] \triangleq \begin{bmatrix} 0 & -e_3 & e_2 \\ e_3 & 0 & -e_1 \\ -e_2 & e_1 & 0 \end{bmatrix}. \tag{4}$$

On the other hand, the attitude kinematics model can be expressed as (Wertz, 1978)

$$\mathbf{q}(k+1) = \mathbf{\Phi}(k)\mathbf{q}(k), \tag{5}$$

with

$$\mathbf{\Phi}(k) \triangleq \exp\left\{ \frac{1}{2} \begin{bmatrix} -[\boldsymbol{\omega}(k)\times] & \boldsymbol{\omega}(k) \\ -\boldsymbol{\omega}(k)^{\mathrm{T}} & \mathbf{0}_{1\times3} \end{bmatrix} \Delta t \right\}, \tag{6}$$

where $\boldsymbol{\omega}(k)$ is the S_{B} representation of the angular velocity of S_{B} w.r.t. S_{R} and Δt is the sampling period.

Finally, let the rate-gyro measurement $\check{\boldsymbol{\omega}}(k) \in \mathbb{R}^3$ at instant k be modeled by

$$\check{\boldsymbol{\omega}}(k) = \boldsymbol{\omega}(k) + \delta\boldsymbol{\omega}(k), \tag{7}$$

where $\{\delta\boldsymbol{\omega}(k)\} \in \mathbb{R}^3$ is a zero-mean white Gaussian sequence with covariance $\mathbf{Q}(k)$.

The main problem of the present paper is to recursively estimate $\mathbf{q}(k)$ using the kinematic model Eq. (5), the measurement models Eq. (2) and Eq. (7), the sequence of rate-gyro measurements $\{\check{\boldsymbol{\omega}}(1), \check{\boldsymbol{\omega}}(2), ..., \check{\boldsymbol{\omega}}(k)\}$, and the sequence of vector measurements $\{\mathcal{V}(1), \mathcal{V}(2), ..., \mathcal{V}(k)\}$.

3 PROBLEM SOLUTION

The present section proposes a solution to the attitude determination problem defined in Section 2 using the REQUEST algorithm (Bar-Itzhack, 1996). First, the Wahba problem as well as the QUEST method are reviewed.

3.1 The Wahba Problem and the QUEST Algorithm

The Wahba problem for computing the quaternion estimate $\hat{\mathbf{q}}(k)$ at instant k can be stated as the minimization of

$$J(\mathbf{q}(k)) = \frac{1}{2}\sum_{i=1}^{n} a_i \|\check{\mathbf{b}}_i(k) - \mathbf{D}(\mathbf{q}(k))\check{\mathbf{r}}_i(k)\|^2, \tag{8}$$

subject to $\|\mathbf{q}(k)\| = 1$, where $\left(\check{\mathbf{b}}_i(k), \check{\mathbf{r}}_i(k)\right)$ is the pair of vector measurements (defined in Section 2), $\mathbf{D}(\mathbf{q}(k))$ is the attitude matrix corresponding to the quaternion $\mathbf{q}(k)$ (see Eq. (3)), a_i is a positive weight associated with the ith measurement pair, and n is the number of vector measurements available at instant k.

The minimization problem of Eq. (8) can be replaced by the maximization of (Shuster and Oh, 1981)

$$G(\mathbf{q}(k)) = \mathbf{q}(k)^{\mathrm{T}}\mathbf{K}(k)\mathbf{q}(k), \tag{9}$$

where

$$\mathbf{K}(k) \triangleq \begin{bmatrix} \mathbf{S}(k) - \sigma(k)\mathbf{I}_3 & \mathbf{z}(k) \\ \mathbf{z}(k)^{\mathrm{T}} & \sigma(k) \end{bmatrix} \in \mathbb{R}^{4\times4}, \tag{10}$$

$$\mathbf{S}(k) \triangleq \mathbf{B}(k) + \mathbf{B}(k)^{\mathrm{T}} \in \mathbb{R}^{3\times 3}, \tag{11}$$

$$\sigma(k) \triangleq \frac{1}{m(k)} \sum_{i=1}^{n} a_i \check{\mathbf{b}}_i(k)^{\mathrm{T}} \check{\mathbf{r}}_i(k) \in \mathbb{R}, \tag{12}$$

$$\mathbf{B}(k) \triangleq \frac{1}{m(k)} \sum_{i=1}^{n} a_i \check{\mathbf{b}}_i(k) \check{\mathbf{r}}_i(k)^{\mathrm{T}} \in \mathbb{R}^{3\times 3}, \tag{13}$$

$$\mathbf{z}(k) \triangleq \frac{1}{m(k)} \sum_{i=1}^{n} a_i \left[\check{\mathbf{b}}_i(k)\times\right] \check{\mathbf{r}}_i(k) \in \mathbb{R}^{3}, \tag{14}$$

and

$$m(k) \triangleq \sum_{i=1}^{n} a_i. \tag{15}$$

The solution $\hat{\mathbf{q}}(k)$ to the maximization of $G(\mathbf{q}(k))$ in 9 is given by the following eigenvalue/eigenvector equation:

$$\mathbf{K}(k)\hat{\mathbf{q}}(k) = \lambda \hat{\mathbf{q}}(k), \tag{16}$$

where λ is the maximum eigenvalue of $\mathbf{K}(k)$. In other words, the solution $\hat{\mathbf{q}}(k)$ is the eigenvector corresponding to the maximum eigenvalue of $\mathbf{K}(k)$. Reference Shuster and Oh (1981) presents an efficient algorithm for solving the above eigenvalue/eigenvector problem; this is the well-known QUEST algorithm. The same paper also shows that λ is close to 1 (for noise-free measurements, it is exactly 1). In short, the cited work shows that the optimal Gibbs vector (see Wertz (1978), for information about different attitude parameterizations) is given by

$$\mathbf{p}(k) = \left[(\lambda + \sigma(k))\,\mathbf{I}_3 - \mathbf{S}(k)\right]^{-1} \mathbf{z}(k), \tag{17}$$

and the corresponding quaternion is given by

$$\hat{\mathbf{q}}(k) = \frac{1}{\sqrt{(1 + \mathbf{p}(k)^{\mathrm{T}}\mathbf{p}(k))}} \begin{bmatrix} \mathbf{p}(k) \\ 1 \end{bmatrix}. \tag{18}$$

3.2 The REQUEST Algorithm

The formulation presented here is based on the original work Bar-Itzhack (1996). Particularly, in the present work, since the visible landmarks can change throughout the MAV motion, we consider that a variable number $n(k)$ of vector measurements is taken at the kth algorithm iteration.

Let us denote by $\mathbf{K}_{j|k}$ the QUEST $\mathbf{K}$ matrix at instant j, but constructed with vector measurements taken up to instant k. Assume that $\mathbf{K}(k|k)$ is given at instant k. According to the REQUEST method, the prediction of the $\mathbf{K}$ matrix is given by

$$\mathbf{K}(k+1|k) = \check{\mathbf{\Phi}}(k)\mathbf{K}(k|k)\check{\mathbf{\Phi}}(k)^{\mathrm{T}}, \tag{19}$$

where $\check{\mathbf{\Phi}}(k)$ is the quaternion transition matrix with the same form of Eq. (6), but computed with the rate-gyro measurement $\check{\boldsymbol{\omega}}(k)$ instead of the true angular velocity $\boldsymbol{\omega}(k)$.

On the other hand, to update $\mathbf{K}$ with new pairs of vector measurements taken at instant $k+1$, one can use

$$\mathbf{K}(k+1|k+1) = \frac{\rho m(k)}{\rho m(k) + \delta m(k+1)} \mathbf{K}(k+1|k) + \frac{1}{\rho m(k) + \delta m(k+1)} \delta\mathbf{K}(k+1), \tag{20}$$

where $0 \le \rho \le 1$ is a fading factor tuned to reduce the effect of rate-gyro measurement errors on the estimation of $\mathbf{K}$ and

$$m(k+1) = m(k) + \delta m(k+1), \tag{21}$$

$$\delta m(k+1) = \sum_{i=1}^{n(k)} a_i, \tag{22}$$

$$\delta\mathbf{K}(k+1) \triangleq \begin{bmatrix} \delta\mathbf{S}(k+1) - \delta\sigma(k+1)\mathbf{I}_3 & \delta\mathbf{z}(k+1) \\ \delta\mathbf{z}(k+1)^{\mathrm{T}} & \delta\sigma(k+1) \end{bmatrix}, \tag{23}$$

$$\delta\mathbf{S}(k+1) \triangleq \delta\mathbf{B}(k+1) + \delta\mathbf{B}(k+1)^{\mathrm{T}}, \tag{24}$$

$$\delta\mathbf{B}(k+1) \triangleq \sum_{i=1}^{n(k)} a_i \check{\mathbf{b}}_i(k+1)\check{\mathbf{r}}_i(k+1)^{\mathrm{T}}, \tag{25}$$

$$\delta\mathbf{z}(k+1) \triangleq \sum_{i=1}^{n(k)} a_i \left[\check{\mathbf{b}}_i(k+1)\times\right] \check{\mathbf{r}}_i(k+1), \tag{26}$$

$$\delta\sigma(k+1) \triangleq \sum_{i=1}^{n(k)} a_i \check{\mathbf{b}}_i(k+1)^{\mathrm{T}} \check{\mathbf{r}}_i(k+1). \tag{27}$$

Finally, the desired quaternion estimate $\hat{\mathbf{q}}(k+1|k+1)$ at instant $k+1$ using information up to instant $k+1$ is obtained in the same way as in the QUEST method (see Eq. (17)-18) as the eigenvector of $\mathbf{K}(k+1|k+1)$ corresponding to the maximum eigenvalue of the same matrix.

4 METHOD EVALUATION

The attitude motion is simulated with the quaternion kinematic equation (Wertz, 1978) excited by a true angular velocity $\boldsymbol{\omega}$ of S_{B} w.r.t. S_{R} that induces a cone motion. The representation in S_{B} of such an angular velocity is given by (Waldmann, 2001)

$$\boldsymbol{\omega}(t) = \begin{bmatrix} -\Omega^p \sin\theta^c \cos(\Omega^p t) \\ -\Omega^p \sin\theta^c \sin(\Omega^p t) \\ \Omega^p (\cos\theta^c - 1) \end{bmatrix}, \tag{28}$$

where t is the continuous time, $\Omega^p \in \mathbb{R}$ is the precession rate and $\theta^c \in \mathbb{R}$ is the cone angle.

The vector measurements are simulated using Eq. (1), taking into account a total of four landmarks, whose positions are represented in S_{R} by the points $P_1 = [0.3\ 0.2\ 1]^{\mathrm{T}}, P_2 = [0.5\ 0.8\ 1]^{\mathrm{T}}, P_3 = [0.7\ 0.3\ 1]^{\mathrm{T}}, P_4 = [0.5\ 0.5\ 1]^{\mathrm{T}}$. Moreover, without loss of generality, the vehicle's center of mass is assumed to be fixed at $P = [0.5\ 0.5\ 0.4]^{\mathrm{T}}$. Therefore, assuming that the S_{R} representation of the vector measurements are noise-free, we have

$$\check{\mathbf{r}}_1 = \begin{bmatrix} -0.2857 \\ -0.4286 \\ 0.8571 \end{bmatrix}, \check{\mathbf{r}}_2 = \begin{bmatrix} 0 \\ 0.4472 \\ 0.8944 \end{bmatrix}, \check{\mathbf{r}}_3 = \begin{bmatrix} 0.3015 \\ -0.3015 \\ 0.9045 \end{bmatrix}, \check{\mathbf{r}}_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{29}$$

The parameter values used in the simulation model are all listed in Table 1.

Table 1: Parameters of the ground-truth model.

Sampling time	$\Delta t = 0.05$ s
Simulation period	$\Delta t \times k_f = 10$ s
Covariance of the vector measurement errors	$\mathbf{R}_i(k) = 1.6 \times 10^{-3}\mathbf{I}_3, \forall i$
Covariance of the rate-gyro measurement errors	$\mathbf{Q}(k) = 7.1 \times 10^{-7}\mathbf{I}_3$
Precession rate	$\Omega^p = 60$ degree/s
Cone angle	$\theta^c = 20$ degree

Here, we use the REQUEST method reviewed in Subsection 3.2 to estimate the vehicle's attitude quaternion throughout the coning motion. The weighting factors associated with the vector measurements are chosen as $a_i = 1, \forall i$.

In order to evaluate the performance of the method, Monte Carlo simulations with 1000 are realized. The principal Euler angle corresponding to the true attitude error is considered as a figure of merit. It is given by

$$\epsilon(k) = \operatorname{acos}\left(\frac{\operatorname{tr}\left(\mathbf{D}(\hat{\mathbf{q}}(k|k))\mathbf{D}(\mathbf{q}(k))^{\mathrm{T}}\right) - 1}{2}\right). \tag{30}$$

The simulation is repeated for different values of the fading factor ρ and different number n of visible landmarks. The sample mean μ_ϵ and standard deviation σ_ϵ of $\epsilon(k)$ at the final discrete time k_f for each combination of ρ and n are registered in Table 2.

Table 2: Monte Carlo simulation results. The quantities μ_ϵ and σ_ϵ are, respectively, the simple mean and sample standard deviation of $\epsilon(k_f)$, where k_f is the final discrete-time instant.

n	ρ	$(\mu_\epsilon, \sigma_\epsilon)$
2	0	$(3.95, 1.93)$
	0.5	$(2.42, 1.12)$
	0.95	$(1.05, 0.33)$
3	0	$(3.40, 1.61)$
	0.5	$(2.13, 0.89)$
	0.95	$(0.98, 0.26)$
4	0	$(3.16, 1.54)$
	0.5	$(1.98, 0.84)$
	0.95	$(0.96, 0.27)$

In Table 1, one can see an improvement in performance as the fading factor ρ and the number of visible landmarks n are increased. However, the observed improvement when n is changed from $n = 3$ to $n = 4$ is not significant, which suggest that $n = 3$ is a good choice. From the equations of the REQUEST method in Subsection (3.2), we conclude that if $\rho = 0$, and therefore no past measurements are considered in the current estimate, this algorithm is degenerated into the

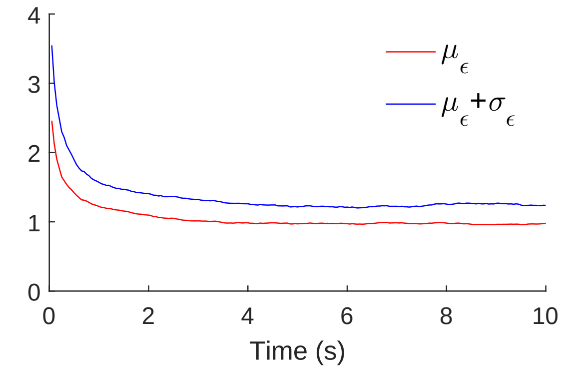

Figure 2: Sample mean and standard deviation of ϵ for $n = 3$ and $\rho = 0.95$.

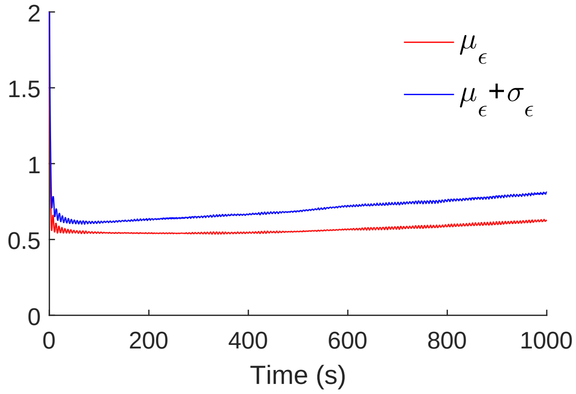

Figure 3: Sample mean and standard deviation of ϵ for $n = 4$ and $\rho = 1$.

QUEST method reviewed in Subsection 3.1. In this case, we observe the worst performance. Figure 2 shows a time plot of $\mu_\epsilon(k)$ and $\mu_\epsilon + \sigma_\epsilon(k)$ for $n = 3$ and $\rho = 0.95$.

It is worth mentioning two important cases that are not shown in Table 2. The first one is when we consider only $n = 1$ visible landmark. In this case, the matrix within the brackets in Eq. (17) becomes singular and then we cannot extract the attitude estimate from the $\mathbf{K}$ matrix. The second case is when we set the fading factor in $\rho = 1$. In this case, there is no fading factor at all and thus the attitude estimates show a divergent behavior, as seen in Fig. 3. This divergence is due to the time propagation of past vector measurements using noisy rate-gyro measurements; the fading factor ρ was introduced in Bar-Itzhack (1996) just to avoid such a divergence.

5 CONCLUDING REMARKS

The classic Wahba problem was recast here for MAV attitude determination setting the vector measurements as the unit vectors pointing from the optical center of a downward-pointing camera to landmarks within its field of view. The REQUEST algorithm was chosen here as the solution method due to its simple implementation and tuning. Moreover, it fits well the problem, since it is originally formulated in a way that permits a variable number of vector measurements over the time.

In this paper, we brought the formalism on attitude determination from the aerospace area to adapt it to the MAV literature. The method presented here was evaluated by Monte Carlo simulations, which showed its effectiveness as well as how its performance varies with respect to the number of visible landmarks and the value of a fading factor parameter. Different from the conventional attitude determination methods for MAVs, which uses magnetometer and accelerometer, here we adopt only a single camera to extract all required vector measurements.

For future works, we investigate the use of new attitude determination algorithms and prepare a bench experiment for generating real data for evaluating them. The experiment consists of a low-cost single-board Linux computer collecting data from triaxial rate-gyro, accelerometer and magnetometer, as well as from a camera. This setup is mounted on a Quanser 3D Hover platform, whose encoders provide a ground-truth for attitude. This experiment will allow a quantitative comparison between camera-based attitude determination and those methods based on magnetometer and accelerometer.

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