Optimizing Power Distribution Network - Phase II submission

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1 Introduction

Power-grid optimization (PGO) [1][2][3] is one of the most popular optimization problems in the electrical power industry. Solutions to such problem can reduce the cost of power production, decrease energy loss in distribution networks which can lead to a reduction of harmful impact on the environment. The PGO problem is generally formulated as a large-scale mixed integer nonlinear problem and solving it is very difficult due to the nonlinear cost function and the combinatorial nature of the set of feasible solutions. It has been proven that PGO is not only NP-hard but also NP-complete, so it is impossible to develop an algorithm with polynomial computation time to solve it. With the increasing number of energy sources, PGO problem poses a hard challenge making it crucial to develop effective methodologies to tackle this challenge.

Power-grid optimization is concerned with minimizing the total operational cost to meet an estimated power demand of customers over a given time horizon. The optimized solution needs to take care of various factors such as the energy demand of different customers, the capacity of different nodes in the network, and the transmission and distribution losses. As the problem is NP-complete, the number of decision variables involved in this problem grows exponentially with the size of the network, making it difficult for classical computers to solve the problem efficiently. As quantum mechanics allow us to have super-position of states, quantum computers can handle combinatorial optimization better than classical computers.

The model used for this challenge is IEEE-14 Bus system. From a set of units U the amount of power generated by each unit i to satisfy a given load requirement L is represented by p_i. The fuel cost f_i of the committed unit i is usually formulated as a quadratic polynomial with A_i, B_i, and C_i being the coefficients of this polynomial. The PGO problem for a single time period is mathematically represented by a mixed-integer quadratic programming problem in Eqn.(1).

$$\begin{aligned} \min & \sum_{i \in U} f_i \\ \text{s.t. } & f_i = A_i y_i + B_i p_i + C_i p_i^2 \\ & \sum_{i \in U} p_i = L \\ & P_{\min,i} y_i \leq p_i \leq P_{\max,i} y_i \quad \forall i \in U \\ & y_i \in \{0,1\} \end{aligned} \tag{1}$$

Here binary variable y_i represents whether the corresponding unit i is online. The generated power limits of each unit i are imposed by lower bound $P_{min,i}$ and an upper bound $P_{max,i}$. As the cost function includes several binary variables and some continuous variables, we use the Quantum

Approximate Optimization Algorithm (QAOA) for optimization of such variables. QAOA provides a scalable advantage over classical algorithms for combinatorial optimization problems with binary variables. To simultaneously optimize continuous variables we discretize those variables and convert them into a binary variable. As QAOA optimize over discrete binary variables, it can avoid some major issues such as barren plateau problem[4] that are unavoidable in gradient decent and quantum machine learning approaches that work with continuous variables.

2 Quantum Approach to Scenario 1

The Mixed-Integer Linear Programming (MILP) can be solved for few number of decision variables using classical computer. As the complexity grows, the problem can not be solved using classical computers, hence researchers are looking for a quantum algorithm to solve such problems. Though the quantum computer can solve problems effectively, it is important to understand that the qpus can handle binary variables only. If there are any continuous or discrete decision variables in the optimization, the problem has to be restated in terms of binary variables. Hence we implement a way to convert the MILP into a Binary Quadratic Model (BQM) [5]. To convert the continuous variables into binary variables we discretize the continuous variable by partitioning the full range into a finite number of bins with equal width. For example, a decision variable P with range $(0, 20)$ can be partitioned into $N = 5$ bins, $(0, 4), (4, 8), (8, 12), (12, 16), (16, 20)$. Each of the sets will be assigned a binary variable $z_i \in \{0, 1\}$. So if the optimized value falls into one of these range, that particular binary variable z_i will be active ($z_i = 1$) and other decision variables are (z_0). Using this idea we can discretize the continuous variable in the PGO as follows,

$$\min \quad \textstyle\sum_{i\in U} f_i \tag{2}$$

$$\text{s.t.} \quad h_i = \left(\frac{P_{\max,i} - P_{\min,i}}{N}\right) \tag{3}$$

$$p_i = \textstyle\sum_{k=1}^{N+1} \left(P_{\min,i} + (k-1)h_i\right) z_{ik} \tag{4}$$

$$f_i = A_i\,(1 - v_i) + B_i p_i + C_i p_i^2 \tag{5}$$

$$v_i + \textstyle\sum_{k=1}^{N+1} z_{ik} = 1 \quad \forall i \in U \tag{6}$$

$$\textstyle\sum_{i\in U} p_i = L \tag{7}$$

$$v_i, z_{ik} \in \{0, 1\} \tag{8}$$

Where h_i in Eqn.(4) is the width of the generator i and p_i in Eqn.(5) is the discretized continuous variable. $z_{i,k}$ is the binary decision variable of a particular bin in the generator i. Variable $v_{i,k}$ is the set of auxiliary variables, which is 0 if a particular generator is active and 1 is particular generator is inactive.

These set of equations constitutes what is knows are binary quadratic model (BQM) that can be solved in a quantum computer. The BQM is first converted into a Ising Hamiltonian. The mapping is done by eigenstates of Pauli Z operator. The map is to encode the 0 and 1 binary variables to eigenstates $|0\rangle$ and $|1\rangle$ and the decision variables to Z operators. This will convert the classical cost function with weights into a quantum Ising Hamiltonian and the weights into linear and quadratic coefficients h and J. Finally, the ground state of the Hamiltonian can be found using exact diagonalization, variational quantum eigensolver (VQE) [6] and quantum annealing [7]. For a small Hamiltonian, exact diagonalization methods can be done easily, for large size Hamiltonian, we have to resort for VQE or annealing, which is really quick in getting the ground state of the Hamiltonian. In our solution, we are going to use quantum approximate optimization algorithm (QAOA) [8] to get the ground state of the Hamiltonian. The first step in QAOA is to choose a quantum circuit ansatz that parameterizes the quantum state. Typically, this involves alternating layers of two types of quantum gates: the problem unitary $U(C)$ (evolving under the Ising Hamiltonian) and the mixer unitary $U(B)$ (introducing entanglement). The expectation value of the Hamiltonian is calculated using the parametrized state. Then the expected value is minimized using classical gradient optimization method to find the optimal parameters in the ansatz. This is iteratively done with updated parameter values until it reaches minimum cost. Putting the parameter value corresponding to the minimum cost into QAOA ansatz gives us the ground state of the Hamiltonian.

For scenario 1, the number of decision variables in our problem is $4(numberofbins)+4(generators)+4(auxiliaryvariables) = 20)$. Hence we used the Aria 1 by IonQ which has 25 qubits. The results of the problem with 200 measurement shots returns most probable solution and the code output returns the optimization solution, that satisfies all the constraints and the bounds in the original optimization problem. Below is the solution, the first line tells us which generators are active bus system and next few lines gives us the active power lines in the IEEE $-$ 14 bus system, which gives the expected topology in the scenario 1.

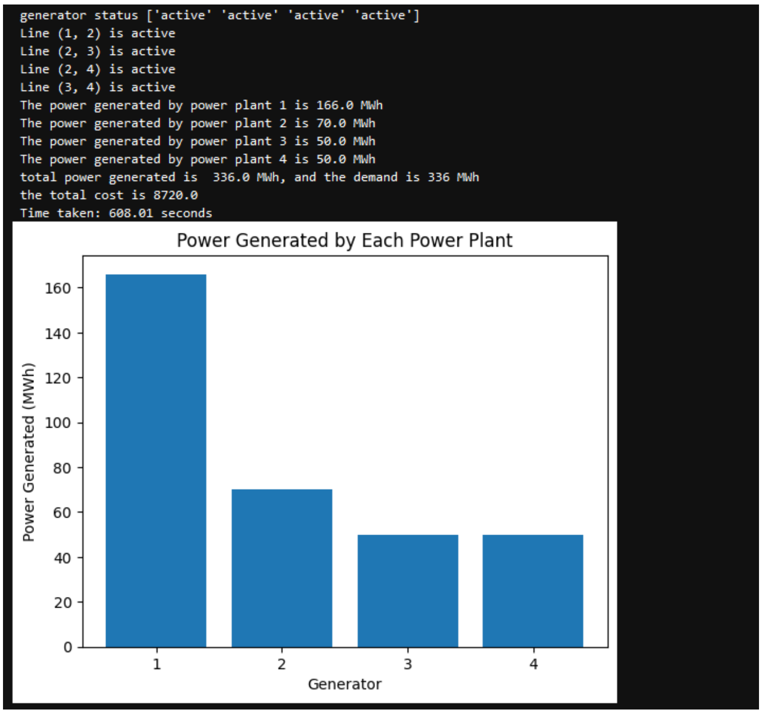

Figure 1: (
The quantum solution for the scenario 1 obtained using IONQ Aria 1 qpu which consists of 25 qubits. The number of shots performed on the qpu = 200.)

3 Solution for Scenario 2

The solution 2 for scenario 2, slightly more complicated than Scenario 1. The scenario 2 is to find the optimal topology and power generation for IEEE $-$ 14 bus system if corrective actions such as generation redispatch or transmission line switching is happened. The extra constraints needed to be added is

- Generator re-dispatch constraint [9]:

$$p_i(new) = p_i + (r_{i,up}\Delta g_{i,up}) - (r_{i,down}\Delta g_{i,down}). \tag{9}$$

  where $r_{i,up/down} \in \{0,1\}$ is the re-dispatch status and $r_{i,up} + r_{i,down} = 1$ should satisfy.

- The new powers should satisfy the demand constraint

$$\sum_{i\in U} p_i(new) = L. \tag{10}$$

  This ensures that the even beyond the generator is running its limits, the total power equals the demand.

- Transmission line switching constraint:

$$\sum_{k=1}^{N+1} z_{mk} + \sum_{k=1}^{N+1} z_{nk} = 1, \quad \forall (m,n) \in \text{lines} \tag{11}$$

  where m and n are the buses connected in the power line. This condition ensures that particular faulty line is not activated. An alternate path is chosen by the algorithm and an optimized solution is provided.

The number of decision variables in is this problem is $2 \times 2(\text{numberofbins}) + 5 \times 2(\text{generators}) + 5 \times 2(\text{auxiliaryvariables}) = 24)$. The problem is again solved using IONQ Aria 1 qpu with 15 shots. The results are given below.

The bus splitting constraint (For constraint expressions, refer to our phase 1 submission) can be implemented by introducing new variables $w_{i,l}$. The number of variables to be introduced for IEEE-14 bus system is $14 \times 14 = 196$ new variables and new continuous 14 variables for Voltages at bus i, discretizing this continuous variables to $N = 3$ requires additionally 42 new variables. Hence a total of 262 qubits are required to solve the bus split corrective action. The highest number of qubits available to us 256 based on reservation basis. Hence this problem can not be solved using Amazon Bracket services provided currently. So we restrict ourselves to only redispatch and the transmission line switching problem and obtained the results.

4 Conclusion

The power grid optimization problem can be solved using a quantum computer. The solution for the scenario is very accurate because of less number of variables and more shots of measurements can be performed on the qpu. The solution for the scenario is very close to the demand constraint and the solution accuracy can be further improved by increasing the number of discretized bins (which requires more number of qubits). The optimal topology and power generation for both scenario 1 and scenario 2 are obtained. For decision variables less than 5, classical algorithms performs better and shows results faster than quantum algorithm. This is because the problem is constructed as a QAOA and solved using gate based quantum computer. To actually see the speed up, we have to resort to annealer which shows actual speedup in finding the optimazation problems which are not available using Amazon Braket services. But as we increase the number of variables to 12, the

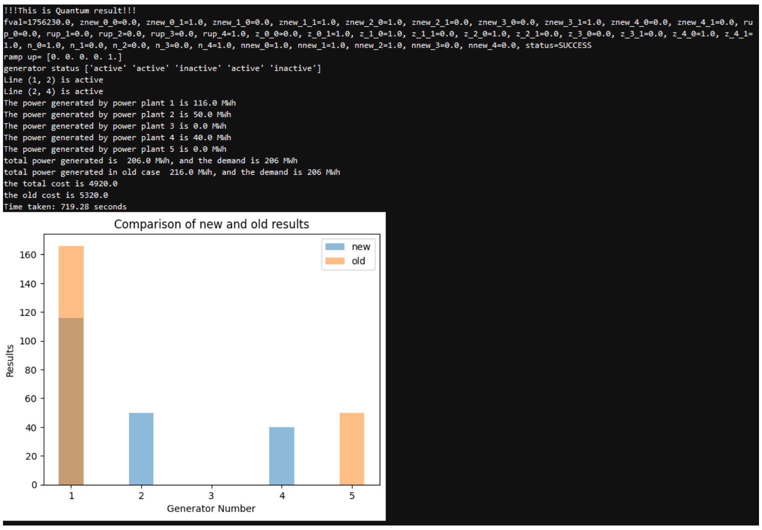

Figure 2: (
The quantum solution for the scenario 1 obtained using IONQ Aria 1 qpu which consists of 25 qubits. The number of shots performed on the qpu = 25. This is just for the redispatch constraint.)

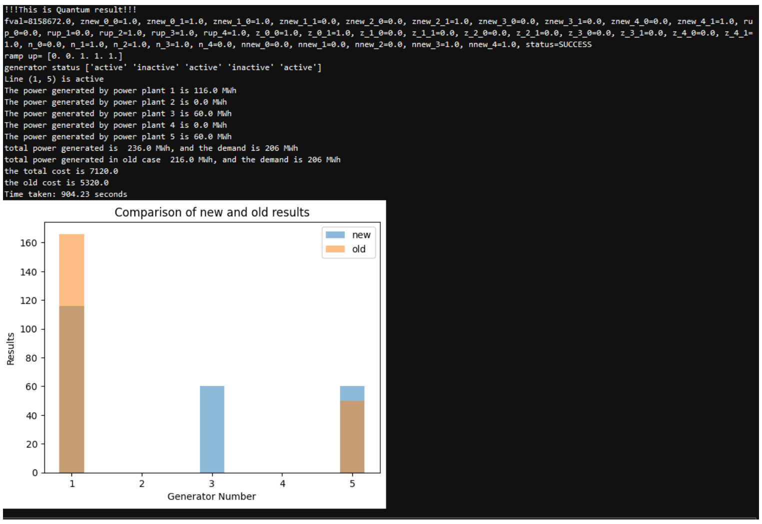

Figure 3: (
The quantum solution for the scenario 1 obtained using IONQ Aria 1 qpu which consists of 25 qubits. The number of shots performed on the qpu = 25. This is for both redispatch and line transmission switching constraint, where line (1,2) is not active.)

classical solution is not available due to the unavailability of enough memory (the program showed memory error and timeout error). Hence the problem can not be solved with more than 12 variables in our classical computer, but with qpus, we solved the problem with 24 variables showing quantum computing a potential candidate for solving MILP problem with a large number of variables. For scalability, and to include many real world scenarios such as AC analysis, more number of variables and constraints to be included to solve the problem.

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