Optimizing Power Distribution Network - Phase I submission

Muthumanimaran V, Abhisek Panda,
Deeksha Singh, Harikrishnan K J, Aswath S

January 2024

1 Introduction

The optimization of power distribution networks holds paramount significance in the contemporary landscape of energy management and sustainability. As the global demand for electricity continues to surge, the efficiency and reliability of power distribution systems become increasingly crucial. An optimized power distribution network ensures the seamless and economical flow of electricity from generation sources to end-users [1], mitigating energy losses [2], and reducing environmental impact.

In an era characterized by a growing reliance on electronic devices, smart technologies, and renewable energy sources, the need for a robust and finely tuned power distribution network is more pronounced than ever. Through careful planning, advanced technologies, and data-driven strategies, optimization addresses challenges such as voltage fluctuations, line losses, and peak demand management. This not only enhances the operational performance of the distribution network but also contributes significantly to the sustainability goals of both utility providers and end-users.

Quantum optimization [3] holds the potential to revolutionize power distribution network management by leveraging the unique properties of quantum computing to solve complex optimization problems more efficiently than classical computers. Quantum optimization algorithms, such as Quantum Approximate Optimization Algorithm (QAOA) [4] and Variational Quantum Eigensolver (VQE) [5], offer a novel approach to address the intricacies of optimizing power distribution networks.

In essence, the optimization of power distribution networks aligns with the broader objectives of achieving energy security, environmental sustainability, and economic efficiency. As we navigate a future characterized by increasing energy demands and a growing awareness of environmental stewardship, the optimization of power distribution networks emerges as a pivotal element in building a resilient and sustainable energy infrastructure.

2 IEEE 14 bus system

Researchers and engineers use the IEEE 14-bus system to study and test different aspects of power grid systems, and optimization. This model provides a balance between complexity and simplicity, making it suitable for a wide range of studies without becoming overly intricate. Simulations on the IEEE 14-bus system help researchers analyze the behavior of the power distribution network under different conditions, investigate voltage stability, assess the impact of load changes, and test the effectiveness of control strategies. In the next subsection, we will propose the complete problem for scenario 1 of TCS quantum computing challenge - Optimising Power Distribution Network.

2.1 Scenario 1

The optimization problem to be optimized for scenario 1 is as follows

2.1.1 Cost Function

The scenario 1 is very simple case. Given the quadratic costs limits, generation limits and other demands, what is the optimal power generation can be achieved. The cost function is quadratic in real power generation [2]

$$J(g_i) = \sum_{i}^{N_g} a_i g_i^2 + b_i g_i + c_i, \tag{1}$$

where $J(g_i)$ is the cost function to be minimized. g_i is the optimal generated power for the i^{th} generator. a_i, b_i and c_i are the quadratic co-efficients. When the generator generates power more than its capacity, it has been observed that the cost goes quadratically, the co-efficient a_i captures this quadratic cost . The linear cost is simply cost proportional to the amount of power generation given by b_i. The maintenance cost is given by c_i. The cost function is subjected to following constraints given in next subsection.

2.1.2 Constraints:

- Real power generation limits [2]:

$$g_{min,i} \leq g_i \leq g_{max,i}. \tag{2}$$

- Reactive power limits [2]:

$$Q_{min,i} \leq Q_{g,i} \leq Q_{max,i} \tag{3}$$

- Active power flow limits on each line [6]:

$$-M.z_{il} \leq f_{il} \leq M.z_{il} \tag{4}$$

where z_{il} is the status of the line (obtained from initial admittance matrix A.) and f_{il} [1] is defined as

$$f_{il} = V_i V_l (\mathrm{Re}(Y_{il}) \cos(\theta_i - \theta_l) + j\mathrm{Im}(Y_{il}) \sin(\theta_i - \theta_l)). \tag{5}$$

where θ_i the phase angle of the i^{th} bus. V_i is the voltage magnitude at bus i. Y_{il} are the elements of admittance nodal matrix [7] defined as

$$Y_{il} = \frac{1}{R_{il} + jX_{il}}, \quad \text{for } i \neq l \tag{6}$$

R_{il} and X_{il} are the resistance and reactance of the lines. For the diagonal elements [7]

$$Y_{ii} = \sum_{l}^{N_l} Y_{il} + B_{il}. \tag{7}$$

where N_l is the number of lines and B_{il} is the susceptance.

- Real power demands [1]:

$$P_i(V, \theta) = g_i - P_{D,i} \tag{8}$$

where $P_{D,i}$ is the real power demand on each line and $P_i(V, \theta) = \sum_l^{N_l} f_{il}$.

- Reactive power demands[1]:

$$Q_i(V, \theta) = Q_{g,i} - Q_{D,i} \tag{9}$$

where $Q_{g,i}$ reactive power generation, $Q_{D,i}$ is the reactive power demand and $Q_i(V, \theta)$ [1] defined as

$$Q_i(V, \theta) = Q_i \sum_{l}^{N_l} Q_l (\mathrm{Re}(Y_{il}) \cos(\theta_i - \theta_l) - j\mathrm{Im}(Y_{il}) \sin(\theta_i - \theta_l)). \tag{10}$$

2.1.3 Decision variables

- g_i - real generated power at generator i.
- $Q_{g,i}$ - reactive power at generator i.

The complete problem for scenario 2 is given in next subsection.

2.2 Scenario 2

The cost function with constraints for the scenario 2 is as follows

2.2.1 Cost function:

$$J(g_i) = \sum_{i}^{N_g}(a_i g_i^2 + b_i g_i + c_i) + \lambda_1 \sum_{i}^{N_r} r_i + \lambda_2 \sum_{l}^{N_s}(1 - z_{il}) + \lambda_3 \sum_{i}^{N_b}(1 - w_{il}) \quad (11)$$

where w_{il} is the vector selecting one fo three power transfer scenarios for the line. z_{il} is the status of the line and r_i is the re-dispatch status of generator.

2.2.2 Constraints:

- All constraints in **scenario 1**.
- Voltage magnitude limits [8],[9]:

$$V_{min,i} \leq V_i \leq V_{max,i} \quad (12)$$

- Phase angle limits [8],[9]:

$$\theta_{min,i} \leq \theta_i \leq \theta_{max,i} \quad (13)$$

note: V_i and θ_i are mentioned explicitly in the data. In the scenario 2, the corrective actions will change the topology of IEEE14 system, hence the optimized V_i and θ_i has to be found.

- Bus splitting constraint [8]:

$$\begin{aligned} w_{il} + w_{jl} &= 1 \quad (14) \\ f_{il} &= w_{il} . f_{il}^{scenario} \quad (15) \end{aligned}$$

- Generator re-dispatch constraint [10]:

$$g_i = g_{i,previous} + (r_i \Delta g_{i,up} - r_i \Delta g_{i,down}) \quad (16)$$

where $\Delta g_{i,up}$ and $\Delta g_{i,down}$ are ramp up and ramp down voltages.

- Transmisslion line switching constraint [9]:

$$z_{il}^{(t+1)} \in \{0, 1\} \quad (17)$$

where the argument $(t + 1)$ indicates the new scenario and (t) is for the old scenario.

- Limiting simultaneous line switching actions:

$$\sum_{l} |z_{il}^{(t+1)} - z_{il}^{(t)}| \leq N_{max,i} \quad (18)$$

where $N_{max,i}$ is the maximum allowable number of lines that can be switched simultaneously at bus i.

2.2.3 Decision Variables

- All decision variables mentioned in **Scenario 1**.
- V_i is voltage magnitude at bus i.
- θ_i is the voltage phase angle at bus i.
- r_i is the re-dispatch status.
- z_{il} is the status of the line.
- w_{il} is the power transfer scenarios.

3 Roadmap to Quantum Solution

Classically the problem can be solved using minimizing the cost function with constraints in subsection (2.1.1-2) and (2.2.1-2). The code for scenario 1 is attached in the github folder in phase 1 (filename: classicalsolutionscenario1). Scenario 2 can also be solved using classically. But one can get efficient optimized solution using quantum optimization. The entire optimization problem can be solved using **Quadratic Approximate Optimization Algorithm (QAOA)** [4] [11] which can handle both binary and non-binary variables. A sample program for optimization problem with non-binary decision variables is solved using QAOA and the code is attached in phase 1 folder named "sample-qaoacode1". We use similar gate based approach to construct the cost function with constraints and find the optimal decision variables using QAOA technique.

Problem with machine learning technique: Since the data provided is very less. It is not useful to go for machine learning protocols.

4 Amazon Credits Requirement

Since the Quantum Approximate Optimization Algorithm (QAOA) is a hybrid algorithm, it needs to be executed on a Quantum Processing Unit (QPU) that supports hybrid algorithms. Amazon Braket offers several QPUs, including Oxford Quantum Circuits - Lucy, IonQ - Harmony, IonQ - Forte 1, QuEra - Aquila, and Rigetti - Aspen-M-3.

Considering the number of variables, Scenario 1 necessitates 5 generators, requiring a minimum of 3 qubits to construct the quadratic function, thus demanding 15 qubits (not including reactive decision variables). For Scenario 2, the qubit requirement depends on the corrective action, assuming 10 decision variables, which translates to an additional 30 qubits. In total, Scenario 2 requires 45 qubits. While Scenario 1 can be implemented on IonQ Forte 1, Scenario 2 necessitates running the hybrid algorithm on Aquila or Aspen-M-3.

The associated cost for running the algorithm is 0.30 USD per task plus 0.01 USD per shot for Aquila and 0.30 USD per task plus 0.00035 USD per shot for Rigetti - Aspen-M-3.

The result's accuracy is contingent on the number of shots, representing measurements to obtain the probability distribution. In Rigetti - Aspen-M-3, the maximum allowable number of shots is 1,000,000, with a default of 1,024 shots. Assuming an increase to 10,000 shots, the cost per task becomes 350 USD per algorithm. To ensure result reliability, it is prudent to run the algorithm two or three times on the QPU for fact-checking, requiring a minimum of 700 USD to $1,000$ USD for phase 2.

References

[1] Georgios B Giannakis, Vassilis Kekatos, Nikolaos Gatsis, Seung-Jun Kim, Hao Zhu, and Bruce F Wollenberg. Monitoring and optimization for power grids: A signal processing perspective. *IEEE Signal Processing Magazine*, 30(5):107–128, 2013.

[2] Lucian Ioan Dulau. Optimal power flow analysis of ieee 14 system with distributed generators. *Journal of Electrical and Electronics Engineering*, 9(1):9, 2016.

[3] Akshay Ajagekar and Fengqi You. Quantum computing for energy systems optimization: Challenges and opportunities. *Energy*, 179:76–89, 2019.

[4] Jaeho Choi and Joongheon Kim. A tutorial on quantum approximate optimization algorithm (qaoa): Fundamentals and applications. In *2019 International Conference on Information and Communication Technology Convergence (ICTC)*, pages 138–142. IEEE, 2019.

[5] Jules Tilly, Hongxiang Chen, Shuxiang Cao, Dario Picozzi, Kanav Setia, Ying Li, Edward Grant, Leonard Wossnig, Ivan Rungger, George H Booth, et al. The variational quantum eigensolver: a review of methods and best practices. *Physics Reports*, 986:1–128, 2022.

[6] J. Zhu. *Optimization of Power System Operation*. IEEE Press Series on Power Engineering. Wiley, 2009.

[7] H. Saadat. *Power System Analysis*. Number v. 1 in McGraw-Hill series in electrical and computer engineering. WCB/McGraw-Hill, 1999.

[8] Yuqi Zhou, Ahmed S. Zamzam, Andrey Bernstein, and Hao Zhu. Substation-level grid topology optimization using bus splitting, 2021.

[9] Burak Kocuk, Santanu S. Dey, and Xu Andy Sun. New formulation and strong misocp relaxations for ac optimal transmission switching problem. *IEEE Transactions on Power Systems*, 32(6):4161–4170, 2017.

[10] Kennedy Mwanza and You Shi. Congestion management: Re-dispatch and application of facts. 2006.

[11] Anh Phuong Ngo, Christan Thomas, Hieu Nguyen, Abdullah Eroglu, and Konstantinos Oikonomou. Evaluate quantum combinatorial optimization for distribution network reconfiguration. In *2022 North American Power Symposium (NAPS)*, pages 1–6, 2022.