TCS CONSULTANCY SERVICES

Building on belief

TCS Quantum Challenge
Challenge 3- Optimizing Fleet Allocation (Phase1)

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Literature Survey

Classical: In literature, classically there are many papers on Aircraft fleeting and Routing problems, they either solve using heuristics approach or by making mathematical MILP model. Some of the papers which we referred are:

- Network flow-based approaches for integrated aircraft fleeting and routing [1]:

  They investigated network flow-based heuristic approaches and used multi-commodity network flow model for the aircraft fleeting and routing problem.

- A new approach to fleet assignment and aircraft routing problems [2]:

  In this paper, a new mathematical formulation is developed for fleet scheduling problems (i.e the combination of fleet assignment and aircraft routing problems) in single hub & spoke systems.

Literature Survey

Quantum: There are few papers which tried to solve these kind of Aircraft Fleet Assignment/ Tail-Assignment type problems in quantum using Gate circuit-based models or by making QUBO and annealing method. Some of the papers which we referred are:

- Applying the Quantum Approximate Optimization Algorithm to Tail-Assignment Problem[3] :

  The data instances are reduced to fit on quantum devices with 8, 15, and 25 qubits. The reduction procedure leaves only one feasible solution per instance, which allows us to map the tail-assignment problem onto the exact-cover problem.

- A QUBO Model to the Tail Assignment Problem [4]:

  Tail Assignment Problem was framed as a Quadratic Unconstrained Binary Optimization (QUBO) model and was solved using a classical and two hybrid solvers. They concluded that, for the considered datasets, there was a higher probability of obtaining better solutions for this problem using one of the hybrid solvers when compared with a classical heuristic algorithm such as Simulated Annealing (SA).

- Quantum alternating operator ansatz for solving the minimum exact cover problem [5]:

  They took the Minimum Exact Cover (MEC) problem as an example, applied Quantum alternating operator ansatz algorithm to Non-Trivial Feasible Solution Problems. They transformed MEC into a multi-objective constrained optimization problem, where feasible space consists of independent sets that are easy to find.

Solution Approach

- We solve the problem statement using Classically to benchmark and compare our Quantum results.
- In Classical and also for Quantum, we have done Data preprocessing.
- We have made Routes from the given Flights (Data Preprocessing) and done Route Assignment to Aircraft.
- We combined sequential flights from data to make a Route (1st Flight of a Route start at Base airport and last flight of a Route ends at Base airport. By this, we took care of Aircraft continuity constraint also and the last flight should finally return to the same Aircraft base constraint also. Route Forecasted Seats is the maximum of Forecasted seats of all flights in a route.
- We decompose our full approach to solve for a particular day (as schedule is repetitive mostly on weekdays), and then, solved Day wise.
- Further, we decompose to solve at Base Airport to further reduce decision variables and number of qubits required in circuit.

Classical Approach

- In Mathematical Model, Binary Integer Program (BIP) has been made whose Objective is to Optimize route assignment with minimum operating cost and maximize aircraft utilization.
- Beta weightage parameter is given for demand fulfilment and Gamma weightage is given for aircraft utilization (negative is used as objective function is to minimize and we have to maximize these components of it).
- Constraint 1 takes care of Flight covering or Route covering constraint ie. each schedule flight is flown by exactly or atmost 1 aircraft.
- Constraint 2 takes care that only 1 aircraft or atmost 1 aircraft is assigned to all overlap routes.
- Constraint 3 takes care of constraint on maximum number of flights flown by aircraft on a day.

Mathematical model:

R - set of Routes in a day, indexed by r

A - set of Aircrafts available in a day, indexed by a

A_r - set of Aircrafts that can be assigned to route r (based on day, maintenance and airport base constraints)

R_o - set of overlap Routes (whose time of the day overlaps)

R_d - set of distinct Routes (whose time of the day doest not overlaps)

C_a - Operating cost of aircraft a

D_r - Forecasted seats of route r

Q_a - Capacity of aircraft a

N_{fr} - Number of flights in route r

N_a - Maximum flights that can be issued to aircraft a in a day

β - Weightage parameter for demand fulfilment

γ - Weightage parameter for aircraft utilization

Decision Variables:

x_{ra} - Indicates if Aircraft a is assigned to Route r or not

Objective:

$$\min \sum_{r \in R} \sum_{a \in A_r} (N_{fr} C_a + \beta(D_r - Q_a) - \gamma) x_{ra}$$

Constraints:

$$\sum_{a \in A_r} x_{ra} \leq 1 \quad \forall r \in R \tag{1}$$

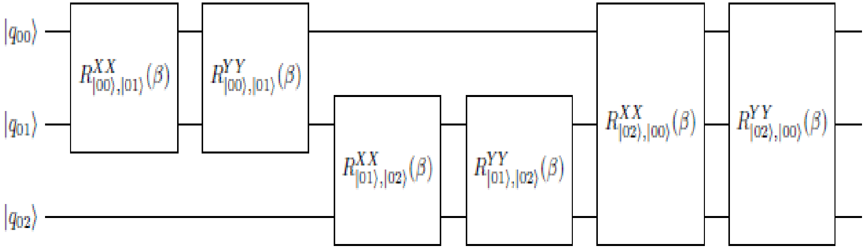
$$\sum_{r \in R_o} x_{ra} \leq 1 \quad \forall a \in A \tag{2}$$

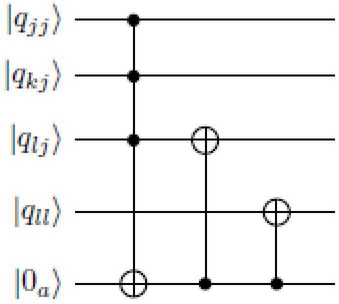
$$\sum_{r \in R_d} N_{fr} x_{ra} \leq N_a \quad \forall a \in A \tag{3}$$

$$x_{ra} \in \{0, 1\} \quad \forall r \in R, \forall a \in A_r$$

Quantum Approach

- In Quantum, Qiskit Quantum Approximate Optimization Algorithm (QAOA) algorithm is used.
- The objective function of the BIP is encoded as the cost function for QAOA.
- The constraints have been incorporated as mixers.
- QAOA with Mixers is used, Swap Mixers and Controlled Bit flip mixer for constraint feasibility.
- Decision Vars of BIP model as qubits, Objective is same.
- Initial Feasible State is allocating any available aircraft to routes.
- **Swap Mixers** (Ring XY Mixers): made using XX-YY gates, it is used to go to next feasible state (Route get allocated to different aircraft), for Constraint 1 and 2 of mathematical model.
- **Controlled Bit-Flip Mixer**: made using multi-controlled Toffoli gates, it is used to avoid infeasible state, infeasible combination, for Constraint 3 of mathematical model.





Data Preprocessing

- Data Preprocessed and Routes made from Flights

RouteNumber	RouteName	RouteStartTime	RouteEndTime	BaseAirport	NumofFlights	RouteForecastedSeats
1	1111_1112_1113_1114_	625	2335	LGW	4	225
2	1115_1116_1117_1118_	650	2320	LTN	4	225
3	1119_1120_1121_1122_1123_1124_	650	2300	LGW	6	225
4	1125_1126_1127_1128_1129_1130_	630	2350	LGW	6	225
5	1131_1132_1133_1134_	525	2330	LGW	4	220
6	1135_1136_1137_1138_	630	2300	LTN	4	230
7	1139_1140_1141_1142_1143_1144_	600	2300	LGW	6	200
8	1145_1146_1147_1148_1149_1150_	600	2350	LGW	6	200
9	1151_1152_1153_1154_1155_1156_	600	2300	LGW	6	225
10	1157_1158_1159_1160_1161_1162_	600	2350	LGW	6	200
11	1163_1164_1165_1166_	625	2335	LTN	4	225
12	1167_1168_1169_1170_	735	2320	LGW	4	225
13	1171_1172_1173_1174_1175_1176_	650	2300	MAN	6	230
14	1177_1178_1179_1180_1181_1182_	630	2350	MAN	6	225
15	1183_1184_1185_1186_	650	2320	MAN	4	225
16	1187_1188_1189_	930	1700	LGW	3	220
17	1190_1191_1192_	1005	2320	LTN	3	225
18	1193_1194_1195_	930	1700	LTN	3	185
19	1196_1197_1198_1199	1200	2350	MAN	4	225

Results

- Classically and in QAOA with mixers optimal results were achieved for a day (29th Nov) at base airport (LTN).
- For this sample data for a day, at base airport LTN (5 Routes (18 Flights) and 8 Aircrafts were available) and we have to assign these aircrafts to routes.
- Beta = 120 , Gamma = 48000 (found by tuning using hit and trial in classical to get optimal results)
- Objective optimal cost: -116960 and Route Assignment to Aircraft is as follows:
- x_2_G-AC=1, x_6_G-AA=1, x_11_G-AE=1, x_17_G-CB=1, x_18_G-CD=1. It means for Routes 2,6,11,17 and 18, Aircrafts G-AC, G-AA, G-AE, G-CB and G-CD were allocated respectively.
- Results Postprocessing : Extract Flights from Routes (Full Schedule), Routes name is having Sequence of Flights i.e. Flights allocated to an Aircraft for that day in that sequence.
- Evaluation Metric: Operating Costs of an aircraft, less weightage on Demand Fulfilment (Beta) and more weightage on Aircraft utilisation (Gamma) and On-time Performance of Flights.

Assumptions:

- More or less schedule is repititive for each day, thus showing approach for a day.
- Flights can be delayed and Empty Flights to Base airport can be inserted if sufficient Aircrafts are not avaiable and for Demand Fulfilment of Routes (or Flights). Optimize route assignment with minimum operating cost and miximize aircraft utilization and minimum delay.

Approach to solve for Full data (Phase2)

- Aircraft Maintenance and Airport Curfew constraints will be taken care in Data Preprocessing and while making Decision Variables.
- Routes are more than Aircraft available at some Base Airport for some day (Flights can be delayed and Empty Flights can be inserted).
- We are planning to use Metaheuristics, Large Neighborhood Search (for removing and adding flights to different Routes) to solve full problem classically,
- Hybrid approach in Quantum (Classical + QAOA) for these adjustments and to get best solution. Classically data will be preprocessed to get feasible allocation and it will be optimized using QAOA with Mixers.

AWS Infrastructure Estimate for Phase2:

- # Qubits on an average for Quantum Circuits : 30-40 Qubits.
- Cost for solving at a base airport for 1 day will be around 3 USD.

```
[248]: print("Task Summary")
print(t.quantum_tasks_statistics())
print('Note: Charges shown are estimates based on your Amazon Braket simulator and quantum processing unit (QPU) task usage. Estimated charges
print(f"Estimated cost to run this example: {t.qpu_tasks_cost() + t.simulator_tasks_cost():.2f} USD")

Task Summary
{'arn:aws:braket:::device/quantum-simulator/amazon/sv1': {'shots': 393, 'tasks': {'COMPLETED': 69}, 'execution_duration': datetime.timedelta(s
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Note: Charges shown are estimates based on your Amazon Braket simulator and quantum processing unit (QPU) task usage. Estimated charges shown
may differ from your actual charges. Estimated charges do not factor in any discounts or credits, and you may experience additional charges ba
sed on your use of other services such as Amazon Elastic Compute Cloud (Amazon EC2).
Estimated cost to run this example: 3.06 USD

[ ]:
```

- Cost for solving 1 day around 10 USD.
- Cost to solve for full data (30 Days) around 300 USD.
- As shots increases and T1 is used instead of SV1, cost will increase.
- Hybrid approach for full data also increase our costs.
- AWS Credits estimate : 500

References

1. Haouari, Mohamed et al. "Network flow-based approaches for integrated aircraft fleeting and routing". Eur. J.Oper. Res. 193 (2009):591-599.
2. Unal, Yusuf & Sevkli et al. "A new approach to fleet assignment and aircraft routing problems". Transportation Research Procedia (2021).
3. Vikstal, Pontus et al. "Applying the Quantum Approximate Optimization Algorithm to the Tail-Assignment Problem". Physical Review Applied (2019).
4. Martins, L., Rocha, A. and Castro, A. "A QUBO Model to the Tail Assignment Problem". In Proceedings of the 13th International Conference on Agents and Artificial Intelligence (ICAART 2021) - Volume 2, pages 899-906.
5. Wang, Sha-Sha & Liu, et al. "Quantum Alternating Operator Ansatz for Solving the Minimum Exact Cover Problem"(2023). 10.2139/ssrn.4458977.

Thank you