TCS Quantum Challenge 3 - Optimizing Fleet Allocation : Team 03360112

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1 Problem Statement

Airline scheduling aims to find the most profitable schedule that is reliable and satisfies all operational constraints. This problem is concerned with the assignment of individual aircraft to a pre-determined network of flights operated by the airline. Given that different assignments result in different amounts of profit or incur different costs, the objective is to maximise total profit taking into consideration factors such as passenger demand, seating capacities, operational costs, maintenance requirements, and governmental regulations. Flowchart in Figure 1 summarizes overview of problem statement. Given a schedule of flights to be flown, the aircraft fleeting and routing problem consists of determining a minimum-cost route assignment for each aircraft so as to cover each flight by exactly one aircraft while satisfying maintenance requirements and other activity constraints.

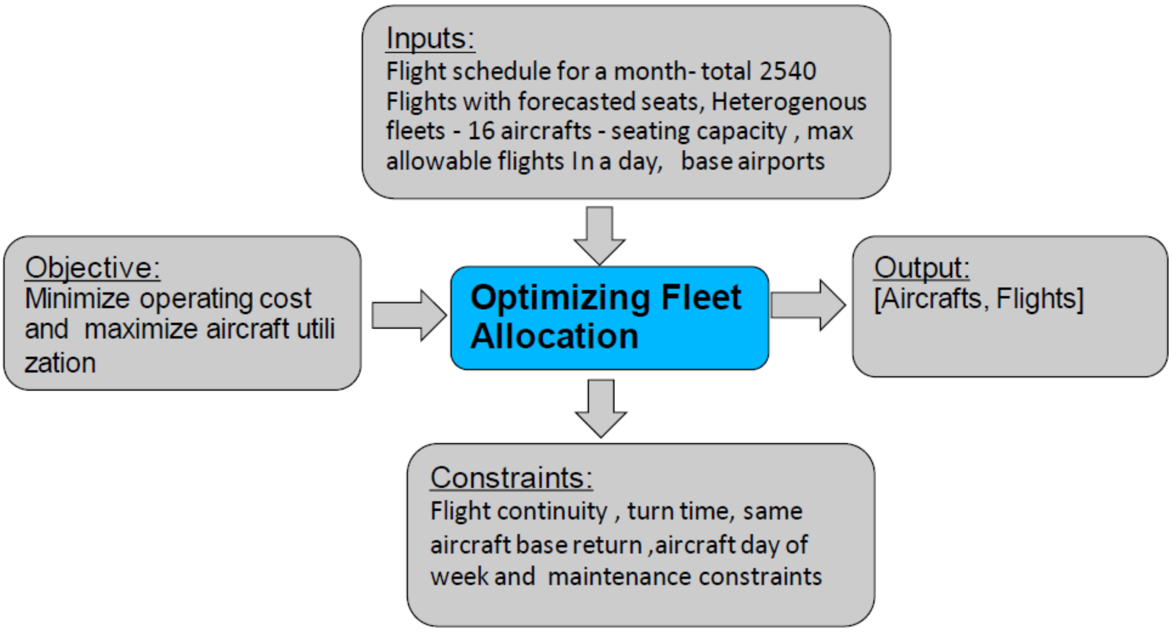

Figure 1: Overview of Problem Statement

2 Literature Survey

In literature, there are many papers on aircraft fleeting and routing problems, they either solve using heuristics approach or mathematical MILP model. Haouari et al.[1] investigated network flow-based heuristic approaches and used multi-commodity network flow model for the aircraft fleeting and routing problem. Unal et al [2] presented a new mathematical formulation for fleet scheduling problems (i.e. the combination of fleet assignment and aircraft routing problems) in single hub & spoke systems.

There are few papers which tried to solve these kind of aircraft fleet assignment/ tail-assignment type problems in quantum. Vikstal et al.[3] applied the Quantum Approximate Optimization Algorithm to the tail assignment problem, the data instances are reduced to fit on quantum devices with 8, 15, and 25 qubits. The reduction procedure leaves only one feasible solution per instance, which allows us to map the tail-assignment problem onto the exact-cover problem. After mapping, it solved the problem using Quantum Approximate Optimization Algorithm. Martins et al.[4] framed tail assignment problem as a Quadratic Unconstrained Binary Optimisation (QUBO) model and was solved using a classical and two hybrid solvers. They concluded that, for the considered datasets, there was a higher probability of obtaining better solutions for this problem using one of the hybrid solvers when compared with a classical heuristic algorithm such as Simulated Annealing (SA). Wang et al.[5] applied the Quantum alternating operator ansatz for solving the minimum exact cover problem, they took the Minimum Exact Cover (MEC) problem as an example, appliesd Quantum alternating operator ansatz algorithm to non-trivial feasible solution problems. They transform MEC into a multi-objective constrained optimization problem, where feasible space consists of independent sets that are easy to find.

3 Solution Approach

We have used hybrid approach in Quantum to solve our problem. We are preprocessing data classically for quantum input and we used Quantum Alternating Operator Ansatz (QAOA) in quantumto get solution. Classically data will be preprocessed to get feasible allocation and it will be optimized using QAOA with Mixers. We will be explaining more about in below sections. Data correction: There is some typo errors in data we observed and corrected them, In Planned schedule for flights 1115, 1167, 1190, 1193, changed Arrival Airport from "EDI" to "MAN" for continuity constraint in the legs of routes given. In Maintenance sheet also, for G-BF changed EndTime from 600 to 1600.

Data Preprocessing: We made routes from the given flight legs by combining sequential flights (legs). Routes are made from legs given in input data such that the first and last flight of a route starts and ends at the base airport. We combined sequential flights from data to make a route (1st Flight of a route start at base airport and last flight of a route ends at base airport. By this, we took care of aircraft continuity constraint also and the last flight should finally return to the same aircraft base constraint also. In routes, if aircraft if flight is not available from base airport of aircraft we inserted empty flights at start and end of each route. Route forecasted seats is the sum of all forecasted seats of all flights in a route.

We made routes from flights and done route assignment to aircraft. We have solved for each date, for full data of 28 days, and accumulate all results for full data results. Classical model is solve to internal benchmark our results and compare our quantum results.

3.1 Classical Approach

In mathematical model, Binary Integer Program (BIP) has been made whose objective is to optimize route assignment with minimum operating cost and maximize aircraft utilization. β weightage parameter is given for demand fulfilment and γ weightage is given for aircraft utilization (negative is used as objective function is to minimize and we have to maximize these components of it).

Mathematical model, BIP:

R - set of Routes in a day, indexed by r
A - set of Aircrafts available in a day, indexed by a
C_a - Operating cost of aircraft a
D_r - Forecasted seats of route r
Q_a - Capacity of aircraft a
N_{fr} - Number of flights in route r
N_a - Maximum flights that can be issued to aircraft a in a day
β - Weightage parameter for demand fulfilment
γ - Weightage parameter for aircraft utilization

Decision Variables:

x_{ra} - Indicates if Aircraft a is assigned to Route r or not

Objective:

$$\min \sum_{r\in R}\sum_{a\in A}(N_{fr}C_a + \beta(D_r - N_{fr}Q_a) - \gamma)x_{ra}$$

Constraints:

$$\sum_{a\in A} x_{ra} \leq 1 \;\; \forall r \in R \tag{1}$$

$$\sum_{r\in R} x_{ra} = 1 \;\; \forall a \in A \tag{2}$$

$$x_{ra} \in \{0,1\} \;\; \forall r \in R, \forall a \in A$$

Constraint 1 takes care of flight covering or route covering constraint ie. each schedule flight is flown by exactly or atmost 1 aircraft. Constraint 2 takes care that each aircraft will be allocated to only 1 route. We took this as quality constraints as in data preprocessing, we analysed that aircrafts are always less than number of routes to allocate. Aircraft maintenance and airport curfew constraints will be taken care in data preprocessing and while making decision variables.

We choose $\beta = 100$, $\gamma = 100000$, after some experiments to get optimal results in classical for this model.

We also solved connection network model given in [1]. We got same number of flights aswe got from our BIP model as given above. Thus, we concluded that routes given in input data are optimized. Moreover due to qubit limitations in quantum, we can not implement large quantum circuit and in connection network model there are large number of consecutive pairs of flights which we take as qubits in quantum. Thus, we settled with our Hybrid approach instead of connection network model in quantum.

From inital experiments and preprocessing, we observed that 2 major constraints driving the assignment of flights to an aircraft: Aircraft maximum flights in a day, base airport start and end for an aircraft . Aircraft day of the week and maintenance constraints will be taken care in data preprocessing and while making decision variables for BIP.

We also checked for feasibilty of a route manually while preprocessing. This took care of following constraints; flight continuity constraint , turn time, the last flight should finally return to the same aircraft base . Further, routes takes care of constraint on maximum number of flights flown by aircraft on a day. We decompose our problem to solve at Base Airport to further reduce decision variables. We filter on both the factors, aircrafts max allowable flights and base airports - LGW, LTN, MAN to reduce reduce decision variables. Below table shows the routes which we allowed to allocate by aircrafts max allowable flights.

Number of Flights in Routes	Aircrafts Max allowable Flights
3	3
4	4,5
6	6,7

3.2 Quantum Approach

The Quantum Alternating Operator Ansatz (QAOA) algorithm is a variational quantum algorithm. In a hybrid quantum-classical setting, the quantum processor prepares and measures a parametrized quantum state to estimate the expectation value of the problem Hamiltonian. Using these measurement outputs, the classical processor updates the parameters to minimize the cost function. QAOA have specific mixing Hamiltonian (as per the hard constraints of an optimization problem) to restrict the search in the feasible subspace only. The objective function of the BIP is encoded as the cost function for QAOA.

The constraints have been encorporated as mixers. QAOA with mixers is used Controlled Bit-flip mixer for constraint feasibility. Decision variables of BIP model as qubits. All feasible combinations as qubits. Qubit is a decision variable indicates aircraft allocated to route, modeled objective function as aircraft cost and difference in forecasted seats and aircraft capacity with weightage.

Objective function is to minimise the total operating cost and maximise aircraft utilisation. Objective function formula is same that of above BIP model. Objective : Aircraft cost + β (deficit in seats)+ γ * flights counts

Rotation mixer, Rx gate, β angle parameter is used to change qubit state. In Controlled Bit-Flip Mixer, we used CNOT gates, to avoid infeasible state (infeasible combination). Infeasible combinations for our problem such as when 2 aircrafts get assigned to 1 route or when 1 route get allocated to 2 aircrafts. We have shown QAOA ansatz in Figure 2 and Controlled Bit-Flip Mixer in Figure 3 for small example of 2 Aircraft allocation to 3 Routes, whose lists are given below. We can see how Cbf3 mixer (3 qubit Controlled Bit-Flip Mixer) in Figure 4 avoid infeasible state like 2 routes allocation to 1 aircraft or 2 aircraft allocation to 1 route.

ListAircrafts = ['G-AC', 'G-AE'] , ListRoutes = [1, 2, 5]

Decision Variable	Qubit
x_G-AC_1	q_0
x_G-AE_1	q_1
x_G-AC_2	q_2
x_G-AE_2	q_3
x_G-AC_5	q_4
x_G-AE_5	q_5
'ancilla_{0}	q_6

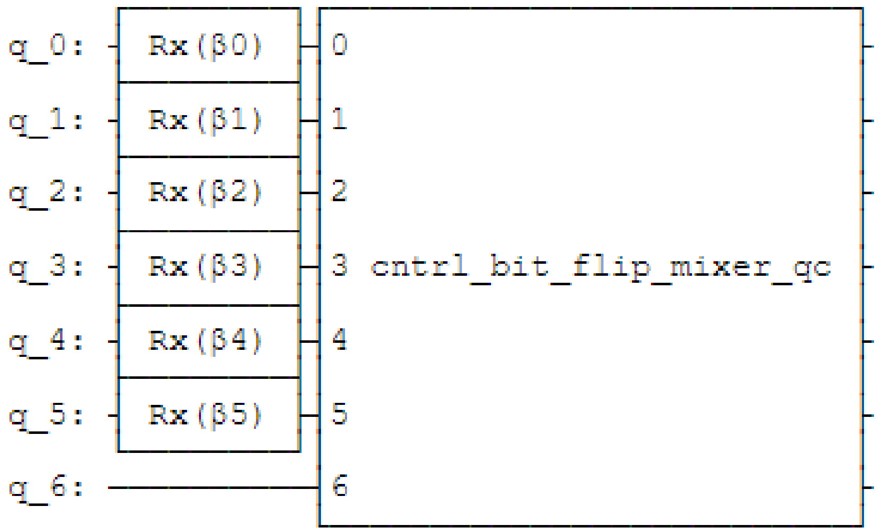

Figure 2: QAOA Ansatz

In sample mathematical optimisation model of the problem given in document, below it is mentioned how we are fulfilling them in our QAOA approach.

- Each scheduled flight should be flown by exactly one aircraft - By Control Bit flip mixer (Each route containing flights flown by atmost one aircraft).
- Aircraft continuity constraint: If a flight is arriving at a certain airport assigned to an aircraft, the next flight (if any) assigned to that aircraft must depart from the same airport - By data preprocessing, in making routes from flights, checked for this continuity constraint
- For consecutive flights assigned to an aircraft, the turn-around time in the schedule should not be less than the turn-around time of that airport. By data preprocessing, in making routes from flights, checked for the turn-around time constraint
- Demand Fulfilment constraint: Aircraft Type is assigned based on Aircraft capacity and Forecasted seats/demand for the day. a. If a suitable Aircraft based on Forecasted seats is not available for the scheduled time, then assign an Aircraft with lower capacity (vice-versa). This is because On-Time-Performance and Aircraft Utilization are more important than demand fulfilment. - Took β parameter in objective function, it's weightage can be set and demand fulfilment constraint can be taken care, we are assigning an Aircraft with lower capacity, if a suitable aircraft not available.
- The last flight should finally return to the same Aircraft base - By data preprocessing, in making routes from flights, we are taking care that start and end of route be the aircraft base, moreover, we are decomposing and choosing from feasible solution.
- The number of flights/trips assigned to an Aircraft for a particular day should not exceed the predefined limit - We are decomposing and giving feasible Routes (flights) to aircrafts.

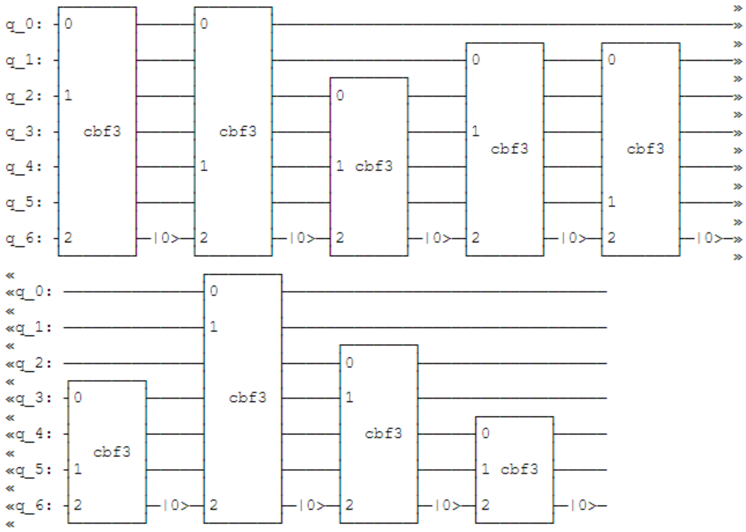

Figure 3: Controlled Bit Flip Mixer

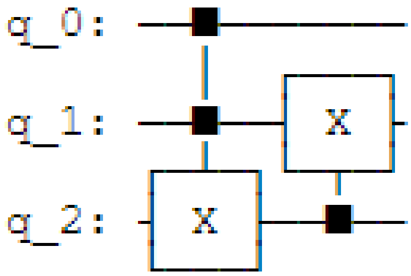

Figure 4: cbf3

- Curfew time constraints: The Flight schedules shall address the following constraints—Airport open and close time, Day of Week, Curfew time of the Airport- By data preprocessing, in taking available aircrafts for allocation took Day of Week constraint, when routes are formed airport curfew time was checked in preprocessing.

- Aircraft Maintenance constraint: The flight schedules shall address the following constraints—Unavailability of Aircraft for scheduling due to Aircraft maintenance - By data preprocessing, in taking available aircrafts for allocation we took this aircraft maintenance constraint.

Aircraft maintenance constraint taken care in preprocessing for simplicity and also be handled post processing - by inserting flights in available time of the day.

4 Results

To get final results, we have done postprocessing of our QAOA results, we exracted flights from routes and come up with aircraft allocation to flights for full data day by day and get full schedule. Our evaluation metrics are: operating costs of an aircraft, less weightage on demand fulfilment (β) and more weightage on Aircraft Utilisation (γ) and on-time Performance of flights (no delay of flights). We calculated Aircraft Cost, Number of Allocated Flights and Deficit in Seats (Flights forecasted seats and aircrafts seats difference) for full data and compared with internal classical benchmark of BIP results.

- Quantum Results :

Number of Allocated Flights -	1460
Number of Empty Flights -	162
Aircraft Cost -	10274000
Deficit in Seats -	9691

- Classical Results: Number of Allocated Flights, Number of Empty Flights, Aircraft Cost in Classical is same as that of quantum results. Deficit in Seats = 8681. We get less deficit in seats compared to quantum. Here, quantum does not perform better in Deficit of seats metric compared to Classical.
- These results are for 20 shots, 1 layer, by local simulator, MPS (' Matrix Product State') and COBYLA optimizer in QAOA.
- We are able to assign 1460 out of 2540 flights for full data. We inserted 162 empty Flights, from and to the Base airport at Start and End of some routes to fulfill base airport constraint and get schedule. Please note that after aircraft day of week and maintenance constraint, the maximum number of flights we can assign for full data are 1794. So, we are able to assign 1622 (1460+162) flights out of possible 1794 flights allocation.
- If we do time execution comparison, docplex solver for BIP takes less time compared to MPS simulator for QAOA.For a particular day, docplex solver can solve BIP in seconds but MPS simulator solve QAOA in minutes. Maybe on AWS, it can execute faster.

If number of shots increases, we have observed improvements in QAOA results, i.e. Deficit in seats metric come same as Classical for 100 shots, it just takes more time to execute that's why we took 20 shots. We can also give delay to unallocated flights to see adjustments in aircraft, replace empty flights with unalloacted flights having same OD pair with delay, these types of heuristically postprocessing can be done to get flexible schedules and to allocate more flights. We took advantage of routes given in input data and done allocation of those routes to aircrafts but we also can make our routes to get more flights in them. There is large number of cancellation of flights (unallocation) due to operational constraint , aircraft maximum flight allowed in a day constraint. Operating cost is same as all possible aircrafts are utilized, we trying to fit maximum flights for all aircrafts, almost all day demand (number of flights) is more that supply, aircraft, aircraft cost is immaterial. We can always play with evaluation metrics, objective function parameters, total operating costs of an aircraft and aircraft utilization (surplus or deficit capacity, maximum allowable flights, occupancy time utlilisation of aircraft) to get flexible schedules.

5 Conclusion

We were able to allocate 1460 flights and we get on time performance i.e. no delay of all allocated flights. Different evaluation metrics results like Number of Allocated Flights, Number of Empty Flights, Aircraft Cost in Quantum is same as that of Classical results but for Deficit in Seats metric Classical gives better results. Thus, we can conclude that for this approach, we do not get any advantage in solution quality or execution time in solving this problem in Quantum compared to Classical.

We also experimented with Connection network model classically, but it gives same number of flights assignment as BIP model for Classical, that's why we finalised with BIP model for Classical. Connection network model can be at Route Leg level or Flight leg level, Flight leg level gives large number of consecutive pairs of flights, which we can't solve on quantum due to qubit limitations, that's why we settled for our solution approach.

Regarding scalability of our quantum approach, if we have more number of qubits, we do not to get feasible options also in our solution approach, will solve the whole problem at one go for a day. Even, we can try to solve connection network model in quantum, if we can handle all consecutive pairs of flights pairs in one go for a day and if there are hundreds of qubits available to denote that possible consecutive pairs of flights.

6 References

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5. Wang, Sha-Sha & Liu, et al. "Quantum Alternating Operator Ansatz for Solving the Minimum Exact Cover Problem"(2023). 10.2139/ssrn.4458977.