TATA CONSULTANCY SERVICES

Building on belief

TCS Quantum Challenge
Challenge 3- Optimizing Fleet Allocation (Phase1)

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Literature Survey

- Classical: - Heuristics approach or mathematical MILP model

- Network flow-based approaches for integrated aircraft fleeting and routing [1]
    - Solves aircraft fleeting and routing problem.
    - Network flow-based heuristic approaches
    - Multi-commodity network flow model

- A new approach to fleet assignment and aircraft routing problems [2]
    - Fleet scheduling problems in single hub & spoke systems
    - Solves the combination of fleet assignment and aircraft routing problems
    - Novel Mathematical formulation

Literature Survey

- Quantum - Aircraft Fleet Assignment/Tail-Assignment, Gate circuit-based models or by making QUBO and annealing method

- Applying the Quantum Approximate Optimization Algorithm to Tail-Assignment Problem[3]
  - Reduced data instance to fit on quantum devices with 8, 15, and 25 qubits
  - The reduction has only one feasible solution per instance, which maps the tail-assignment problem onto the exact-cover problem

- A QUBO Model to the Tail Assignment Problem [4]
  - Tail Assignment Problem was framed as QUBO solved using a classical and two hybrid solvers
  - For the considered datasets, hybrid solvers worked better when compared with a classical SA

- Quantum alternating operator ansatz for solving the minimum exact cover problem [5]
  - Applied QAOA to Minimum Exact Cover problem to get non-trivial feasible solution problems
  - Transformed MEC into a multi-objective constrained optimization problem, where feasible space consists of independent sets that are easy to find.

Solution Approach

- Preprocessed data
  - Made routes from the given flights by combining sequential flights
  - Such that first and last flight of a route starts and ends at base airport respectively.
  - This took care of following constraints
    - Aircraft continuity constraint
    - The last flight should finally return to the same Aircraft base
  - Route forecasted seats = Max (forecasted seats of all flights in a route)

- Decomposed the problem to reduce decision variables and number of qubits required.
  - Solve for each base airport for a day
  - Repeat for each base and day of week to solve entire problem

- Modeling
  - Modeled and Solved the decomposed problem using classical mathematical optimization
  - Used QAOA with mixer circuits to solve on quantum

Classical Approach

- Modeled problem as Binary Integer Program (BIP)
- Objective – Minimize operating cost and maximize aircraft utilization
- Weights balancing for demand fulfilment and aircraft utilization.
- Constraint 1 - Flight covering or route covering constraint i.e., each schedule flight is flown by exactly or at most 1 aircraft.
- Constraint 2 - At most 1 aircraft is assigned a route and two overlapping routes are not assigned to same aircraft
- Constraint 3 - Maximum number of flights flown by aircraft on a day.

Mathematical model:

R - set of Routes in a day, indexed by r

A - set of Aircrafts available in a day, indexed by a

A_r - set of Aircrafts that can be assigned to route r (based on day, maintenance and airport base constraints)

R_o - set of overlap Routes (whose time of the day overlaps)

R_d - set of distinct Routes (whose time of the day doest not overlaps)

C_a - Operating cost of aircraft a

D_r - Forecasted seats of route r

Q_a - Capacity of aircraft a

N_{fr} - Number of flights in route r

N_a - Maximum flights that can be issued to aircraft a in a day

β - Weightage parameter for demand fulfilment

γ - Weightage parameter for aircraft utilization

Decision Variables:

x_{ra} - Indicates if Aircraft a is assigned to Route r or not

Objective:

$$\min \sum_{r \in R} \sum_{a \in A_r} (N_{fr} C_a + \beta(D_r - Q_a) - \gamma) x_{ra}$$

Constraints:

$$\sum_{a \in A_r} x_{ra} \leq 1 \quad \forall r \in R \tag{1}$$

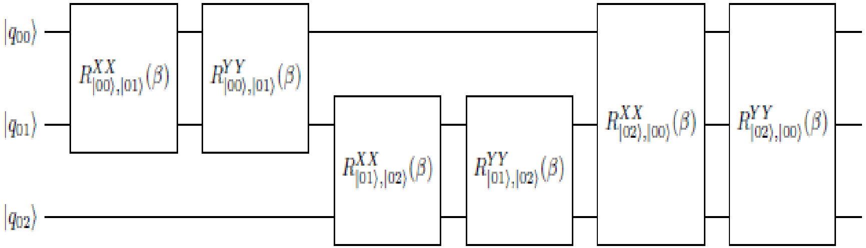
$$\sum_{r \in R_o} x_{ra} \leq 1 \quad \forall a \in A \tag{2}$$

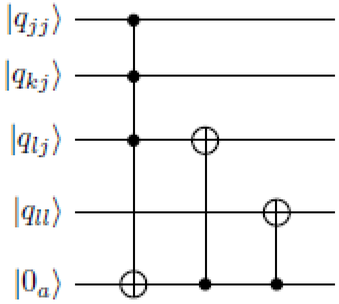
$$\sum_{r \in R_d} N_{fr} x_{ra} \leq N_a \quad \forall a \in A \tag{3}$$

$$x_{ra} \in \{0, 1\} \quad \forall r \in R, \forall a \in A_r$$

Quantum Approach

- Used Quantum Approximate Optimization Algorithm (QAOA) with Mixers
- Modeled objective function of BIP as cost function; decision vars of BIP as qubits; and constraints of BIP as mixers.
- Used Swap Mixers and Controlled Bit flip mixer for constraint feasibility.
- Initial feasible solution is obtained by allocating any available aircraft to routes sequentially
- **Swap Mixers** (Ring XY Mixers): Used XX-YY gates, to go to next feasible state (Route get allocated to different aircraft), for constraint 1 and 2 of BIP.
- **Controlled Bit-Flip Mixer**: Used multi-controlled Toffoli gates, to avoid infeasible state, infeasible combination, for constraint 3 of BIP.





Data Preprocessing

- Data Preprocessed and Routes made from Flights

RouteNumber	RouteName	RouteStartTime	RouteEndTime	BaseAirport	NumofFlights	RouteForecastedSeats
1	1111_1112_1113_1114_	625	2335	LGW	4	225
2	1115_1116_1117_1118_	650	2320	LTN	4	225
3	1119_1120_1121_1122_1123_1124_	650	2300	LGW	6	225
4	1125_1126_1127_1128_1129_1130_	630	2350	LGW	6	225
5	1131_1132_1133_1134_	525	2330	LGW	4	220
6	1135_1136_1137_1138_	630	2300	LTN	4	230
7	1139_1140_1141_1142_1143_1144_	600	2300	LGW	6	200
8	1145_1146_1147_1148_1149_1150_	600	2350	LGW	6	200
9	1151_1152_1153_1154_1155_1156_	600	2300	LGW	6	225
10	1157_1158_1159_1160_1161_1162_	600	2350	LGW	6	200
11	1163_1164_1165_1166_	625	2335	LTN	4	225
12	1167_1168_1169_1170_	735	2320	LGW	4	225
13	1171_1172_1173_1174_1175_1176_	650	2300	MAN	6	230
14	1177_1178_1179_1180_1181_1182_	630	2350	MAN	6	225
15	1183_1184_1185_1186_	650	2320	MAN	4	225
16	1187_1188_1189_	930	1700	LGW	3	220
17	1190_1191_1192_	1005	2320	LTN	3	225
18	1193_1194_1195_	930	1700	LTN	3	185
19	1196_1197_1198_1199	1200	2350	MAN	4	225

Results

- Reduced data Instance has
    - 1 day (29th Nov) , 1 base airport (LTN)
    - 5 routes (with 18 Flights) , 8 aircrafts are available
- Beta = 120 , Gamma = 48000 i.e. given more weightage to aircraft utilization

- Result
    - Objective optimal cost: -116960 and Route Assignment to Aircraft is as follows:
    - x_2_G-AC = 1, x_6_G-AA = 1, x_11_G-AE = 1, x_17_G-CB = 1, x_18_G-CD = 1
    - Routes 2, 6, 11, 17 and 18, Aircrafts G-AC, G-AA, G-AE, G-CB and G-CD were allocated respectively.
- Results Postprocessing
    - Extract flights from routes (full schedule), Routes name is having sequence of Flights i.e. Flights allocated to an Aircraft for that day in that sequence.
- Evaluation Metric:
    - Operating Costs of an aircraft and On-time Performance of Flights

Approach to solve for Full data (Phase2)

- Aircraft Maintenance and Airport Curfew constraints will be taken care in data preprocessing and while making decision variables.
- At some base airports routes are more than aircraft available for some day – will incorporate delaying of flights or adding empty flights
- Planning to use Metaheuristics, Large Neighborhood Search (for removing and adding flights to different Routes) to solve full problem classically.
- Hybrid approach in Quantum (Classical + QAOA) for these adjustments and to get best solution. Classically data will be preprocessed to get feasible allocation and it will be optimized using QAOA with Mixers.

AWS Infrastructure Estimate for Phase2:

- # Qubits on an average for Quantum Circuits : 30-40 Qubits.
- Cost for solving at a base airport for 1 day will be around 3 USD.

```
[248]: print("Task Summary")
       print(t.quantum_tasks_statistics())
       print('Note: Charges shown are estimates based on your Amazon Braket simulator and quantum processing unit (QPU) task usage. Estimated charges
       print(f"Estimated cost to run this example: {t.qpu_tasks_cost() + t.simulator_tasks_cost():.2f} USD")
```

```
Task Summary
{'arn:aws:braket:::device/quantum-simulator/amazon/sv1': {'shots': 393, 'tasks': {'COMPLETED': 69}, 'execution_duration': datetime.timedelta(s
econds=2444, microseconds=692000), 'billed_execution_duration': datetime.timedelta(seconds=2444, microseconds=692000)}}
Note: Charges shown are estimates based on your Amazon Braket simulator and quantum processing unit (QPU) task usage. Estimated charges shown
may differ from your actual charges. Estimated charges do not factor in any discounts or credits, and you may experience additional charges ba
sed on your use of other services such as Amazon Elastic Compute Cloud (Amazon EC2).
Estimated cost to run this example: 3.06 USD
```

```
[ ]:
```

- Cost for solving 1 day around 10 USD.
- Cost to solve for full data (30 Days) around 300 USD.
- As shots increases and T1 is used instead of SV1, cost will increase.
- Hybrid approach for full data also increase our costs.
- AWS Credits estimate : 500

References

1. Haouari, Mohamed et al. "Network flow-based approaches for integrated aircraft fleeting and routing". Eur. J.Oper. Res. 193 (2009):591-599.
2. Unal, Yusuf & Sevkli et al. "A new approach to fleet assignment and aircraft routing problems". Transportation Research Procedia (2021).
3. Vikstal, Pontus et al. "Applying the Quantum Approximate Optimization Algorithm to the Tail-Assignment Problem". Physical Review Applied (2019).
4. Martins, L., Rocha, A. and Castro, A. "A QUBO Model to the Tail Assignment Problem". In Proceedings of the 13th International Conference on Agents and Artificial Intelligence (ICAART 2021) - Volume 2, pages 899-906.
5. Wang, Sha-Sha & Liu, et al. "Quantum Alternating Operator Ansatz for Solving the Minimum Exact Cover Problem"(2023). 10.2139/ssrn.4458977.

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Thank you