
Research interests

Keywords Probabilistic modelling and inference; Monte Carlo methods; applications to machine learning, signal processing, particle physics, and cell biology.

Academic positions

- Feb. 2015- **Chargé de recherche**, *CNRS & CRISAL, Université de Lille*, Lille (France).
CRISAL [link] is the department of computer science, signal processing and automatic control.
Permanent full-research position, employed by the French national centre for scientific research.
- 2013-2015 **Postdoctoral fellow**, *Department of Statistics, University of Oxford*, Oxford (UK).
2020 Science fellowship of the EPSRC, working with Chris Holmes on large-scale Markov chain Monte Carlo methods, motivated by biological data analysis.
2020 science [link] is a network of computational scientists creating effective approaches to answer fundamental life science questions.

Education

- 2009-2012 **Doctorat (Ph.D.)**, *Université Paris-Sud XI*, Orsay (France), *très honorable*.
Computer science, *Towards adaptive learning and inference - Applications to hyperparameter tuning and astroparticle physics*, under the supervision of Balázs Kégl [link].
Très honorable is the highest honours you can get in Université Paris-Sud. My thesis reviewers were Éric Moulines and Christian Robert. Feel free to ask me for their reports.
- 2008-2009 **Master (M.Sc.)**, *Ecole Normale Supérieure*, Cachan (France), *highest honours*.
Mathematics, computer vision and machine learning.
Master programme 'MVA'; this programme is considered the best machine learning master nationwide.
- 2008 **Agrégation**, *Université Louis Pasteur*, Strasbourg (France), *national rank 82*.
Mathematics major, probability & statistics minor.
The *agrégation* is a national competitive exam, originally meant to hire teachers for undergraduate levels. A large proportion (2800+ candidates that year for 252 positions) of French students in mathematics take it, as it is also considered a sesame to the Ph.D. programme.
- 2005-2009 **Magistère**, *Université Louis Pasteur*, Strasbourg (France), *highest honours*.
Applied and pure mathematics.
Magistères are selective university programmes made of traditional B.Sc./M.Sc. programmes with privileges such as small-group exercise sessions and advanced complementary courses.

Management of funding resources

- 2016-2020 **ANR JCJC**, *rôle*: PI, **172k€**, 4 people.
Project BoB "Bayesian inference on a budget".
Annual call for starting grants by ANR, the French funding agency (13% acceptance rate).
- 2016 **CNRS PEPS JCJC**, *rôle*: PI, **10k€**, 2 people.
Project DPPMC "Monte Carlo with determinantal point processes".
PEPS JCJC is a call issued by CNRS for short-term exploratory research projects by young researchers.
- 2015 **CNRS PEPS JCJC**, *rôle*: co-PI, **7k€**, 2 people.
Project PROMO "Projected Monte Carlo".
PEPS JCJC is a call issued by CNRS for short-term exploratory research projects by young researchers.

Awards

2013 **2nd prize of the *Gilles Kahn* award**, *Société Informatique de France*.

Awarded by the French society of Computer Science for the best French Ph.D. of the academic year.

Recent (2016-) international seminars and visits

- ▷ University of Bristol, UK (3 days, 2017)
- ▷ University of Aalborg, Denmark (7 days, 2017)
- ▷ Max Planck Institut Tübingen, Germany (2 days, 2016);
- ▷ University of Kent, UK (2 days, 2016);
- ▷ Microsoft Sunnyvale, USA (2 days, 2016);
- ▷ University of Utrecht, Netherlands (3 days, 2016);
- ▷ Harvard University, USA (3 days + 1 week 2016);
- ▷ University of Oxford, UK (several visits totalling 1 month since 2016);

Recent (2016-) invited talks in special sessions of international workshops/conferences

- ▷ MLSS *African machine learning summer school*, Algiers, Algeria, June 2018.
- ▷ SSP conference on *Statistical signal processing*, Freiburg, Germany, June 2018.
- ▷ BNPSI workshop on *Bayesian nonparametrics for signal and image processing*, Bordeaux, France, June 2018.
- ▷ Workshop on *Cardiac modelling* of the Royal Statistical Society, Chicheley, UK, February 2017.
- ▷ MCQMC conference *Monte Carlo and quasi-Monte Carlo methods*, Stanford Univ., Palo Alto, USA, August 2016.
- ▷ Workshop on *High-Dimensional Statistical Models & Big Data*, Alan Turing Institute, London, UK, February 2016.
- ▷ MCMSki conference on *Monte Carlo methods*, Lenzerheide, Switzerland, January 2016.

Selected recent (2016-) invited talks in national workshops/conferences

- ▷ Invited speaker at the French *Académie des Sciences* for a mini-workshop on determinantal point processes, June 2018.
 - ▷ Invited speaker at the Physics colloquium of ENS Lyon, France, March 2018.
 - ▷ Plenary speaker at the GRETSI conference, September 2017.
- GRETSI is the main French event on signal processing, held every other year since 1967, with 400+ regular participants. Plenaries are prestigious and usually given by more senior academics.*

Teaching Experience

CNRS positions come with no teaching duty. Out of personal inclination, I still maintain a small teaching activity at the master level.

2016-2018 **Lecturer**, ENSAE, Paris (France).

9 hours per year, on Bayesian nonparametrics (master-level students in statistics and econometrics).

2015-2018 **Lecturer**, École centrale de Lille, Lille (France).

15 hours per year, on practical machine learning with applications to bankruptcy prediction (master-level engineering students).

2013-2014 **Lecturer**, Univ. Oxford, Oxford (UK).

8 hours, teaching half of the course *Advanced simulation* on Monte Carlo methods (4th year statistics students).

2013 **Class tutor**, Univ. Oxford, Oxford (UK).

14 hours, tutoring for the course *Advanced simulation* on Monte Carlo methods (4th year statistics students).

- 2009–2012 **Teaching assistant**, *Univ. Paris-Sud XI*, Orsay (France).
64 hours per year. Covered topics include Linear and Nonlinear Programming (L3), Stochastic processes (M1), C programming (L2) and orientation of students (L1).
[L1, L2, and L3 correspond to the first, second, and third year that lead to a bachelor's degree, M1 and M2 to the two years of master.](#)
- 2008–2009 **Examiner**, *Lycée J.-B. Say*, Paris (France).
Maths examiner for undergraduate students from “classes préparatoires”.
“Classes préparatoires” are competitive undergraduate programmes.

Scientific responsibilities and research management

- 2009– Reviewer for journals such as *Annals of Statistics*, *Journal of the Royal Statistical Society B*, *Bernoulli*, *Journal of Machine Learning Research (JMLR)*, *IEEE Transactions on Pattern Analysis and Machine Intelligence (TPAMI)*, *IEEE Transactions on Signal Processing (TSP)*, *IEEE Transactions on computers (TC)*, *Journal of Computational and Graphical Statistics (JCGS)*, and conferences such as *International Conference on Machine Learning (ICML)*, *Advances in Neural Information Processing Systems (NIPS)*, *International conference on learning theory (COLT)*.
- 2013– I have co-organised several national workshops in France and in the UK, and one international *workshop* [\[link\]](#) at ICML'14, one of the major international machine learning conferences.

Supervision

Percentages display my own share of the supervision.

- 2017– **Ph. D. student**, *Ecole Centrale de Lille*, Lille (France), 50%.
I am co-supervising Ayoub Belhadji's Ph.D. with Pierre Chainais (Prof. Centrale Lille & CRIStAL), on *Determinantal point processes for dimension reduction in signal processing*.
- 2017 **Master student**, *Ecole Centrale de Lille*, Lille (France), 50%.
I have co-supervised Ayoub Belhadji's internship with Pierre Chainais (Prof. Centrale Lille & CRIStAL), on *Determinantal point processes for dimension reduction in signal processing*. Ayoub went on to start a PhD with us.
- 2016– **Ph. D. student**, *Ecole Centrale de Lille*, Lille (France), 70%.
I am co-supervising Guillaume Gautier's Ph.D. with Michal Valko (HDR, Inria Lille & CRIStAL), on *Fast sampling of determinantal point processes*.
- 2016 **Master student**, *Ecole Centrale de Lille*, Lille (France), 100%.
I have supervised Guillaume Gautier's internship (master MVA, ENS Cachan), on *Determinantal point processes in statistics and machine learning*. Guillaume went on to start a PhD with me.
- 2016 **Master student**, *Ecole Centrale de Lille*, Lille (France), 100%.
I have co-supervised Souhail Toumadi's internship (master-level engineering student, majoring in “data analysis and decision-making” at Ecole Centrale de Lille, and following in parallel a master in probability and statistics, Université de Lille), on *Sampling uniform spanning trees*. Souhail then went on to start another prestigious master (MVA; ENS Cachan).
- 2014–2018 **Ph. D. student**, *Univ. Oxford*, Oxford (UK), 25%.
I have co-supervised Ross Johnstone's Ph.D. with Gary Mirams and David Gavaghan (Oxford Computer Science), with industrial collaborators at *Roche labs* (Basel, Switzerland) on *Uncertainty characterisation in action potential modelling for cardiac drug safety*.
- 2012 **Master student**, *Univ. Oxford*, Oxford (UK), 33%.
I have co-supervised Joseph Page's internship with Mike Bonsall (Oxford Zoology) and Maria Bruna (Oxford Maths), on *The effect of migration on the resilience and behaviour of coral reefs*.
- 2012 **Master student**, *Univ. Paris-Sud XI*, Orsay (France), 50%.
I have co-supervised Ahmed Lasmar's master internship with Balázs Kégl, on *Modelling the electro-magnetic component of the Auger tank signal*.

Computer skills

Tools.

I have a working knowledge of C, Python, Matlab and Mathematica, and I am a regular user of CPU grids and social coding platforms such as *Github* [link]

Reproducible research.

I am running a small workgroup on *reproducible research* in Lille, where we transfer software engineering skills to researchers to achieve high standards of reproducibility. See our *tutorial* [link] for instance, or an *concrete example* [link] of our standards, our software package on DPP sampling for machine learning. More software activity on my *GitHub account* [link].

Languages

French	Fluent	<i>Mother language.</i>
English	Fluent	<i>Main working language.</i>
German	Fluent	<i>“Abitur” with highest honours (German equivalent to the French “baccalauréat” or British A-levels).</i>

Interests

Music I play the piano, keyboards and synthesizers, and I am teaching myself guitar and drums. I am currently especially interested in free improvised music.

Track Record

In this section, I present my research interests in a loose chronological order.

Adaptive inference methods. During my thesis, I have been particularly interested in self-tuning inference and optimization methods, also denoted as *adaptive stochastic search* methods. Indeed, for today’s complex and large-scale models of natural phenomena, inference pipelines are sophisticated arrangements of algorithmic blocks, which cannot realistically be manually tuned. My main contribution has been to propose, apply and study an adaptive Metropolis algorithm with online relabeling [25, 10, 5], that performs Markov chain Monte Carlo simulation with adaptive proposals in models with symmetries, such as mixture models. This algorithm is novel in that it is adaptive both in its proposal mechanism and in the definition of the target distribution, which makes its theoretical study both challenging and interesting, combining tools from stochastic approximation and vector quantization.

Hyperparameter tuning in machine learning. My contributions in machine learning (ML) are in the subfield of hyperparameter optimization. Still with the idea in mind to deliver turn-key algorithms, I first studied and proposed an improvement to Bayesian optimization algorithms [12], a useful paradigm for hyperparameter tuning. In 2011, I was invited to spend a month with Yoshua Bengio’s group at Université de Montréal, to develop methods that apply to the automatic tuning of deep learning networks [11]. Deep learning algorithms are a class of state-of-the-art ML algorithms which can have tens of hyperparameters, the tuning of which is usually referred to as black magic by non-experts, thus justifying the development of automatic tuning methods. Motivated by the positive results we obtained in [11], I have used dataset similarity to improve on automatic hyperparameter tuning [14].

Applications to particle physics. During my thesis, I was a member of the Pierre Auger collaboration, a team of 300+ researchers and technicians devoted to the construction, maintenance, and data analysis of the Auger cosmic ray observatory. I have thus participated in several internal collaboration conferences, discussed and coauthored a number of journal papers with the full collaboration (see publication list). My direct and personal contributions include probabilistic modelling of the low-level signal [27] and a complex empirical Bayes inference procedure [26]. More marginally, I made technical contributions such as radio measurements [28] and spent a month on

the site of the observatory in Argentina in 2011 on a shift, monitoring the telescopes of the experiment.

Scalable Bayesian inference. Loosely speaking, Bayesian inference for big data faces two main challenges: big data can be *tall* – it contains an overwhelming number of individual data points – and/or it can be large – each data point contains a large number of components. Since my postdoc, I have had a keen interest into scaling up Bayesian inference to tall datasets while maintaining the theoretical properties that make Bayesian inference desirable in the first place. I have given the first MCMC algorithm that both relies on subsamples of the original dataset and still provably leads to the same convergence properties as vanilla MCMC [8]. A side-product of this work has been the proof of a novel concentration inequality [4]. Since then, I have improved the above algorithm and given a widely cited critical review of the recent abundant literature on the topic in [3].

Applications to biology. When moving to Oxford in 2013 I joined the 2020 science network – a UK-based network of young computational scientists. I have started transferring statistical tools to computational biology, in particular cardiac modelling [32, 30, 29] and cell tissue tracking [31]. More marginally, the whole 2020 science network put together their experiences in a paper [33] with generic recommendations for successful cross-disciplinary collaborations.

Determinantal point processes. Determinantal point processes (DPPs) are point process models that naturally encode diversity or repulsiveness between the points of a given realization, using algebraic arguments. They arise in many fields, e.g. random matrix theory, combinatorics, or quantum physics. Our landmark results was to show that DPPs lead to fast Monte Carlo integration [35, Under review for *Annals of Applied Probability*], leading to a natural stochastic version of Gaussian quadrature, and we have proven faster-than-Monte-Carlo central limit theorems for our estimators. See also [13] for a different description of the train of thought that took us to the results in [35]. This opens the way towards more efficient Monte Carlo integration for expensive-to-evaluate integrands, such as those arising in inference for complex biological systems. For historical reasons, the computational aspects of DPPs have been largely understudied, and we have started investigating inference for DPPs [7] and fast sampling [6].

Applications to signal processing. After a stimulating *talk* [link] by Patrick Flandrin, I was intrigued by the seemingly repulsive point process formed by the zeros of the short-time Fourier transform of white noise. We proved [1] that this process is an instance of zeros of a Gaussian analytic function, an object of recent booming interest in the random matrix community. This opened a probabilistic toolbox full of theorems that apply directly to the design of filters for signal processing. This first result sparked a lot of interest and raised new deep questions. Along with our follow-up work [34], it gives yet another viewpoint a stimulating new field of application for repulsive point processes such as DPPs.

Publications

My research is multidisciplinary, and publishing habits differ among the relevant disciplines.

In **machine learning**, top journals are *Journal of Machine Learning Research* and *Machine Learning*. However, it is considered as important and prestigious to publish in the following conferences: *NIPS (Neural and Information Processing Systems)*, *ICML (International Conference in Machine Learning)*, *International Conference on Learning Theory (COLT)* (they are all refereed, with an acceptance rate around 20%). Short author lists are more common, with the first author being outlined.

In **probability and statistics**, journal papers are more valued than conference papers, and major journals are more numerous. Short author lists are the rule, with the first author being outlined. *Annals of applied probability* is one of the top 3 journals in probability, *Bernoulli* in the top 5 for statistics.

The process of publication in **experimental physics** is pretty different. Journal papers are more

valued, but they often have long author lists. As I was a member of the 300+-member Auger Collaboration, I coauthored journal papers that were the result of the work of the full collaboration. *Astroparticle Physics* is the main journal in the field of Astroparticles, to which the Auger experiment is dedicated. My main personal contributions to Auger (modelling and inference) are underlined in “short-author-list” conference proceedings and internal papers (GAP notes). The latter are shown here under technical reports, and are not available outside of the Auger Collaboration.

In **biology**, conferences usually do not issue proceedings either, so that journal publications are more valued as well, similarly to physics. Long author lists are more common as well, with the first (“main”) and last (“senior”) authors being outlined.

My publications in a nutshell, and before giving the full list:

- ▷ 37 papers in international journals: 5 in *computational statistics* (CS; used here loosely as a proxy for “probability, statistics, machine learning and signal processing”), 27 in physics, 5 in biology. Out of this, 9 are not co-authored by my PhD advisor (4, 0, 5 respectively).
- ▷ 3 more journal papers in applied maths are under review and available as preprints.
- ▷ 14 papers in international peer-reviewed conferences: 7 in CS, 7 in physics. Out of this, 3 are not co-authored by my PhD advisor (3, 0 respectively).
- ▷ 1 book chapter (applied statistics for experimental physicists),
- ▷ 3723 citations (of which 1000+ in applied maths) as of September 2018 on *Google Scholar* [link],
- ▷ my h-index is 29,
- ▷ A full list of publications is on *Google Scholar* [link], and I maintain a trimmed one on *HAL* [link].

CS journals

- [1] **R. Bardenet**, J. Flamant, and P. Chainais. On the zeros of the spectrogram of white noise. *Applied and Computational Harmonic Analysis*, 2018.
- [2] **R. Bardenet**, F. Lavancier, X. Mary, and A. Vasseur. On a few statistical applications of determinantal point processes. *ESAIM: Proceedings and Surveys*, 60:180–202, 2017.
- [3] **R. Bardenet**, A. Doucet, and C. Holmes. On Markov chain Monte Carlo methods for tall data. *Journal of Machine Learning Research (JMLR)*, 2017.
- [4] **R. Bardenet** and O.-A. Maillard. Concentration inequalities for sampling without replacement. *Bernoulli*, 2015.
- [5] **R. Bardenet**, O. Cappé, G. Fort, and B. Kégl. Adaptive MCMC with online relabeling. *Bernoulli*, 21(3):1304–1340, 2015.

CS refereed proceedings

- [6] G. Gautier, **R. Bardenet**, and M. Valko. Zonotope hit-and-run for efficient sampling of projection dpps. In *International Conference on Machine Learning (ICML)*, 2017.
- [7] **R. Bardenet** and M. K. Titsias. Inference for determinantal point processes without spectral knowledge. In *Advances in Neural Information Processing Systems (NIPS)*, pages 3375–3383, 2015.
- [8] **R. Bardenet**, A. Doucet, and C. Holmes. Towards scaling up MCMC: an adaptive subsampling approach. In *Proceedings of the International Conference on Machine Learning (ICML)*, 2014. <http://jmlr.org/proceedings/papers/v32/bardenet14-sup.pdf>.

- [9] **R. Bardenet**, M. Brendel, B. Kégl, and M. Sebag. Collaborative hyperparameter tuning. In *International Conference on Machine Learning (ICML)*, 2013.
- [10] **R. Bardenet**, O. Cappé, G. Fort, and B. Kégl. An adaptive Metropolis algorithm with online relabeling. In *Proceedings of the International Conference on Artificial Intelligence and Statistics (AISTATS)*, volume 22, pages 91–99, April 2012.
- [11] J. Bergstra, **R. Bardenet**, B. Kégl, and Y. Bengio. Algorithms for hyperparameter optimization. In *Advances in Neural Information Processing Systems (NIPS)*, volume 24. The MIT Press, 2011.
- [12] **R. Bardenet** and B. Kégl. Surrogating the surrogate: accelerating Gaussian-process-based global optimization with a mixture cross-entropy algorithm. In *Proceedings of the 27th International Conference on Machine Learning (ICML)*, 2010.

CS refereed workshops

- [13] **R. Bardenet** and A. Hardy. From random matrices to Monte Carlo integration via Gaussian quadrature. In *Proceedings of the IEEE Statistical Signal Processing workshop (SSP)*, 2018.
- [14] **R. Bardenet**, M. Brendel, B. Kégl, and M. Sebag. SCoT: surrogate-based collaborative tuning for hyperparameter learning that remembers the past. NIPS workshop on Bayesian Optimization and Decision Making, 2012.
- [15] **R. Bardenet**, B. Kégl, and G. Fort. Relabeling MCMC algorithms in Bayesian mixture learning. Snowbird Learning workshop, 2011.
- [16] **R. Bardenet** and B. Kégl. Sampling-based optimization with mixtures. NIPS workshop on Optimization for Machine Learning, 2009.

CS book chapters

- [17] **R. Bardenet**. *Proceedings of the 2012 IN2P3 School of Statistics*, chapter Monte Carlo methods. EDP Sciences, 2013.

Physics Journals (selected papers)

- [18] P. Abreu et al. The lateral trigger probability function for UHE cosmic rays showers detected by the Pierre Auger Observatory. *Astroparticle Physics*, 35:266–276, 2011. Auger Collaboration paper.
- [19] P. Abreu et al. Anisotropy and chemical composition of ultra-high energy cosmic rays using arrival directions measured by the Pierre Auger Observatory. *Journal of Cosmology and Astroparticle Physics*, page 022, 2011. Auger Collaboration paper.
- [20] P. Abreu et al. Advanced functionality for radio analysis in the Offline software framework of the Pierre Auger Observatory. *Nuclear Instruments and Methods in Physics Research A*, 635:92–102, 2011. Auger Collaboration paper.
- [21] P. Abreu et al. The Pierre Auger Observatory scaler mode for the study of solar activity modulation of galactic cosmic rays. *JINST*, 6:01003, 2011. Auger Collaboration paper.
- [22] P. Abreu et al. Search for first harmonic modulation in the right ascension distribution of cosmic rays detected at the Pierre Auger Observatory. *Astroparticle Physics*, 34:627–639, 2011. Auger Collaboration paper.

- [23] P. Abreu et al. The exposure of the hybrid detector of the Pierre Auger Observatory. *Astroparticle Physics*, 34:368–381, 2011. Auger Collaboration paper.
- [24] P. Abreu et al. Update on the correlation of the highest energy cosmic rays with nearby extragalactic matter. *Astroparticle Physics*, 34:314–326, 2010. Auger Collaboration paper.

Physics refereed proceedings

- [25] **R. Bardenet** and B. Kégl. An adaptive Monte Carlo Markov chain algorithm for inference from mixture signals. In *Proceedings of ACAT'11, Journal of Physics: Conference series*, 2012.

Physics technical reports

- [26] B. Kégl, R. Busa-Fekete, K. Louedec, **R. Bardenet**, X. Garrido, I.C. Mariş, D. Monnier-Ragaine, S. Dagoret-Campagne, and M. Urban. Reconstructing $N_{\mu 19}(1000)$. Technical reports 2011-054, Auger Project Technical Note, 2011.
- [27] **R. Bardenet**, B. Kégl, and D. Veberic. Single muon response: The signal model. Technical Report 2010-110, Auger Project Technical Note, 2010.
- [28] R. Dallier, **R. Bardenet**, S. Gambetta, H. Lyberis, and C. Macolino. Radio spectrum measurements at Auger, part 2. Technical Report 2010-032, Auger Project Technical Note, 2010.

Biology Journals

- [29] K. A. Beattie, A. P. Hill, **R. Bardenet**, Y. Cui, J. I. Vandenberg, D. J. Gavaghan, T. P. de Boer, and G. R. Mirams. Sinusoidal voltage protocols for rapid characterization of ion channel kinetics. *Journal of Physiology*, 2018.
- [30] R. H. Johnstone, **R. Bardenet**, D. J. Gavaghan, and G. R. Mirams. Hierarchical Bayesian inference for ion channel screening dose-response data. *Wellcome Open Research*, 2016.
- [31] J. Kursawe, P. Brodskiy, C. Narcisso, J. J. Zartmann, R. E. Baker, **R. Bardenet**, and A. G. Fletcher. Robust cell tracking in epithelial tissues through identification of maximum common subgraphs. *Journal of the Royal Statistical Society Interface*, 2016.
- [32] R. H. Johnstone, E. T. Y. Chang, **R. Bardenet**, T. P. De Boer, D. J. Gavaghan, P. Pathmanathan, R. H. Clayton, and G. R. Mirams. Uncertainty and variability in models of the cardiac action potential: Can we build trustworthy models? *Journal of molecular and cellular cardiology*, 96:49–62, 2016.
- [33] B. Knapp, **R. Bardenet**, M. Bernabeu, R. Bordas, M. Bruna, B. Calderhead, J. Cooper, A. G. Fletcher, D. Groen, B. Kuijper, J. Lewis, G. McInerny, T. Minssen, J. Osborne, V. Paulitschke, J. Pitt-Francis, J. Todoric, C. A. Yates, D. Gavaghan, and C. M. Deane. Ten simple rules for a successful cross-disciplinary collaboration. *PLoS Computational Biology*, 11(4):e1004214, 2015.

Under review

- [34] **R. Bardenet** and A. Hardy. Time-frequency transforms of white noises and Gaussian analytic functions. *arxiv preprint: arxiv:1807.11554*, 2018.
- [35] **R. Bardenet** and A. Hardy. Monte Carlo with determinantal point processes. *Under revision for Annals of Applied Probability; arXiv preprint arXiv:1605.00361*, 2016.