import numpy as np  
import math  
  
np.random.seed(seed=0)  
A = np.random.randint(10, size = (5,3))  
print(A)

## [[5 0 3]  
## [3 7 9]  
## [3 5 2]  
## [4 7 6]  
## [8 8 1]]

A

## array([[5, 0, 3],  
## [3, 7, 9],  
## [3, 5, 2],  
## [4, 7, 6],  
## [8, 8, 1]])

np.linalg.eig(A.dot(A.transpose())) # I think this works

## (array([ 3.74765993e+02, 4.79827176e+01, 1.82512898e+01, -4.48728322e-15,  
## 1.62799148e-14]), array([[ 0.21225332, 0.09958505, -0.95484953, 0.16771422, 0.12272347],  
## [ 0.56724169, -0.6194619 , 0.00926204, -0.06655454, -0.5306983 ],  
## [ 0.31008733, 0.13326169, 0.24689917, 0.9059928 , 0.3580154 ],  
## [ 0.51393848, -0.19265757, 0.11251445, -0.23573212, 0.67345991],  
## [ 0.52239686, 0.74261306, 0.12065608, -0.30174467, -0.34867604]]))

Λ1, U1 = np.linalg.eig(A @ A.transpose()) # this works like above but I like notation better  
  
Λ1, U1

## (array([ 3.74765993e+02, 4.79827176e+01, 1.82512898e+01, -4.48728322e-15,  
## 1.62799148e-14]), array([[ 0.21225332, 0.09958505, -0.95484953, 0.16771422, 0.12272347],  
## [ 0.56724169, -0.6194619 , 0.00926204, -0.06655454, -0.5306983 ],  
## [ 0.31008733, 0.13326169, 0.24689917, 0.9059928 , 0.3580154 ],  
## [ 0.51393848, -0.19265757, 0.11251445, -0.23573212, 0.67345991],  
## [ 0.52239686, 0.74261306, 0.12065608, -0.30174467, -0.34867604]]))

Λ2, U2 = np.linalg.eig(A.transpose() @ A)  
  
Λ2, U2

## (array([374.76599263, 18.25128978, 47.98271759]), array([[-0.51284918, -0.60635883, -0.60771267],  
## [-0.68691355, 0.71443602, -0.13315757],  
## [-0.51491309, -0.34915631, 0.78291403]]))

U, S, Vh = np.linalg.svd(A)  
  
U

## array([[-0.21225332, -0.09958505, 0.95484953, 0.01530755, -0.18182245],  
## [-0.56724169, 0.6194619 , -0.00926204, -0.49002922, 0.23300087],  
## [-0.31008733, -0.13326169, -0.24689917, -0.22348396, -0.88044471],  
## [-0.51393848, 0.19265757, -0.11251445, 0.82787418, -0.0267421 ],  
## [-0.52239686, -0.74261306, -0.12065608, -0.15593687, 0.36980152]])

S

## array([19.35887374, 6.92695587, 4.27215283])

Vh  
  
# The Λs and 𝑆𝑇∗𝑆 all the same - eigenvalues

## array([[-0.51284918, -0.68691355, -0.51491309],  
## [-0.60771267, -0.13315757, 0.78291403],  
## [ 0.60635883, -0.71443602, 0.34915631]])

Λ1[0:3] - Λ2

## array([-1.70530257e-13, 2.97314278e+01, -2.97314278e+01])

Λ2 - S.dot(S)

## array([ -66.23400737, -422.74871022, -393.01728241])

U1 - U # expecting near zero, but not all???

## array([[ 0.42450663, 0.1991701 , -1.90969906, 0.15240667, 0.30454592],  
## [ 1.13448338, -1.2389238 , 0.01852407, 0.42347468, -0.76369917],  
## [ 0.62017466, 0.26652337, 0.49379834, 1.12947676, 1.23846012],  
## [ 1.02787696, -0.38531514, 0.22502891, -1.06360631, 0.70020201],  
## [ 1.04479373, 1.48522612, 0.24131215, -0.1458078 , -0.71847756]])

U2 - Vh.transpose()

## array([[-5.55111512e-16, 1.35383227e-03, -1.21407150e+00],  
## [ 2.22044605e-16, 8.47593598e-01, 5.81278449e-01],  
## [-1.11022302e-16, -1.13207034e+00, 4.33757713e-01]])

U1, U, U2, Vh

## (array([[ 0.21225332, 0.09958505, -0.95484953, 0.16771422, 0.12272347],  
## [ 0.56724169, -0.6194619 , 0.00926204, -0.06655454, -0.5306983 ],  
## [ 0.31008733, 0.13326169, 0.24689917, 0.9059928 , 0.3580154 ],  
## [ 0.51393848, -0.19265757, 0.11251445, -0.23573212, 0.67345991],  
## [ 0.52239686, 0.74261306, 0.12065608, -0.30174467, -0.34867604]]), array([[-0.21225332, -0.09958505, 0.95484953, 0.01530755, -0.18182245],  
## [-0.56724169, 0.6194619 , -0.00926204, -0.49002922, 0.23300087],  
## [-0.31008733, -0.13326169, -0.24689917, -0.22348396, -0.88044471],  
## [-0.51393848, 0.19265757, -0.11251445, 0.82787418, -0.0267421 ],  
## [-0.52239686, -0.74261306, -0.12065608, -0.15593687, 0.36980152]]), array([[-0.51284918, -0.60635883, -0.60771267],  
## [-0.68691355, 0.71443602, -0.13315757],  
## [-0.51491309, -0.34915631, 0.78291403]]), array([[-0.51284918, -0.68691355, -0.51491309],  
## [-0.60771267, -0.13315757, 0.78291403],  
## [ 0.60635883, -0.71443602, 0.34915631]]))