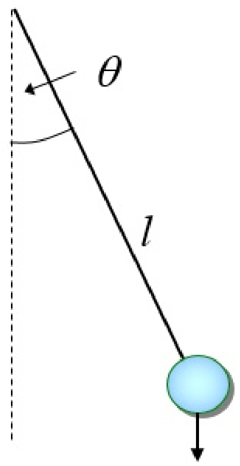
HW6 (due on May/15 12:30PM)

1. Run and understand pendulum.m

The model is constructed based on the following equations.



$$\frac{d^2\theta}{dt^2} + \omega^2 \sin\theta = 0$$

where $\omega^2 = \dfrac{g}{l}$

1) Modify the model to simulate a swing starting from the start point on the left (10 points)

2) Modify the model to simulate a swing of the pendulum on the Moon. Would it run faster or slower than its swing on Earth? (10 points)

2. Using TDP.m to plot the trajectory of the end point of the second pendulum in two different scenarios: 1) the swing is very periodic and regular. 2) the swing stays in a very chaotic manner. (20 points)

3. Run and understand lorenzattractor.m

The model is constructed based on the following equations

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = rx - y - xz$$
$$\frac{dz}{dt} = xy - bz$$

Modify parameter r to see what the critical value of r is to determine the stability and behavior of the system: from two strange attractors state to one attractor state? (20 points)

4. Run and understand lotkavolterra.m

The model is constructed based on the following equations. The four parameters are all selected as 1.

$$\frac{dx}{dt} = x(\alpha - \beta y)$$

$$\frac{dy}{dt} = -y(\gamma - \delta x)$$

The first equation describes that the rate of change of the prey (x) 's population is given by its own growth rate minus the rate at which it is preyed upon. The second equation expresses that the rate of change of the predator (y) 's population depends upon the rate at which it consumes prey, minus its intrinsic death rate.

1. Plot the changes of x and y against time and a phase plot between changes of x and y (10 points)
2. Please provide some thoughts to explain why the populations of x and y become more and more after each die/revive cycle. (10 points)

5. Based on the following K-M model,

$$\frac{dS}{dt} = -\beta\, I(t)S(t)$$

$$\frac{dI}{dt} = \beta\, I(t)S(t) - \gamma\, I(t)$$

$$\frac{dR}{dt} = \gamma\, I(t)$$

Please plot a scatter plot between the initial number of infected population (I) against the maximum number that the infected population (I) could reach within 300 time steps. The initial condition is (T=0): S+I=1000 and R=0. (20 points).